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Abstract

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Keywords

criterion, failure, soils, anisotropic, formulation, cross

Disciplines

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Formulation of cross-anisotropic failure criterion for soils

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Abstract: Inherently anisotropic soil fabric has a considerable influence on soil strength. To model this kind of inherent anisotropy, a three-dimensional anisotropic failure criterion was proposed, employing a scalar-valued anisotropic variable and a modified general three-dimensional isotropic failure criterion. The scalar-valued anisotropic variable in all sectors of the deviatoric plane was defined by correlating a normalized stress tensor with a normalized fabric tensor. Detailed comparison between the available experimental data and the corresponding model predictions in the deviatoric plane was conducted. The proposed failure criterion was shown to well predict the failure behavior in all sectors, especially in sector II with the Lode angle ranging between 60º and 120º, where the prediction was almost in accordance with test data. However, it was also observed that the proposed criterion overestimated the strength of dense Santa Monica Beach sand in sector III where the intermediate principal stress ratio *b* varied from approximately 0.2 to 0.8, and slightly underestimated the strength when *b* was between approximately 0.8 and 1. The difference between the model predictions and experimental data was due to the occurrence of shear bending, which might reduce the measured strength. Therefore, the proposed anisotropic failure criterion has a strong ability to characterize the failure behavior of various soils and potentially allows a better description of the influence of the loading direction with respect to the soil fabric.

Key words: *cross-anisotropy; soil fabric; failure criterion; triaxial test; torsional shear test*

1 Introduction

Many studies concerning the isotropic failure criteria for various geomaterials, such as the well-known Lade-Duncan criterion (Lade and Duncan 1975), SMP criterion (Matsuoka and Nakai 1974), adaptive criterion (Xiao et al. 2010, 2011; Liu et al. 2010), and generalized nonlinear strength theory (Yao et al. 2004), have been published during the past century. However, nowadays there seems to be a growing interest in investigating the anisotropy within soils. The depositional process often unfavorably induces an anisotropic soil fabric with transverse isotropy (cross-anisotropy) on the bedding plane. According to previous

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investigations by Oda and Nakayama (1989), Kirkgard and Lade (1993), Abelev and Lade (2004), Lee and Pietruszczak (2008), and Zhong et al. (2011), anisotropy has a significant influence on the strength and deformation behaviors of soils.

In order to interpret the cross-anisotropy of soils, a lot of efforts have been made recently to develop an appropriate anisotropic failure criterion. Oda and Nakayama (1989), for example, extended the Drucker-Prager criterion from the micro-structural viewpoint using a newly defined fabric tensor. This criterion has shown a great potential in modeling the failure behavior of soils under general three-dimensional conditions. Dafalias et al. (2004) considered the interaction between the fabric tensor and stress tensor and established a plasticity constitutive model for sand by taking into account the effect of inherent fabric cross-anisotropy on the mechanical response. However, additional experiments need to be conducted to obtain the explicit value of the anisotropic parameter in the model (Dafalias et al. 2004). In addition, Abelev and Lade (2004) proposed a cross-anisotropic failure criterion based on Lade's (1977) isotropic failure criterion. However, this model, as suggested by the authors (Abelev and Lade 2004) themselves, cannot characterize the anisotropic failure behavior when the principal stress axis rotates. Lade (2007, 2008) developed a new general three-dimensional failure criterion for cross-anisotropic soils under the conditions with and without occurrence of the rotation of stress axes by incorporating a microstructure tensor first defined by Pietruszczak and Mroz (2000, 2001. Although this modified model has shown a great ability to describe the failure behavior of soils under a general three-dimensional condition with the rotation of stress axes, it cannot capture some strength behaviors under torsional shear tests. Likewise, Xiao et al. (2012) generalized the SMP criterion (Matsuoka and Nakai 1974) available for granular materials with initially inclined anisotropy by employing the same microstructure tensor (Pietruszczak and Mroz 2000). Mortara (2010) has also proposed a unified failure criterion for both isotropic and anisotropic cohesive-frictional materials; however, his model can only describe the cross-anisotropy of soils when the loading direction coincides with the axis of anisotropy, just as the model proposed by Abelev and Lade (2004) does. Moreover, its formulation consists of so many parameters that are too complicated for practical application.

