Identification and optimisation of hopper-discharge chute systems for bulk granular materials

G. John Montagner
University of Wollongong

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IDENTIFICATION AND OPTIMISATION OF HOPPER-DISCHARGE CHUTE SYSTEMS FOR BULK GRANULAR MATERIALS

by

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Thesis submitted for the Degree
of Doctor of Philosophy

Department of Mechanical Engineering,
The University of Wollongong.

May, 1977
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ABSTRACT

In an effort to decrease the price-performance ratio in the agricultural industry for material handling, and recognising that the hopper-discharge chute forms a part of most such handling systems, optimum performance of this component becomes a necessity. To achieve this goal it is necessary to first obtain a detailed knowledge of its dynamic characteristics.

Operation under uniform flow conditions is analysed and a model incorporating a generalised drag force is used to formulate the design criteria to obtain chute profiles suitable for operation in the 'fast' flow mode and able to achieve a prescribed optimum performance. The design procedure for minimum transit time is presented in the form of a computer programme incorporating a sensitivity analysis on all significant design parameters. The resulting optimum chute's performance was contrasted with the performance of other chute shapes more commonly used.

The transient flow characteristics of a model hopper-discharge chute system under gravity flow are examined. Using the P.R.B.S. and cross-correlation method of identification, the impulse flow responses for the system under varying conditions of initial flow and chute geometries are obtained. With the aid of spectral analysis the magnitude and phase characteristics of the identified hopper-chute model were determined, enabling both the system bandwidth and the behaviour within that bandwidth to be determined.
This information permits the examination of non-linearities in the flow as well as enabling the determination of the dynamic flow response characteristics during prescribed controlled operation of the flow control valve.

This transient analysis was supported by the use of high speed cine photography to determine the flow characteristics associated with the operation of the flow control valve. Techniques for conducting this high speed film analysis are presented.

It is shown that of the chutes tested, the optimum chute has the most favourable overall performance characteristics both under uniform and transient flow conditions.
ACKNOWLEDGEMENTS

The investigation described herein was carried out in the Department of Mechanical Engineering of the University of Wollongong.

The invaluable assistance and guidance given by Professor A.W. Roberts, Dean of Engineering, the University of Newcastle, and by Associate Professor W.H. Charlton, Department of Electrical Engineering, University of Wollongong, in all stages of the work and in the preparation of this report is gratefully acknowledged.

For his many suggestions with the electrical apparatuses and with the assembler language programme, I wish to thank Dr. G.W. Trott of the Department of Electrical Engineering.

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LIST OF PRINCIPAL SYMBOLS

The various symbols introduced in the text are listed here in alphabetical order.

a coefficient
a acceleration
B chute width
f frequency
\( F_q \) drag force on grain stream
\( g \) acceleration due to gravity
G power spectral density
h impulse response or system weighting function
H average depth of grain stream
H(f) system function
\( H_0 \) depth of grain stream at entry to chute
HB depth of grain stream to chute width ratio
j \( \sqrt{-1} \)
k ratio of lateral to normal pressure along chute wall
k_{eo} effective linear pressure gradient down chute wall surface at zero velocity
m mass
n normal co-ordinate
n number of samples
n(t) noise signal
|N| modulus of normal force
N' normal force per unit mass
N number of clock pulses per sequence
Pn  pressure in normal direction
R  correlation function
s  displacement
So  constant power spectral density
\( t \)  time
\( t \)  tangential co-ordinate
\( \Delta T \)  period of clock pulse
V  grain stream velocity
Ve  grain stream exit velocity
Vo  grain stream velocity at chute entry.
x, y  Cartesian co-ordinates
x(\( t \))  input signal
y(\( t \))  output signal
\( \theta \)  chute slope angle with the vertical
\( \theta_f \)  limiting slope angle
\( \tau \)  correlation delay time
\( \tau \)  coefficient of friction of grain on chute surface
\( \tau_e \)  equivalent coefficient of friction on stream element
\( \lambda \)  dummy variable for time
\( \mu \)  coefficient of velocity dependent resistance
\( \rho \)  radius of curvature
\( \omega \)  computation variable
CHAPTER 1: INTRODUCTION

1.1 General

The need to design, install and control highly efficient materials handling systems cannot be denied. Palowitch (Reference 1) estimates that in the U.S.A., for example, materials handling activities cost industry about $125 billion in 1970 representing 25% of the Gross National Product of that country. Thus, even small incremental gains in the efficiency of materials handling can yield significant cost reductions.

The variety of materials currently being handled in bulk is almost endless. The Australian agricultural industry provides a range of such activities. The materials handling function only adds time-position utility to the material; it does not add form-utility usually.

The viability of the total bulk materials handling system and its components is strongly influenced by the characteristics of the material. The ability to delineate the relevant material characteristics and the ability to describe quantitatively the mechanics of flow are essential pre-requisites to ensuring this viability.

In narrowing this investigation to granular agricultural products, one takes cognizance of the present annual Australian wheat production requiring the handling of more than twenty million tonnes per year. For 1973 this amounted to 20,135,000 tonnes with a dollar value in excess of $348 million (Reference 2).

It is recognised that the techniques developed for these agricultural products should quite readily be applicable to other suitable free flowing cohesive and non-cohesive materials.
1.2 Outline of the Problem

The substance of this investigation forms part of a comprehensive research programme in progress at the University of Wollongong to study the bulk handling of granular materials. It is well recognised that the hopper-discharge chute is a component of most material handling systems. Roberts (Reference 3) highlighted the need for determining the dynamic characteristics, with a view to optimising the design for gravity flow under steady state conditions and to enable accurate flow control under transient conditions, such as blending.

In the bulk handling process it is usually necessary to transport the granular material from an elevated storage bin to some laterally displaced lower position. The problem of efficiently utilising the available potential energy, so as to achieve some specified performance objective, has received increased attention by Roberts et al (References 4 to 6).

The main objective of the steady flow problem has been to achieve optimum chute profiles for either minimum transit time or maximum exit velocity in some specified direction, whilst achieving stable 'fast flow'.

Use was made of the Fletcher-Powell constrained minimisation algorithm, to optimise the selection of coefficients for the polynomial approximation method, for evaluating the steady flow problem.

For the controlled flow case where flow transients are caused by the operation of the control valve, the complexity of the problem makes it quite difficult to apply theoretical analysis for the determination of the dynamic model.
The advantages of experimental techniques based on statistical methods are now well known, being more reliable than conventional methods applied to systems that are inherently 'noisy'. This study makes use of the Pseudo-Random Binary Signal and cross-correlation method of analysis to identify the transient characteristics of the hopper-discharge chute system. High speed photographic techniques were used to aid the interpretation of phenomena indicated by the stochastic analysis.

1.3 Review of Previous Research

1.3.1 Granular Materials

A number of schemes (Reference 7) have been used in the past to quantify the steady state 'dynamics' of the hopper-discharge chute system with some success. Although, at first sight, an analytical solution seems straightforward, Roberts (References 6, 7) showed, that because of the distributed nature of the parameters involved, lumped parameter approximations imposed a number of restrictions to a general solution. A feasibility study (Reference 8), indicated that the Pseudo-Random Binary Signal (P.R.B.S.) and cross-correlation method, might prove to be quite effective in identifying the flow transients under controlled flow, because of extraneous system noise insensitivity.

Optimisation studies (References 8 to 13) have concentrated on either minimising transit time through the chute or maximising horizontal exit velocity under steady state conditions for gravity flow, through chutes of varying geometry. Solution methods, based on variational techniques including, Pontryagin's minimum principle, proved awkward since small variations in the initial values of the Lagrange multipliers produced large differences in the solution. A
discrete segment method of solution for the descent curve proved (Reference 10) efficient only when Coulomb friction was ignored. The polynomial approximation outlined by Chiarella (Reference 11) was used to design a chute profile, resulting in improved relative transit times compared to the more commonly used profiles. However, due to the low exit angle, instabilities in flow occurred and the method needed to be modified to incorporate a constraint on the minimum exit or lift angle in the profile.

Parlour (Reference 14) delineated the parameters necessary to quantify the mechanics of granular material flow down inclined straight chutes. The experimentally derived pressure profiles refined the earlier work of Roberts (References 3, 7) in introducing the concept of equivalent friction coefficient providing additional insight into the pressure distribution at the chute boundaries as well as the total drag force.

1.3.2 System Identification

Recent comparative studies (References 15 to 20) of system identification schemes have highlighted the difficulty of finding a general method for system identification, that can yield meaningful results for a broad class of identification problems. However, given the usual unavailability of 'a priori' information of system parameters often encountered in actual plant identification, the P.R.B.S./cross-correlation method has been found to yield the best overall performance, with regard to reliability, accuracy and computation speed; suitably modified for on-line identification (Reference 15).
The many advantages of the P.R.B.S./cross-correlation method of identification must be weighed against the disadvantages of high equipment cost for on-line analysis, and large data reductions for off-line work. The former was not a problem in this case as the necessary equipment was on hand.

In a pilot study (References 8, 21) the application of the P.R.B.S./cross-correlation identification method to bulk handling problems was demonstrated. The inherent noise immunity of this identification method makes it suitable for application to the hopper-discharge chute problem under investigation.

1.4 Scope of this Research

The experimental investigations which form the basis of this research thesis generally extend previous work in this field. The steady flow problem is analysed, and formulated on the basis of a lumped parameter model subjected to drag forces resulting from both Coulomb and velocity dependent friction. The possibility of flow instability, mentioned previously resulting from chute profiles generated by methods based on unconstrained parameter optimisation, is obviated by using an optimising technique after Fletcher-Powell enabling optimisation under parameter constraints. A programme is developed which enables an optimal chute geometry to be calculated given the material characteristics. The inlet conditions and outlet position are such that a nominated parameter can be either minimised or maximised. The cases analysed include minimum descent time and maximum exit velocity in a specified direction.

The main research effort is directed towards the experimental identification of the hopper-discharge chute dynamic model and an experimental test of its validity. Pursuing the analysis of Choda
and Willis (Reference 22), regarding the phenomenon of 'fast' and 'slow' flow regimes, this report concerns itself to the analysis of the 'fast' flow phase. In the 'fast' flow mode the material makes contact with the chute bottom and side walls but does not make contact with the top of the chute. In the 'slow' flow mode the chute operates completely full, with the material making contact across the complete perimeter of the chute. 'Fast' flow is more efficient with the flow rate out of the hopper being entirely governed by the characteristics of the hopper orifice. In 'slow' flow additional retardation effects are encountered which impair the flow rate and the chute becomes, in effect, an extension of the hopper.

It is shown that the P.R.B.S./cross-correlation method of model identification, is suitable to the granular materials handling field, yielding both a time domain and via Fourier transforms, a frequency domain description of the dynamic model. Because of the noise immunity characteristics of this technique, it is possible to carry out on-line identification by superimposing low energy random signals onto the plant under test whilst the plant is in normal operation. A side benefit of this method yielded an accurate determination of in-chute transit time.
PART 1: UNIFORM FLOW
CHAPTER 2: GRANULAR MATERIAL FLOW DYNAMICS

2.1 General

The designed transfer of granular solids by gravity flow through discharge chutes requires the establishment of satisfactory geometrical characteristics of the chute profile. Roberts (Reference 3) showed that for efficient operation, the most desirable mode of flow is the 'fast' flow mode in which the grain stream thickness tapers towards the exit end of the chute.

Occasions arise whereby a discharge chute is required to achieve some prescribed optimised flow performance. A case in point is that of finding the profile which yields a minimum transit time. This implies a minimum average stream thickness which is a desirable feature of discharge chute operation. Another often cited example is that of maximising the kinetic energy to achieve a maximum discharge velocity, particularly in a horizontal direction to impart a maximum throw to the granular material.

Blending and mixing processes incorporating gravity flow bins and discharge chutes require the transient flow be controlled. The operation of the control valves cause flow transients, which in the case of critical blending operations with unavoidable transport blending lags, give rise to unnecessary out-of-specification mix.

A necessary pre-requisite to solving these problems is the determination of the dynamic model. The complexity of this problem was alluded to by Roberts (Reference 8) in proposing a pilot study based on experimental methods, rather than purely theoretical analysis, to establish the dynamic model. On the basis of the results of that pilot study, this experimental investigation based on experimental system identification techniques was undertaken.
2.2 **Description of the Apparatus**

The experimental apparatus is designed to investigate the gravity flow of the selected granular material through chutes of different geometries, at various discharge rates and chute inclinations. The design allows for recirculating operations, with the grain discharging from an elevated fixed 'funnel flow' hopper, through an orifice chute system under test to the receiving hopper, which is the feed hopper for the grain conveyor. Figure 1 shows the general arrangement of the apparatus.

Construction details of the apparatus, and other ancillary fabricated equipment are described below.

**Main Rig Framework**

The main rig (Plate 1) is fabricated from steel angles welded to form a rigid framework, in which the chutes and other auxiliary components may be erected as required for each test. The model 'funnel flow' hopper, of approximately 0.1m³ capacity, has the front wall made of perspex to facilitate grain movement observations. To the base is fitted a steel orifice assembly with provision for static flow control setting as well as a dynamic perturbing valve operated by a Philips (PR 9270) solenoid. The recycling ability is clearly seen (in Figure 1 and Plate 1).

The rig accommodates straight chutes of approximately one and a quarter metres in length at inclinations ranging from the stalling angle to approximately 30° from the vertical depending on cross-section. Chutes of curved geometries passing through the same end points are also accommodated.
1.1 General arrangement of the testing apparatus

FIGURE 1 The testing apparatus
FIGURE 1: The testing apparatus

1.2 Hopper discharge characteristics
Plate 1  Main testing rig
Control-Perturbing Valve

The control-perturbing valve assembly (Plate 2) is bolted to the bottom of the elevated hopper and effectively controls the datum discharge rate through a variable sized rectangular orifice and additionally allows the perturbation of this main flow with the fluctuating pseudo-random binary signal. A general assembly of the valve is shown in Figure 2.

The two gates slide in slots under the orifice assembly butting against each other in the closed position. The slides are so arranged in close contact with the orifice plate to obviate jamming by whole grains. The main gate is manually set and locked with a locking screw in the test position. The perturbing gate is operated by a Philips electro-dynamic vibration exciter (type PR 9270) powered from a Philips power amplifier (Type GM 5535). To the perturbing gate is also fitted a Hewlett Packard displacement transducer of the 7DCDT series which monitors the actual movement of this gate. The nominal stroke of the Philips solenoid of ±4mm could be reduced by the adjustment of a mechanical stop. The chute supports fitted were fabricated from perspex enabling visual and photographic observation of flow from the hopper orifice.

Chutes

Chutes were fabricated from 5mm perspex to give an open chute of rectangular cross-section with an internal width of 25mm and a wall height of approximately 90mm. The curved chutes were cut using a bandsaw from full sized drawings glued to the perspex protective sheet.
Figure 2  HOPPER-ORIFICE ARRANGEMENT
Plate 2  Underside view of hopper
Inclinometer

An inclinometer was used to measure the angle of inclination of the test straight chute to the horizontal. The device is located at the base of the chute and a plumb bob enables inclinations to be read off directly from graduations marked on the perimeter of the quadrant.

Flowmeter

The design of a number of direct reading electrical output flowmeters was attempted (see section 10.1). For the present tests a cantilevered beam fitted with a rectangular flap and strain gauges was used. Plate 3 shows the flowmeter assembly with its multiaxis adjustment.

2.3 Model

2.3.1 Introduction

Choda and Willis (Reference 22) nominated two flow regimes, 'fast' and 'slow' flow. Roberts (Reference 3) initially and more recently Parlour (Reference 14) proposed approximate theories to account for the flow behaviour. The theories, based on dynamic analysis, gave rise to generalised flow equations, which were nonlinear in form. An equivalent friction coefficient was introduced by Roberts (Reference 3) to account for the drag on the chute bottom and side walls, based on a pressure distribution as shown in Figure 3.
Plate 3  Flowmeter
Parlour (Reference 14) improved the approximation by measuring the bottom and side wall pressure distribution and found that the distribution was as shown in Figure 4. It can be seen that the bottom pressure distribution is approximately uniform across the chute while the side wall pressure increases nonlinearly with depth of grain.

**Figure 3** Pressure distribution on chute walls (Roberts)
'Fast' flow conditions represent the more efficient mode of operation and will be the basis of modelling. The three researchers mentioned worked with millet seed flowing through perspex chutes, and advantage is taken of the availability of their published parameters to conduct this study.

2.3.2 Lumped Parameters

Although the physical problem is concerned with a distributed discrete system, Roberts (References 3, 7) has shown that, for the 'fast' flow mode, the bulk flow characteristics may be modelled in terms of a representative single particle moving subject to tangentially directed resisting forces.
On the basis of the pressure distribution shown in Figure 3 in which it is assumed that the vertical pressure is equal to the hydrastatic pressure, this being consistent with shallow bed operation, Roberts (Reference 7) derived the following expression for the equivalent coefficient of friction

$$\tau_e = \tau (1 + k \frac{H}{B})$$

where $k$ = ratio of lateral to vertical pressure

$\frac{H}{B}$ = stream depth to width ratio.

Using this concept of equivalent coefficient of friction it was shown that the resistance to flow was a combination of Coulomb and speed dependent friction forces.

In the same reference, analysis of the energy loss in the flowing stream was undertaken and the ratio of work done due to grain-on-grain sliding and grain-on-chute sliding resulted in the following proportions.

$$U_g : U_s : U_b = 9.4\% : 8.3\% : 82.3\%$$

where $U_g$ = work done due to grain-on-grain sliding

$U_s$ = work done due to grain-on-chute sidewall sliding

$U_b$ = work done due to grain-on-chute bottom sliding.

This showed that the major energy loss was against the chute bottom with the contributions of $U_g$ and $U_s$ being quite small. The lumped model takes into account the friction drag force based on $U_s$ and $U_b$ and by working with the average velocity the grain-on-grain sliding has a relatively minor contribution and was ignored.

