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The fourier spectrum analysis of optical feedback self-mixing signal under weak and moderate feedback

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Keywords

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Abstract

The spectrum characteristics of self-mixing signals observed in optical feedback self-mixing interferometry (OFSMI) is studied in this paper. The purpose is to provide guidance for the design of pre-processing techniques for eliminating noise or disturbance. The influence of two important parameters of the OFSMI system on the spectrum, that is, the optical feedback factor and the linewidth enhancment factor, are measured by means of Discrete Fourier transform (DFT). The simulated results show that, at a weak feedback, the OFSMI signals are strictly band limited in nature, with the cut-off frequencies being the vibration frequency and the fringe frequency respectively. Also increasing broadens the bandwidth but has little influence on the bandwidth of the signal. At moderate feedback, the OFSMI signals are still band limited with much higher cut-off frequencies, and there is a considerable spectrum spread over a very wide range in the high frequency end. The presented analysis provides sufficient information for applying signal processing to OFSMI signals such as the filter design.

1. Introduction

In the last decade, optical feedback self-mixing interferometry (OFSMI) has attracted many researchers and extensive studies of using OFSMI have been conducted for various applications [1-3]. An OFSMI system employs a single semiconductor laser diode with a build-in photodiode and thus is much simpler in contrast to conventional interferometry techniques. A significant conclusion regarding the performance of OFSMI systems is that resolution of $2/\lambda$ is considered to be achievable for displacement measurement by means of fringe-counting method under weak or moderate feedback regime, where λ is the operating wavelength of the laser [4]. To achieve a higher resolution, other more advanced techniques must be utilized [5-9].

Signal processing plays an important role in the OFSMI systems. Due to various factors signal waveforms observed in OFSMI systems always contain noise, sparkle disturbance and amplitude fluctuation. In order to achieve accurate measurement, the raw signals must be pre-processed before any measurement technique is applied. As the signal waveform carries useful information, it is always desired that pre-processing is able to eliminate the noise and disturbance and at the same time to keep the waveform shape unchanged. Out of other signal processing techniques, filtering is always an effective tool for noise elimination for OFSMI signals. Design and implementation of suitable filters is an important issue which requires the knowledge and characteristics of the signals. However, to the best knowledge of the authors this issue has not been thoroughly addressed by other researchers so far.

The main purpose of this paper is to study characteristics of OFSMI signals in frequency domain. To achieve this, a theoretical model of OFSMI is described in Sections 2 and the spectral properties of simulated OFSMI signals are presented in Section 3. The discussion and conclusion are given in the Section 4.

2. The Background of OFSMI

The Optical Feedback Self-Mixing interference is introduced by intentionally allowing a laser beam to be reflected back into laser cavity, which induces output power fluctuation to the laser light. This fluctuation is monitored by a photodiode within the laser package. The optical output power with optical feedback can be expressed through following formulas [10, 11]:

$$\phi_f(t) = \phi_0(t) - C\sin[\phi_f(t) - \arctan(\alpha)] \qquad (1)$$

$$g(t) = \cos[\phi_f(t)] \qquad (2)$$

$$P_f(t) = P_0[1 + mg(t)] \qquad (3)$$

where $\phi_f(t)$ and $\phi_0(t)$ are the laser phase shift with and without feedback respectively, α is Linewidth Enhancement factor, C is optical feedback level factor. In case of $C \leq 1$, power fluctuations at laser output are similar to the sinusoidal oscillation, and the laser diode is working at weak feedback regime. When C becomes greater, the fluctuation becomes more saw-tooth-like. P_0 is the laser emitted intensity without external feedback. m is modulation index depending with the reflection coefficient of the reflector.

In the Equation (1), the phase shift $\phi_0(t)$ of external target is determined by:

$$\phi_0(t) = \frac{4\pi L(t)}{\lambda_0} \qquad (4)$$

where $L(t)$ is the external cavity length and λ_0 is the wavelength of the laser without optical feedback. In case that the external cavity is subject to sinusoidal vibration, $L(t)$ is expressed as:

$$L(t) = L_0 + \Delta L \sin(2\pi f_v t) \qquad (5)$$

where L_0 is the length external cavity when the object is at the equilibrium position of the vibration and ΔL is the amplitude of vibration respectively, f_v is the vibration frequency. Thus $\phi_0(t)$ can be rewritten as:

$$\phi_0(t) = \frac{4\pi}{\lambda_0}L_0 + \frac{4\pi}{\lambda_0}\Delta L \sin(2\pi f_v t) \qquad (6)$$

Obviously, as ϕ_0 is periodic with a fundamental frequency f_v, the OFSMI waveform $g(t)$ will also be periodic with the same fundamental frequency.

