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Deformable Object Simulation with Poisson Equation

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Abstract – This paper presents a new methodology for the deformation of soft objects by drawing an analogy between Poisson equation and elastic deformation. The potential energy stored in an elastic body as a result of a deformation caused by an external force is propagated among mass points by the Poisson equation. An improved Poisson model is developed for propagating the energy generated by the external force in a natural manner. A method is presented to derive the internal forces from the potential energy distribution. This proposed methodology not only deals with large-range deformations, but also accommodates both isotropic and anisotropic materials by simply changing the constitutive coefficients. Examples are presented to demonstrate the efficiency of the proposed methodology.

Index Terms – deformation, soft objects, Poisson equation, surgery simulation and analogous systems.

I. INTRODUCTION

Virtual reality based surgery simulation is expected to provide benefits in many aspects of surgical procedure training and evaluation. To this end, a significant amount of research efforts have been dedicated to simulating the behaviours of deformable objects. These researches can be divided into two types. One is focused on real-time simulation such as mass-spring models [1, 2] and spline surfaces used in deformation [3, 4]. The advantage of this method is that the computation is less time consuming and the algorithm is easier to be implemented. However, the method does not allow accurate modelling of material properties, and more importantly, increasing the number of springs leads to a stiffer system. The other method is devoted to accurate deformation modelling such as Finite Element Method (FEM) [5, 6] and Boundary Element Method (BEM) [7, 8]. In FEM or BEM, rigorous mathematical analysis based on continuum mechanics is applied to accurately model the mechanical behaviours of soft objects. However, these methods are computationally expensive and are typically simulated off-line. The pre-calculation [7], matrix condensation [9] and the space and time adaptive level-of-detail [10] techniques are used to enhance the computational performance.

In general, most of the existing methods for soft object deformation are fully built on an elastic model that is an ideal tool to describe the deformation. However, the behaviours of soft objects such as human tissues and organs are extremely nonlinear [11, 12]. The common deformation methods, such as mass-spring, FEM and BEM, are mainly based on linear

elastic models because of the simplicity of linear elastic models and also because linear elastic models allow reduced runtime computations. However, linear elastic models cannot accommodate large-range geometric deformations and they only allow the displacements to be less than 10% of the deformable object size [11, 13]. Although the few methods based on the nonlinear elastic model can handle the large-range deformations [14], the use of quadric strains generally requires a very expensive computation for real-time simulation. In addition, extra work often needs to be performed for anisotropic deformations.

This paper presents a Poisson based methodology for deformable object simulation. The deformation is formulated as a Poisson equation. The potential energy stored due to a deformation caused by an external force is calculated and treated as the source injected into the Poisson system, as described by the law of conservation of energy. An improved Potential model is developed for propagating the energy among mass points in a natural manner. A method for deriving the internal forces from the potential energy distribution is also presented. This methodology can not only deal with large-range deformations, but it can also accommodate both isotropic and anisotropic materials easily through simply changing the constitutive coefficients.

There are several investigations that combine Poisson equation with deformable object modeling. However, these are mainly focused on fluid and gas modeling [15, 16, 17] and the Poisson equation is only used for the computational purpose. Recently, a method was reported for mesh editing through manipulating the Poisson gradient [18]. However, this work is only focused on mesh processing without any discussions on physically-based deformations. To the best of our knowledge, this study is the first to directly use Poisson equation to mimic the deformation of soft objects under externally applied loads.

II. DESIGN OF POISSON MODEL

A. Physical Analogy

We formulate the deformation of soft objects as Poisson equation. The generalized Poisson equation involving an isotropic medium is

$$\begin{aligned}\rho \nabla^2 \phi &= q \\ \vec{r} &= -\rho \nabla \phi\end{aligned}\tag{1}$$

where ∇^2 is the divergence operator, ∇ is the gradient operator, ρ is a constitutive coefficient, ϕ is a scalar

potential function, q is a source function and $\vec{r}$ is a flux function. The Poisson equation describes many steady-state application problems in heat transfer, mechanics and electromagnetics, and provides a natural manner for energy propagation.

In addition, the Poisson equation is independent of time. It allows the environment models to be updated incrementally and has fast response to changes in the environments. Therefore, the dynamic environment can be changed real-time during execution. This feature is very suitable for real-time deformation due to the variable external force.

