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Mathematical Model to Predict Solids Content of Water Treatment Residuals during Drying

Ali Gharaibeh¹; Muttucumaru Sivakumar²; and Dharmappa Hagare³

Abstract: Dewatering and drying of residuals are extremely energy intensive processes, which are necessary to reduce the quantity of wet residuals produced from the water and wastewater treatment operations. Meteorological conditions are a major factor in the drying of residuals, which can greatly affect the drying period. A mathematical model is developed for the process of drying of water treatment residuals. A steady-state heat-balance equation is applied for a control volume of residuals that takes into account the heat transfer by radiation, convection, and evaporation. The mathematical model was validated using drying experiments conducted in a wind tunnel as well as other experiments conducted in an open environment equipped with a weather monitoring station. Good agreement was obtained between model predictions and experimental observations. The model can be used to predict the drying time of a given application of water treatment residuals with the knowledge of meteorological conditions.

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Introduction

Water treatment has become an increasingly important requirement in potable water supplies. Water treatment operations produce high-quality drinking water as well as wet residuals as a by-product. The residuals mainly include suspended solids, any organics found in water, and chemicals used in the treatment process such as coagulants, coagulant aids, and filter aid polymers.

Residuals are conventionally dried in lagoons and sand drying beds that are open to atmosphere. Residuals can be removed when dried or can be stored for a prolonged period in these lagoons. Sand drying beds are the most widely used method of dewatering water treatment residuals. Land availability and cost limit their use. Weather is a major factor in the drying of residuals, which may take from many days to a few months. The dried residuals are easy to handle and transport if the solids content is approximately 30–50% (wet basis).

Drying of water treatment plant residuals is a complex process, which involves heat and mass transfer occurring simultaneously. Drying of wet solids has been studied extensively and the phases of drying were identified as constant and falling rate periods. Fresh residuals applied on sand drying beds normally

have a concentration of 1–4% solids content (wet basis). Most of the free-water in the residuals can be drained easily (Tsang and Vesilind 1990), and a portion evaporates into the atmosphere. Two distinct drying periods were clearly identified in a previous study (Gharaibeh et al. 2001), the first period is below 15% solids content and the second period is above 15%.

In recent times very few research papers have been published in modeling the drying of water treatment plant residuals. However, there were two previous attempts to develop mathematical models for the alum water treatment residuals on sand drying beds based on preliminary work by other researchers. Clark (1970) developed mathematical equations to enable the design engineer to predict the drying bed area. An equation was developed to predict the drainage time on sand beds and another equation to predict the evaporation component with the knowledge of the critical moisture content. The critical moisture content is dependent on the nature of the material under investigation and not on the weather conditions. Lo (1971), the second researcher, studied the effect of rain on the rate of drainage of water treatment residuals on the sand drying beds; the constant rate period was approximated by the drying rate of a free-water surface. For the falling rate period, the drying equations developed by Nebiker (1967) and Clark (1970) were used by Lo (1971). The drying rate of the falling rate period depends upon the equilibrium moisture content. The equilibrium moisture content of a material varies as it dries (Henderson 1952), which makes it difficult to predict. The critical moisture content, which is the main feature of the above-mentioned models, varies with the thickness of the material and with the rate of drying, as reported by McCabe and Smith (1976).

McCabe and Smith (1976) reported that if the initial moisture content of the solid is below the critical moisture content, the constant rate period does not occur. In practice, residuals might be applied in the drying beds when they are above the critical moisture content and will definitely be removed before reaching the equilibrium moisture content around 30–50% solids content. The drying of residuals has been studied through a series of experiments performed in a laboratory wind tunnel as well as field experiments in experimental sand drying beds. A mathematical

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model was developed to predict the water evaporation from a control volume of a given residuals application thickness and area. The model was formulated using a heat balance approach with the use of ambient temperature, residuals surface temperature, solar radiation, relative humidity, and wind speed. The model predicts the solids content of the given application thickness with time by the knowledge of the prevailing meteorological conditions. It is shown that the model predicts the drying process reasonably well up to 50% solids content (wet basis).

Theoretical Model

Basic Heat Balance

Consider a control volume (open system exposed to air) of residuals having an area A (m^2) and a small thickness s (m). The water evaporates and the solids remain within the control volume, so the moisture content X ($\text{kg}_{\text{water}}/\text{kg}_{\text{dry solids}}$) becomes less with time t (s). The temperature is assumed uniform throughout since the Biot number is less than (0.1). Heat can be transferred in and out of the control volume via convection, radiation, evaporation, and conduction. Water vapor leaves the control volume through the control surface area A .