More recently Parlour (Reference 14) having determined the actual pressure distribution to be as shown in Figure 4 was able to refine the expression for the equivalent coefficient of friction. He
found that on this basis the ratio of work done by the various components was

$$U_g : U_S : U_b = 10.4\% : 11.3\% : 78.3\%$$

which further supports the assumption of a lumped parameter model.

Figure 5 shows the co-ordinates with respect to which the particle dynamics are formulated. The gravitational force acts in the direction of the positive y axis.
Parlour found that for millet seed in an open channel perspex chute the following parameters are applicable:

\[ \tau = \text{average coefficient of friction} \]
\[ = 0.306 \]

\[ \tau_e = \text{effective coefficient of friction (includes friction between grain base and walls of the chute and the equivalent friction due to dynamic internal stresses).} \]

\[ \tau_e = \tau [1 + k_{eo} \frac{H}{B} (1 + cv^2)] \]  \hspace{1cm} (2.1)
\[ = \tau [1 + k_{eo} \frac{H}{B} + k_{eo} \frac{H}{B} cv^2] \]

\[ \text{Since } \frac{v_o}{v} = \frac{H}{H_0} \]
\[ \therefore \quad H = \frac{v_o H_0}{Bv} \text{ (uniform flow conditions)} \]
\[ \therefore \quad \tau_e = \tau [1 + k_{eo} \frac{v_o H_0}{Bv} + k_{eo} \frac{v_o H_0}{Bv} cv^2] \text{ (for } v > 0) \]
\[ \equiv \tau [1 + \frac{c_1}{v} + c_2v] \text{ (for } c_2 = 0 \text{ same as Roberts')} \] \hspace{1cm} (2.2)

where
\[ c_1 = \frac{k_{eo} v_o H_0}{B} \]
\[ c_2 = \frac{k_{eo} v_o H_0}{B} c \]

Now for millet seed:

\[ k_{eo} = 0.346 \]
\[ c = 0.0215 \text{ (sec/ft}^2) \]
\[ = 0.001997 \text{ (sec/m}^2) \]
\[ \therefore \quad c_1 = 0.346 \frac{v_o}{B} H_0 \]
\[ c_2 = 6.9096 \times 10^{-4} \times \frac{v_o H_0}{B} \]
\[ \tau = 0.306 \]

\[ \therefore \quad \tau_e = 0.306 \left[ 1 + 0.346 \frac{V_o H_o}{B V} + 6.9096 \times 10^{-4} \times \frac{V_o H_o}{B} v \right] \]

(2.3)

The limiting angle of theta, \( \theta_f \), is given by:

\[ \theta_f = 90^\circ - \tan^{-1}(\tau_e) \]

(2.4)

For the present \( \theta_f \) will be interpreted to be the angle at which a straight chute would begin to stall or choke as its inclination with the vertical was increased whilst 'fast' flow was in progress. Later it will be shown that this angle constitutes a major constraint in the design of chutes for trouble-free stable flow.

2.3.3 Statistical Evidence

The validity of the lumped parameter model was investigated, so as to gain confidence in the optimal chute profiles to be predicted from it. Visual and photographic observations of the stream flow within the chute and upon leaving the chute were made.

The test rig was arranged as depicted in Figure 1 so as to measure the transit time for a single grain particle moving down the chute. In order to circumvent the difficulty of recording the impact of a single grain due to transducer sensitivity limitations, approximately ten grains were released simultaneously. Observation revealed little interference throughout the flight down the chute with the grains sliding on the bottom surface and striking the flowmeter with an impact pattern shown by trace 2 on Plate 4.
The required number of grains were placed on the perturbing valve within the empty hopper and the valve was then stepped open using a square wave function generator as a driving signal to the solenoid. The motion of the perturbing valve, as measured by the linear displacement transducer, was used to trigger a dual trace storage cathode ray oscilloscope (CRO). The striking of the flowmeter by the grains provided the second trace, delayed by an amount equal to their transit time. Plate 4 shows a typical test enabling the transit time to be measured as the difference between the two traces.
In order to confirm or otherwise, the validity of the lumped parameter model, tests were carried out as discussed above. The mean descent time was found to be 0.704 seconds* for the model chute with geometry corresponding to the constrained optimisation specification given elsewhere (Section 2.3.1). This compares with 0.700 seconds calculated by estimating the step response of the chute. It will be shown (Section 5.2) that the cross-correlation determined under these conditions approximates the impulse response of the chute which, when integrated, yields the step response. The transit time was taken to be the delay time to the midpoint of the step rise, as shown in Figure 6.

* Since it is realised that this study concerns itself with incremental gains in performance, the necessity arises to quote experimental results to a rather large number of significant figures in order to report these incremental differences.
2.4 Uniform Flow

2.4.1 Introduction

A rigorous analysis of the uniform steady state flow of granular material, down chutes is a necessary pre-requisite to the implementation of an optimal design. The uniform flow case is of particular interest, as it is felt that this constitutes the activity in which chutes are presently most frequently utilised. The aim here is to present a general solution to the problem of establishing stable 'fast' flow in chutes of prescribed geometry. Given certain known characteristics of the material to be transported the physical constraints of the chute entry and exit conditions, we seek a general solution to the chute geometry such that a choice can be made as to the optimisation of any given parameter. A single parameter will be chosen for optimisation from the great many possible variables to illustrate the general solution method.

2.4.2 Apparatus

All the uniform flow tests were carried out on the model hopper-chute rig shown in Figure 1. Chutes with profiles as shown in Figure 7 and Plate 5 were, in turn connected to the testing rig such that all of them received from and discharged to the same end points. That is, the straight chute at 40° to the vertical, the parabolic chute of the form

\[ x = cy^2 \]

where \( c = \frac{x_0}{(y_0)^2} \)

and both the unconstrained and the constrained (as detailed later) profile chutes were tested with identical inlet and outlet
Figure 7  Chute profiles for model chute
Plate 5  Comparison of Chute Profiles
co-ordinates. As can be seen the hopper, chute and grain elevator form a closed loop enabling the static capacity of the model hopper to be effectively increased by recycling the grain under test. The motor speed controller on the grain elevator enabled the auger lifting capacity to be varied and, if desired, to be set equal to the discharge rate from the hopper, effectively maintaining a constant head above the hopper orifice.

2.4.3 Procedure

In each case millet seed was set cycling round the testing loop to condition the chute surface, and to filter out husks which appeared in the body of the material. The husks adhered to the perspex surfaces and had to be removed to enable proper observation of the stream flow. The cross-correlation equipment was used to measure in-chute transit times. This gave an accurate measure, by way of the transport delay, of the time taken for the grain stream to pass from the perturbing gate to the flowmeter. The main 'flow' gate was set to an orifice opening of 25mm, corresponding to a flow rate of 4 kg/min. whilst the perturbing gate stroke was tested at both ±2mm and ±4mm. The capability of the correlation method to cope with this very poor signal to noise ratio was clearly evident.

As a crosscheck, stream velocity at exit was measured using both a visual and a photographic technique. Visually, the grain stream was observed to fall in an arc in front of a card with a grid and co-ordinate system centred on the chute axis. Using the equation for free fall

$$V_e = \left[ \frac{yx^2}{2\cos^2 \alpha (y-x \tan \alpha)} \right]^{\frac{1}{2}}$$

(2.5)

and the observed x, y co-ordinates the exit velocity of the mean stream flow could be calculated.
Photographically the velocity could be determined knowing the film scaling factor and the framing rate. In all photographic sequences, a scale is included near the field of interest, and the camera enabled timing pips, derived from a timing light generator, to be placed on the film edge.

In all tests visual observation confirmed that the flow was 'fast' flow with stream depth $H$ decreasing uniformly towards the chute runout.

2.4.4 Equations of Motion

Using the parameters obtained by Parlour (Reference 14) and taking axes as shown in Figure 8 we proceed to define the equations of motion of a particle model moving down a chute of arbitrary radius of curvature $\rho$. As can be seen in Figure 5 granular material enters the open channel curved chute of constant rectangular cross-section with velocity $v_0$. At an arbitrary cross-section the average velocity of the grain stream is $v$ and the mean stream thickness is $H$. 
An energy balance between two successive cross-sections, (1) and (2) yields

\[
\text{Kinetic Energy (2) = Potential Energy (1+2) - Work done (1+2) + Kinetic Energy (1)}.
\]

\[
K.E._2 = P.E._{1+2} - W.D._{1+2} + K.E._1
\]

i.e., \[ \frac{1}{2}mv_2^2 = mgy - W.D._{1+2} + \frac{1}{2}mv_1^2 \] (2.6)

Define a drag force \[ F_D = \tau \theta |N| + uv \] (2.7)

where \[ N = dm g \sin \theta + dm a_n \] (2.8)

**Figure 8** Acceleration model - force balance
Now
\[ a_n = \rho \omega^2 \]
\[ = \rho (\frac{d\theta}{dt})^2 \]
\[ = \rho (\frac{d\theta}{dt}) (\frac{d\theta}{dt}) = \rho \left[ -\frac{1}{p} \frac{ds}{dt} \frac{d\theta}{ds} \right] \] (For this orientation \( ds = -\rho d\theta \))
\[ = -v^2 \frac{d\theta}{dy} \cos \theta \] (2.9)

Substituting Equation 2.9 into 2.8
\[ N = dm \cdot g \cdot \sin \theta + dm \cdot (v^2 \frac{d\theta}{dy} \cos \theta) \] (2.10)

Dividing by \( dm \), Equation 2.10 becomes
\[ N^1 = g \cdot \sin \theta - v^2 \frac{d\theta}{dy} \cos \theta \] (2.11)

where
\[ N^1 = \text{normal force/unit mass} \]

Substituting this into Equation 2.7 yields
\[ F_D \triangleq t_e [g \cdot \sin \theta - v^2 \frac{d\theta}{dy} \cos \theta] + \mu v \] (2.12)

\( \equiv \) generalised drag force per unit mass combining

Coulomb and Velocity dependent friction components

Now
\[ \text{W.D.}_{1+2} = \int_1^2 F_D \, ds \] (2.13)

Let \( ds \) be an element of arc at the cross-section under consideration.
\[ ds = \sqrt{1 + (\frac{dx}{dy})^2} \, dy \]
\[ = \sqrt{1 + \tan^2 \theta} \, dy \]
\[ = \sec \theta \, dy \] (2.14)
Substituting into Equation 2.13 yields

\[ W.D.1 \rightarrow 2 = \int_1^2 F_d \frac{dy}{\cos \theta} \]

i.e. Equation 2.6 becomes

\[ \frac{1}{2} v_2^2 = g y - \int_0^y \frac{F_D}{\cos \theta} \, dy + \frac{1}{2} v_1^2 \tag{2.15} \]

re-arranging

\[ v_2^2 = 2gy - 2 \int_0^y \frac{F_D}{\cos \theta} \, dy + v_1^2 \tag{2.16} \]

and finally

\[ v_2 = \sqrt{2gy - 2 \int_0^y \frac{F_D}{\cos \theta} \, dy + v_1^2} \tag{2.17} \]

Equation 2.17 expresses the velocity based on a lumped parameter model of the grain stream at an arbitrary cross-section for granular material moving under the influence of gravity subject to a generalised drag force combining Coulomb and velocity dependent friction components.

For millet seed, substituting Equation 2.3 into Equation 2.12 yields

\[ F_D = 0.306 \left[ 1 + 0.346 \frac{y_0 H_o}{B y} + 6.9096 \times 10^{-4} \times \frac{y_0 H_o v}{B} \right] \left[ g \sin \theta - v^2 \frac{d \theta}{dy} \cos \theta \right] + yv \tag{2.18} \]

which needs to be substituted into Equation 2.17 to calculate, for example, the velocity at an arbitrary cross-section (2).

From a force balance at an arbitrary cross-section in Figure 8, and using moving co-ordinates defined by angle \( \theta \) and radius of curvature \( \rho \), the dynamic equilibrium of an element within the stream flow results in a generalised equation of motion.
Using the symbols as defined above, a force balance yields:

\[ \tau_e |N| + \mu v + \frac{dm}{dt} \ddot{A}_t = \frac{dm}{dt} g \cos \theta \]  
(2.19)

and as before

\[ N = \frac{dm}{dt} g \sin \theta + \frac{dm}{dt} A_n \]
(2.8)

repeated

combining and expressing all variables per unit mass.

\[ \tau_e \left[ g \sin \theta + A_n \right] + \mu v + A_t - g \cos \theta = 0 \]  
(2.20)

Now

\[ A_t = \frac{d}{dt} (v) \]

\[ = \frac{d}{dt} \left[ \rho \left( \frac{d\theta}{dt} \right) \right] \]

\[ = \rho \frac{d^2 \theta}{dt^2} + \frac{d\rho}{dt} \cdot \frac{d\theta}{dt} \]  
(2.21)

and \[ A_n = \rho \left( \frac{d\theta}{dt} \right)^2 \]  
(2.22)

substituting Equation 2.21 and Equation 2.22 into Equation 2.20 yields:

\[ \tau_e \left[ g \sin \theta + \rho \left( \frac{d\theta}{dt} \right)^2 \right] + \mu v + \rho \frac{d^2 \theta}{dt^2} + \frac{d\rho}{dt} \cdot \frac{d\theta}{dt} - g \cos \theta = 0 \]  
(2.23)

re-arranging

\[ \frac{d^2 \theta}{dt^2} + \frac{1}{\rho} \left( \frac{d\rho}{dt} \cdot \frac{d\theta}{dt} \right) + \tau_e \left( \frac{d\theta}{dt} \right)^2 + \frac{g}{\rho} \left( \tau_e \sin \theta - \cos \theta \right) + \frac{\mu v}{\rho} = 0 \]  
(2.24)

This is the generalised equation of motion for a lumped parameter model, expressed in moving co-ordinates and incorporating a generalised drag force combining Coulomb and velocity dependent friction forces.

These equations form the basis for the calculation of chute geometries chosen to optimise some yet to be specified parameter subject to a number of chosen constraints.
3.1 General

The necessity/desirability of making incremental improvements in the efficiency of the materials handling task has already been alluded to in Section 1.1 and once the material parameters have been ascertained, the additional cost of designing a chute with optimum geometry is considered to be small compared to the integrated gain of even a minor efficiency improvement. Capital and operational costs incurred in changing chutes to suit the product being gravity feed would seem feasible.

There are numerous situations known to occur in practice which require the flow through discharge chutes to be optimised. In discharging material from a hopper into a rail car, one may wish to maximise material throw, i.e., maximise exit velocity. In some cases it may be desirable to minimise transit time in order to obtain the most favourable flow pattern, i.e., 'fast' flow, which helps to make the chute self-clearing. The constraint of matching exit speed and direction of material falling onto a conveyor belt has been cited for its potential in minimising conveyor energy losses.

In a recent application (Reference 23) use was made of what was essentially a double-sided tetrahedral chute for the controlled aerial spreading of crop fertilizer so as to achieve a specified spreading pattern on the ground, i.e., maximum uniform dispersion.

For practical reasons a designer may choose not to implement an optimal chute geometry but such knowledge enables him to work 'with certainty'. In effect, the optimal geometry becomes his 'yardstick' against which to compare his compromised solution.
3.2 Model Formulation

3.2.1 Minimum Descent Time

Whilst variational methods of optimisation are indirect and involve the finding of the extremum of a functional it is possible to reformulate this problem into an equivalent 'pseudo-static' problem in which direct methods of optimisation can be applied. In attempting to set up and solve this problem one encounters difficulties with classical methods (Reference 12). The physical model is analogous to a bead on a wire and the mathematical description of the friction force requires the modulus of the normal reaction force. This complicates the analysis and the overall problem is numerically difficult since a two-point boundary value problem arises in the application of the calculus of variations or Pontryagin's Minimum Principle. Solution methods based on the polynomial approximation outlined by Chiarella (References 10, 11) are found to be particularly suitable to this class of optimisation problem.

The choice of co-ordinates for the optimal model is an important consideration; investigations have shown that moving co-ordinates θ and y are the most satisfactory. θ(y) is the chute slope with the vertical, a function of y and is the dependent or control variable while y is the independent variable. Referring to the system model shown in Figure 8 and recalling Equation 2.17:

\[ v_2 = \sqrt{2gy - 2\int^y \frac{PD}{\cos \theta} \, dy} + v_1^2 \]  \hspace{1cm} (2.17)

repeated

The descent time between two successive cross-sections A and B is given by:

\[ t_{AB} = \int_A^B dt \]
\[ = \int_A^B \frac{ds}{v} \]  \hspace{1cm} (3.1)
Now \( ds = \sec \theta \ dy \) (from Equation 2.14)

\[ t_{AB} = \int_{A}^{B} \frac{\sec \theta}{v} \ dy \]  \hspace{1cm} (3.2)

Since the end points \((X, Y)\) are fixed, the function \( \theta(y) \) must satisfy

\[ \int_{0}^{Y} \tan \theta \ dy = X \]  \hspace{1cm} (3.3)

In order to combine Equations 2.17 and 3.2 to arrive at an expression for the descent time define a quantity:

\[ \omega(y) = -\int_{0}^{Y} \left[ \frac{\mu v + \tau_{e} |N|}{\cos \theta} \right] \ dy \]  \hspace{1cm} (3.4)

Note that \( \omega(y) \) satisfies the differential equation

\[ \frac{d\omega}{dy} = -\left[ \frac{\mu v + \tau_{e} |N|}{\cos \theta} \right] \]  \hspace{1.5cm} (3.5)

with \( \omega(0) = 0 \).