The purpose of this work is to establish a new general anisotropic failure criterion for various soils by employing a normalized fabric tensor (Gao et al. 2010) and the recently proposed adaptive criterion (Xiao et al. 2010, 2011). As will be shown in the following sections, through interaction between the fabric tensor and stress tensor, the newly presented model can well capture the inherent cross-anisotropy within soils.

2 Isotropic failure criterion

The strength mechanism of geomaterials is very complicated due to various components, fabrics, and fissures in soils, and complex environmental influences (Yang et al. 2008; Azami et al. 2009), and there has been much interest in understanding and quantifying it. According to recent studies by Xiao et al. (2010) and Liu et al. (2010), a new general isotropic failure criterion (adaptive criterion) has been proposed, which can well characterize the failure behavior of general soils in the deviatoric plane. The function of the adaptive criterion in the three-dimensional stress space is formulated as follows:

$$\frac{I_1^3 + \mu I_1 I_2}{I_3} = \frac{\left(3 - \sin\varphi_0\right)^3 + \mu\left(9 - \sin^2\varphi_0\right)\left(1 - \sin\varphi_0\right)}{1 - \sin\varphi_0 - \sin^2\varphi_0 + \sin^3\varphi_0} \tag{1}$$

where μ is the adaptive parameter, and if $\mu = 0$, Eq. (1) corresponds to the Lade-Duncan criterion (Lade and Duncan 1975), while it yields to the SMP criterion (Matsuoka and Nakai 1974) when $\mu \to +\infty$; φ_0 denotes the friction angle in the triaxial compression condition; and I_1, I_2, and I_3 are the first, second, and third principal stress invariants, respectively. As shown in Eq. (1), the adaptive parameter μ is not a concrete number when $\mu \to +\infty$, and there are, unfavorably, too many values from 0 to $+\infty$ for engineers to use in practice. To facilitate further application, a modified expression was developed by analogy with Yao et al. (2004) as follows:

$$\frac{\mu I_1^3 + (1-\mu) I_1 I_2}{I_3} = \frac{\mu\left(3 - \sin\varphi_0\right)^3 + (1-\mu)\left(9 - \sin^2\varphi_0\right)\left(1 - \sin\varphi_0\right)}{1 - \sin\varphi_0 - \sin^2\varphi_0 + \sin^3\varphi_0} \tag{2}$$

where μ varies from 0 to 1 instead of simply remaining positive in Xiao et al. (2010), and is determined by the ratio of soil strength under drained triaxial compression to that under drained triaxial extension. The SMP criterion is a particular case of Eq. (2) corresponding to $\mu = 0$, while the Lade-Duncan criterion corresponds to $\mu = 1$. Furthermore, the modified adaptive failure criterion represented by Eq. (2) is linear in the meridian plane, which slightly departs from experimental results by Kirkgard and Lade (1991, 1993) as well as Yu et al. (2002). To move the analysis further, Eq. (2) is modified by some necessary substitution and transformation procedures as follows:

$$q = pM_0 g(\theta)\left(\frac{p}{P_{\mathrm{a}}}\right)^n \tag{3}$$

where P_{a} is the atmospheric pressure; q and p are the deviatoric stress and mean stress, respectively; n is an exponent that helps to control the curvature of the failure curve influenced by p in the meridian plane; θ is the Lode angle; M_0 is determined by the friction angle φ_0, i.e., $M_0 = \dfrac{6\sin\varphi_0}{3 - \sin\varphi_0}$; and the shape function $g(\theta)$ in the deviatoric plane according to Xiao et al. (2010) is

$$g(\theta) = \frac{\cos\left[\dfrac{1}{3}\arccos\left(-\dfrac{3\sqrt{3}L_3}{2L_2^{3/2}}\right)\right]}{\cos\left[\dfrac{1}{3}\arccos\left(-\dfrac{3\sqrt{3}L_3\cos 3\theta}{2L_2^{3/2}}\right)\right]} \tag{4}$$

in which L_2 and L_3 can be expressed as

$$\begin{cases} L_2 = \dfrac{\mu E(\varphi_0) + (1-\mu)\left[H(\varphi_0) - G(\varphi_0)\right]}{3\mu E(\varphi_0) + 3(1-\mu)H(\varphi_0) - 27(1+2\mu)G(\varphi_0)} \\ L_3 = \dfrac{2\mu E(\varphi_0) + 2(1-\mu)H(\varphi_0)}{27\mu E(\varphi_0) + 27(1-\mu)H(\varphi_0) - 243(1+2\mu)G(\varphi_0)} \end{cases} \tag{5}$$

with $\begin{cases} E(\varphi_0) = (3 - \sin\varphi_0)^3 \\ H(\varphi_0) = (9 - \sin^2\varphi_0)(1 - \sin\varphi_0) \\ G(\varphi_0) = 1 - \sin\varphi_0 - \sin^2\varphi_0 + \sin^3\varphi_0 \end{cases}$.