In the proposed Poisson equation analogy, the deformation of soft objects by an external force is treated as a Poisson equation with a source at the contact point. The steady-state of the Poisson equation corresponds to an equilibrium state of the deformation under an external force. The external force caused deformations is treated as the source to generate energy. The energy generated by the source is propagated among mass points through the Poisson equation and a potential field is further developed on the object surface. The flux that originates from the source always flows from the point with a high potential energy to the point with a low potential energy. If the flow of the flux is further thought to be caused by the internal forces, such a Poisson equation with the source, the flux and the flow of the flux can be seen as a communication medium among an external force, internal forces and deformation.

B. Poisson Model

The Poisson model is designed as a 2D Poisson equation in theory while it is constructed in 3D object surface with the 3D boundary condition. Therefore, the Poisson model is between 2D and 3D, and is called the $2\frac{1}{2}$ -dimensional Poisson model.

The Poisson equation (1) in 2D is

$$\rho \left(\frac{\partial^2 \varphi}{\partial u^2} + \frac{\partial^2 \varphi}{\partial w^2} \right) = q \quad (2)$$

$$\vec{r} = -\rho \nabla \varphi$$

For anisotropic materials, Eq. (2) becomes:

$$\rho_u \frac{\partial^2 \varphi}{\partial u^2} + \rho_w \frac{\partial^2 \varphi}{\partial w^2} = q \quad (3)$$

$$\vec{r} = -\nabla \rho \varphi$$

For the sake of convenient description, the deformable object refers to an isotropic deformable object unless we declare it as anisotropic.

To solve the Poisson equation (2), it is necessary to determine the boundary conditions. The boundary conditions determine the character of the energy interchange over the boundary. In the case of soft object deformation, the boundary can be treated as the part of the soft object that connects/contacts with other objects. For simplicity, we

choose the Dirichlet boundary conditions, i.e. the specified potential on the boundary :

$$\varphi = \varphi_\Omega \quad \text{on} \quad (4)$$

where φ_Ω is the given boundary potential function.

III. CONSTRUCTION OF POISSON MODEL

A. Source Formulation

When a soft object is deformed under an external force, there is a displacement observed. The deformation is the consequence of the work done by the external force. According to the law of conservation of energy, the work done by the external force can be transformed into an equivalent potential energy at the contact point. Therefore, the source q that provides the potential density is:

$$q = \frac{\vec{F} \bullet \vec{U}}{A_F} \quad (5)$$

where $\bullet$ is the dot product, $\vec{F}$ is the external force, $\vec{U}$ is the displacement and A_F is the area on which the external force is applied.

If the external force is applied to a point, or the area that the force is applied on is very small, the source q may be regarded as the elastic strain energy density at the contact point:

$$q = \frac{1}{2} \sigma \varepsilon = \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \sigma_{ij} \varepsilon_{ij} \quad (6)$$

where σ is the stress tensor and ε is the strain tensor at the contact point. The commonly used and simple strain tensor is linear Cauchy strain tensor [19] described by:

$$\varepsilon = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (7)$$

A straight forward approach that leads to a linear relationship between these two tensors is provided by the Hooke's law:

$$\sigma = C \varepsilon \quad (8)$$

For isotropic materials, Eq. (8) can be further written in the form of components:

$$\sigma_{ij} = \delta_{ij} \lambda \sum_k \varepsilon_{kk} + 2\mu \varepsilon_{ij} \quad (9)$$

where δ is the Kronecker delta, λ and μ are the two láme constants.

From Eqs. (8) and (9), it can be seen that the displacement caused by the external force can be deduced and subsequently the source q can be calculated.

In the ordinary Poisson equation, the source at each point has the same value. For our purpose, the energy generated by the external force is treated as a source and propagated to

other mass points along the object surface. Therefore, the obtained source is set only at the contact point of the external force, and the source values at other points are set to zero.

B. Constructing the Poisson Model on a Rectangular Net

The construction of the Poisson model on a rectangular net is straightforward. The Poisson equation for each internal node can be established using a finite-difference scheme. For the point $\vec{P}_{i,j}$ shown in Fig. 1, the Poisson equation is shown in Eq. (10).