In terms of this variable, Equation 2.17 resulting from the application of work and energy principles, becomes:

\[ v_{2} = \sqrt{v_{1}^{2} + 2(gy + \omega)} \]  \hspace{1cm} (3.6)

substituting into Equation 3.2 for the descent time yields

\[ t_{AB} = \int_{0}^{Y} \frac{1}{\sqrt{v_{1}^{2} + 2(gy + \omega) \cos \theta}} \ dy \]  \hspace{1cm} (3.7)

The problem now becomes; find \( \theta(y) \) so as to minimise \( t \) whilst at the same time, satisfying the end condition constraint of Equation 3.3.
3.2.2 Maximum Exit Velocity

Sometimes it occurs that a maximum possible exit velocity is desired for granular material flowing in chutes under gravity. The loading of railcars with wheat is a case in point. Within this framework one may aim to optimise either the exit velocity or some component of the exit velocity such as the horizontal component.

The required profile shape of gravity flow discharge chutes to achieve maximum exit velocity involves minimising losses.

Formulation along similar lines to that of the particle model used previously assumes that the initial velocity of the particle is $v$, and an energy balance reveals:

Change in kinetic energy = change in potential energy - frictional losses;

i.e., $\frac{1}{2} m (v_2^2 - v_1^2) = mgy - \text{frictional losses}$  \hspace{1cm} (3.8)

The frictional loss in going from the origin $(0,0)$ to a general point $s(x, y)$ is

$\text{loss} = \int_{0}^{s} \tau_{e} |N| + \mu v \, ds$  \hspace{1cm} (3.9)

where $ds$ is an incremental arc length.

Now $ds = \sec \theta \, dy$

$\therefore \text{loss} = \int_{0}^{s} \tau_{e} |N|^1 \, \frac{dy}{\cos \theta}$  \hspace{1cm} (3.10)

substitution into 3.8 yields after re-arrangement:

$m (v_2^2 - v_1^2) = 2mgy - 2\int_{0}^{s} \tau_{e} |N| + \mu v \, \frac{dy}{\cos \theta}$  \hspace{1cm} (3.11)

Recall that a force balance after resolving into normal components resulted in:

$N = dm \, \sin \theta + dm \, A_n$
\[ N^1 = g \sin \theta - v_2^2 \cos \theta \frac{d\theta}{dy} \quad (2.11) \]

substituting 2.11 into 3.11 yields:

\[ (v_2^2 - v_1^2) = 2gy - \int_0^S \left\{ \tau_0 (g \sin \theta - v_2^2 \cos \theta \frac{d\theta}{dy}) + \mu v \right\} \frac{dy}{\cos \theta} \quad (3.12) \]

Now, depending on specified end constraints, that is, entry and exit angles, entry and exit co-ordinates one could complete this formulation and solve to obtain the optimum profile to maximise the exit velocity for the exit angle specified. In the present study attention is directed towards solving the minimum descent time problem as defined in Equation 3.7.

3.2.3 Additional Constraints

In order to provide a practical general solution to the problem of minimising the descent time, a number of design constraints and certain system constraints need to be considered. The designer may wish to specify the inlet grain stream velocity and its direction, to comply with the physical arrangement of the hopper-discharge system. To ensure flow stability, and make the chute self-clearing, the angle that the chute bottom makes with the vertical, \( \theta \), needs to be constrained. This angle normally increases towards the exit or runout and can exceed the limiting angle, \( \theta_L \). For 'fast' flow this angle determines the stability of the chute system. Should the stream flow be momentarily blocked, initiated by a single sticking grain, the chute would stall and choke or at the very least flow would change from 'fast' flow to 'slow' flow. Once stalled, the chute would not self-clear, requiring manual grain removal before flow can be re-established. This phenomenon became obvious when the results of Roberts (Reference 8) for the unconstrained minimum descent time were
verified. Plate 6 shows the range of responses resulting from a momentary grain blockage. Initially the grain stream is flowing in the 'fast' flow mode as shown in Frame 1, when a minute disturbance in the stream causes the flow to experience a sudden transition to the 'slow' flow mode, shown in Frame 2. The bunching up of the grain stream here is clearly evident. In order to emphasise the seriousness of this instability problem, Frames 3 and 4 respectively show statically the initiation of grain flow instability and the fully choked chute. These were obtained by immediately shutting off the flow control valve as soon as a flow disturbance was detected whilst the chute was discharging under a light flow rate. A momentary delay in shutting off the flow control valve resulted in the fully choked condition shown in Frame 4. The inability of the chute to self-clear is evident since the grain stream is in a stable stationary state.

The designer needs to identify this limiting angle, $\theta_f$, for the material being handled and to ensure that the combination of inlet and outlet co-ordinates, chute geometry is chosen so as to avoid exceeding this limit. The programme package developed in Section 3.4 enables the designer to generate appropriate chute geometries constrained so as to meet this stability criterion. Testing showed that the unconstrained solution represents an insignificant reduction in descent time and in a practical situation, of course, the stability requirement would rule out any consideration of such a profile. For these reasons the unconstrained chute profile was, from here on, excluded from any further analysis.
TRANSITION TO 'SLOW' FLOW MODE

FULLY CHOKED CHUTE

FRAME 3

FRAME 1

FRAME 4

FRAME 2

INITIAL FORMATION OF GRAIN FLOW INSTABILITY

'FAST' FLOW MODE

GRAIN BUNCHING

Plate 6  Chute stalling (unconstrained)
3.3 Methods of Solution

3.3.1 Introduction

Solution methods to the unconstrained formulation have been presented in the literature. Charlton (Reference 5) ignored frictional losses arising from the curvature term and used variational methods to obtain a solution to the problem approximation. Chiarella (Reference 10) applied a method, analogous to the finite element technique for numerical solution of variational problems, which he called a discrete segment solution method. The advantages of this method were quickly superseded by a polynomial approximation method presented by Charlton (Reference 12) which was effective in handling the general resisting force. That is, a resisting force comprising Coulomb drag as well as a velocity dependent drag component. This approach will be used to develop a solution for the minimum descent time problem as defined in Equation 3.7 subject to the intrinsic constraint contained in Equation 3.3.

3.3.2 Polynomial Approximation

Having selected a moving co-ordinate system for the equations of motion with co-ordinates $y$ and $\theta$, the problem solution involves approximations for $\theta(y)$ or $\tan \theta(y)$ by a polynomial in $y$.

i.e., $\tan \theta(y) = A_0 + A_1 y + A_2 y^2 + \ldots + A_m y^m$

$$= \sum_{m=0}^{M} A_m y^m$$  \hspace{1cm} (3.13)
An expression for \( \frac{d\theta}{dy} \), required in Equation 2.18 is obtained by differentiating Equation 3.13

\[
i.e., \sec^2 \theta \frac{d\theta}{dy} = \sum_{m=1}^{M} m A_m y^{m-1}
\]

re-arranging

\[
\frac{d\theta}{dy} = \cos^2 \theta \sum_{m=1}^{M} m A_m y^{m-1}
\]

Now substituting Equation 3.13 into the end condition Equation 3.3 and carrying out the integrations yields the approximation:

\[
\sum_{m=0}^{M} \frac{A_m y^{m+1}}{m+1} - X = 0
\]

for which

\[
A_M = \frac{M+1}{Y^{M+1}} \left[ X - \sum_{m=0}^{M-1} \frac{A_m y^{m+1}}{m+1} \right]
\]

where \((X, Y)\) are the exit co-ordinates.

This means that only the coefficients \(\{A_0, A_1, A_2, \ldots A_{M-1}\}\) are independent. That is, the descent time \(t\) is a function of these \(M\) coefficients.

i.e., \(t = t(A_0, A_1, A_2, \ldots A_{M-1})\) \hspace{1cm} (3.18)

The problem of minimising the transit time, now reduces to one of finding a set of coefficients, \(\{A_0, A_1, A_2, \ldots A_{M-1}\}\) which minimise the expression for \(t\) (Equation 3.7) subject to the constraint Equation 3.3.

To specify the initial direction one takes note that if \(y = 0\), Equation 3.13 becomes:

\[\theta_0 = \tan^{-1} A_0\]
This implies that $A_0$ must be specified, in which case one minimises $t$ with respect to the $(M-1)$ coefficients remaining

i.e., $t = t (A_1, A_2, A_3, \ldots, A_{M-1})$ \hspace{1cm} (3.19)

The actual number of terms to be considered will depend on a number of factors including the order of accuracy required, the computational effort considered appropriate, and the total number of constraint equations specified in the particular problem formulation. Some preliminary test runs with $M$ equal to 2, 4, 8 revealed that no significant improvement resulted by considering $M$ greater than 4. Thus, in all the final design calculations $M$ was set equal to 4.

3.4 Fletcher-Powell Algorithm

3.4.1 Description

For the constrained minimisation problem considered here, experience has shown that the Fletcher-Powell algorithm as outlined in Kuester (Reference 24) provides a fast, computationally efficient solution method. Basically, it is a hill climbing technique, using a penalty function to drive the solution back into the feasible solution space, should it cross the constraint boundary.

The algorithm was programmed by Haarhoff (Reference 24) and finds the minimum of a multivariable nonlinear function subject to nonlinear equality constraints.

i.e., Minimise $F(x, x_2 \ldots, x_N)$

subject to $G_K(x, x_2 \ldots, x_N) = 0$

where $K = 1, 2, 3, \ldots, M$.  

(3.20)
The method incorporates the constraints into a modified, unconstrained objective function which is then optimised by the unconstrained minimisation technique of Fletcher and Powell. Derivates of the objective function with respect to the independent variables are thus required. Inequality constraints can be treated by use of slack variables and transformations. The algorithm proceeds as follows:

1) A new unconstrained objective is formulated from the original function and constraints,

\[ \Phi = F - \sum_{K=1}^{M} \lambda_K G_K + B \sum_{K=1}^{M} G_K^2 \]  

where \( \lambda_K \) and \( B \) are constants.

2) A starting point is selected (feasible or nonfeasible) and the value and derivatives of \( F \) and values of \( G_K \) are determined. The derivatives can be either analytical or numerical approximations. The \( H \) matrix in the Fletcher and Powell procedure is set equal to the identity matrix for the first iteration. The \( \lambda_K \) values are determined from the following system of equations,

\[ \sum_{i=1}^{N} \sum_{j=1}^{M} (\lambda_j \frac{\partial G_j}{\partial x_i} \frac{\partial G_K}{\partial x_i}) = \sum_{i=1}^{N} (\frac{\partial G_K}{\partial x_i} \frac{\partial F}{\partial x_i}) \]  

where \( K = 1, 2, \ldots, M. \)

The \( B \) value is set equal to some positive number; the authors suggested a value of 30. Testing showed that the algorithm was relatively insensitive to variations in \( B \) within the range of 30-100.

3) A series of search directions and one dimensional search steps are then determined per the unconstrained Fletcher and Powell general procedure for the modified objective function with updating of the
\( \lambda_k \) values for each new iteration. When convergence is achieved, \( G_k = 0 \) and \( F = \emptyset \) and the required function \( F \) has been optimised.

3.4.2 Programming Considerations

To use the algorithm to minimise the descent time \( t \), as a function of the polynomial coefficients, it is necessary to write a number of subroutines, which define the objective function and the constraining equations.

Briefly, the programme package consists of a main programme controlling calculations and able to call on fifteen subroutines. Initial values for the chute parameters are read in, including estimates of the polynomial coefficients. The constrained minimisation is then carried out to yield the minimised descent time \( t \) for certain convergence criteria. Stability of the package when seeking low convergence criteria of the order of \( 10^{-12} \), was improved considerably by using double precision variables. Advantage was taken of a recently acquired plotter and software package to plot the chute profiles. This proved invaluable in drawing across-the-board observations of trends not readily discernable from the raw printed data.

Further details and operating procedures of the programme can be found in Section 10.5.
CHAPTER 4: RESULTS AND DISCUSSION - UNIFORM FLOW

4.1 Comparison of Model Results with Theoretical Calculation

During the course of the research programme the need for improved methods for a non-contact velocity detector was considered to be of great importance. The concept of an ultrasonic transducer was pursued as it has obvious attractive features. The development of this device which was carried out in parallel with the main experimental programme resulted in a prototype (discussed in Section 10.1.4) being completed and tested towards the end of the research programme. This ultrasonic velocity detector enabled quick determination of stream velocity at, say, inlet to and outlet from the chute. Having established that the exit velocity from the model hopper into the chute entry was approximately 0.5 metre per second and using the previously derived design parameters (Sections 2.4.4 and 3.2.1), a chute with appropriate minimum time geometry was generated. Table 1 summarises the comparison of the differences between the experimental results and the assumed theoretical criteria. For millet seed, the limiting angle, \( \theta_f \), is approximately 60° and Parlour's value for the equivalent coefficient of Coulomb friction, \( \tau_e \), is used. This leaves the coefficient of viscous drag, \( \mu \), and the exit velocity, \( V \), as unknowns. By assuming a range of values for the viscous coefficient, \( \mu \), it is possible to generate a number of chute geometries. Each such chute would be associated with its particular descent time and exit velocity. If it is assumed that there is no viscous drag, then the percentage difference in descent time between the theoretical and experimental chute is 30% and the percentage difference in exit velocity is 44%.

However, it is recognised that although the variation in the coefficient of viscous drag has little effect on the chute profile at
this model scale (see Section 4.5 for a detailed sensitivity analysis) this variation does affect both the descent time and the exit velocity. In fact, if it is assumed that $\mu = 2.0$ the relative percentage differences reduce to:

descent time difference $= 11\%$
exit velocity difference $= <2\%$

**Table 1: Comparison of model results with theoretical parameters**

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>EXPERIMENTAL</th>
<th>THEORETICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Velocity (VINIT)</td>
<td>$\frac{4}{2}$ 0.5 m/s</td>
<td>0.5 m/s</td>
</tr>
<tr>
<td>Coulomb Coefficient ($\tau$)</td>
<td>$\frac{4}{2}$ 0.306</td>
<td>0.306</td>
</tr>
<tr>
<td>(based on Parlour)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height to Breadth Ratio (H.B.)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Limiting Theta ($\theta_f$)</td>
<td>60°</td>
<td>60°</td>
</tr>
<tr>
<td>Coefficient of Viscous Drag $\mu$</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Descent Time</td>
<td>.70</td>
<td>.48</td>
</tr>
<tr>
<td>Exit Velocity $V(\text{exit})$</td>
<td>2.3 m/s</td>
<td>4.1 m/s</td>
</tr>
<tr>
<td>Design Coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1=-.6108006$</td>
<td>7.16043</td>
<td></td>
</tr>
<tr>
<td>$A_2= 20.8306$</td>
<td>-27.32309</td>
<td></td>
</tr>
<tr>
<td>$A_3=-43.8373$</td>
<td>41.2757</td>
<td></td>
</tr>
<tr>
<td>$A_4= 26.46906$</td>
<td>-19.312</td>
<td></td>
</tr>
<tr>
<td>Equation of Profile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = \frac{A_1y^2 + A_2y^3 + A_3y^4 + A_4y^5}{2 \cdot 3 \cdot 4 \cdot 5}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.2 Check on the Value of Viscous Drag

At this stage it was recognised that an experimental check would need to be made on the actual amount of viscous drag present in the model chute. Using a straight chute for this, simplifies slightly the equations to be integrated and affords a certain amount of cross-check on the curved, optimally generated chute results. Since, for this chute experimental results were available for inlet velocity, exit velocity and descent time, as well as the length of the chute, two related approaches were taken. Firstly, acceleration down the chute was formulated in terms of $dv/dt$, and then as a check, it was formulated in terms of $v \frac{dv}{ds}$. Figure 9 depicts the acceleration model resulting from a force balance on an element sliding down the open chute subject to friction on the chute bottom and walls. It is assumed here that this frictional drag force can be adequately represented by the two lumped components - Coulomb and viscous drag forces.

![Figure 9 Force Balance](image-url)
A force balance on the element yields

\[ \frac{dv}{dt} = g \cos\theta - T_e N - \mu v \]  
\[ = g \cos\theta - T_e g \sin\theta - \mu v \]  
\[ = g \cos\theta - \tau(1 + \frac{C_1}{v} + C_2 v) g \sin\theta - \mu v \]  
\[ (4.1) \]
\[ (4.2) \]

Note that \( \frac{dv}{dt} = 0 \) for terminal velocity.

For a given experimental set up Equation 4.2 can be simplified to

\[ \frac{dv}{dt} = A - B - C \frac{v}{v} - D v - \mu v \]

where

\[ A = g \cos\theta \]
\[ B = \tau g \sin\theta \]
\[ C = \tau g \sin\theta \, C_1 \]
\[ D = \tau g \sin\theta \, C_2 \]
\[ E = A - B \]
\[ F = D + \mu \]

which yields

\[ \frac{dv}{dt} = E - \frac{C}{v} - F v \]  
\[ (4.3) \]

Integrating both sides

\[ \int_1^2 dv = \int_1^2 (E - \frac{C}{v} - F v) dt \]

After re-arranging

\[ \int_1^2 \frac{dv}{(E - \frac{C}{v} - F v)} = \int_1^2 dt \]  
\[ (4.4) \]
In order to maintain the flow of this analysis, the balance of the solution of this equation continues in Section 10.6.1 and ultimately results in
\[
\left( \frac{2Fv_1 + E(v_1 + v_2) - 2Fa(v_2 - v_1) + 2c}{2Fv_1v_2 - E(v_1 + v_2) + 2Fa(v_2 - v_1) + 2c} \right)^{\frac{E}{4F^2a}} \left( \frac{Fv_1^2 - Ev_1 + c}{Fv_2^2 - Ev_2 + c} \right)^{\frac{1}{2F}} \\
= e^{(t_2 - t_1)} \quad (4.7)
\]

Now given \(v_1, v_2\) and \(t\) we seek a value of \(\mu\), the viscous drag coefficient, such that Equation 4.7 is satisfied. Use was made of a Rosenbrock (Constrained Rosenbrock Hill Algorithm - Reference 24) hill climbing subroutine to find the value of \(\mu\) to minimise Equation 4.7 expressed as L.H.S. - R.H.S. = 0. Some difficulties were encountered as the extremum point is not very pronounced and may in fact be a saddle point.

It was decided to operate on the function squared since this sharpens the extremum point and stabilises the numerical solution with \(v_1 = 0.5\) m/s
\(v_2 = 2.7\) m/s
\(t = 0.735\) sec.