At any time t , rate of thermal energy Q (W) enters the control volume by convection from the bulk of air above it and/or by direct and diffuse solar radiation. The thermal energy also leaves the control volume by convection, radiation, evaporation, and conduction. The rate of thermal energy stored in the control volume is given by Q_{stored} (W). However, there will be no energy generated and negligible heat conduction lost to the ground when the control volume is well insulated.

The aim is to find moisture content X of the residuals at any time t for various weather conditions. To achieve this goal, the expanded heat balance equation can be written over the control volume in the following form:

$$(Q_{\text{radiation}} + Q_{\text{convection}})_{\text{in}} = (Q_{\text{radiation}} + Q_{\text{convection}} + Q_{\text{evaporation}})_{\text{out}} + Q_{\text{stored}} \quad (1)$$

where $Q_{\text{radiation}}$, $Q_{\text{convection}}$, and $Q_{\text{evaporation}}$ = rate of thermal energy transferred by radiation, convection, and evaporation (W).

Radiative Heat Transfer

The rate of radiative heat absorbed by the control surface on the left-hand side of Eq. (1), which was expanded by Cengel (1997), can be written as follows:

$$(Q_{\text{radiation}})_{\text{in}} = A\alpha_s(G_{\text{direct}} \cos \theta + G_{\text{diffuse}}) + A\alpha_l\epsilon_{\text{atmospheric}}\sigma T_{\text{sky}}^4 \quad (2)$$

where G_{direct} , G_{diffuse} , and G_{sky} = direct, diffuse, and sky incident solar radiation (W/m^2); α_s and α_l = short- and long-wave solar radiation absorptivities of residuals surface (dimensionless); θ = angle of incidence of solar radiation; $\epsilon_{\text{atmospheric}}$ = atmospheric emissivity under clear sky (dimensionless); σ = Stefan-Boltzmann constant ($5.6697 \times 10^{-8} \text{ W}/\text{m}^2 \text{ K}^4$); and T_{sky} = sky temperature in degrees (K).

The rate of the heat emitted by radiation from the control surface on the right-hand side of Eq. (1) can be written as:

$$(Q_{\text{radiation}})_{\text{out}} = A\epsilon\sigma T_{\text{surface}}^4 \quad (3)$$

where T_{surface} = surface temperature of residuals in degrees (K); and ϵ = emissivity of the residuals surface (dimensionless). The sky temperature is measured by an empirical relation found by Berdahl and Martin (1984), which is related to the dew temperature T_{dew} ($^{\circ}\text{C}$), the ambient temperature T_{∞} in degrees (K), and where t = time (h) measured from midnight, as the starting point, by the following equation:

$$T_{\text{sky}} = T_{\infty} [0.711 + 0.0056T_{\text{dew}} + 0.000073T_{\text{dew}}^2 + 0.013 \cos(15t)]^{0.25} \quad (4)$$

The dew temperature can be estimated using the following equation (Palanz 1984):

$$T_{\text{dew}} = \frac{237.7 \ln(p_{\infty}) - 430.22}{19.08 - \ln(p_{\infty})} \quad (5)$$

where p_{∞} = partial pressure of water vapor at the ambient temperature (kg/m^2). The water vapor partial pressure can be expressed in the following equation:

$$p_{\infty} = \text{RH} \times p_{\text{saturated}} \quad (6)$$

The saturated partial pressure $p_{\text{saturated}}$ in (kg/m^2) can be calculated from the following expression (Murray 1967):

$$p_{\text{saturated}} = 611 \times 10^{(7.5T_{\infty}/237.7 + T_{\infty})} \quad (7)$$

Brutsaert (1982) expressed the atmospheric emissivity under clear skies by the following formula:

$$\epsilon_{\text{atmospheric}} = 1.24 \left(\frac{p_{\infty}}{T_{\infty}} \right)^{1/7} \quad (8)$$

where the partial pressure of water vapor p_{∞} is in millibars. When drying occurs in an enclosure, the sky temperature is replaced by the ambient temperature as it was expressed by Cengel (1997) in the following equation:

$$(Q_{\text{radiation}})_{\text{in}} = \alpha_s A (G_{\text{direct}} \cos \theta + G_{\text{diffuse}}) + A\alpha_l\sigma T_{\infty}^4 \quad (9)$$

Convective Heat Transfer

Heat is supplied to the control volume by convection or lost by convection according to Newton's law of cooling

$$(Q_{\text{convection}})_{\text{in}} = hA(T_{\infty} - T_{\text{surface}}) \quad (10)$$

where h = heat transfer coefficient ($\text{W}/\text{m}^2 \text{ K}$).