3. Spectrum Analysis of OFMSI Signals

3.1. Basic Considerations

Consider the case that the OFSMI signal $g(t)$ is sampled at the rate of $f_s = 1/T$ sample per second and $t = nT = n/f_s$, a discrete sequence $g(n) = g(nt)$ is obtained. Note that $g(n)$ should be periodic if $\frac{f_s}{f_v}$ is an integer, that is $\frac{f_s}{f_v} = K$ and a signal fundamental period has K samples.

The spectrum of above can be obtained by DFTs as follows:

$$G(k) = \frac{1}{N}\sum_{n=0}^{N-1} g(n)e^{\frac{-i2\pi kn}{N}} \qquad (7)$$

Note that the first $N/2+1$ DFT components correspond to frequency components equally spaced between zero and $f_s/2$. In addition, as $g(n)$ is periodic, we should choose the length of DFT N to be an integer multiple of K, e.g. $N = KM$. In such a way the N samples cover M fundamental periods of the signal waveform $g(n)$. The spectrum resolution associated with the DFT in this case is $\frac{f_s}{N}$. Note that longer DFT should be used if we want to achieve higher frequency resolution.

3.2. Spectrum Analysis of OFSMI Signals under Weak Feedback

In order to analyze the spectrum of OFSMI signal, we consider a case where the external target is subject to a simple harmonic vibration with $\Delta L/\lambda_0 = 2$ and $L_0/\lambda_0 = 20$ respectively. The signal waveform is sampled at the rate of 256 times higher than the vibration frequency, that is, $K = \frac{f_s}{f_v} = 256$. In this case we have:

$$\phi_0(n) = 80\pi + 8\pi\sin(\frac{2\pi n}{256}) \qquad (8)$$

Note that we used the normalized magnitude and frequencies for convenient of manipulation.

The length of DFT is chosen as $N = 4096$, and hence $M = 16$ implying that the data covers 16 vibration cycles of the external target and that the frequency resolution is $f_v/16$.
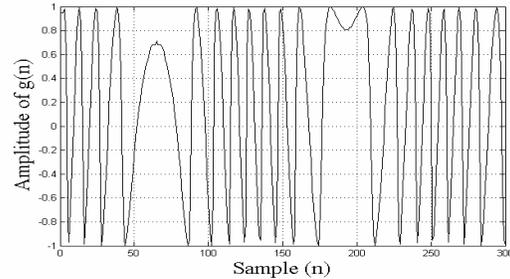

Figure 1. The simulated OFSMI signal with $C = 0.7$ and $\alpha = 3$

Under the weak feedback regime, the waveform of OFSMI signal is similar to a distorted sinusoidal waveform. As an example, the first 300 samples of simulated signal $g(n)$ are plotted in Figure 1.

The corresponding spectrum of OFSMI signal is shown in Figure 2. It is seen that the spectrum exhibits a harmonic structure where there is a peak component at the fundamental vibration frequency plus many harmonic components. It is noticed that there is a group of harmonic components between 14th and 24th order of vibration frequency.

In time domain, each interferometric fringe appears when the displacement varies over a range of $\lambda_0/2$, or equivalently, $\phi_0(t)$ varies over a 2π. As the result, the number of fringes in the half vibration period of OFSMI signal $g(n)$ is determined by $\frac{4\Delta L}{\lambda_0}$. For the example being considered, there are about 16 fringes in each period of $g(n)$ as shown in Figure 1. In frequency domain, these fringes will appear as harmonic components with the frequencies of around 16 times higher than the vibration frequency, that is, the 16th order harmonics of the vibration frequency. Figure 2 shows that the strongest peak (at $18f_v$) is close to fringe frequency, which is consistent to what are expected.
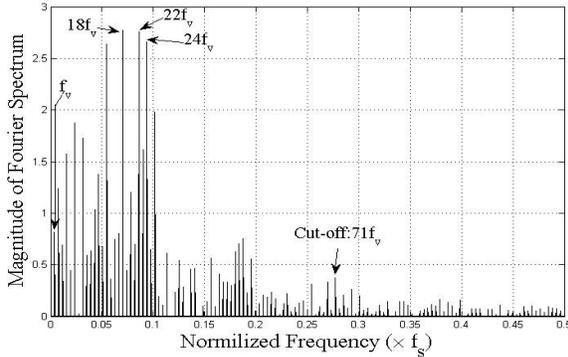