Fig. 1 Poisson model on a rectangular net

$$\begin{aligned} & \frac{2\rho\varphi_{i+1,j}}{\Delta u_i(\Delta u_{i-1} + \Delta u_i)} + \frac{2\rho\varphi_{i-1,j}}{\Delta u_{i-1}(\Delta u_{i-1} + \Delta u_i)} \\ & + \frac{2\rho\varphi_{i,j+1}}{\Delta w_j(\Delta w_{j-1} + \Delta w_j)} + \frac{2\rho\varphi_{i,j-1}}{\Delta w_{j-1}(\Delta w_{j-1} + \Delta w_j)} \\ & - \frac{2\rho\varphi_{i,j}}{\Delta u_{i-1}\Delta u_i} - \frac{2\rho\varphi_{i,j}}{\Delta w_{j-1}\Delta w_j} = q_{ij} \end{aligned} \quad (10)$$

$$\begin{aligned} \Delta u_{i-1} &= \left\| \vec{P}_{i-1,j} \vec{P}_{i,j} \right\| & \Delta u_i &= \left\| \vec{P}_{i,j} \vec{P}_{i+1,j} \right\| \\ \Delta w_{j-1} &= \left\| \vec{P}_{i,j-1} \vec{P}_{i,j} \right\| & \Delta w_j &= \left\| \vec{P}_{i,j} \vec{P}_{i,j+1} \right\| \end{aligned}$$

where $\varphi_{i,j}$ is the potential at the point $\vec{P}_{i,j}$, and $\left\| \vec{P}_{i-1,j} \vec{P}_{i,j} \right\|$ and other similar terms represent the magnitudes of the vector $\vec{P}_{i-1,j} \vec{P}_{i,j}$ and other similar vectors.

For anisotropic materials, since the constitutive coefficient at each node is different, the Poisson model at each internal node may be described by the following:

$$\begin{aligned} & \frac{2\rho_{i+1,j}\varphi_{i+1,j}}{\Delta u_i(\Delta u_{i-1} + \Delta u_i)} + \frac{2\rho_{i-1,j}\varphi_{i-1,j}}{\Delta u_{i-1}(\Delta u_{i-1} + \Delta u_i)} \\ & + \frac{2\rho_{i,j+1}\varphi_{i,j+1}}{\Delta w_j(\Delta w_{j-1} + \Delta w_j)} + \frac{2\rho_{i,j-1}\varphi_{i,j-1}}{\Delta w_{j-1}(\Delta w_{j-1} + \Delta w_j)} \\ & - \frac{2\rho_{i,j}\varphi_{i,j}}{\Delta u_{i-1}\Delta u_i} - \frac{2\rho_{i,j}\varphi_{i,j}}{\Delta w_{j-1}\Delta w_j} = q_{ij} \end{aligned} \quad (11)$$

where $\rho_{i,j}$ is the constitutive coefficient at the point $\vec{P}_{i,j}$.

C. Constructing the Poisson Model on a Triangular Net

For constructing the Poisson model on a triangular net, a finite volume method [20] is used to establish the Poisson equation at each node. The finite volume method subdivides the domain into a finite number of non-overlapping cells or

control volumes over which the energy conservation is enforced in a discrete sense. Fig. 2 shows the finite volume (surrounded by the dash lines) constructed around the point $\vec{P}_0$. The finite volume consists of the centroids of the triangles adjacent to point $\vec{P}_0$ and the midpoints of the edges adjacent to $\vec{P}_0$. The discrete Poisson equation at the point $\vec{P}_0$ is

$$\begin{aligned} & \frac{N_{JA} + N_{AB}}{L_{01}}(\varphi_1 - \varphi_0)\rho + \frac{N_{BC} + N_{CD}}{L_{02}}(\varphi_2 - \varphi_0)\rho \\ & + \frac{N_{DE} + N_{EF}}{L_{03}}(\varphi_3 - \varphi_0)\rho + \frac{N_{FG} + N_{GH}}{L_{04}}(\varphi_4 - \varphi_0)\rho \\ & + \frac{N_{HI} + N_{IJ}}{L_{05}}(\varphi_5 - \varphi_0)\rho = q_0 S \end{aligned} \quad (12)$$

where $L_{nm} = \left\| \vec{P}_m \vec{P}_n \right\|$ is the distance between two points $\vec{P}_m$

and $\vec{P}_n$, N_{KQ} is the distance of two adjacent vertices **K** and **Q** of the finite volume, and S is the measure of the finite volume.

In the similar way as Eq. (11), the Poisson model at point $\vec{P}_0$ for anisotropic materials can be given as

$$\begin{aligned} & \frac{N_{JA} + N_{AB}}{L_{01}}(\rho_1\varphi_1 - \rho_0\varphi_0) + \frac{N_{BC} + N_{CD}}{L_{02}}(\rho_2\varphi_2 - \rho_0\varphi_0) \\ & + \frac{N_{DE} + N_{EF}}{L_{03}}(\rho_3\varphi_3 - \rho_0\varphi_0) + \frac{N_{FG} + N_{GH}}{L_{04}}(\rho_4\varphi_4 - \rho_0\varphi_0) \\ & + \frac{N_{HI} + N_{IJ}}{L_{05}}(\rho_5\varphi_5 - \rho_0\varphi_0) = q_0 S \end{aligned} \quad (13)$$

where ρ_i is the constitutive coefficient at the point $\vec{P}_i$.