Evaporative Heat Transfer

Heat can be released from the control surface of the residuals by the change in phase of liquid water into water vapor. Heat rate by evaporation can be expressed by the following equation (Mujumdar 1987; Vaxelaire et al. 1999):

$$Q_{\text{evaporation}} = h_{fg} m_{\text{solids}} \frac{dX}{dt} \quad (11)$$

where $Q_{\text{evaporation}}$ = rate of heat transfer by evaporation of water (W); h_{fg} = latent heat of vaporization (J/kg); m_{solids} = mass of solids in the residuals application (kg); and dX/dt = change of moisture content with respect to time (s^{-1}).

Fig. 1. Wind tunnel experiment setup

Heat Stored

The heat stored within the control volume of the residuals can be estimated from the following equation (Strumillo and Kudra 1986):

$$Q_{\text{stored}} = \frac{d}{dt} [(m_{\text{solids}} c_{\text{solids}} + m_{\text{water}(i)} c_{\text{water}}) T_{\text{surface}}] \quad (12)$$

where c_{solids} and c_{water} = specific heat of solids and water (J/kg K); and $m_{\text{water}(i)}$ = mass of water at the initial moisture content (kg). Substituting Eqs. (2), (3), and (10)–(12) into Eq. (1) gives us:

$$\begin{aligned} & \alpha_s A (G_{\text{direct}} \cos \theta + G_{\text{diffuse}}) + A \alpha_l \varepsilon_{\text{atmospheric}} \sigma T_{\text{sky}}^4 \\ & + Ah(T_{\infty} - T_{\text{surface}}) - A \varepsilon \sigma T_{\text{surface}}^4 - h_{fg} m_{\text{solids}} \frac{dX}{dt} \\ & = \frac{d}{dt} [(m_{\text{solids}} c_{\text{solids}} + m_{\text{water}(i)} c_{\text{water}}) T_{\text{surface}}] \end{aligned} \quad (13)$$

Nondimensional Heat Transfer Coefficient

In order to predict the moisture content of the residuals at any time t using Eq. (13), the variables in the equation have to be either measured or obtained from the literature. The measured variables are the ambient temperature, surface temperature, and weight of residuals. The constants found in the literature are absorptivity, emissivity, and latent heat of vaporization. An attempt was made to calculate the heat transfer coefficient for the water treatment plant residuals by limiting the number of varying parameters. Laboratory experiments were conducted indoors, therefore, the short-wave heat radiation can be considered negligible.

The heat-stored term can be ignored since it is very small. Eq. (13) can be reduced in terms of the heat transfer coefficient to the following equation:

$$h = \frac{[\varepsilon \sigma T_{\text{surface}}^4 - \alpha_l \sigma T_{\infty}^4 + h_{fg} m_{\text{solids}} (dX/dt)]}{A(T_{\infty} - T_{\text{surface}})} \quad (14)$$

Eq. (14) can be used to calculate the heat transfer coefficient assuming both the emissivity and absorptivity values are 0.9; these values may change with temperature but the change is negligible. All other parameters on the right-hand side of Eq. (14) can be measured.

In practice, heat transfer coefficient is expressed as a nondimensional quantity and is usually related to other known nondimensional numbers. This relationship can be obtained from dimensional analysis. The convective heat transfer coefficient is a strong function of wind speed. Other factors that have influence on the heat transfer coefficient could be the difference of temperature between the residuals and the drying medium (air), relative humidity, the characteristic length, and the thickness of the application. Using the Buckingham Pi theorem (Munson et al. 1994) a relationship was formulated to empirically predict the heat transfer coefficient in the following form:

$$\text{Nu} = \frac{hL}{k} = \gamma R^a \text{Gr}^b \left(\frac{L}{s} \right)^c \text{RH}^d \quad (15)$$

where γ , a , b , c , and d = empirical constants; k = thermal conductivity of humid air (J/m² K); L = characteristic length of the application of residuals (m); Nu = dimensionless Nusselt number = hL/k ; R = Reynolds number = uL/ν ; u = wind speed of air (m/s); ν = kinematic viscosity of air (m²/s);