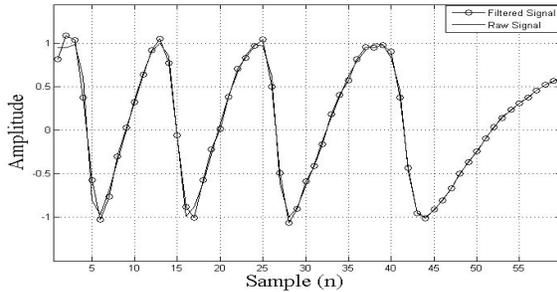**Figure** 2. Fourier spectrum of the signal

Figure 3. The raw signal $g(n)$ with solid line and filtered signal $\tilde{g}(n)$ with cycle dot line

The spectrum in Figure 2 also shows that the OFSMI signal is band-limited with most of the energy falling within the range between the vibration frequency f_v and the fringe frequency respectively. If we want to eliminate noise by means of a band-pass filter, the filter should have the pass-band covering the frequency band of the signal. However, the performance of filtering will depend on the distribution of spectrum. We wish that the signal to be strictly band-limited in that little power spreads outside the pass-band. In order to check the influence of band-pass filters, we assume that an ideal band pass filter is applied which has the cut-off frequencies f_1 of ($f_1 < f_v$) and f_2 respectively. The filtered output is obtained by setting all components outside the range $[f_1, f_2]$ into zeros, and then calculating the inverse DFT. Figure 3 shows the comparison between the original signal (the solid line) and the filtered one (the dotted line) for the case when f_1=0.002686 and f_2=0.2773 respectively.

In order to further check the effect of bandpass filtering, we use means square error (MSE) to measure the difference between the filtered output and original signals as follows:

$$\sigma = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[\tilde{g}(n) - g(n)\right]^2} \qquad (9)$$

where $\tilde{g}(n)$ and $g(n)$ are the filtered and raw signal respectively. By keeping the above MSE a constant value, we work out the minimal f_2 which is considered as the upper cut-off frequency of the filter. Table 1 shows the results for the case of $\sigma = 0.004$ with C varying from 0.1 to 0.9.

Table 1. Spectrum analysis of OFSMI signal weak feedback regime with $\alpha = 3$

C	Upper frequency f_2 for band pass filter	MSE of error residual σ
0.1	0.1094($28f_v$)	0.004
0.7	0.2773($71f_v$)	0.004
0.9	0.3984($102f_v$)	0.004

Table 2. Spectrum analysis of OFSMI signal at weak feedback regime with $C = 0.7$

α	Upper frequency f_2 for band pass filter	MSE of error residual σ
0.7	0.2773($71f_v$)	0.004
2	0.2773($71f_v$)	0.004
4	0.2773($71f_v$)	0.004
6	0.2773($71f_v$)	0.004

It shows an increasing trend in bandwidth when increasing C from 0.1 to 0.9. On the other hand, when C is fixed, the influence of α to the power spectrum distribution is revealed by Table 2. An interesting feature observed is that with the increasing of α, the bandwidth of simulated signal remains almost unchanged.

3.3. Spectrum Analysis at Moderate Feedback Regime

In case of moderate feedback, the signal exhibits a sawtooth-like waveform [10-12]. Figure 4 illustrates a single period of simulated signal $g(n)$ with $C = 2.6$ and $\alpha = 3$. The corresponding spectrum is plotted in Figure 5. Compared with the spectrum represented in Figure 2, the fundamental frequency becomes the dominant component. On the other hand, fringe frequency components become much weaker.
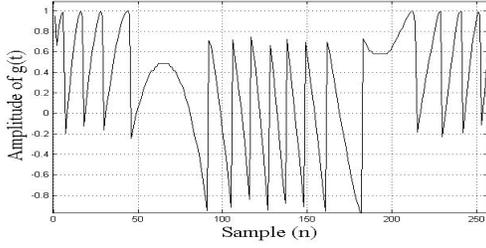