The minimisation revealed a value of \(\mu\) of 1.85. It is well recognised that experimental equipment for measuring these parameters, including the ultrasonic velocity detector, have a resolution limit. This result also needs to be considered in the context that all the parameters could not be measured at the same time. A sensitivity analysis based on a variation of \(\pm 3\%\) in just the three variables above was considered mandatory. This revealed the spread in the value of \(\mu\) to be from 1.78 to 1.88 using combinations of these parameter variations to yield the 'best case' and 'worst case' value of \(\mu\). This result would seem to support the assumption that for the test environment the
coefficient of viscous drag, \( \mu \), is of the order of 2.

As mentioned earlier a cross-check can be made by considering the force balance to yield an acceleration term as a function of velocity and displacement rather than velocity and time.

Using the same right hand side of Equation 4.3 we rewrite the acceleration expression as

\[
\frac{dv}{ds} = \frac{E - c - Fv}{v} = \frac{Ev - c - Fv^2}{v}
\]

i.e., \( v^2 \frac{dv}{ds} = Ev - c - Fv^2 \) (4.8)

Re-arranging

\[
\frac{v^2dv}{Ev - c - Fv^2} = ds
\]

\[
\frac{v^2dv}{F(Ev - c - Fv^2)} = ds
\]

\[
\frac{v^2dv}{-F(v^2 + \frac{c}{F} - \frac{Ev}{F})} = ds
\]

\[
\frac{v^2dv}{(v^2 + \frac{c}{F} - \frac{Ev}{F})} = -Fd ds
\] (4.9)

Again in order to maintain the flow of the analysis, the development of this equation continues in Section 10.6.2 and ultimately results in

\[
e^{(v_2 - v_1)} \left[ \frac{Fv_2^2 + c - Ev_2}{Fv_1^2 + c - Ev_1} \right] \frac{E}{2F} \frac{1}{2a} \left( \frac{E^2}{2F} - \frac{c}{F} \right)
\]

\[
= e^{-F(s_2 - s_1)}
\] (4.15)

Given \( v_1, v_2 \) and \( (s_2 - s_1) \) we seek a value of \( \mu \) the viscous drag coefficient such that Equation 4.15 is satisfied. Use was again made of the Rosenbrock Hill climbing algorithm to minimise Equation 4.15 expressed as \( (L.H.S. - R.H.S.)^2 = 0 \) to avoid any similar problems with insensitive extremum conditions.
The minimisation with

\[ v_1 = 0.5 \text{ m/s} \]
\[ v_2 = 2.7 \text{ m/s} \]
\[ s_2 - s_1 = 1.145 \text{ metres} \]

As mentioned earlier a sensitivity analysis based on a variation of ±3% in just these three parameters revealed the spread in the value of \( \mu \) to be from 2.22 to 2.33 again using combinations of these parameter variations to yield the 'best case' and 'worst case' value of \( \mu \). These results are summarised in Table 2. Clearly under the testing environment the experimental results support the assumption of a coefficient of viscous drag equal to approximately 2 (\( \mu \approx 2.0 \)).

**Table 2: Variation in Viscous Drag - \( \mu \)**

<table>
<thead>
<tr>
<th></th>
<th>Acceleration based on ( \frac{dv}{dt} )</th>
<th>Acceleration based on ( \frac{dv}{ds} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>1.83*</td>
<td>2.28**</td>
</tr>
<tr>
<td>max. ( \mu )</td>
<td>1.88</td>
<td>2.33</td>
</tr>
<tr>
<td>min. ( \mu )</td>
<td>1.78</td>
<td>2.22</td>
</tr>
</tbody>
</table>

* converged for initial \( \mu \) between 1.4 to 5.

** converged for initial \( \mu \) between 0.5 to 4.

4.3 Comparison of Model with Chutes of Other Profiles

In order to compare the performance of the model chute with chutes of known profile, transit times were determined as described previously from cross-correlation transport delay times. Advantage was taken of the availability of a hardware correlator as well as the package programme (Section 10.5) to determine the cross-correlations
simultaneously using both procedures. Figure 10 illustrates a typical experimentally derived impulse function for the hopper-chute system. The transport delay, 'dip' and impulse peak are clearly evident. The transport delay was taken to be the time elapsed to the centroid of the impulse response as represented by the cross-correlation.

Figure 10 Transport delay from the cross-correlation response

Table 3 summarises the comparison of descent time based on the above interpretation for chutes of known form. The descent time for 'single grains' is included to contrast it with the design flowrate of 4 kg/min. For the 'single grain' case the improvement in descent times for the constrained chute is approximately 1% compared to the parabola and 2% compared to the straight chute. For the designed flowrate of 4 kg/min the improvement in descent time calculated by taking the average of the three measurement methods is 2% compared to the parabola and just under 5% compared to the straight chute.
It can be seen from the above that even at this small scale level significant improvements in descent times are possible for the optimally designed chutes when compared to the more commonly used chute forms.

**TABLE 3: COMPARISON OF CHUTE PROFILES**

<table>
<thead>
<tr>
<th></th>
<th>DESCENT TIME BASED ON 4kgm/min</th>
<th>EXIT VELOCITY m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single Grain</td>
<td>Step Response</td>
</tr>
<tr>
<td>Straight Chute</td>
<td>.719</td>
<td>.763</td>
</tr>
<tr>
<td>Parabolic Chute</td>
<td>.712</td>
<td>.707</td>
</tr>
<tr>
<td>Constrained Chute</td>
<td>.704</td>
<td>.700</td>
</tr>
</tbody>
</table>

4.4 Sensitivity Analysis of Model Parameters

Results could be relied upon with confidence if a sensitivity analysis on the design parameters revealed that parameter variations correspond to a prescribed change in chute performance. In order to assess this performance variation each of the design parameters was varied over a range that was considered practicable for the model. Table 4 indicates the range of variation in parameters considered. For millet seed, the limiting angle, $\theta_f$, is known to be approximately $60^\circ$ and hence a variation in this parameter of $\pm 5^\circ$ was considered adequate. Subsequently a variation of $\pm 1^\circ$ was used to assess any influence on chute geometry resulting from the inability of measuring this angle precisely. For this analysis and that of Section 4.5 the $\pm 1^\circ$ variation resulted in an insignificant change in chute geometry and hence performance, and the results from the $60^\circ$ case only are reported here.
The assumed range for initial velocity \( (V_0) \), height to breadth ratio, \((HB)\), and coefficient of viscous drag, \((\mu)\), were considered adequate to cover all likely combinations. The range in Coulomb coefficient \((\tau)\) considered included Roberts' value of \( \tau = 0.464 \) based on the approximate model of Figure 3 and Parlour's value of \( \tau = 0.306 \) based on the more precise model of Figure 4. In addition, for comparative purposes, a value midway between the difference of the two and Parlour's value \( (\tau = 0.35) \) was used.

For each analysis one variable was held constant whilst each of the remaining variables was, in turn, varied through its respective range. The results are presented in graphical form in Figures 11 to 15. Since the chutes of parabolic form are known to have favourable performance characteristics, parabolic profiles as defined in Section 2.4.2 are included in each plot for comparative purposes.

**Table 4: Range of Parameter Variations in Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>55°</th>
<th>60°</th>
<th>65°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limiting Angle ( (\theta_f) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Velocity ( (V_0) ) (m/s)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Coulomb Coefficient ((\tau))</td>
<td>.464</td>
<td>.306</td>
<td>.35</td>
</tr>
<tr>
<td>Height to Breadth Ratio ((H/B))</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Coefficient of Viscous Drag ((\mu))</td>
<td>0</td>
<td>0.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Some general observations can be made regarding the resulting model chute geometries.

1. There is little change in chute geometry with changes in viscous drag \((\mu)\) between 0.5 and 1.5. However, this variation in \( \mu \) does
Figure 11  Variation in limiting angle $\theta_e$
Figure 12  Variation in initial velocity
Figure 13  Variation in Coulomb coefficient ($\tau$)

- $\bigcirc = 0.464$
- $\triangle = 0.306$
- $\ast = 0.35$
- $\ast = $ PARABOLIC CHUTE
Figure 14  Variation in height to breadth ratio
Figure 15  Variation in Viscous Drag ($\mu$)
have a significant effect on the exit velocity and on the descent
time. The maximum spread in exit velocity was 34% while the
maximum spread in the descent time was 21%.

2. There are slight changes only in chute geometry with change in
limiting angle ($\theta_f$) between 55° and 65°, together with only minor
variations in exit velocity and descent time. The spread in exit
velocity was 1% and the spread in descent time was only 0.3%.

3. Large variations in chute geometries result from changes in inlet
velocity ($V_0$) becoming increasingly steeper or more concave with
an increase in inlet velocity. This effect is non-linear and
results in a spread in exit velocity of 1.8% and a spread in
descent time of 16%.

4. Large variations in chute geometries result from changes in
Coulomb coefficient ($\tau$) but with only a slight spread in exit
velocity and descent time. The spread in exit velocity was 6.5%
while the spread in descent time was 2.2%.

5. Large variations in chute geometries result from changes in
height to breadth ratio (HB) but are associated with only a slight
spread in exit velocity and descent time. The actual spread in
exit velocity was 2.8% and the spread in descent time only 0.9%.

Confidence in these observations would obviously be increased
if they were seen to hold for larger scale chutes. While it was not
possible at this stage to carry out an experimental study of large scale
chutes, a theoretical analysis of such a scale up was considered
essential.
4.5 Theoretical Scale Up Analysis

Scale up analysis was considered by way of comparing the model geometry with chutes whose outlets were (3 x 4) metres and (5 x 5) metres away from their inlet, as shown in Figure 16. These orientations were chosen firstly because it was felt they would be realistic sizes for production chutes and secondly to contrast equal leg with the unequal leg type chute geometry.

![Figure 16 Scaleup of chute geometries](image)

For each of these chute end conditions similar profiles were generated as shown in Figures 17 and 18 and included a straight chute, and a parabolic chute, and profiles for both the unconstrained and the constrained optimised chute. Each of these chute profiles was in turn subjected to a sensitivity analysis similar to the model chute using the same range of parameter variations as described in Table 4.
Figure 17  Chute profiles for (3x4) metre chute
Figure 18  Chute profiles for (5x5) metre chute
Figures 19 to 28 detail, in graphical form, the variations in a chute's geometry that result from this sensitivity analysis. Again the parabolic chute form is included in each plot for comparative purposes.

Some general observations across the range of chutes considered can be made with Table 5 summarising the variations of descent time and exit velocity.

**Table 5: Spread in parameter variation with scale up**

<table>
<thead>
<tr>
<th>PARAMETER BEING VARIED</th>
<th>DESCENT TIME</th>
<th>EXIT VELOCITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(.75x.9)m Chute</td>
<td>(3x4)m Chute</td>
</tr>
<tr>
<td>Viscous Drag Coefficient</td>
<td>17.6%</td>
<td>33.4%</td>
</tr>
<tr>
<td>Limiting Angle</td>
<td>.3</td>
<td>.18</td>
</tr>
<tr>
<td>Inlet Velocity</td>
<td>16</td>
<td>8.8</td>
</tr>
<tr>
<td>Coefficient of Coulomb Friction</td>
<td>2.3</td>
<td>.72</td>
</tr>
<tr>
<td>Height to Breadth Ratio</td>
<td>.92</td>
<td>.08</td>
</tr>
</tbody>
</table>

1. With increasing scale up the coefficient of viscous drag (μ) increasingly affects the geometry of the chute particularly with μ > 1.0. The descent time variation increases from a spread of 17.6% to a spread of 46.1% for the (5 x 5) metre chute. At the same time the spread of exit velocity increases from 34.2% to 75.4%.
Figure 19  Variation in limiting angle $\theta_f$ for (3x4) metre chute
Figure 20  Variation in initial velocity for (3x4) metre chute
Figure 21  VARIATION IN COULOMB COEFFICIENT ($\tau$) FOR (3x4) METRE CHUTE
Figure 22  Variation in height to breadth ratio for (3x4) metre chute
Figure 23  Variation in viscous drag ($\mu$)
for (3x4) metre chute
Figure 24: Variation in limiting angle $\theta_f$ for (5x5) metre chute.
Figure 25  Variation in Initial Velocity for (5x5) Metre Chute
Figure 26  VARIATION IN COULOMB COEFFICIENT (γ) FOR (5x5) METRE CHUTE
Figure 27  VARIATION IN HEIGHT TO BREADTH RATIO FOR (5x5) METRE CHUTE
**Figure 28**  
Variation in Viscous Drag (μ) for (5x5) Metre Chute
2. For changes in the limiting angle ($\theta_f$) between 55° and 65° there is little change in the geometry across the size range considered. The descent time spread remains insignificant with a maximum of 0.5%, while the exit velocity spread has a maximum across the range of 1.6%.

3. Changes in inlet velocity ($V_0$) from 0.5 to 1.5 m/s across the size range of the chutes have a decreasing effect on the geometry. The descent time spread drops from 16% to 7.4% and the exit velocity spread drops from 1.8% to a minimum of 0.28% for the (3 x 4) metre chute.

4. Changes in the coefficient of Coulomb friction ($\tau$) from 0.464 to 0.306 across the size range of the chutes have a decreasing effect on the optimally constrained geometry. The descent time spread drops from 2.3% to a minimum of 0.72% for the (3 x 4) metre chute and the exit velocity spread drops from 6.5% to a minimum of 0.64% for the (3 x 4) metre chute.

5. Changes in the height to breadth ratio (HB) from 0.5 to 1.5 across the size range of the chutes have a decreasing effect on the optimally constrained geometry with only a slight spread for the (5 x 5) metre chute. The descent time spread is insignificant across the whole range being always less than 1%. The exit velocity spread drops from 2.8% to a minimum of 0.49% for the (3 x 4) metre chute.

4.6 Design Procedure

Consideration was given at this stage to the possibility of formulating a set of design guides. This might take the form of nomographs cross directing the designer through the design process for chutes operating under uniform flow conditions in the 'fast' flow
mode. Because of the large number of variables involved, any such scheme would need to represent a compromise in order to avoid making the use of the nomographs unwieldly due to the sheer number of reference points. In contrast the exact solution using the design programme package is relatively straightforward. The required input parameters are read into and depending on the computer used, the execution time and cost are relatively small. In the case reported here, the execution time of a complete sensitivity analysis, as detailed in Section 4.4, was of the order of 5 minutes. The programme package itself is programmed in the FORTRAN language and could be readily implemented on any modest computer with a FORTRAN compiler.

For these reasons, perhaps the most satisfactory approach for solving a particular chute problem is to use the programme package with the complete sensitivity analysis. This provides the designer with the complete range of possible solutions enabling him to choose the solution that best fits his physical and environmental constraints.

4.7 Conclusions

From the preceding analysis of the operation of the hopper-discharge chute under uniform flow conditions, the following general observations can be made.

1. For hopper-discharge chutes operating under uniform flow conditions the work of Roberts and Parlour has been extended, to incorporate the limiting angle, $\theta_f$, in their design, resulting in chute geometries suitable for stable 'fast' flow operation.
2. The lumped parameter model has been found to be adequate for analysing the uniform flow of millet through chutes subject to generalised drag forces.

3. For the model, the generalised drag force included a viscous drag component with a coefficient of approximately 2 ($\mu \approx 2.0$).

4. Even at the small scale tested the optimum chute geometry exhibited favourable performance characteristics (minimum transit time) whilst maintaining the desired 'fast' flow pattern compared to chutes commonly used. These incremental improvements in performance represent potential operating cost savings.

5. A design procedure in the form of a package programme has been presented which enables specific solutions for chute design to be obtained incorporating facilities for a sensitivity analysis on all the significant design parameters.

Overall the investigation has shown the potential of discharge chutes as flow controlling devices in the gravity flow of bulk granular solids. For steady flow operation it is possible to design chute profiles to achieve prescribed optimum performance by the use of direct mathematical methods.
PART 2: TRANSIENT FLOW
CHAPTER 5: TRANSIENT FLOW

5.1 System Identification

The necessity of determining the transient performance of chutes was alluded to in Section 2.1. The delineation of a mathematical model of the chute's dynamic behaviour would enable the prediction of chute performance under forced operation of the flow control gate, and highlight those parameters which affect the frequency response of the chute.

Knowledge of the transient phase during blending operations would enable one to determine what proportion of the mix is out of specification and hence subject to waste or reprocessing. In a multi-input blending operation knowledge of the longest transient time constant would determine the appropriate earliest time to take a mix sample for compliance to specification. If the blending operation involves control-valve-chute subsystems with widely differing time constants consideration could be given to the timing order of operation of these subsystems when changing product mix so as to minimise the waste. Pressure for better quality control and increased resource utilisation would seem to indicate a need for these incremental gains in efficiency.

Experimental identification of the transient performance of the hopper-discharge chute system was undertaken. Use was made of the P.R.B.S./Cross-correlation method for reasons given below.

The field of identification and process-parameter estimation has developed rapidly during the past decade. In a survey paper; Åström (Reference 25) reviewed the state-of-the-art/science of system identification schemes, stating that what was needed were comparative tests performed on similar data. Isermann (Reference 26)
compared the performance, computation time and overall reliability of six recursive identification and parameter estimation methods, using three simulated processes. Sardis (Reference 27) compared the computational and convergence properties of six popular on-line parameter identification algorithms, by compiling and evaluating the results of tests on two fourth-order discrete, dynamic systems.

In each case the results are similar and according to Isermann (Reference 26, page 99)

"For general linear processes, COR shows most advantages ...; very good performance, shortest computation time, 100 per cent overall reliability with no problems with poor convergence or instability..."