Table 1. Summary of the Wind Tunnel Experiments

Experiment number	Wind speed (m/s)	Application thickness (mm)	Relative humidity (%)	T_{∞} (°C)	T_{surface} (°C)	Heat transfer coefficient [Eq. (14)] (W/m ² K)	Heat transfer coefficient (predicted) (W/m ² K)
34	1	10	80	22.4	17.1	9.9	8.7
15	1	20	77	21.4	14.4	7.7	6.8
27	1	40	70	25.4	15.4	4.7	5.9
28	1	60	74	25.5	16.4	7.3	5.6
33	1	60	72	29.5	18.5	4.4	5.1
10	1	80	72	22.8	18.1	13.2	11.6
35	2	10	81	22	16.3	16.2	18.3
16	2	20	73	22.3	13.7	12.7	14.1
26	2	40	68	25.8	15.9	12.5	14.5
29	2	60	77	25.8	17.7	13.2	13.4
12	2	80	71	25.4	18.6	15.1	16.9
14	2	80	82	25.0	18.7	10.5	13.1
36	3	10	82	21.5	16.1	25.2	27.6
17	3	20	70	21.0	12.7	29.9	27.8
25	3	40	71	25.3	16.6	26.8	23.9
30	3	60	73	27.3	19.8	28.4	25.7
9	3	80	66	26.8	20.2	37.0	35.4
37	4	10	76	23.9	17.6	45.8	43.5
18	4	20	69	20.7	12.2	43.9	41.1
24	4	40	65	23.5	14.0	45.4	42.5
31	4	60	72	28.3	23.7	63.9	61.0
13	4	80	75	25.0	18.2	42.8	40.1
38	5	10	78	24.5	16.4	35.7	38.5
19	5	20	70	21.4	12.3	44.8	43.7
23	5	40	69	21.4	14.1	53.4	53.2
32	5	60	73	24.7	18.3	56.3	53.9
8	5	80	67	26.3	21.1	65.7	67.5
20	7	20	68	25.7	14.9	63.1	61.5
22	7	40	65	25.4	14.5	62.2	64.2
21	7	60	65	25.9	15	68.1	65.1
11	7	80	57	29.3	19.8	58.5	60.9

Gr =Grashof number= $\beta g L^3 \Delta T / \nu^2$; β =coefficient of volume expansion= $1 / [(T_{\infty} + T_{\text{surface}}) / 2]$ (K^{-1}); g =gravitational acceleration (9.8 m/s^2); ΔT =temperature difference between the air and the residuals surface temperature in degrees (K); s =application thickness (m); and RH =relative humidity of air expressed as decimal.

Wind Tunnel Experiments

Experimental Setup

The experimental setup consists of a wind tunnel unit (A572, P. A. Hilton Ltd.) equipped with a variable speed fan in order to control the wind speed. The wind speed is monitored using a Pitot tube. The wind tunnel is also equipped with air heaters and immersion heaters in a water reservoir in order to produce humidity in the air stream and an air conditioning unit to dehumidify the air. The dry, wet bulb, and residuals surface temperatures were measured using three temperature probes and logged in a data logger (MONITOR LOGGER 40), which can be downloaded via a laptop. Then the data, which are logged on at 6 min intervals,

can be exported into an MS Excel spreadsheet. The wet bulb temperature is measured by keeping one of the temperature sensors wet using a porous wet cloth, which is immersed in a small water reservoir, and in order to keep it full, it is connected to another bigger water reservoir via a small tube. Relative humidity is continuously calculated using the wet and dry bulb temperatures.

The residuals are placed to dry in a tray inside a modified test section, which is fabricated from Perspex and assembled at the exit of the wind tunnel in the direction of the wind flow (Fig. 1). A top-loading balance (AND HP-22K with an accuracy of $\pm 1 \text{ g}$) is used to measure the weight of the tray, which is logged on a continuous basis into the supervisory control and data acquisition system (SCADA) of the water treatment plant via an RS-232-C port.