According to Eq. (2), failure curves of the above isotropic failure criteria in the deviatoric plane are schematically shown in Fig. 1, in which σ_1, σ_2, and σ_3 are the first , second, and third principal stresses, respectively. By adjusting the parameter μ, a whole range of failure curves between the Lade-Duncan criterion and SMP criterion can be readily obtained.

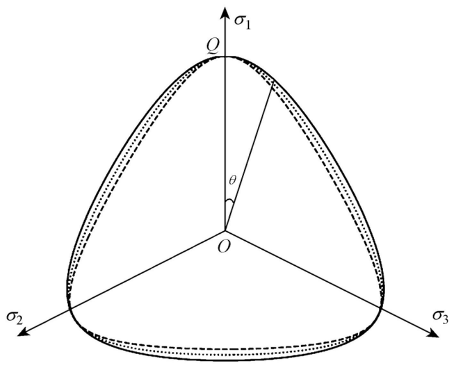

Fig. 1 Failure curves of isotropic failure criteria in deviatoric plane

3 Anisotropic variable based on normalized fabric tensor

In order to describe the inherent anisotropy due to the preferred orientation of constituent particles, Oda and Nakayma (1989) proposed a fabric tensor $\boldsymbol{F}$ and successfully established a new yield function, which could well describe the effects of cross-anisotropy in geomaterials. Instead of giving a detailed derivation of the fabric tensor, presented here is just the ultimate formula for cross-anisotropy:

$$\boldsymbol{F} = \begin{bmatrix} \dfrac{1-\varDelta}{3+\varDelta} & 0 & 0 \\ 0 & \dfrac{1+\varDelta}{3+\varDelta} & 0 \\ 0 & 0 & \dfrac{1+\varDelta}{3+\varDelta} \end{bmatrix} \tag{6}$$

where $\varDelta$ is a measurable quantity which captures the intensity of anisotropy; it ranges from 0, corresponding to the isotropic material, to 1, associated with the maximum anisotropic feature.

Although Δ is a measurable parameter, it is not easy to quantify its accurate value without conducting complicated experiments and calculations. Thus, to facilitate the analysis of the cross-anisotropy of soils without loss of generality, a normalized deviatoric fabric tensor $\boldsymbol{f}$ first introduced by Gao et al. (2010) is employed here as follows:

$$f_{ij} = \frac{d_{ij}}{\sqrt{\sum_{n=1}^{3}\sum_{m=1}^{3} d_{mn} d_{mn}}} \tag{7}$$

where $d_{ij} = F_{ij} - \frac{1}{3}F_{kk}\delta_{ij}$ and $d_{mn} = F_{mn} - \frac{1}{3}F_{kk}\delta_{mn}$ are the deviatoric fabric tensors, and δ_{ij} is the Kronecker function. f_{ij} is defined conventionally in reference to the axis of cross-anisotropy with the normal direction of the bedding plane (represented by the bias on the surface of cubes in Fig. 2) being rotated by an angle of ξ relative to the vertical direction as shown in Fig. 2, in which the stress directions of σ_x, σ_y, and σ_z are fixed with the reference Cartesian coordinate system for convenience. A similar illustration has been presented by Ochiai and Lade (1983).