COR here is defined as correlation analysis with least squares parameter estimation. Whilst this paper presents a comprehensive delineation of the advantages of the correlation method, Sastri (Reference 28) draws attention to some of its disadvantages, and how they may be overcome. In the practical application of the cross-correlation technique a number of errors can arise due to the existence of imperfect input transducer characteristics, non-ideal test input characteristics and wide band and output drift. His findings are of importance in the selection of parameters as described in Section 5.2.3 and are summarised as follows:

1. Make sure that the ratio of pseudo-random binary coded signal bandwidth to that of the system bandwidth is of the order of 14:1 and is bias corrected.
2. Set the digit interval (ΔT) to one quarter of the smallest time constant of interest.
3. Compute the cross-correlation for at least two periods of the
test signal, and use the reference phase. Additional comparative inferences were drawn from References 29 to 33.

Roberts (References 8, 21) showed, in a pilot study, that by making use of the P.R.B.S./cross-correlation technique's many advantages, it appeared to be suitable for application to the bulk handling field. In particular, it appeared suitable for use in determining the transient flow behaviour of granular material discharging from a hopper through a discharge chute. Support for its application here was gained by its successful application to widely diverse fields including impulse response testing of reactor systems, identification of flotation plant dynamics, and for process control in the steel industry (References 34 to 39).

5.2 P.R.B.S./Cross-Correlation Method

5.2.1 Theoretical Consideration

While most aspects of the theory of random signal analysis and system identification are well documented (References 40 to 42), these are not generally in a form directly applicable to an experimental programme. The salient aspects relating to the present experimental investigation are outlined below as a self-contained coherent presentation.

Figure 29 shows, schematically, the system under test, with input signals $x(t)$ and input noise $n(t)$ and with an output signal or response of $y(t)$. The hopper-discharge chute is, in our case, the system or equipment whose dynamic characteristics we seek. For the present, it is assumed that the system is linear and deterministic. The input signal $x(t)$ is the test signal (P.R.B.S.) which is injected into the system at some predetermined point (flow perturbing gate). The signal $n(t)$ represents an extraneous noise signal which could
Figure 29  System identification model
arise in one of a number of ways. This includes via the normal operating signal and any random noise signal originating from some unknown source that appears as an input to the system.

Since it is assumed that the system is linear, it follows the superposition principle and the response is unique for a given input. The system response, given by the convolution integral is

$$y(t) = \int_{0}^{\infty} x(t - \lambda) h(\lambda) \, d\lambda$$

(5.1)

Where $h(\lambda)$ in 5.1 is the system's impulse response or 'weighting function'. The time variable $\lambda$ is often referred to as the age variable. In the case of random signals, the statistical properties are described by correlation functions. For the system identification problem, two correlation functions are of importance, the autocorrelation function $R_{XX}(\tau)$ of the input signal and the cross-correlation function $R_{XY}(\tau)$ of the input and output signals $x(t)$ and $y(t)$.

For the input signal $x(t)$ the autocorrelation function is expressed by

$$R_{XX}(\tau) = E \{x(t) \cdot x(t + \tau)\}$$

(5.2)

where $E$ denotes the 'expected value'.

Alternatively

$$R_{XX}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) \cdot x(t + \tau) \, dt$$

(5.3)

In a similar manner, the cross-correlation function is expressed by

$$R_{XY}(\tau) = E \{x(t) \cdot y(t)\}$$

(5.4)
Combining Equations 5.1, 5.2, 5.4 and simplifying this becomes:

\[ R_{xy}(\tau) = \int_{0}^{\infty} R_{xx}(\tau - \lambda) h(\lambda) \, d\lambda \]  

(5.5)

It is common in engineering applications to deal with discrete signals of finite length. For such cases Equation 5.5 can be expressed in digital form as

\[ R_{xy}(j) = \sum_{i=1}^{j} R_{xx}(j - i + 1) h(i) \Delta\lambda \]  

(5.6)

for \( j = 1, 2, 3 \ldots k \) where

\( k \) is normally \( \frac{n}{10} \)

\( n \) being the number of data values.

Equation 5.6 can be expressed in matrix form as follows

\[ \overline{R}_{xy} = [R_{xx}] \overline{h} \Delta\lambda \]  

(5.7)

where

\( \overline{R}_{xy} \) = cross-correlation column vector
\( [R_{xx}] \) = lower triangular autocorrelation matrix vector
\( \overline{h} \) = system weighting function column vector
\( \Delta\lambda \) = time interval between correlation estimates.

For any arbitrary random signal input \( x(t) \) one can identify the system's weighting function \( h(t) \) by solving the matrix equation (see NOVA programme).

\[ \overline{h} = \frac{\overline{R}_{xy}}{[R_{xx}] \Delta\lambda} \]  

(5.8)

If the input signal approximates 'white noise*', often taken to mean that the input signal bandwidth is very much greater than the

* Strictly, 'white noise' is practically unattainable since it implies constant power at all frequencies.
bandwidth of the system, the identification problem is greatly simplified. The autocorrelation of a 'white noise' signal is an impulse function. Namely

\[ R_{xx} = S_0 \delta(\tau) \] (5.9)

where \( S_0 \) represents the constant value of the power spectral density for that frequency. Substitution into Equation 5.5 and simplifying leads to

\[ R_{xy}(\tau) = S_0 h(\tau) \] (5.10)

re-arranging

\[ h(\tau) = \frac{R_{xy}(\tau)}{S_0} \] (5.11)

Thus the system's 'weighting function' can be obtained directly from the cross-correlation function.

Figure 29 illustrates the addition of extraneous noise signals \( n(t) \) and the respective 'weighting function' of the signal and the noise. Here the cross-correlation is given by

\[ R_{xy}(\tau) = \int_0^\infty R_{xx}(\tau - \lambda) h(\lambda) \, d\lambda + \int_0^\infty R_{xn}(\tau - \lambda) h_n(\lambda) \, d\lambda \] (5.12)

where \( R_{xn}(\tau - \lambda) \) is the cross-correlation function relating the signals \( x(t) \) and the noise input \( n(t) \). Now provided these two signals are statistically independent, then their cross-correlation function reduces to the product of their mean values. Namely,

\[ R_{xn}(\tau - \lambda) = \overline{x} \overline{n} \] (5.13)

Making this assumption and considering the input to approximate 'white noise' Equation 5.12 simplifies to

\[ R_{xy}(\tau) = S_0 h(\tau) + c \] (5.14)
where $c$ is a constant depending on the mean values $\bar{x}$ and $\bar{n}$.

Now if the mean value of either or both signals is zero then this equation reduces to the simple form of Equation 5.10, since $c$ would then be zero.

Thus by ensuring that the input signal has a zero mean the system's impulse function can be determined in the presence of input noise, that is uncorrelated with the test signal. The unsatisfactory requirement of long averaging times for 'white noise', makes it generally unsuitable for practical identification studies. An alternate signal, having the desirable characteristic of an autocorrelation function approximately an impulse, or delta function, is the pseudo-random binary signal (P.R.B.S.). The literature relating to test signal selection is voluminous but the following were found useful (References 43 to 55).

Figure 30 illustrates a P.R.B.S. which is readily generated using a feedback shift register having simple plus-minus amplitude. The mark-space switch occurs in a random manner at various multiples of a clock period $\Delta T$. In this case $N\Delta T$ the sequence time is $15\Delta T$ and the signal pattern repeats itself after this time interval.

The autocorrelation function and corresponding spectral density of this P.R.B.S. are also shown in Figure 30. Leary (References 56, 57) and Brown (References 58 to 60) provide a good introduction into the identification techniques based on the above formulation.

5.2.2 Hardware Implementation

Two hardware implementations were used in the experimental investigation. Plate 7 shows the Hewlett Packard equipment package comprising a 3722A noise generator, a 3721A correlator and 3720A
Figure 30 Test signal characteristics
PLATE 7  IDENTIFICATION HARDWARE
spectrum display. This enabled correlation tests to be carried out quickly with an immediate result visible on the C.R.O. screens. This feature is important avoiding time wasted on data reduction for an experimental test run that was faulty, particularly when carrying out tests to determine parameters to be used.

Use was made of a NOVA 1200 minicomputer to implement the alternate identification scheme as shown in Figure 31. Here both system input and output signals are sampled on-line via an analogue to digital converter. The assembler language package, written for the mini, enabled all the relevant correlation functions to be calculated. Additionally, the power spectral density functions, the system's impulse function, the system's frequency response and direct convolution analysis could be calculated and plotted. The latter is particularly useful for testing the accuracy of the identified model, enabling the comparison of the actual system output with that obtained by convoluting the system's input with its impulse function; in other words, the predicted output.

In order to impose the P.R.B.S. onto the perturbing gate a power amplifier, Philips GM5535, and on electromechanical solenoid, Philips PR9270 were used. The possibility of input contamination by this power train was obviated by measuring the actual displacement of the perturbing gate, with a linear displacement transducer, H.P. 7DTDC, and using this as the P.R.B.S. input to the system in preference to that obtained directly from the noise generator. This pseudo-random binary displacement signal was checked to ensure that its autocorrelation function had suitable delta function characteristics.
Figure 31  Test rig for NOVA identification
5.2.3 Parameter Selection

To obtain a reasonable estimate of the system 'weighting function', it is necessary to examine the effects of the various parameters involved. In order that Equation 5.11 be applicable it is necessary for the clock pulse ($\Delta T$) of the P.R.B.S. generator to be selected such that $(1/\Delta T)$ is approximately ten times the bandwidth of the system. Since the system bandwidth is initially unknown, it is necessary to establish, by systematic trials, the minimum value of $\Delta T$ for a consistent $R_{xy}(\tau)$ to be obtained. Here the quick-look facility of the H.P. correlator was extremely useful. There is a conflicting practical limitation imposed by the frequency response characteristics of the electromechanical perturbing gate system.

The sequence length, or P.R.B.S. period, $N\Delta T$ where $N$ is the number of clock pulses per period, has to be chosen in relation to the 'settling time'** of the system. Exploratory investigations indicate that a sequence length of about five times the longest time constant of interest in the system is necessary. While longer sequence lengths may be used, correspondingly longer computation or averaging times are necessary which may introduce errors, particularly in cases where the system parameters vary with time.

The delay time increment ($\Delta \lambda$) between correlation estimates, should be sufficiently small to permit minor variations in the

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** The settling time is often defined as the time for a signal to settle to within either $\pm 2$ or $\pm 5\%$ of its steady state value including any dead time.
'weighting function' to be detected, but should not be greater than the Nyquist period* in order to prevent errors due to aliasing (Reference 40). Experience in the hopper chute identification study indicated that more reliable results are achieved when (Δλ) is set equal to (ΔT).

The remaining parameter in the analysis is the selection of the correlation experiment time (nΔλ), where n is the number of samples taken. A sampling time of at least one sequence must be employed (i.e., nΔλ = NΔT) but in practice longer experiment times of approximately two or three sequences are generally more satisfactory, particularly where the signal to noise ratio is very poor. This may occur where the perturbing amplitude has to be kept low, compared to the normal operating signal in order to prevent out-of-specification product material being produced whilst the plant is operating normally.

5.3 Programme Development

5.3.1 Introduction

The necessity of having available a computer programme to perform the large statistical data reductions is obvious. During the period of this study the computing facilities available at the University of Wollongong varied enormously. This development is, in part, responsible for the programme development.

* The Nyquist or folding frequency is defined as (1/(2ΔT)) and is the maximum frequency that can be detected from data sampled at time spacing ΔT (seconds).
The I.B.M. 1620 available initially severely limited the scope of any statistical identification programme attempted. This was soon complemented by a small mini, i.e., NOVA 1200 with 8K-16 bit words of core. At about this time, the hardware correlator became faulty and had to be returned to the manufacturer. These last two facts provided a strong incentive to write an identification package that duplicated the capabilities of the hardware correlator providing a cross-reference and extend them to include system computations such as convolution. Because of the small core size and with a view to the eventual use in a closed-loop control system, assembler language was used. Just prior to completing this project a large Univac 1106 became available, which offered the ability of performing very large data reductions because of its, by comparison, enormous core storage (132K-36 bit words).

5.3.2 On-Line NOVA Package

An assembler language programme written for the NOVA line of mini computers has been developed which enables the following functions to be computed:

- $R_{xx}$ - autocorrelation of the input signal.
- $R_{yy}$ - autocorrelation of the output signal.
- $R_{xy}$ - cross-correlation function.
- $G_x$ - power spectral density of the input signal.
- $G_y$ - power spectral density of the output signal.
- $G_{xy}$ - cross-spectral density function.
- $H$ - impulse function or 'weighting function'.
- $G(j\omega)$ - Bode frequency response, magnitude and phase.
- $X$ - predicted input by inverse convolution of the model with the actual system output.
\[ Y \] predicted output by convolution of the model with the actual system input. This enables a specific control policy to be implemented.

Additionally, all the necessary supporting and peripheral driver software had to be developed. This package, of course, with its versatility, has a utility beyond the present problem, evidenced by its presentation at a computer conference (Reference 61). The programme is arranged so that an operating system (Figure 32) can call upon as many subroutines as there are specified tasks. Once started, the operating system asks a series of questions in hierarchial sequence to define all the subroutines that will be called to perform the tasks.

To enable subroutine modifications to be made with a minimum of re-assembling, the programme has been relocatably written. Assembly language made possible the minimisation of the programme size, so that the maximum amount of memory was available for data storage, and was convenient for writing input-output operations requiring a mixture of interrupt and non-interrupt routines.

Data input subroutines have been written for the analogue to digital converter, the teletype reader, the high speed paper tape reader and the cartridge magnetic tape unit. The A to D subroutine accepts analogue signals from any two, out of 16, consecutive channels, representing the input to the system and the response from it. At the initialisation stage the operator specifies the number of samples to be taken and the sampling time interval. The former is only limited by the core size, while the latter is limited by the maximum frequency of the real time clock.
Figure 32  Generation of Operating Runstream
Since it was felt that trends could be more readily discerned by reference to a plot rather than to a listing, a plotting routine was written so that any of the results could be plotted on a standard teletypewriter unit. The programme scales the data amplitude to limit the ordinate to six and a half inches across the page. The abscissa representing either time of frequency increments, can be any length, as the paper feed is in continuous roll form.

Data collection and manipulation is handled by two subroutines. Firstly, the data is loaded into arrays, counted, and the number of correlations set at one tenth of this number. Then the unbiased signal mean is calculated and the data normalised to zero mean to simplify subsequent formulae and calculations, as detailed in Section 5.2.1.

The statistical functions for signal analysis and system computation are related according to the flow chart shown in Figure 33 and have been programmed from the digitised versions of the formulae derived in Section 5.2.1. Details can be found in Section 10.5.1.

In implementing an experimental programme based on the preceding linear theory one is conscious of the need to adequately consider this linear constraint. During the testing programme it was realised that the chute system had non-linear effects due to the motion of the flow control valve. In formally identifying the model, as detailed in the next section, the experimental programme was designed to check this out.
Figure 33  Operating system subroutine hierarchy
CHAPTER 6: CORRELATION TESTS

6.1 General

An experimental programme was designed to use the P.R.B.S. cross-correlation technique to identify the system model, in this case, the dynamics of the hopper-discharge chute system. Using random perturbations of the hopper flow valve and the P.R.B.S. cross-correlation analysis the impulse flow responses for the system under varying conditions of initial flow and chute shapes were obtained. Information obtained indicated that certain flow non-linearities were associated with the motion of the flow control valve. Techniques were developed to enable the examination of these flow non-linearities as well as enabling the determination of the dynamic flow response characteristics during prescribed controlled operation of the flow control valve.

An important adjunct was the development of the system identification package which enabled prediction of system performance under prescribed input conditions. Once the experimental run had been completed and the data collected, the package also enabled the inclusion of any necessary data manipulations, such as digital filtering, the removal of any bias; and made the data itself available in a form suitable for later evaluation, stored on magnetic or paper tape.

6.2 Apparatus

Plate 7 shows the identification hardware which, together with the apparatus and other ancillary equipment shown in Figure 1 and Plate 1, were necessary to carry out the cross-correlation tests. This arrangement is similar to that used to determine the transit times for the uniform flow case. The flow control gate was used to
set the required datum flow of material, while the perturbing gate was actuated by the P.R.B.S. generator via the power amplifier and the electromechanical solenoid. The perturbing gate input signal \( x(t) \) was measured by a displacement transducer, while the flow meter located near the end of the chute detected momentum changes of the material in transit \( y(t) \). The cross-correlation was obtained for these two signals, \( R_{xy}(\tau) \), using the above identification hardware and also using the NOVA package as a cross-check.

Both the cross-correlation signal \( R_{xy}(\tau) \) from the correlator and by means of the complementary Spectrum Display unit, the power spectral density \( G_{xy}(f) \) were recorded graphically using an x-y recorder. Because of the relatively good noise immunity of this identification technique, the grain elevator could be operated in the closed circuit mode whilst carrying out the identification without noticeably affecting the results. This was of significance since the experimentation times in some tests needed to be much longer than the capacity of the hopper would allow. Also the maintenance of a relatively constant head above the orifice was desirable to eliminate any extraneous effects due to a low grain head.

The use of the NOVA identification package with the NOVA situated in another building required special considerations. The programme package's data collection needed to be co-ordinated with the initiation of the P.R.B.S. testing. The ability of the programme package to be remotely started facilitated this situation. This ability of being able to programme the mini computer at what amounts to the hardware level, assembler language, was seen as one of the advantages of using the low level language.
The Noise Generator (H.P. 3722A) provided the requisite pseudo-random binary signal. Each of the parameters sequence length (N), clock period (ΔT) and amplitude (A) could be varied over a wide range. This low energy signal was amplified in the Philips power amplifier (GM 5535) before passing to the Philips solenoid (PR 9270). This was capable of exerting a maximum force of 3.5 newtons per amp (r.m.s.) on the perturbing valve.

The momentum signal y(t) measured by the strain gauge flowmeter required a carrier amplifier for operation. For this a Tektronix (3C66) dynamic bridge amplifier with a carrier frequency of 25kHz was used.

The correlator (H.P. 3721A) had a bandwidth of D.C. to 250kHz and enabled quick determination of correlation functions with 100 delay points being displayed at any one time and with a total delay capability of 1000 delay points.