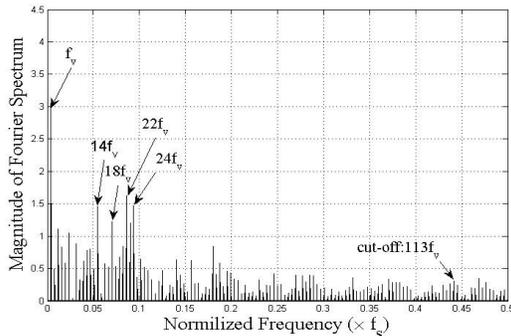**Figure** 4. First 256 samples of simulated OFSMI signal with $C = 2.6$ and $\alpha = 3$

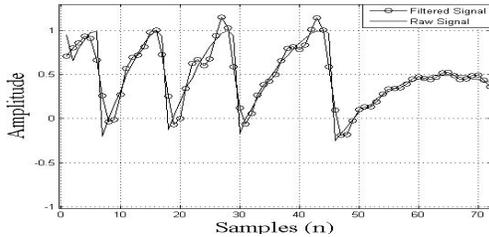**Figure** 5. Fourier spectrum of the OFSMI signal

Figure 6. The raw signal $g(n)$ with solid line and filtered signal $\tilde{g}(n)$ with cycle dot line

It was noticed that some small peaks, appeared in higher frequency area in Figure 5. This is because sharp edges appear in the waveform. As these sharp edges contain useful information, the stopband and cut-off frequency should be chosen with more care.
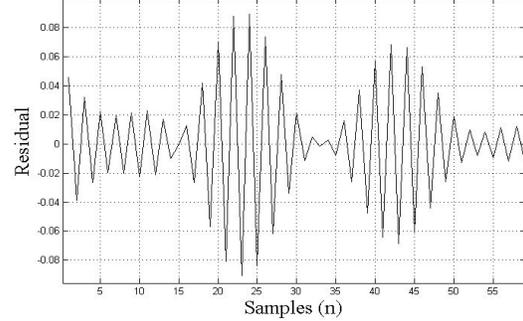

Figure 7. The error residual of filtered signal $\tilde{g}(n)$

We also checked the influences of band-pass filtering to the signals using the same approach presented above for the cases of weak optical feedback. For this case being studied, the pass band of filter is chosen to cover the frequency range between $f_1 = 0.0027$ and $f_2 = 0.4414$. Figure 6 shows the comparison between the filtered signal (in dotted lines) and unfiltered signals (in solid lines). The error residual is plotted in Figure 7. As it can be seen, although σ is quite small with the value of 0.004, significant difference between two signals can still be found at the sharp edge locations. This implies that in practice it is difficult to use a band-pass filter that is able to eliminate noise while keeping the waveform shape unchanged at the same time.

Table 3. Spectrum analysis of OFSMI signal moderate feedback regime with $\alpha = 3$

C	Upper frequency f_2 for band pass filter	MSE of error residual σ
1.6	0.4609 ($118 f_v$)	0.004
2.6	0.4414 ($113 f_v$)	0.004
3.6	0.4063 ($104 f_v$)	0.004
4.6	0.3867 ($99 f_v$)	0.004

Table 4. Spectrum analysis of OFSMI signal at moderate feedback regime with $C = 2.6$

α	Upper frequency f_2 for band pass filter	MSE of error residual σ
0.7	0.4285 ($109 f_v$)	0.004
2	0.4453 ($114 f_v$)	0.004
4	0.4414 ($113 f_v$)	0.004
6	0.4492 ($115 f_v$)	0.004

Table 3 shows the variance of the upper frequency of ideal band-pass filter with respect to C value when we keep the MSE constant (i.e. 0.004). It is seen that f_2 is much higher than that in case of weak optical feedback. When C is fixed, the variation of bandwidth with respect to α is shown in Table 4. It shows that α has a slight increase in the pass band of the filter.

4. Conclusion

The spectrum analysis was carried out for OFSMI signals based on simulation with respect to the values of C and α. In case of weak feedback regime, the feedback parameter C has significant influence to the bandwidth of OFSMI signal in that increasing C will broaden the bandwidth, but α has little influence on the bandwidth of the signals. Also as the signals are strictly band-limited, we can always design a band-pass filter for pre-processing the signal in order to eliminate noise while keeping the signal waveform. However at moderate feedback regime, although the signals are still close to be band limited, the spectrum spreads over a much wider range at high frequency area. In this situation use of linear bandpass filters are problematic in that it always results in signal waveform change. In other words, it is a challenging issue to extract OFSMI signal waveform buried in noise in the case of moderate optical feedback.

5. References

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