Fig. 2 Poisson model on a triangular net

IV. INTERNAL FORCE DERIVATION AND MODEL DYNAMICS

A. Internal Force Derivation

Potential functions provide an effective way of describing internal forces based on point positions. For a potential function ψ , the force exerted on a point $\vec{P}_i$ is due to the

gradient of the potential energy ψ with respect to the change in position, as described below

$$\vec{f} = -\nabla_{\vec{P}_i} \psi \quad \nabla_{\vec{P}_i} \psi = \left(\frac{\partial \psi}{\partial \bar{P}_{ix}}, \frac{\partial \psi}{\partial \bar{P}_{iy}}, \frac{\partial \psi}{\partial \bar{P}_{iz}} \right) \quad (14)$$

The potential field developed by the Poisson equation describes the energy distribution on the object surface. The associated potential function is $\phi(u, w)$. According to the law of conservation of energy, the potential at each point can be transformed into an equivalent elastic potential. Therefore, the internal elastic force at a point can be represented by the negative gradient of the potential, i.e. the flux at this point;

$$\vec{f} = -\rho \nabla_{\vec{P}_i} \phi \quad (15)$$

For anisotropic materials, Eq. (15) becomes

$$\vec{f} = -\nabla_{\vec{P}_i} \rho \phi \quad (16)$$

Since the flux flows along an edge from a high potential point to a low potential point, an internal force exists between any two adjacent points and the internal force at a point is derived from the neighbouring points of this point. The force between any adjacent points is calculated as follows:

For any two adjacent points $\vec{P}_i$ and $\vec{P}_j$, and assuming that the potentials at these two points are $\phi_{\vec{P}_i}$ and $\phi_{\vec{P}_j}$, the potential at any point $\vec{P}$ between these two points is regarded as a function of the distance between the point $\vec{P}_i$ and the point $\vec{P}$. Therefore, the following relationships can be written:

$$\phi = \phi(l) \quad l = \|\vec{P} - \vec{P}_i\| \quad (17)$$

Thus:

$$\nabla_{\vec{P}_i} \phi = \frac{d\phi}{dl} \nabla_{\vec{P}_i} l = -\frac{\phi_{\vec{P}_j} - \phi_{\vec{P}_i}}{\|\vec{P}_j - \vec{P}_i\|} \overrightarrow{P_i P_j} \quad (18)$$

where $\overrightarrow{P_i P_j} = \frac{\vec{P}_j - \vec{P}_i}{\|\vec{P}_j - \vec{P}_i\|}$ and $|\phi_{\vec{P}_j} - \phi_{\vec{P}_i}|$ is the magnitude of

the potential change between the point $\vec{P}_i$ and the point $\vec{P}_j$.

Therefore, the force between the point $\vec{P}_i$ and the point $\vec{P}_j$ is:

$$\vec{f}_{ij} = \frac{|\phi_{\vec{P}_j} - \phi_{\vec{P}_i}|}{\|\vec{P}_j - \vec{P}_i\|} \overrightarrow{P_i P_j} \quad (19)$$

The internal force $\vec{g}_i$ at a given point $\vec{P}_i$ is the sum of the internal forces from all connected neighbouring points of the point $\vec{P}_i$.

$$\vec{g}_i = \sum_{j \in N(\vec{P}_i)} \vec{f}_{ij} \quad (20)$$

where $N(\vec{P}_i)$ is the connected neighbouring points of $\vec{P}_i$, $\vec{f}_{ij}$ is the force between the point $\vec{P}_i$ and its neighbouring point $\vec{P}_j$.

B. Model Dynamics

When an external force is applied to a soft object, the contact point of the external force is replaced with a new position. As a result, the other points not influenced by the external force are in an unstable state. The energy generated by the external force is propagated among mass points through the local connectivity of cells to establish a new equilibrium state by generating the corresponding internal forces. Effectively, the new position of each point is obtained based on the equilibrium state. The physically dynamic process is governed by the Lagrangian equation of motion of each node:

$$m_i \frac{d^2 \vec{P}_i}{dt^2} + \gamma_i \frac{d\vec{P}_i}{dt} + \vec{g}_i = \vec{F}_i \quad (21)$$

where $\vec{P}_i$ is the position vector of the node i, m_i and γ_i are the mass and damping constant of the node i, respectively. $\vec{g}_i$ is the net internal force applied to the node i at time t, and $\vec{F}_i$ is the external force applied to the node i at time t.