The complementary Spectrum Display Unit (H.P. 3720A) was particularly useful enabling the frequency domain conversion of the correlator results to be displayed in any combination of linear or log, versus linear or log, scales. The frequency interval displayed was directly tied to the selected correlation delay time and the total range of frequency displayed was always two decades. Both the correlator and the spectrum display unit were fitted with direct signal outputs to drive an x-y recorder having an automatic pen-lift facility.
6.3 Procedure

Using the test rig shown in Figure 1, identification investigations were performed for chutes of rectangular cross-section, of various straight and curved shapes. A wide range of datum flows and straight chute slope angles were incorporated in the test programme.

Preliminary testing was necessary to ascertain appropriate equipment parameters and operating procedures. In each case the main flow control gate was preset to a fixed datum flow rate and the perturbing gate was operated in a pseudo-random binary mode via the power train described above.

The quick-look facility of the identification hardware enabled repeated testing of a given chute configuration until one was satisfied that the result was stable and repeatable. At this point the discrete correlation function in dot form and the spectral function in combinations of log. and linear scales were recorded. For many of the tests, the final experimental run using the correlator package was accompanied by a simultaneous analysis using the NOVA identification package. This duplication made the results not only useful for cross-checking but also made them complementary; the NOVA Identification package having system computation capabilities beyond those of the correlator package.

Since it is recognised that the experimental identification programme involved stepped movement of the flow control valve, some insight into the flow characteristics associated with this valve motion necessitated supplementary investigations. These took the form of step responses and high speed cine photography.
6.4 Step Responses

Step response tests were performed on the hopper-discharge chute system and the results compared to those obtained from the cross-correlation tests. In each case the perturbing gate was stepped, open and shut, while the main flow control gate was open allowing the rated throughput through the chute.

The apparatus as shown in Figure 1 was used to perform the step responses. The perturbing gate system was activated by a low frequency square wave generator, while the flow response was recorded using a high frequency ultraviolet (U.V.) recorder. Both cases of no main flow and with a main flow of 4 kg/min were tested. In order to maximise the trace on the U.V. recorder in registering the step responses, the galvanometer drive amplifier was offset to ignore the D.C. level of the main flow for that test. To improve the keeping properties of the resulting U.V. record, it was sprayed with a fixing solution.

The relative magnitude of the parameter signal compared to the 'grain noise' signal necessitated the pre-filtering of the signal from the strain gauge bridge amplifier to the U.V. recorder. This filtering was achieved using a variable frequency Butterworth filter (design details in Section 10.2) which provided maximally flat filtering with a 40 dB/decade attenuation beyond the selected cut-off frequency.

The results of this step testing will be contrasted in Section 7.4 with the predicted performance of hopper chute system based on its identified model.

In support of these results for the step testing, high speed
HYCAM SPEED CURVES: 400 FOOT MODELS
PICTURES PER SECOND—FEET OF FILM—TIME IN SECONDS

REGULATED RANGE: SETABILITY 5%—FLATNESS OF CURVE AFTER ACCELERATION 1%
NON-REGULATED RANGE: SETABILITY 10%—REPEATABILITY 5%
ALL CURVES ARE THE SAME FOR .115 AND 230 V.A.C. MODELS

Fig. 34
Camera Performance Details
cine photography was used to observe the characteristics of the grain flow through the bin orifice during the motion of the perturbing gate.

6.5 **High Speed Cine Photography**

A 16 mm Hycam high speed motion picture camera, model K2004E-230, was used. The camera incorporates electronic speed control and has a high speed rotating prism optical head to provide framing rates of up to 11000 frames/second. Performance details are shown in Figure 34.

The stop motion projector (Seimens model 2000) was arranged as shown in Figure 35. By adjusting the relative position of the projector and mirror tilt angle, a correctly oriented magnified image could be projected onto the glass top table. Since a reference grid was included in all filming sequences, a specific magnification ratio could readily be obtained, 1X, 2X, 4X being commonly used. As the film was projected one frame at a time onto the glass top table a sheet of tracing paper was used to plot successive positions of selected grains for each time increment. The framing rate was precisely known since the film edge had timing pips as detailed below and thus the velocity of specific grains could be calculated.
Philips quartz halogen and agrapho filament lamps were used for illumination with approximately 4 kw being normally required.

To facilitate timing measurements the camera was fitted with neon discharge tubes that produce timing marks on either side of the film edge. A timing light generator to provide one lamp with 50Hz pulses and the other with 1000Hz pulses was designed and built (details in Section 10.3). Exposure settings were estimated using a Weston Mark IV light meter of the reflective reading type. The more appropriate spot type light meter was not available necessitating some trial and error to obtain reasonable exposures using the Weston meter.

Using this information a series of tests were designed to observe the grain stream at the orifice exit, at the chute exit and, as discussed below, in a 'two-dimensional' hopper. The film was analysed using a number of devices including the stop-motion projector mentioned above, as well as a 3M micro-film reader-printer (model 500) which enabled magnified photocopies to be obtained suitable for later analysis. Film spools of 30 and 120 metres length were used depending on the framing rate required and the observation time sought. For example, to photograph three complete cycles of step openings using a framing rate of 1000 frames/second required approximately 110 metres of film. Details of exposure selection and camera technique can be found in Section 10.4.

The signal from the H.P. displacement transducer was used to trigger one of the timing lights (described above) in the high speed camera and yielded a stripe on the film edge corresponding to the opening and shutting of the perturbing gate. This provided a timing reference for the step responses.
The high speed cine photographic analysis of the flow through the bin orifice during the perturbing gate motion confirmed the observations of Parlour (Reference 14). The grain stream flow issuing from a hopper is asymmetrically influenced by the sudden opening and closing of the flow control gate. In order to lessen the effect of the perturbing gate end section, it was sharpened as shown in Figure 36 and its stroke was halved. The effect persisted and the high speed film revealed that the grain profile flowing out of the hopper orifice was still asymmetrical and linked.

Figure 36 Perturbing gate end-section modification

Figure 37 and Plate 9 record the shadow profiles comprising the effects of gate opening and closing.
Plate 9

Grainflow profiles showing effect of valve perturbations
In an attempt to emphasise the difference between these two perturbations a plot of stream thickness versus time has been extracted from the grain profiles shown in Figure 37. The overshoot accompanying a closing perturbation is clearly evident in Figure 38.
Further analysis will be deferred for the present being continued in Section 7.5 after the correlation identification results have been interpreted.

In order to pursue this asymmetrical behaviour and gain further insight into its causes it was decided to construct a 'two-dimensional' transparent hopper.

6.6 'Two-Dimensional' Transparent Hopper

In order to observe the flow pattern above and around the bin orifice, a small scale, 70 mm front wall to back wall transparent hopper, as shown in Plate 8, was constructed. The step opening of the perturbing gate was activated by a 240 volt washing machine solenoid and return spring system. A mixture of approximately 50% ink blackened and plain millet seed was used to aid in the photographic analysis, providing a better contrast.

The testing programme included similar stepped opening and shutting of the perturbing gate on the '2D' hopper as was used for the
main hopper-discharge chute system. Both cases of no main flow, i.e., only the perturbation, and with main flow present were tested. Using a reference grid behind the falling grain enabled the grain paths to be plotted before, during and after a gate opening and closing.
The analysis techniques used here were similar to those used to analyse the grain flow from the main hopper orifice. In addition, grain movement adjacent to the front perspex wall could be observed and photographed. The shearing planes for various perturbing valve positions were observed and velocity profiles along specified reference axes were computed.

Depth-of-field is a function of 'f' stop and lens diameter. The selection of the required framing rate using a particular film imposes an upper limit to the depth-of-field for a given intensity of illumination. The availability of approximately 4 kw of suitable lighting necessitated a narrow depth of field. This often resulted in the particular grain that was being plotted on the glass top table disappearing from view into the body of the hopper, requiring the sequence to be restarted with an adjacent grain; one of the most frustrating aspects of this experimental investigation.

It is recognised that this narrow hopper distorts to some extent the flow pattern within the funnel flow bin but this was considered an acceptable compromise in order to gain additional insight into the hopper's influence on the chute's dynamic performance. Having recognised the importance of these related investigations to the cross-correlation identification scheme, their further analysis will however be deferred until after the main experimental analysis; the cross-correlation test.
CHAPTER 7: RESULTS AND DISCUSSION - TRANSIENT FLOW

7.1 Transient Behaviour of Straight Inclined Chutes

In order to confirm the suitability of the cross-correlation method of process system identification as applied to the hopper-chute system, a series of tests were performed on the model equipment as outlined in Section 6.3. At this stage it is assumed that the flow behaviour through the chute is quasi-linear. That is, for small variations about the datum flow, the variation in flow is linearly proportional to the position of the control valve.

It is assumed that any noise present is statistically uncorrelated with the test signal so that the simpler Equation 5.14 for the cross-correlation applies.

Identification of the impulse function was performed for a range of chute inclination angles for the straight chute each with eight initial flow settings. This linear modelling in the time domain was supplemented by a frequency domain analysis.

Assuming the process to be ergodic* the cross spectral density $G_{xy}(f)$ is obtained from the Fourier transform of the cross-correlation function $R_{xy}(\tau)$.

that is, $G_{xy}(f) = \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-j(2\pi ft)} d\tau$

Under the conditions embraced by Equation 5.14 in which $h(\tau) \propto R_{xy}$, the system function is proportional to the cross spectral density function, $H(f) \propto G_{xy}(f)$.

*If the time average of a random signal is equal to the ensemble average then the process is called ergodic. (Reference 42, p.87)
7.1.1 Impulse Function for Straight Inclined Chutes

Figures 39 to 41 show the impulse response or weighting function $h(t)$ curves obtained by cross-correlation analysis for the straight inclined chute under datum flows ranging from 12mm to 57mm valve opening corresponding to 1.6 kg/min to 10 kg/min flow rate respectively for inclinations of 35°, 40° and 45°. Some general observations can be made.

1. As the inclination of the chute with the vertical (θ) increases the descent time increases as indicated by the transport lags.

2. For all flow rates and inclinations tested the flow experiences a 'dip' or negative going pulse prior to the positive going pulse normally anticipated for a linear system. The phenomenon is apparently associated with the re-distribution of the grain paths during the transient flow through the orifice of the bin and is analysed in Section 7.5.

3. As the main flow is increased relative to a constant P.R.B.S. step the response becomes more oscillatory indicating the presence of non-linear characteristics of the transient flow.

4. For the 35° inclination with its relatively high flow velocity the response shows more oscillatory behaviour for all flow rates.

5. As can be seen there is a variation in the form of the response for a given inclination at various flow rates indicating non-linear characteristics of the transient flow. However, as is also apparent the pattern is substantially constant for small variations in the preset flow. This last fact supports the quasi-linear assumption about a datum flow.
Figure 39  Impulse function curves for chute inclination $\theta = 35^\circ$
Figure 40  IMPULSE FUNCTION CURVES FOR CHUTE INCLINATION $\theta = 40^0$
Figure 41

Impulse function curves for chute inclination \( \theta = 45^\circ \)
6. All \( h(\tau) \) response curves show the same constant transport delay time for a given inclination which is associated with the time of travel through the chutes. The approximate transit times for the three inclinations 35°, 40° and 45° are 0.70, 0.77 and 0.89 seconds respectively. In this respect the flow perturbation and cross-correlation analysis is a useful method for determining the average velocity of travel through the chute system. The utilisation of this fact for an open channel bulk granular material flow meter is indicated.

7. Since the flow through the bin orifice under opening and closing perturbations of the valve has asymmetric characteristics, the initial flow reduction depicted in the \( h(\tau) \) curves may be accentuated by the closing perturbations of the valve when its motion is controlled by the P.R.B.S. generator.

In order to facilitate comparisons between inclinations Figure 42 shows the \( h(\tau) \) records of the straight chute for a flow rate of 4.0 kg/min for inclinations of 35°, 40° and 45°. Clearly the pattern of the impulse response is substantially the same. The varying transport delay with increasing inclination is also clearly delineated.

7.1.2 Bode Diagrams for Straight Inclined Chutes

Additional information concerning the transient flow characteristics may be gained from cross spectral density analysis, under the conditions previously described (Section 7.1), where the system function \( H(f) \) is a function of the cross spectral density \( G_{xy}(f) \). \((H(f) = G_{xy}(f)).\)

In order to assess the effect of chute inclinations on the frequency response of the hopper-discharge chute system use was made of
FIGURE 42 STRAIGHT INCLINED CHUTE (TIME DOMAIN)
the hardware correlator to obtain the Bode magnitude and phase versus frequency plots for the straight inclined chute at inclinations to the vertical of 35°, 40° and 45°.

Figure 43 illustrates the resulting Bode magnitude and Bode phase plots.

1. The frequency response based on the -3dB point for each inclination is essentially the same at 11Hz. Using the low frequency level as the zero asymptote the spread in the -3dB points was from 10.5Hz to 11.2Hz for inclinations of 45° to 35° respectively. This means that step flow adjustments with a period of less than 91 msec (i.e., \( \frac{1}{11} \) Hz) would be significantly attenuated.

2. As indicated the numerical Fourier transformation implemented both in the hardware correlator and the identification package tends to introduce end effect errors at the higher frequency end of the curves. This results as a consequence of Shannon's Sampling Theorem (Reference 41, p.188) limiting the frequency discernable for a given sampling time interval.

3. The attenuation corresponding to each inclination varies from 4.6dB to 7.5dB for inclinations of 45° and 40° respectively. These attenuation interpretations are relative to the D.C. level of the datum flow rate and are influenced by equipment parameter selection particularly the strain gauge flow meter offset.

4. The phase versus frequency plot indicates that it is a direct consequence of the delay lag which contributes a term in the response of the form \( e^{\jmath \omega t} \). Consequently this type of plot does not contribute significantly to the understanding of the model dynamics and will be dropped in the remaining Bode diagrams.
FIGURE 43  STRAIGHT INCLINED CHUTE (FREQUENCY DOMAIN)
5. Ignoring the end effects mentioned in 2. above, the high frequency asymptote suggests a minimum phase system of about fourth order.

The results above confirm that the P.R.B.S. cross-correlation method of process system identification can effectively be used to determine the hopper-chute system model. Since it is known that the valve motion produces asymmetric flow patterns as a result of its disturbing influence on the granular particles falling through the bin opening, the identified system weighting function needs to be used with care. The system response to a prescribed movement of the control valve can be predicted using this system model and convolution, provided that the control valve movements are not so large as to be influenced by the non-linear characteristics of the chute system. In addition the transport lags inherent in the weighting function curves provide a useful measure of the transit time and hence average velocity of travel through the chute.

With the aid of spectral analysis it is possible to determine the magnitude and phase characteristics of the system function \( H(f) \), enabling both the system bandwidth and the behaviour within that bandwidth to be analysed.

7.2 Transient Behaviour of Curved Chutes of Known Form

In order to compare the transient behaviour of the straight chute with chutes of known curved profiles, cross-correlation tests were performed on the parabolic chute and the optimally generated chute mentioned earlier. As was established in the uniform flow case the parabolic chute has favourable performance characteristics justifying its inclusion in this comparison. Each of the three chutes were tested with the same initial and end points.
7.2.1 Impulse Comparison of Straight Chute with Chutes of Known Form

Figure 44 shows a comparison of the impulse functions for chutes of known curved form each operating at a datum flow rate of 4.0 kg/min. Some general observations can be made.

1. The transport lags as depicted indicate that the optimal chute has a marginally faster descent time, as expected from the results on the uniform flow case.

2. The parabolic chute response is the most oscillatory; not a particularly favourable characteristic with regard to stability.

3. All chutes exhibit the characteristic 'dip' in flow as mentioned for the straight chute case.

4. The optimum chute impulse response function shows the most favourable rise-time performance with a smooth step transition from the 'dip' to the peak value; a favourable characteristic for transient operation. In blending this would minimise out of specification mix. It appears that in generating a chute profile to minimise the descent time under uniform flow conditions one obtains a chute that performs well under transient conditions. This is probably due to the fact that the design is based on energy considerations constrained by boundary forces such that lumpiness or grain stream bunching is avoided to sustain stable 'fast' flow as previously defined.

Clearly the optimum chute profile generated for minimum transit time has favourable transient characteristics when compared to the more commonly used straight and parabolic chute profiles.
FIGURE 44  COMPARISON OF IMPULSE RESPONSE
7.2.2 Bode Diagrams for Curved Chutes

Again recourse is made to the cross spectral density analysis to gain additional insight into the curved chute transient performance. Figure 45 depicts a frequency domain comparison of the curved chute performance characteristics. Some observations can be delineated for a flow rate of 4.0 kg/min.

1. The parabolic chute has the highest bandwidth having an upper frequency cut-off of approximately 14Hz. This is consistent with its oscillatory impulse response function.

2. The straight chute has the least attenuation at low frequencies up to approximately 4.0Hz being -3dB down at 4.5Hz and at 8.8Hz. The slight difference in characteristics for the straight chute between Figures 43 to 45 are a consequence of improvements in experimental technique between the first results taken in 1973 and the later ones taken in 1976.

3. The optimum chute has the highest relative attenuation for low frequencies and the lowest frequency bandwidth of 6.8Hz if the straight chute's 'dip' at 5Hz is ignored. This is seen as a consequence of the mobilisation of the available kinetic energy to achieve the constrained geometry for minimum transit time. Clearly the parabolic chute exhibits favourable transient characteristics with the optimum chute not significantly inferior. It should be remembered here that the optimum chute had significantly better transit time characteristics under uniform flow conditions.
Figure 45 Frequency domain comparison of chutes
7.2.3 Comparison of the Optimum Chute at Varying Flow Rates

Recognising that the chute flow characteristics are non-linear it was felt desirable to obtain impulse function responses for a range of flow rates. Some observations from Figure 46 follow:-

1. For all flow rates tested the characteristic 'dip' is again present.

2. For low flow rates with consequent relatively high perturbing flow rate compared to main flow the impulse function is of characteristic delta form indicating a sharp rise-time with little or no oscillatory components.