Thirty-one wind tunnel experiments were conducted over an 8 month period at different wind speeds (1, 2, 3, 4, 5, and 7 m/s). Initial solids content for all experiments were varied from 5 to 10% solids content (wet basis); all experiments were stopped after achieving 50% solids content. Initial solids content was measured using a moisture analyzer (Sartorius MA 30 with an accuracy of

Fig. 2. Wind tunnel experiments

± 0.001 g). Five different application thicknesses varying from 10 to 80 mm were conducted for different wind speeds. Residuals were applied in a foil tray (330 mm by 220 mm) on top of a flat aluminum plate; a polystyrene foam sheet separated the tray and the plate in order to minimize the heat loss from the residuals. The thickness of the residuals cake drops with time; the whole tray and the plate are lifted up using the jack to offset the drop. The foil tray in the front and back of the direction of the wind is folded down to minimize the wind turbulence at the surface whenever around 5 mm drop in thickness occurs.

Experimental Results and Discussion

Performing linear regression analysis of the logarithmically transformed data obtained from 23 wind tunnel experiments ($R^2=0.968$). The coefficients of Eq. (15) were determined and given as:

$$\frac{hL}{k} = 890.36(R^{1.192})(Gr^{0.927})\left(\frac{L}{s}\right)^{0.0628}(RH^{-1.997}) \quad (16)$$

The heat transfer coefficient calculated from Eq. (16) is then inserted into Eq. (14) where moisture content can be calculated with respect to time. The physical properties of air and water used for the calculations were adapted from Incropera and Dewitt (1996). Applying the finite-difference technique, Eq. (14) is used to calculate the moisture content of the residuals for the wind tunnel experiments. Starting from the initial moisture content, the calculated moisture content for each time increment is deducted from the previous reading until the desired final moisture content is achieved. The moisture content is transformed into percentage solids content (SC) using the following equation:

$$SC = \frac{100}{1 + X} \quad (17)$$

Table 1 shows a summary of all the wind tunnel experiments. The data are sorted for increasing wind speed and then increase in application thicknesses. A comparison between the calculated and measured average heat transfer coefficients is shown in Table 1.

Fig. 2 shows four of the eight experiments used for verification of the model. The predicted and experimental solids content curves are plotted versus time for the four selected wind tunnel experiments. The predictions show good agreement with experimental results up to 30% solids content in most of the experiments. The deviation mainly occurs at the end of the drying period, where experimental results were 39, 49, 42, and 51% compared to 56, 46, 46, and 49% for Experiments 14, 37, 28, and 22, consecutively. At 30% solids content the surface of residuals becomes fully cracked and a hard crust is formed on top. Above 30% solids content, there will be temperature variations between the hard crust and the remaining wet residuals; resistance of moisture migration within the residuals in order to escape from the hard crust increases. The cracks and the temperature variations may create turbulence at the surface, and hence, affect temperature measurement, which is affecting the prediction of moisture content from Eq. (14).

Field Experiments

Experimental Sand Drying Bed with Drainage and Rain

A weather station is used to monitor the weather conditions as well as the residuals surface temperature. The weather parameters

Fig. 3. Experimental sand drying bed

measured are ambient temperature, wind speed, solar radiation, relative humidity, and rainfall. Relative humidity, wind speed, and solar radiation are measured on an hourly basis. Ambient temperature, residuals surface temperature, and rainfall are measured on a 6 min basis. These parameters are stored in a data logger (MONITOR LOGGER 40) and later downloaded via a laptop computer with special software compatible with MS Excel. The data logger is powered by a built-in rechargeable battery, which is charged by a solar panel mounted on a 5 m mast on top of the weather station.

The experimental sand drying bed surface dimensions are 500 mm wide and 500 mm long with a depth of 600 mm. It is fitted with a drainage tap via a hose to a 20 L container. The bed is filled with a supporting layer of gravel (10 mm in diameter), sand (200 mm), and the top section (200 mm), was left for residuals applications as shown in Fig. 3. The bed's residuals temperature is continuously measured using a leaf temperature sensor supported with a piece of polystyrene on top of the residuals surface.

The residuals are applied in the experimental drying bed straight from the residuals thickener from the local water treatment plant, where the solids content of the thickened residuals range from 1.0 to 3.5%. Samples were taken daily to measure the moisture content using a moisture analyzer (Sartorius MA 30 with an accuracy of ± 0.001 g). The underdrained water is collected and measured daily in a 20 L container. When the solids content reached nearly 50%, most experiments were halted, and residuals were removed from the bed.