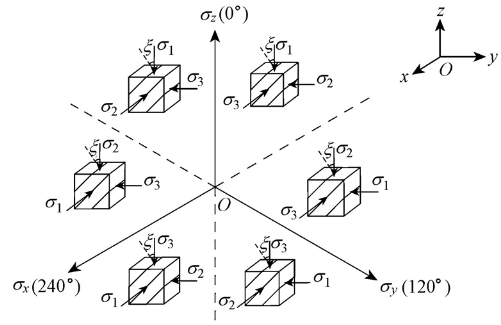

Fig. 2 True triaxial tests of specimen with initially inclined axis of cross-anisotropy

Instead of quantitatively reflecting the intensity of cross-anisotropy, the normalized tensor is only a measure of the orientation of anisotropy within soils. On the other hand, a unit deviatoric stress tensor $\boldsymbol{n}$ (Dafalias et al. 2004) is similarly adopted by recalling the definition of the intermediate principal stress ratio $b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ as

$$\boldsymbol{n} = \frac{1}{\left[6\left(b^2 - b + 1\right)\right]^{1/2}} \begin{bmatrix} 2-b & 0 & 0 \\ 0 & 2b-1 & 0 \\ 0 & 0 & -(1+b) \end{bmatrix} \tag{8}$$

where n_{ij} is defined in reference to the axes of σ_1, σ_2, and σ_3. In addition, many experiments (Lam and Tatsuoka 1988; Hong and Lade 1989; Lade and Kirkgard 2000) previously conducted usually set one axis of isotropy in coincidence with an axis of the Cartesian coordinate system, for example, the x axis was assumed in this paper, and let the normal direction of cross-anisotropy rotate by an angle ξ with respect to the vertical direction in the y-z plane, so as to facilitate the further analysis of the interaction between the stress tensor and fabric tensor. Components of the fabric tensor are subjected to an orthogonal transformation as follows:

$$f'_{ij} = f_{kl}\beta_{ik}\beta_{jl} \tag{9}$$

where β_{ik} and β_{jl} are the cosines of the angles between two corresponding axes: the one describing cross-anisotropy and the other describing the applied stress. Then, based on the work by Dafalias et al. (2004), a scalar-valued anisotropic variable is defined by correlating the stress tensor with the fabric tensor as follows:

$$A = \boldsymbol{f}' : \boldsymbol{n} \tag{10}$$

Based on the analysis above, values of the anisotropic variable A in six sectors of the deviatoric plane can be deduced. For a better understanding of the variable A, here, by extending what has been proposed by Gao et al. (2010), formulations of A in all sectors are as follows:

$$A = \begin{cases} \dfrac{3(b-1)\cos^2\xi + (1-2b)}{2\sqrt{b^2-b+1}} & \text{sector I } \left(0° \le \theta \le 60°\right) \\ \dfrac{3(1-b)\cos^2\xi + (b-2)}{2\sqrt{b^2-b+1}} & \text{sector II } \left(60° \le \theta \le 120°\right) \\ \dfrac{3\cos^2\xi + (b-2)}{2\sqrt{b^2-b+1}} & \text{sector III } \left(120° \le \theta \le 180°\right) \\ \dfrac{3b\cos^2\xi + (1-2b)}{2\sqrt{b^2-b+1}} & \text{sector IV } \left(180° \le \theta \le 240°\right) \\ \dfrac{-3b\cos^2\xi + (b+1)}{2\sqrt{b^2-b+1}} & \text{sector V } \left(240° \le \theta \le 300°\right) \\ \dfrac{-3\cos^2\xi + (b+1)}{2\sqrt{b^2-b+1}} & \text{sector VI } \left(300° \le \theta \le 360°\right) \end{cases} \tag{11}$$

According to Eq. (11) and the fundamental relationship between b and θ, i.e., $b = \frac{1}{2}\left(\sqrt{3}\tan\theta + 1\right)$, the variation of A with regard to θ at different values of ξ is characterized in Fig. 3. Obviously, A varies between −1 and 1, and in particular when $\xi = 0°$, A varies from −1, relative to the conventional triaxial compression shear mode, to 1, relative to the conventional triaxial extension shear mode.

Generally, when dealing with hollow cylinder torsional shear experiments, the same pressure inside and outside the cylinder was applied in a cell (Hong and Lade 1989; Lade and Kirkgard 2000), which means that the axial stress σ_r and circumferential stress σ_θ could be treated as equivalent, and furthermore, $b = \sin^2\xi$. In this type of situation the relationship between A and b can be redefined as a function between A and ξ, which corresponds to the solid line in Fig. 4. A similar plot has been given by Gao et al. (2010). Fig. 4 further clarifies the detailed relationship between the anisotropic variable A and the rotation angle ξ. It is observed that A covers the whole range between −1 and 1 by choosing proper values of b.