3. The descent time increases marginally with increasing flow rate indicating the non-linear operation mentioned.

4. At the designed flow rate the impulse function characteristics are still well behaved in terms of rise-time and oscillatory characteristics.

5. With increase in flow rate beyond the design point the impulse function exhibits increasing oscillatory behaviour. This characteristic has a direct consequence on the stability of the chute flow when coupled with the usual requirement of variable flow rate capability through the chute. It is suggested that for stable flow the designed flow rate would need to be the maximum required flow rate for the optimally generated chute profile.
Figure 46
Optimum chute \((\tau) = 0.0\)

Amplitude \(v^2/\text{DIV}\)

<table>
<thead>
<tr>
<th>Time ((\tau)) Sec.</th>
<th>Valve Opening MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>V1</td>
</tr>
<tr>
<td>0.25</td>
<td>V2</td>
</tr>
<tr>
<td>0.5</td>
<td>V3</td>
</tr>
<tr>
<td>0.75</td>
<td>V4</td>
</tr>
<tr>
<td>1.0</td>
<td>V5</td>
</tr>
<tr>
<td>1.25</td>
<td>V6</td>
</tr>
<tr>
<td>1.5</td>
<td>V7</td>
</tr>
</tbody>
</table>

\(A = \frac{v}{2}\)
7.2.4 Bode Diagrams for the Optimum Chute with Varying Flow Rate

Figure 47 illustrates the fluctuation in chute bandwidth with increasing main flow. Some observations:-

1. The design flow rate corresponds to the highest attenuation and the lowest frequency bandwidth. For flow rates of 1.6, 4.0, 6.4 and 8.4 kg/min the respective bandwidths are 12.6, 7.0, 11.1 and 9.2Hz.

2. For small flow rates, e.g., at 1.6 kg/min the bandwidth is a maximum at a value of 12.6Hz.

3. At approximately ±50% of the designed flow rate the bandwidth is similar and at least 50% higher than at the design point.

4. The frequency response to the breakpoint is flattest for the designed flow rate; a desirable characteristic.

The results above contrasting chutes of curved form with the straight inclined chute show the straight chute to be inferior in a number of respects. The optimum chute profile has the fastest transit time and the more favourable impulse function characteristic when compared to the parabolic and straight chute forms. The parabolic chute has the highest bandwidth and the most oscillatory response and is considered preferable to a straight chute under these conditions.

7.3 Step Responses

It is well recognised that additional insight into the flow transient behaviour not obvious from the h(τ) curves, can be gained by integrating the impulse responses to obtain the equivalent step response. Rise-time, overshoot, and oscillatory responses become readily evident. From a practical point of view, process changes in
Figure 47  Frequency Domain Results for Optimum Chute
flow would normally be implemented by stepping the control valve to the new position. Furthermore, experimental testing programmes normally include step response testing, which by means of convolution, can be the basis of system performance prediction.

7.3.1 Step Response for the Straight Chute

Figures 48 to 53 represent the integral of the impulse functions for the straight chute, cross-comparing chute inclinations with initial flow settings. These cross comparisons are particularly useful in highlighting similarities and differences not immediately apparent in the $h(\tau)$ system weighting function curves.

In Figure 48 for a chute inclination of $\theta = 35^\circ$ at a 12mm datum flow setting the step response indicates some initial oscillatory tendencies. At a flow rate corresponding to a height to breadth ratio of one, with a 25mm datum flow setting, the step response shows a favourable rise-time characteristic with little oscillatory tendency. The 38mm datum flow is relatively sluggish and would represent significant out-of-specification mix.

In Figure 49 for a chute inclination of $\theta = 40^\circ$ the general trends observed above again apply; the 'dip' is much more pronounced and further to the right indicating the longer transit time. The significant difference being that the 38mm datum flow rate now exhibits extremely oscillatory behaviour, and could lead to unstable flow patterns. Again the 25mm datum flow rate corresponds to the steepest rise-time with an attractive step response.

Figure 50 shows that for a chute inclination of $\theta = 45^\circ$ the 12mm datum flow has the most attractive step characteristic. The 25mm datum flow corresponds to minimum descent time but the levelling
Figure 48

Step response straight chute $\theta = 35^\circ$
Figure 49
TIME (τ) SEC.

○ = 12 MM
△ = 25 MM
+ = 38 MM

STEP RESPONSE STRAIGHT CHUTE θ = 40°
Figure 50  Step response straight chute $\theta = 45^\circ$
Figure 51  Step response straight chute 12 mm flow
Figure 52  Step response straight chute 25 mm flow
Figure 53  Step response straight chute 38 mm flow
off half-way to the step indicates initiation of oscillatory influences in the impulse response. The 38mm datum flow rate was particularly troublesome and required extensive prefiltering to reduce the very large oscillatory behaviour and should not strictly be compared to the rest of these diagrams.

In Figure 51 for variable inclinations at an initial datum flow rate of 12mm, the increasing prominence of the 'dip' is clearly evident for increasing chute inclination. For all inclinations the responses indicate oscillatory behaviour which would not usually be very attractive for design considerations.

In Figure 52 for an initial datum flow rate of 25mm the increasing time delay with increasing chute inclination is clearly indicated. The response for the $\theta = 45^\circ$ inclination indicates onset of increasing oscillatory behaviour at mid step.

In Figure 53 for an initial datum flow rate of 38mm the increasing oscillatory behaviour is evident and has implied consequences in designing chutes with height to breadth ratios greater than one. It should be noted that the curve for $45^\circ$ inclination is the same one as was mentioned earlier and because of the necessary pre-filtering should not really be considered here.

The results above clearly indicate the desirability of operating straight chutes under transient conditions with a datum flow rate corresponding to a height-to-breadth ratio of no more than one. For a height-to-breadth ratio of 1.5 the step response was increasingly oscillatory with increasing inclination angle, $\theta$. 
These equivalent step responses complement the impulse response \( h(t) \) curves and transient characteristics such as rise-time, overshoot, the 'dip', and oscillatory behaviour becomes readily apparent. It is suggested that any subsequent work in this area, or in other areas using the P.R.B.S./cross-correlation identification technique, should consider this type of equivalent step response. It is calculated by integrating the system's weighting function and provides a lucid delineation of the system's transient characteristics.

7.3.2 Step Responses for Chutes of Known Form

In Figure 54 step responses for curved chutes of known form as compared previously are contrasted. The overall performance of the optimum chute is clearly superior. It has the smoothest, steepest, rise-time with the minimum time delay.

The rise-time measured as defined in Figure 6 for the optimum chute is half that for the straight chute inclined such that the end conditions are at the same co-ordinates. The oscillatory behaviour of the parabolic chute and the sluggishness of the straight chute are again evident.

7.3.3 Step Response for the Optimum Chute

The step responses shown in Figure 55 compare the optimum chute's performance for various initial flow settings. The increasing time delay with increasing flow rate indicates the non-linear characteristics of this chute profile. Significantly, the least oscillatory response corresponds to the designed flow rate condition of 25mm datum flow setting or 4.0 kg/min. The designed flow rate exhibits the minimum 'dip' and minimum overshoot; clearly favourable criteria for a system subjected to transient operation. The 12mm datum flow rate,
Figure 54  Step response chutes of known form
Figure 55  Step response optimum chute
corresponding to -50% of the designed criterion has the next most favourable rise-time and oscillatory characteristics. The increasing oscillatory behaviour with increasing initial datum flow setting again implies that for 'fast' stable flow the design point ought to be near the maximum anticipated capacity of the chute.

The results above for the equivalent step responses highlight the usefulness of this technique and support the earlier conclusions comparing chutes of known geometric form. The straight chute's oscillatory behaviour when operating with a datum flow corresponding to a height-to-breadth ratio greater than one has direct consequences for the designer who wishes the chute to operate significantly in a transient mode.

The overall performance of the optimum chute was clearly superior under these transient conditions and, considered together with its superior performance in achieving minimum transit time under stable 'fast' flow conditions provides the designer with a clear 'benchmark' or 'yardstick' for his chute design.

7.4 Predicting Flow Transients

As outlined in Section 5.2.1 the cross-correlation function or impulse function represents the dynamic model of the system under test, in this case, the hopper-discharge chute system. At this stage it would be informative to compare the actual step responses with those obtainable by convolution with the derived system model.

Although the hopper-discharge chute system behaviour is strictly non-linear in as much as the impulse response functions show varying characteristics for variations in the datum flow, the assumption of linearity for small disturbances about a particular control point is
quite satisfactory. Based on this assumption it is possible, via linear systems theory, to compute the response of the system to any prescribed motion of the flow controlling valve. This can be achieved by using Equation A5.14 repeated here

\[ y(0) = (x(0) H(0)) \Delta \lambda \]

\[ y(1) = \frac{x(0) H(1) + x(1) H(0)}{2} \Delta \lambda \]

\[ y(j) = \left[ \frac{x(0) H(j) + x(j) H(0)}{2} + \sum_{i=1}^{j-1} x(i) H(j-1) \right] \Delta \lambda \]

\[ j = 2, 3, \ldots m-1 \]  \hspace{1cm} (A5.14) repeated

In the case of a step input \( x(j) \) will be constant throughout. Using Equation A5.14 the computed step response for the straight 40° chute is shown in Figure 56. It compares favourably with the measured step response obtained experimentally as detailed in Section 6.4 and is also shown in the same figure. The 'dip' and the rise-time characteristics are clearly evident.

A significant aspect of the flow behaviour is that a sudden opening of the valve, causes an initial reduction in flow, following the transport lag, before the flow finally increases to its new steady state value.

It is possible then, using convolution techniques, to compute the response to any input. The corollary, for predicting the required input to yield a desired output, inverse convolution, has proved a little more difficult to implement but offers certain attractions. The numerical difficulties are associated with the use of Equation A5.15 repeated here

\[ x(j) = \frac{1}{H(0)} \left[ \frac{y(j)}{\Delta \lambda} - \sum_{k=0}^{j-1} H(j-k) x(k) \right] \]

\[ j = 0, 1, \ldots m-1. \]  \hspace{1cm} (A5.15) repeated
Figure 56  Comparison of computed and measured step responses for straight inclined chute
When $H(0)$ is very small or zero the predicted input approaches infinity; obviously an impractical possibility. Using Equation A5.15 and the impulse function model for the optimum chute for the designed flow rate of 4.0 kg/min, the required control valve motion to produce a step output response was computed.

Figure 57 shows the predicted valve motion to achieve a unit step increase in flow above a given datum flow rate. The curve shows that the control valve needs to be stepped open 60 milliseconds before the required step response by an initial amount equal to approximately three times this required step and then be closed down to the target unit step according to the position-time history shown. This ramping action occurs during a period of 210 milliseconds. The shutting transient gives rise to a momentary increase in flow which is compensated by the slight overshoot in the control valve position shown before settling down to the new steady state value corresponding to the unit step increase in flow rate.

Linear systems theory in the form of the convolution integral has been shown to satisfactorily predict the transient performance of the hopper-discharge chute system. If large step changes above a datum flow rate are contemplated the technique needs to be used with care as a quasi-linear system model is assumed. Within the framework of this limitation however, the prediction of the transient behaviour of the hopper chute system enables the controlled blending of granular material to be effected 'with certainty'; out-of-specification product being clearly delineated.

It is possible now, using convolution techniques, to compute the response of the system to any given input. While inverse convolution techniques enable the required input to achieve a desired
FLOW CONTROL VALVE TIME-HISTORY

HOPPER ORIFICE

INITIAL DATUM

TARGET DATUM

PREDICTED INITIAL OPENING

PERTURBING VALVE MOVEMENT

Figure 57 Predicted flow transient
response to be determined. Further, should optimal control be the objective, it is possible to achieve this by establishing an appropriate performance index based on an error signal representing the difference between the desired and the actual hopper chute response. This performance index would then be maximised or minimised as appropriate to the given problem formulation.

7.5 Flow Non-Linearities

7.5.1 Bin Outlet Flow Resulting from Valve Perturbations

As discussed in Section 6.5 high speed cine photography was used to examine the effects of the perturbing valve operation on the grain flow from the bin orifice. Figure 37 and Plate 9 depict the differences in grain stream profiles resulting from an opening perturbing valve motion and a closing motion. For the opening perturbation, that is a sudden increased opening of the valve, the grain stream increased almost uniformly to the new steady state value corresponding to the greater flow. However, for the closing perturbation, that is a sudden decrease in the datum setting of the valve, the grain stream is deflected or bent as shown. At the same time the stream thickness in reducing to the new steady state value has a small 'overshoot', in that the thickness first decreases to a minimum value before the steady state condition is reached.

In an attempt to identify the cause of this asymmetrical behaviour the perturbing valve end section was ground down to a fine edge as shown in Figure 36 so that it presented the least possible across-the-stream surface area. Additionally the perturbing stroke was halved to a value of ±2mm of movement. Retesting revealed that the asymmetry persisted, and is probably due to the re-organisation of the grain forces within the hopper and discharge orifice resulting from
the horizontal shearing effects of the perturbing valve motion.

In order to pursue this asymmetric effect further a 'two-dimensional' transparent hopper was constructed and the results are discussed below.

In Figure 58 the positions of individual grains falling from the bin have been plotted as described in Sections 6.5 for typical opening and closing perturbations of the valve. Representative flow contours at the bin orifice are plotted for the grain stream flow immediately following the valve movement with successive positional contours spaced five milliseconds apart.

From grain path records such as these grain particle velocities at the bin orifice have been computed and four sets of velocity profiles are shown in Figure 59. The four flow regimes depicted (top to bottom) are:

1. Steady state flow, with the perturbing valve in innermost position.
2. Transient flow immediately after the perturbing valve is moved to the outermost position.
3. Steady state flow, with the perturbing valve in the outermost position.
4. Transient flow immediately after the perturbing valve is moved to the innermost position.

For each flow regime two profiles are shown, the short dashed line profile representing the velocity distribution across the stream 5 milliseconds after exit from the bin outlet and the long dashed line profile representing the velocity distribution across the stream after falling for 20 milliseconds from the bin outlet. Some general
Figure 58  Grain paths and flow profiles at bin outlet resulting from valve perturbations
Figure 59  Grain stream velocity profiles at bin outlet
characteristics are indicated.

1. For steady state flow the velocity profiles are symmetrical whereas for the transient flow phases following the perturbing valve movements, the velocity profiles are asymmetric.

2. The velocity variation across the stream is more marked for the 5 millisecond reference lines than for the 20 millisecond reference lines. In the latter case the profiles have much 'flatter' characteristics as a result of the grains becoming increasingly influenced by gravity and air resistance.

3. The average grain stream velocity is approximately 1 metre/sec for all cases after the grains have fallen for 20 milliseconds with the control valve in one of its two datum positions.

4. Immediately following the closing perturbation the velocity of the far side grain stream is approximately 10% higher than the near side grain stream, indicating a slight increase in flow during the shutting transient.

High speed cine photography has enabled the behaviour of the grain flow from the bin orifice to be determined before, during and after a perturbing valve movement. The grain stream velocities accompanying this perturbing valve movement have been clearly delineated.

7.5.2 'Two-Dimensional' Hopper Results

As mentioned previously, the problem of the asymmetric characteristics of the flow during perturbations of the flow control valve were pursued by using a so-called 'two-dimensional' hopper. It is recognised that the narrow funnel bin contemplated introduces flow distortions not experienced in a full depth hopper. However, it was
felt that any additional insight into the flow patterns immediately above, through and out of the hopper would be of benefit in interpreting the transient characteristics of the hopper orifice.

Visual inspection of the high speed film confirmed that the valve motion influenced the grain flow well into the body of the model hopper. After a transient perturbation this influence causes the body flow velocity to momentarily oscillate. This was alluded to by Parlour (Reference 14). Initially a closing perturbation causes the body flow to stop completely for approximately 24 milliseconds at a cross-section 50mm above the orifice and then to oscillate briefly.

Figure 60 shows a full size representation of the shear planes, illustrating the shift in the right hand shear plane when the perturbing valve assumes its two steady state positions. Relative slip occurs between the two pairs of lines associated with the same datum point at the orifice. As for the case discussed in Section 7.5.1, the path traced by the individual grains was plotted for successive frames and from these the velocity profiles across nominated horizontal sections within the '2-D' hopper were calculated.

Four flow regimes are considered:
1. Before closing - with the perturbing valve in the open position.
2. At closing - with the perturbing valve moving to the closed position.
3. After closing - with the perturbing valve in the closed position.
4. At opening - with the perturbing valve moving to the open position.

Figure 61 shows the velocity profiles at a cross-section 100mm above the bin orifice. The approximate uniform velocity profile before and after perturbation is clearly evident. The disturbance to this grain movement due to valve perturbations shows a slight asymmetry.
Figure 60 Slip planes for '2-D' model hopper
Figure 61  Velocity profiles 100 mm above orifice
to the right hand side or perturbing valve side. However, the disturbances produce essentially complementary influences.

Figure 62 shows the velocity profiles at a cross-section 50mm above the bin orifice and the effect of the valve motion is becoming more pronounced, resulting in a drop in the average centre line velocity of 44% of the pre-perturbation centre line velocity. The steady state velocity profile with the perturbing valve closed now shows a flatter profile.

Figure 63 shows three of the velocity profiles at the bin orifice to be essentially flat with a slight asymmetric influence on the perturbing valve side, indicating that there is little relative movement between grains across the stream flow. This is expected as the grains are leaving the influence of the hopper and are becoming predominantly influenced by gravity and air resistance. The velocity profile corresponding to the closing perturbation has a centre line velocity of 0.75 metres/second. For this the grain stream nearest the perturbing valve experiences a drop in velocity of 18% while the far side grain stream velocity drops only 7%, compared to the centre line velocity. It should be noted that the shutting perturbation is again accompanied by a slight increase in the average grain stream velocity. At the centre line this increase is approximately 7% of the pre-perturbation centre line velocity. At opening the slight drop in grain stream velocity nearest the perturbing valve is clearly evident and supports the description of the 'dip' in flow resulting from an opening step.