Single Application Field Experiments

Six experiments were conducted from March 2000 to September 2000. The experiments shown in Table 2 are for single application thickness. Table 2 shows the daily average meteorological conditions as well as the application depth of residuals, initial and final

Fig. 4. Single application field experiment

solids content, drying time, and the total drained water. In order to calculate the moisture content with respect to time for these experiments, the following equation is used:

$$\frac{dX}{dt} = \frac{1}{h_{fg}m_{\text{solids}}} \left(\alpha_s A (G_{\text{direct}} \cos \theta + G_{\text{diffuse}}) + A \alpha_t \epsilon_{\text{atmospheric}} \sigma T_{\text{sky}}^4 \right) + \frac{d}{dt} \left(\frac{m_{\text{drained}}}{m_{\text{solids}}} \right) + \frac{d}{dt} \left(\frac{m_{\text{rain}}}{m_{\text{solids}}} \right) \quad (18)$$

The wind speed values are corrected for measurement height using the empirical power-law wind profile $[u_{\text{ref}} = u(z_{\text{ref}}/z)^{0.31}]$ (Hsu and Meindl 1994), where u_{ref} = wind speed at reference level (m/s); z = height above the ground (m); and z_{ref} = height at reference level (m). The drainage and the rainwater are included in the mass balance equation for the experimental sand drying bed. Unlike the wind tunnel experiments, the field experiments were exposed to direct sunshine and rainfall as well as fluctuations in ambient temperature, wind speed, and relative humidity. Fig. 4 shows some fluctuations in the drying curve of Experiment 16 due to rainfall during the course of the experiment. If the cracks in the residuals surface are deep to the sand surface, the water finds its way easily to be drained through the sand bed layer. The dips in curves soon recover after the rain stops, within a day or two depending on the weather conditions. The drying curve of Fig. 4 has good agreement between the predicted and experimental results up to 30% solids content. Above 30% solids content, the drying curve deviates from experimental results due to unpredictable drying patterns and temperature measurement fluctuations of the hard crust formed at the residuals surface. The final experimental solids content is 39% compared to 44% predicted. The coefficient of determination (R^2) is 0.97 for Experiment 16, as shown in Fig. 4.

Table 2. Single Application Thickness Experiments

Experiment number	Application depth (mm)	Initial solids content (%)	Final solids content (%)	Drying time (days)	Relative humidity (%)	T_{∞} (°C)	T_{surface} (°C)	Total rainfall (mm)	Solar radiation (MJ/m ²)	Wind speed (m/s)	Total drained water (L)
11	150	3.4	66.5	23	91	20.9	19.9	189.2	6.3	1.6	71
12	200	1.6	57.2	34	89	17.1	16.0	73.6	5.7	1.5	52
13	200	3.5	65.0	21	70	12.7	10.8	3.2	8.7	3.0	27
14	200	1.5	54.0	27	76	12.2	9.8	29	6.0	2.1	42
15	150	2.6	64.0	22	64	13.7	11.0	19.3	17.4	2.8	27
16	100	2.7	92.2	18	72	17.8	16.8	14.1	13.0	2.5	16

Table 3. Multiple Application Thickness Experiments

Experiment number	Initial application depth (mm)	Number of applications	Initial solids content (%)	Final solids content (%)	Drying time (days)	Relative humidity (%)	T_{∞} (°C)	T_{surface}	Total rainfall (mm)	Solar radiation (MJ/m ²)	Wind speed (m/s)	Total drained water (L)
17	50	5	1.4	58.4	33	89	19.6	18.7	122.2	12.2	1.8	81
18	100	3	1.5	56.3	21	84	22.4	20.8	82.6	14.1	1.9	72
19	50	5	1.7	58.5	35	87	21.7	20.8	166.4	11.5	1.5	67
20	50	5	2.6	51.5	56	85	16.9	15.9	94.6	5.5	1.8	47

Multiple Application Field Experiments

In an actual sand drying bed, residuals are applied continuously in a rotation manner around the bed until the bed is covered with a small thickness of residuals; then it is left to dry before another layer is applied. Experiments 17, 18, 19, and 20 are multiple applications conducted to simulate actual bed operation. The applications were applied on a daily basis according to the number of applications shown in Table 3. This continues until the 200 mm top section of the experimental bed is full. Eq. (18) is applied to predict the moisture content with respect to time. The thickness term s in the heat transfer Eq. (16), which is inserted into Eq. (18), is updated for each daily application. Fig. 5 shows the drying curve of Experiment 20. There is good agreement between predicted and experimental curves. The final experimental solids content is 48% compared to 54% predicted. The fluctuations in the drying curve for Experiment 20 were due to the affect of rainfall with 0.97 coefficient of determination.