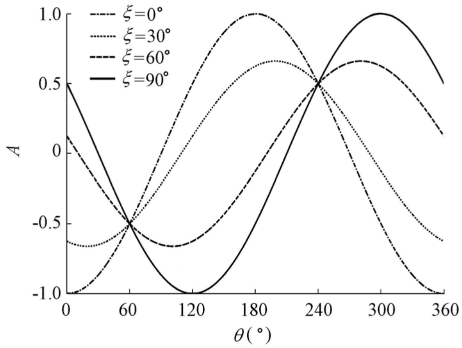

Fig. 3 Variation of anisotropic variable A with respect to Lode angle θ (Gao et al. 2010)

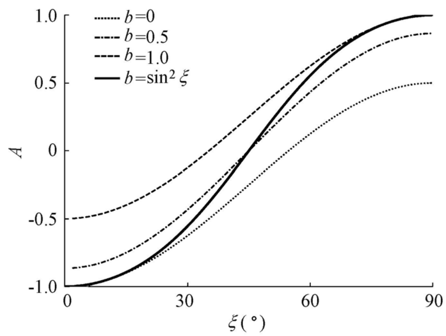

Fig. 4 Relationship between anisotropic variable A and rotation angle ξ

4 Anisotropic failure criterion

Based on what discussed above, presented herein is a new anisotropic failure criterion for soils:

$$q = M_0 z(A) g(\theta) \left(\frac{p}{P_\mathrm{a}} \right)^n \tag{12}$$

where the function $z(A) = \exp\left[\alpha(1+A)^k\right]$ is employed as a modification of M_0. In general, point Q (Fig. 1), with the Lode angle θ being equal to 0, is often set as the reference point. At point Q, A is assumed to be identical to □1, and thus $z(A)\big|_{\theta=0} \equiv 1$, which means that the anisotropic failure criterion and its corresponding isotropic failure criterion have the same prediction performance (Gao et al. 2010). Apart from the anisotropic variable A, which reflects the influence of the loading direction with respect to the soil fabric, other two parameters, α and k, are introduced to enable a better representation of the cross-anisotropy in soils. Here, α serves as a role of measuring the intensity of cross-anisotropy in soils, while k is used for minor adjustment to obtain a better description of the cross-anisotropic feature. In particular, when $\alpha = 0$, it is observed that $z(A) \equiv 1$, and the anisotropic failure criterion yields to a corresponding isotropic failure criterion irrespective of the loading direction. Further discussion of the two parameters above is illustrated in Fig. 5. Eq. (12) becomes the Lade-Duncan criterion-based anisotropic failure criterion if $\mu = 1$, while it yields to the SMP criterion-based anisotropic failure criterion if $\mu = 0$. In the following discussion the parameter μ and friction angle φ_0 are set as constants, i.e., 0.167 and 30°, respectively, and the fabric tensor and stress tensor are coaxial.

In order to analyze the effect of parameter α, three typical values of α, i.e., −0.05, 0, and 0.05 have been chosen. As shown in Fig. 5(a), the failure curve expands outwards with the value of α increasing from negative to positive. In particular, when $\alpha = 0$, the anisotropic failure criterion yields to a corresponding isotropic failure criterion irrespective of the changing of other parameters, such as ξ and k. Similarly, Figs. 5(b) and (c) indicate the effect of k on the failure curve. It is observed from Fig. 5(b) that as the value of parameter *k* becomes larger, the failure curve expands outwards when the Lode angle θ is less than $\pi/2$ or greater than $3\pi/2$ while it shrinks inwards when θ varies between $\pi/2$ and $3\pi/2$. However, an opposite change has been observed when α remains positive, as shown in Fig. 5(c).

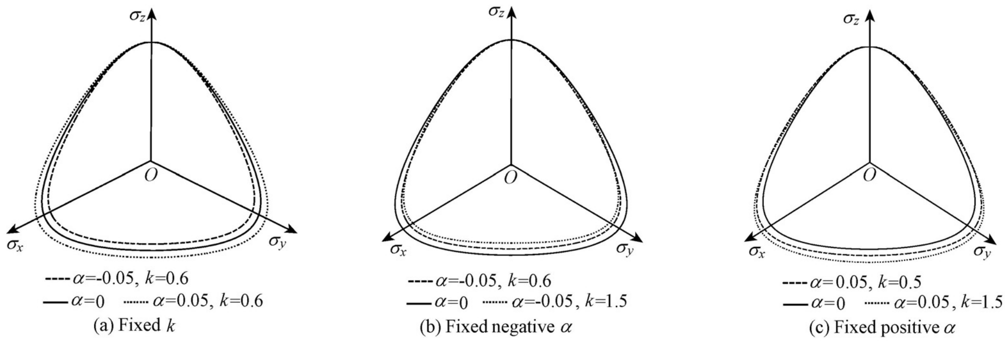

Fig. 5 Failure curves of anisotropic failure criteria in deviatoric plane at fixed *k* and fixed negative and positive α