The solution of Eq. (21) can be computed either by implicit integration scheme or explicit integration scheme. Although the implicit integration has the advantage of being unconditionally stable, which means it allows large time steps to be used, it is computationally intensive and requires inverting a sparse matrix at each iteration [21]. Therefore, the explicit integration scheme is adopted to solve the Eq. (21). The advantage of the explicit scheme is that no rigid matrix inversion is required for updating each vertex.

The second-order differential equations can be divided into a system of first order differential equations by introducing a velocity function $\vec{V}_i$.

Fig. 3 Deformations of isotropic materials

Fig. 4 Deformations of anisotropic materials

$$\frac{d\vec{P}_i}{dt} = \vec{V}_i \quad \frac{d\vec{V}_i}{dt} = \frac{\vec{F}_i - \vec{g}_i - \gamma_i \vec{V}_i}{m_i} \quad (22)$$

The explicit Euler integration scheme is used to numerically solve Eq. (22).

$$\vec{V}_i^{n+1} = \vec{V}_i^n + \Delta t \frac{\vec{F}_i - \vec{g}_i - \gamma_i \vec{V}_i}{m_i} \quad \vec{P}_i^{n+1} = \vec{P}_i^n + \Delta t \vec{V}_i^n \quad (23)$$

V. IMPLEMENTATION AND DISCUSSIONS

A prototype system has been implemented for interactive deformable simulation. Experiments are conducted to investigate isotropic deformations, anisotropic deformations and the non-linear load-deformation of the model.

Fig. 3 illustrates the deformations of an isotropic material modelled with 400 mass points (the constitutive coefficient = 0.8 and the damping = 10.0). Fig. 3(a) shows the undeformed object whose shape is a superquadric. Fig. 3(b), (c), (d) and (e) are different views of the object deformed under a

compressive force. Fig. 3(f) and (g) are the objects deformed under a tensile force.

The behaviour of anisotropic materials can be easily simulated by the proposed model through simply setting different different constitutive coefficients. Fig. 4 illustrates the deformations of anisotropic materials. The red parts of the images have a very small constitutive coefficient and are not deformed by the external forces. Fig. 4(a) and (b) show the different views of the cylinder deformed under a compressive force, where the red parts are not deformed.

The proposed methodology has been tested to verify the non-linear load-deformation relationship. Eight materials that are modelled with different damping constants (from right to left in Fig. 5, the damping is 1.0, 2.0, 4.0, 5.0, 6.0, 7.0, 9.0 and 10.0, respectively) are tested. The deformation is calculated when the force applied to the model is increased at a constant rate. The results in Fig. 5 demonstrate that deformation varies non-linearly with the applied force.

With the deformable modelling examples, it is observed that most of the deformation simulation time is spent on solving the Poisson equation by the refined Gaussian

elimination whose calculation speed is $O(N^2)$. The efficiency of solving the Poisson equation could be further improved to $O(N)$ by a neural network algorithm [22].

Fig. 5 Nonlinear load-deformation relationship

Compared with most of the existing deformation methods such as mass-spring, BEM and linear FEM, the methodology can perform large-range displacements through its non-linear load-deformation relationship. Compared with non-linear FEM [14], the Poisson model is easier to be formulated when compared to the complex non-linear elastic models. Only surface mass points are involved in computation and rendering without any inside points while the interior has to be meshed and calculated in FEM. Further, the methodology can easily accommodate anisotropic materials by simply setting constitutive coefficients while extra work has to be performed in both mass-spring and FEM.

VI. CONCLUSIONS

Presented in this paper is a new methodology to mimic the deformation of soft objects by drawing an analogy between Poisson equation and elastic deformation. The contribution of this paper is that the Poisson equation is solved on a 3D object surface to propagate the energy generated by the external force for extrapolating the internal forces of the deformed object. An improved Poisson model is developed for propagating the energy generated by the external force on the object surface in a natural manner. A method is presented for deriving the internal forces from the potential energy distribution. This proposed methodology can not only deal with large-range deformations due to the natural energy propagation, but also can accommodate both isotropic and anisotropic materials through simply modifying the constitutive coefficients.

Ongoing research includes the improvement of the computational efficiency with a neural network algorithm, the integration of the virtual surgery environment with the Phantom haptic device for force feedback and modelling complex behaviours of soft objects with the variable constitutive coefficient and complex boundary conditions.

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