Conclusions

This study was an attempt to develop a mathematical model to predict the drying time of water treatment plant residuals with the knowledge of meteorological conditions. A steady-state heat balance equation was formulated over a control volume of a residuals application thickness in order to predict the moisture content for a given time. The heat balance takes into account heat transmission by radiation, convection, and evaporation. A heat transfer coefficient correlation was formulated using dimensional analysis.

Two types of experiments were conducted, wind tunnel experiments and field experiments. The wind tunnel experiments were used in the prediction of the heat transfer coefficient correlation. Eight wind tunnel experiments were used to verify the model as well as other experiments performed in the field. The predictions

using the model gave good agreement with experimental work in the wind tunnel and the field experiments of up to 30% solids content. The deviation occurs between 30 and 50% solids content where the residuals are more in the dry zone and the moisture faces resistance in its way to the surface. The model may be used for other types of residuals such as alum residuals with minimal modifications to emissivity and absorptivity constants.

Notation

The following symbols are used in this paper:

- A = surface area (m²);
- a, b, c, d = empirical constants in Eq. (24);
- Bi = Biot number hL/k (dimensionless);
- c = specific heat of residuals (J/kg K);
- c_{water} = specific heat of water (J/kg K);
- c_{solids} = specific heat of solids in residuals (J/kg K);
- g = gravitational acceleration (m/s²);
- G_{direct} = direct incident solar radiation (W/m²);
- G_{diffuse} = diffuse incident solar radiation (W/m²);
- G_{sky} = sky incident solar radiation (W/m²);
- Gr = Grashof number $= \beta g L^3 \Delta T / \nu^2$ (dimensionless);
- h = heat transfer coefficient (W/m² K);
- h_{fg} = latent heat of vaporization (J/kg);
- k = thermal conductivity of humid air (J/m² K);
- L = characteristic length of the application (m);
- m_{drained} = mass of drained water (kg);
- m_{rain} = mass of rain (kg);
- m_{solids} = mass of solids (kg);
- $m_{\text{water}(i)}$ = mass of water at the initial moisture content (kg);
- Nu = Nusselt number $= hL/k$ (dimensionless);
- p_{∞} = water vapor partial pressure at the ambient temperature (kg/m s²);
- $p_{\text{saturated}}$ = saturated water vapor partial pressure at the ambient temperature (kg/m s²);
- Q_{stored} = rate of thermal energy stored (W);
- $Q_{\text{radiation}}$ = rate of thermal energy by radiation (W);
- $Q_{\text{convection}}$ = rate of thermal energy by convection (W);
- $Q_{\text{evaporation}}$ = rate of thermal energy by evaporation (W);
- R = Reynolds number $= uL/\nu$ (dimensionless);
- RH = relative humidity (decimal);
- s = thickness of the residuals application (m);
- SC = wet basis solids content (percentage);
- t = time (s);
- T_{dew} = dew temperature (°C);
- T_{surface} = residuals surface temperature (K);
- T_{∞} = ambient air temperature (K);
- u, u_{ref} = wind speed and wind speed at reference level (m/s);

Fig. 5. Multiple application field experiment

X = moisture content dry basis ($\text{kg}_{\text{water}}/\text{kg}_{\text{dry solids}}$);
and

z, z_{ref} = heights above the ground surface and at
reference height (m).

Greek symbols

α_s = short wave solar radiation absorptivity of
residuals surface (dimensionless);

α_l = long wave solar radiation absorptivity of
residuals surface (dimensionless);

β = coefficient of volume expansion $= 1/[(T_{\infty}$
 $+ T_{\text{surface}})/2]$ (K^{-1});

ϵ = long-wave emissivity of residuals surface
(dimensionless);

$\epsilon_{\text{atmospheric}}$ = atmospheric emissivity under clear skies
(dimensionless);

γ = empirical constant in Eq. (24) (dimensionless);

ν = kinematic viscosity (m^2/s);

θ = angle of incidence; and

σ = Stefan-Boltzmann constant (5.6697
 $\times 10^{-8} \text{ W/m}^2 \text{ K}^4$).

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