As shown in section 3, the interaction between the fabric tensor and stress tensor has equipped the proposed new anisotropic failure criterion with a powerful capacity which can be used to describe the initial inclination of the cross-anisotropy for soils. When the stress tensor and fabric tensor are coaxial, the inclination angle ξ is equal to 0, and the failure curve will be symmetric about the σ_z axis, while, if they are un-coaxial, i.e., $\xi > 0$, the symmetry axis will change. A detailed analysis of the axis inclination of cross-anisotropy is necessarily illustrated in Fig. 6. It is observed that the failure curve is symmetric about the σ_x axis when $\xi = 45°$ and symmetric about the σ_y axis in the vicinity of $\xi = 90°$.

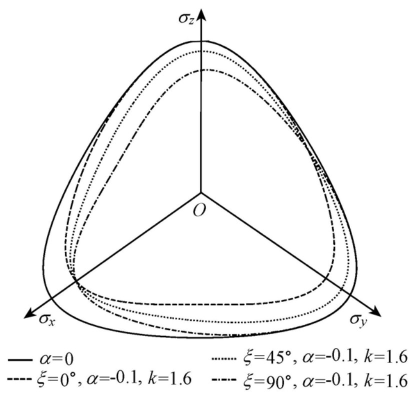

Fig. 6 Effect of inclination of cross-anisotropy on failure curve in deviatoric plane

5 Parameter determination

As previously suggested by Lade (2007), results of triaxial compression tests and conventional triaxial extension tests, especially, the test data obtained near $b = 0$ and/or $b = 1$, can be readily available to determine the model parameters. Thus, by analogy with Lade

(2007), calculations of parameters α and k have been conducted using the experimental data at $\theta = \pi/3$, $2\pi/3$, and π.

It is common to estimate a failure criterion by comparison with the corresponding test data in the φ-b diagram. The relevant formulation for the peak friction angle φ with respect to b is as follows:

$$\begin{cases} \varphi = \arcsin\left\{\frac{1}{2}M_0 l(\theta,A)\left[\sqrt{b^2-b+1}+\frac{1-2b}{6}M_0 l(\theta,A)\right]^{-1}\right\} \\ l(\theta,A) = \dfrac{z(A)\cos\left[\frac{1}{3}\arccos\left(-\frac{3\sqrt{3}L_3}{2L_2^{3/2}}\right)\right]}{\cos\left[\frac{1}{3}\arccos\left(-\frac{3\sqrt{3}L_3\cos 3\theta}{2L_2^{3/2}}\right)\right]} \end{cases} \tag{13}$$

In the case of $\theta = \pi/3$, $b = 1$, and $A = -0.5$, Eq. (13) can be simplified as

$$\begin{cases} \varphi\big|_{\theta=\frac{\pi}{3}} = \arcsin\left\{\frac{1}{2}M_0 l\left(\frac{\pi}{3},A\right)\left[1-\frac{1}{6}M_0 l\left(\frac{\pi}{3},A\right)\right]^{-1}\right\} \\ l\left(\frac{\pi}{3},A\right) = \dfrac{\exp\left(\alpha 0.5^k\right)\cos\left[\frac{1}{3}\arccos\left(-\frac{3\sqrt{3}L_3}{2L_2^{3/2}}\right)\right]}{\cos\left[\frac{1}{3}\arccos\left(\frac{3\sqrt{3}L_3}{2L_2^{3/2}}\right)\right]} \end{cases} \tag{14}$$

In the case of $\theta = 2\pi/3$, $b = 0$, and $A = \frac{3}{2}\cos^2\xi - 1$, Eq. (13) can be simplified as

$$\begin{cases} \varphi\big|_{\theta=\frac{2\pi}{3}} = \arcsin\left\{\frac{1}{2}M_0 l\left(\frac{2\pi}{3},A\right)\left[1+\frac{1}{6}M_0 l\left(\frac{2\pi}{3},A\right)\right]^{-1}\right\} \\ l\left(\frac{2\pi}{3},A\right) = \exp\left[\alpha\left(\frac{3}{2}\cos^2\xi\right)^k\right] \end{cases} \tag{15}$$

In the case of $\theta = \pi$, $b = 1$, and $A = \frac{3}{2}\cos^2\xi - \frac{1}{2}$, Eq. (13) can be simplified as

$$\begin{cases} \varphi\big|_{\theta=\pi} = \arcsin\left\{\frac{1}{2}M_0 l(\pi,A)\left[1-\frac{1}{6}M_0 l(\pi,A)\right]^{-1}\right\} \\ l(\pi,A) = \dfrac{\exp\left[\alpha\left(\frac{3}{2}\cos^2\xi+\frac{1}{2}\right)^k\right]\cos\left[\frac{1}{3}\arccos\left(-\frac{3\sqrt{3}L_3}{2L_2^{3/2}}\right)\right]}{\cos\left[\frac{1}{3}\arccos\left(\frac{3\sqrt{3}L_3}{2L_2^{3/2}}\right)\right]} \end{cases} \tag{16}$$

With the help of Eqs. (14) through (16), values of α and k can be easily worked out. After determining all the required parameters in the proposed anisotropic failure criterion, predictions of soil strength behaviors can be carried out.

6 Comparison with experimental data

Several typical comparisons with the true triaxial test data of dense Santa Mnica Beach from Abelev and Lade (2004) and of Toyoura sand from Lam and Tatsuoka (1988) as well as the torsional shear test data of San Francisco Bay mud by Lade and Kirkgard (2000) are illustrated in Figs. 7 through 9. In order to justify the advantages of the proposed anisotropic failure criterion over those from literature, a detailed comparison with Lade's (2008) criterion is also presented in Fig. 7. For clarity, Figs. 8 and 9 only present the comparison of the results of the proposed anisotropic failure criterion with experimental results. η_0, Ω_1, and m are parameters in Lade's (2008) anisotropic failure formulation.

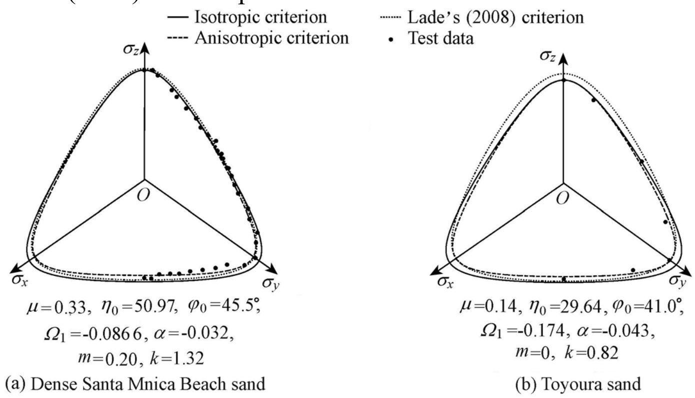

Fig. 7 Results of dense Santa Mnica Beach sand and Toyoura sand predicted by isotropic and anisotropic failure criteria in deviatoric plane and their comparison with test data

As shown in Fig. 7, predictions by the proposed anisotropic failure criterion fit favorably with the experimental results. In contrast, the isotropic failure criterion fails to capture the anisotropic strength of soils in sectors II and III. Also, Lade's (2008) criterion overestimates the soil strength in sector I but has a slightly better performance than the proposed criterion in sector II (Fig. 7(a)). Furthermore, it is observed from Fig. 8 that the anisotropic failure criterion indicates a better curve-fitting ability in all three sectors than the isotropic failure criterion. Unlike the isotropic failure criterion, the anisotropic criterion presented here well captures the φ-b relationship in sectors I and II when compared with the test data of dense Santa Monica Beach sand (Abelev and Lade 2004), especially in sector II where the prediction is almost in accordance with the test data. However, it cannot be denied that in sector III the proposed anisotropic failure criterion slightly overestimates the soil strength when b varies from approximately 0.2 to 0.8, while it slightly underestimates the soil strength when b is between approximately 0.8 and 1. That difference in the middle range of b is probably due to the occurrence of shear banding during the triaxial test (Lade 2011). Actually, Abelev and Lade (2004) pointed out that shear banding could occur in the hardening regime for b varying from approximately 0.18 to 0.85, and thus might reduce the measured soil strength.

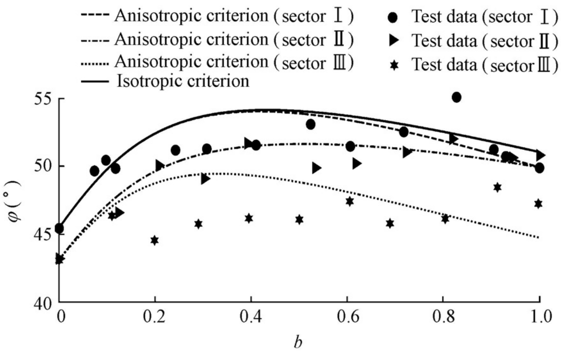

Fig. 8 Comparison of predicted results by failure criteria with test data (Abelev and Lade 2004) in φ-b diagram

Fig. 9 shows a nearly perfect prediction of the peak friction angle with respect to b and ξ by the proposed anisotropic failure criterion in comparison with the test data of K_0-consolidated San Francisco Bay mud (Lade and Kirkgard 2000), while the isotropic failure criterion seems to fail in prediction as b becomes greater than about 0.1. Moreover, as the best curve-fitting performance comes at $\mu = 0$, it is suggested that the SMP criterion-based anisotropic failure criterion is more suitable for predicting the strength behavior of K_0-consolidated San Francisco Bay mud than the Lade-Duncan criterion-based anisotropic failure criterion.

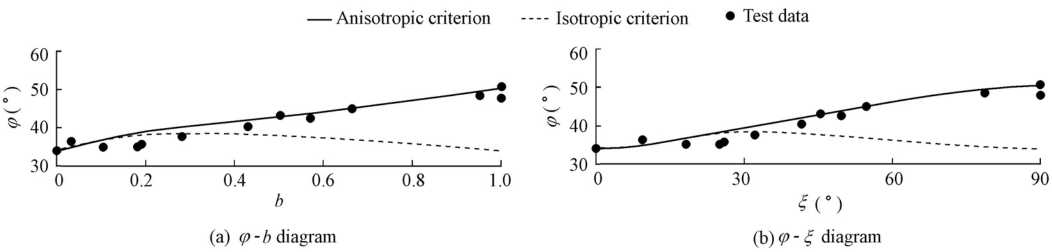

Fig. 9 φ-b and φ-ξ diagrams of K_0-consolidated San Francisco Bay mud predicted by isotropic and anisotropic failure criteria and their comparison with torsional shear test results (Lade and Kirkgard 2000) for $\mu = 0$, $\varphi_0 = 34°$, $\alpha = 0$, and $k = 1.38$

7 Conclusions

Most soils exhibit inherently cross-anisotropic strength behavior due to some specific depositional processes. In some cases, their axes of cross-anisotropy would rotate by an angle with respect to the vertical direction. To characterize this kind of phenomenon, an anisotropic failure criterion for soils has been established in this paper by introducing a normalized fabric tensor. All the parameters within the proposed criterion can be determined by a triaxial compression test. The normalized fabric tensor does not quantitatively reflect the intensity of cross-anisotropy; it is only a measure of the orientation of corss-anisotropy within soils. Detailed formulas of an anisotropic variable A for six sectors were obtained by combining the normalized fabric tensor and the stress tensor.

Compared with the isotropic failure criterion and Lade's (2008) criterion, the proposed anisotropic failure criterion well captures the failure behavior of soils in all sectors, especially in sector II, where the prediction is almost in accordance with test data. It is also suggested that, when predicting the strength behavior of K_0-consolidated San Francisco Bay mud, it should be better to use the SMP criterion-based anisotropic failure criterion rather than the Lade-Duncan criterion-based anisotropic failure criterion. However, it cannot be denied that for the range of the intermediate principal stress ratio b where shear bending occurs, the proposed criterion slightly overestimates the strength of soils.

In summary, the proposed anisotropic failure criterion can well describe the failure behavior of various soils and potentially allows a better description of the influence of the loading direction with respect to the soil fabric.

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