Taking Figures 61 to 63 together one can observe the increase in velocity from the 100mm point to the bin orifice within the shearing section of the body of the hopper. The average centre line velocities being approximately 0.13, 0.27, and 0.7 metres/sec respectively.
Figure 62  Velocity profiles 50 mm above orifice
Figure 63 Velocity profiles at chute orifice
The '2-D' hopper results have shown that the flow behaviour below the bin orifice is influenced by grain force redistribution above the bin orifice during perturbing valve movement. High speed film analysis is an effective technique for identifying the grain flow patterns associated with the control valve movement. The necessary film analysis to plot individual grain paths is time consuming but not difficult.

Overall the transient analysis has confirmed that the hopper-discharge chute system can be modelled using quasi-linear systems theory. The influence of the non-linear characteristics of the flow control valve can be minimised by considering small perturbations about the datum flow. The P.R.B.S./cross-correlation identification scheme implemented effectively ensured this small perturbation about the operating point was used. With this constraint, linear systems theory has effectively identified the hopper-discharge chute model. By using convolution techniques the chute performance under prescribed transient conditions has been successfully predicted.
CHAPTER 8: SUMMARY OF CONCLUSIONS

Uniform Flow

The experimental investigation of the performance of hopper-discharge chutes operating in the 'fast' flow mode under steady uniform flow conditions has delineated the following:-

1. The work of earlier researchers, particularly Roberts and Parlour, has been extended. The design of chutes incorporating the limiting angle, $\Theta_f$, has resulted in chute geometries suitable for 'fast' stable optimum flow operation.

2. The lumped parameter model has been found to be adequate for analysing the uniform flow of millet seed through chutes subject to generalised drag forces.

3. For the model, the generalised drag force included a viscous drag component with a coefficient of approximately 2 ($\mu \approx 2.0$).

4. Optimisation based on minimising the descent time and maximising the exit velocity in a prescribed direction has been formulated using the lumped parameter model. The design of chutes to minimise the descent time has been solved.

5. The combination of Charlton's polynomial approximation method and the optimising algorithm of Fletcher and Powell has resulted in an optimisation computer programme that readily solves the minimisation formulation providing a design procedure incorporating a sensitivity analysis and all the significant design parameters.

6. Even at the small scale tested the optimum chute geometry exhibited favourable performance characteristics (minimum transit time) whilst maintaining the desired 'fast' flow pattern compared to chutes
commonly used. In terms of the minimisation of the descent time the chute profiles were ranked in their order of merit; optimum, parabolic and straight inclined chutes. These incremental improvements in performance represent potential operating cost savings.

7. Overall the investigation has shown the potential of discharge chutes as flow controlling devices in the gravity flow of bulk granular solids. For steady flow operation it is possible to design chute profiles to achieve prescribed optimum performance by the use of direct mathematical methods.

**Transient Flow**

The experimental programme designed to identify the transient characteristics of the hopper-discharge chute system disclosed the following:-

1. The investigation supports the results of Robert's pilot study indicating that the P.R.B.S./cross-correlation method of process system identification can be effectively applied to the hopper-discharge system model.

2. Although the transient flow shows minor non-linear characteristics, the results indicate that an assumed linear or quasi-linear formulation is satisfactory for reasonably small movements of the flow control valve.

3. With the aid of spectral analysis it is possible to determine the magnitude and phase characteristics of the system function, \( H(f) \), enabling both the system bandwidth and the behaviour within that bandwidth to be determined.
4. The transport lags inherent in the chute's weighting function curves provide a useful measure of the material transit time.

5. The optimum chute impulse response function shows the most favourable rise-time performance with a smooth step transition from the 'dip' to the peak value; a favourable characteristic for transient operation. In blending, this would minimise out-of-specification product. It appears that in generating a chute profile to minimise the descent time under uniform flow conditions one obtains a chute that performs well under transient conditions. This is probably due to the fact that the design is based on energy considerations constrained by boundary forces such that lumpiness or grain stream bunching is avoided to sustain stable 'fast' flow.

6. The transient results indicate that straight chutes should not be used with a height to breadth ratio (as previously defined) greater than one for stable transient operation.

7. Linear systems theory in the form of the convolution integral was used to predict successfully the transient performance of the hopper-discharge chute system. If large step changes above a datum flow rate are contemplated the technique needs to be used with care as a quasi-linear model is assumed. Within this framework, however, knowledge of the transient behaviour of the hopper-chute system during controlled blending of granular material enables an operator to work 'with certainty', minimising out-of-specification product.

It is possible then, using convolution techniques, to compute the response of the hopper-chute system to any given flow control valve
movement to achieve a desired flow can be determined. Further, should optimal control be the objective, it is possible to achieve this by establishing an appropriate performance index based on an error signal representing the difference between the desired and the actual hopper-chute response.

8. The transient results generally support those of the uniform flow case in ranking the chutes in the same order of merit; optimum, parabolic and straight inclined chute, particularly when it is recognised that the optimum chute had significantly better transit time characteristics under uniform flow conditions. On the other hand, the results clearly show that the optimum chute needs to be operated at no more than the designed flow rate for stable flow.

9. High speed cine photography of the transient behaviour of the flow control valve generally supported the assumption of a quasi-linear model for the system identification indicating that the non-linearity was relatively minor and the use of the linear systems theory justified. This analysis revealed that the non-linearity was not due to the perturbing valve end section but rather to the dynamic reorganisation of grain forces within the body of the hopper.

10. The overall performance of the optimum chute was clearly superior under these transient conditions and considered together with its superior performance in achieving minimum transit time under stable 'fast' flow conditions provides the designer with a clear 'benchmark' or 'yardstick' for his chute design.
Suggestions for Further Work

The following general areas emerged as possible extensions to this study and would broaden the scope for application of the present findings:

1. An experimental investigation on large scale chutes ought to be conducted to verify that the theoretical scale up analysis presented is supported.

2. An investigation based on a distributed parameter model may result in an analytical solution to the transient behaviour of the hopper-chute system.

3. An experimental investigation using cohesive materials may reveal that only minor modifications to the present model would enable the designer to achieve optimum chute profiles for cohesive materials.
CHAPTER 9: REFERENCES AND BIBLIOGRAPHY


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10.1 Flowmeter for Granular Material

10.1.1 Introduction

The design of a suitable flowmeter has proved quite difficult. The criteria to be met includes a meter capable of measuring the flow of non-cohesive materials flowing in a small scale open channel chute and producing an electrical output signal having relatively insignificant in-meter time delay. The similar related problem of measuring grain stream velocity under the same design criteria was also considered. It is recognised that with an impulsive momentum type transducer measuring \( (mv) \) a constant output could occur with a changing mass flow rate. It was thought that if a suitable velocity detector could be built, the mass flow rate would be determined by dividing the output from an impulsive transducer by the velocity

\[ i.e., (m = \frac{mv}{v}) \]

For at least 70 years attempts have been made to take advantage of the inherent benefits of a solids flow measuring device based on the principle of interpreting momentum changes. Dean (Reference 62), in the mid-Fifties, designed a pneumatic flowmeter shown in Figure A1.1 for granular materials with some success. In devices of this type an impulse sensor is attached to a pivoted beam in a fashion similar to a weighing scale. It can be seen that the falling material generates a torque about the axis of the beam.

The angular motion imparted to the moment arm by the material striking the sensor is opposed by a spring-damper system. This rotation is measured in Dean's device by a pneumatic sensor which, in turn, gives an output signal that for small angular displacements would be linear and proportional to the amount of material flowing. The
Figure A1.1  Dean's flowmeter for granular materials
accuracy of this instrument is influenced by any build-up of material on the plate and more fundamentally by the accuracy with which the falling material is centred on the inclined plate. Clearly a grain stream centred on A would register a different flow rate to that grain stream centred on B.

Most of these problems have been overcome by Nugent et al (Reference 63) by designing a flowmeter that, although it uses the same principle as Dean's flowmeter, has the pivot axis vertical rather than horizontal. Figure A1.2 shows his model, incorporating a Bendix pivot as a frictionless bearing. The Bendix pivot consists of a number of leaf springs arranged so that they will always be under tension, and require a constant bending force which results in a constant friction pivot point that is virtually hysteresis free.

Nolte (References 64, 65) has been associated with the design of a number of devices operating on the principle of measuring the reaction to the material flow after it has fallen a known height. His refinement in ensuring that the sensor moved only vertically and hence became insensitive to the direction of entry of the material resulted in a number of commercial devices being produced and are typified by the device shown in Figure A1.3. In these devices the material is passed through a set of baffles to remove any initial vertical velocity. The material then falls a set height within the meter and impinges on a conical sensor supported on springs and constrained to move in the vertical direction only. The vertical displacement of the sensor is detected by a displacement transducer.

Henderson (References 66, 67) proposed a flowmeter based on the change in angular momentum principle, whereby material flowed into a constant speed rotating vane system, as shown in Figure A1.4. The
Figure A1.2  Granular flowmeter of Nugent et al.
Figure A1.3  Nolte's flowmeter for granular materials
change in angular moment due to the granular material passing through the meter, causes a torque in the supporting shaft which can be measured by strain gauges attached to the shaft.

Material enters the meter and strikes the cone at the top of the meter and is then directed into the top of the rotating vanes near the axis of rotation. The angular rotation of the vanes causes the material to slide radially along the vanes and be thrown into a collecting hopper. The strain gauges are oriented so as to measure torque only and ignore any bending moments which may be present. As with most such systems measuring small torques from rotating shafts with the usual high noise present due to the slip-ring subsystem, considerable filtering is necessary. This filtering usually limits the frequency response of the meter and can introduce a significant time delay.

For the flowmeters analysed above the maximum to minimum flow measuring range is limited and the in-meter transit time would make them generally unsuitable for the present investigation. The overall complexity of some of the devices also precluded their use in this investigation.

The large range of devices designed for closed channel operation such as those mentioned by Carmichael (Reference 68) and those mentioned in References 69 and 70 were reviewed to examine the concepts involved in their operation and their possible use in an open channel grain flowmeter.
Figure A1.4  Henderson’s flowmeter for granular materials
10.1.2 Flowmeters/Velocity Detectors Investigated

A number of flowmeter and velocity detector designs were considered and some of the more promising devices were tested. The desire to meet the design criteria of Section 10.1.1 with a relatively simple and inexpensive transducer required considerable effort.

Flowmeters

A number of impulse type flowmeter transducers were tested. Figure A1.5 shows a piezoelectric system measuring impact pressure caused by the falling grain stream. Whilst this arrangement had adequate sensitivity the associated charge amplifier was subject to drift, requiring frequent recalibration and was thus not considered suitable. The transducer's sensitivity to moisture, requiring special precautions for storage and the need for more than the usual care in handling the signal cables when operating within the grain stream, also made it unsuitable in this application.
Figure A1.6 illustrates a system based on a spring and
displacement transducer. The requirements for this transducer include
the ability to measure small flow fluctuations above a given datum
flow rate. A spring could be chosen to yield an adequate displacement
sensitivity on light loads, but for heavier loads the excessive
deflection away from the datum position resulted in a non-linear
response caused by the shift in the grain stream impact point.
Figure A1.7 depicts a measuring system based on a flapper valve fitted with strain gauges to measure deflection. This design was adopted and details follow in Section 10.1.3.

Figure A1.7  STRAIN GAUGE IMPULSIVE FLOWMETER SYSTEM

Velocity Detectors

Figure A1.8 illustrates a visual velocity measuring system based on cycling lights over a known distance and synchronising the 'travelling' light beam to the grain stream flow. In the present system the lights are cycled using a function generator to drive a decade counter with each digit driving a separate light via switching transistors. Some of the current problems with this system relate to the reflection of light from the perspex chute, the randomness of the
grain flow and inability of switching the incandescent lamps on and off rapidly. Modifications to this system as an 'off-line' velocity detector are being considered.

**Figure A1.8** 'Linear' Strobe Velocity Detector
Roberts (References 3, 7) showed that the velocity of the grain stream at any given cross-section of the chute was a function of the grain head provided the mass flow rate was constant

\[ \frac{H}{H_0} = \frac{V_0}{V} \]

Photodiodes arranged as shown in Figure A1.9 could be used to measure the head and, knowing the inlet conditions and assuming uniform flow, the velocity at that point could be calculated.

**Figure A1.9 Photodiode Height of Grain Stream Sensor**
Problems with light reflections off the perspex chute and with the difficulty of packing enough photodiodes to make the height measurement fine enough resulted in this method not being pursued.

Another possibility considered was the use of a very light free-spinning wire paddle wheel photo-encoder as illustrated in Figure A1.10. This device would enable the surface and subsurface velocities to be measured. Despite the requirements for low friction bearings and low rotational inertia, together with the need for a relatively simple electronic circuit, this device was a distinct possibility. However, the Doppler-effect meter described below overshadowed this unit and further work on the wire paddle wheel unit was deferred.

Figure A1.10  Wire Paddle Wheel Photo-encoder
Hamid (Reference 71) describes a recent development of a microwave Doppler-effect flow monitor suitable for particulate solids. The flowmeter utilises the principle of continuous wave Doppler radar with results suggesting an integration or in-meter time lag of approximately half a second for a grain velocity of 1 m/s. Further investigation of this principle was undertaken and design details for an ultrasonic unit are presented in Section 10.1.4.

The more conventional techniques using photographic methods and static profile scales were also used successfully in this experimental investigation.

10.1.3 Strain-Gauge Cantilever Flowmeter

In order to satisfy the conflicting design criteria of high sensitivity and high natural frequency for the cantilever flowmeter the structural parameters involved were investigated. On the one hand, high sensitivity requires a flexible cantilever, but this implies deflection away from the grain stream for a high datum flow rate compared to a light load causing non-linear operation due to the shift in the striking point. Also, a relatively flexible cantilever may have its natural vibration frequency low enough to interfere with the recording of step changes in flow.

The equation for the natural frequency of a cantilever from Hurty (Reference 72, p.203) is,

$$\omega_n = (β1)_n^2 \sqrt{\frac{E1}{m1^4}}$$

where $β1 = 1.875, 4.694, 7.855$ for $n = 1, 2, 3$ respectively.

The geometric parameters were selected to suit the sizes of readily available bright metal strip so as to obtain an optimum
compromise between natural frequency and sensitivity.

The resulting flowmeter is shown in Figure A1.7 with Philips PR 9833K/03 strain gauges fitted.

Calibration curves using a Tektronix Type 3066 bridge amplifier are shown in Figure A1.11. The measured natural frequency of 34Hz compares favourably with a design target frequency of 37Hz. The strain gauge flowmeter was found to be simple in use, reliable in operation and effectively met the design criteria.

10.1.4 Ultrasonic Velocity Detector

The necessity of implementing a non-contact velocity detector has already been alluded to because of its many attractions. Various devices have been described in the literature (References 69, 73, 74) which rely for their operation on the Doppler shift principle. The high equipment cost associated with both laser and microwave based systems placed them outside the reach of this experimental study. However the many advantages of a Doppler-effect meter were strong incentives to find such a solution technique. The availability in early 1967 of relatively inexpensive ultrasonic transducers (approximately $7.00 a pair) was instrumental in the final realisation of the ultrasonic velocity detector.

The Doppler principle is well known and the velocity of a target is given by

\[
\nu_t = \frac{\nu_w f_d}{2f_w \cos \theta}
\]  

(A1.1)

where \( \nu_t \) = target velocity

\( \nu_w \) = velocity of the emitted wave

\( f_d \) = Doppler frequency

\( f_w \) = frequency of the emitted wave
Figure A1.11 Calibration curves for the strain gauge flowmeter
0 = angle between the centre line of the emitted wave and the
velocity direction.

For a particular system Equation A1.1 can be rewritten as

\[ v_t = K f_d \]  \hspace{1cm} (A1.2)

where \( K \) is a proportionality constant and is a useful measure of
sensitivity and resolution of the instrument.

wavelength \( \lambda = \frac{v_0}{f_w} \)

For high sensitivity and resolution a small value of \( K \) is
required.

Birchenough's (Reference 75) laser meter for measuring the
velocity of solids in gas-solid suspensions had a \( K \) value of
3.5 x 10^{-7} m/sec² when using red light with a wave length of
7 x 10^{-7} m and assuming an incident angle \( \theta \) of 0°. On the other hand
Hamid's (Reference 71) microwave device had a \( K \) value of 1.43 x 10^{-2} m/
sec² which although far below those of the laser system has a
considerable cost advantage. In comparison, using the ultrasonic
transducers mentioned above, a device with a \( K \) value of 4.38 x 10^{-3} m/
sec² was implemented. This is a significant improvement on the
microwave device being better than three times more sensitive.

The prototype was built at a cost of less than $100 with the
assistance of the Electrical Engineering Department.

The testing programme included tests to measure:

1. the range
2. the dispersion of the ultrasonic beam
3. the effect of directing the beams by means of plastic tubes
4. the effect of air turbulence across the ultrasonic beam
5. the possibility of measuring velocity through certain materials
such as sponge rubber, felt and plastic sheeting.

Results of these tests are too lengthy to be presented here and will be published at a later date.

A calibrating bench as shown in Frame 1 of Plate A1 was used for most of these tests. Frame 2 shows the monitoring equipment used and is essentially the same correlation and spectral analysis gear used previously. Frame 3 shows the fitting of the plastic tubes to the transducers and the opto-encoder used as a velocity reference. Frame 4 shows the transducers in position to measure the velocity of a rubber belt with wheat attached to its surface.

Plate A2 shows a second prototype being experimented with at present. Its portability and digital readout are clearly evident in Frame 1 where it is being used to measure the velocity of the same wheat covered rubber belt. Frame 2 shows the layout of the printed circuit boards. The use of a single multisided printed circuit board should enable the device to be miniturised even further.

Overall the ultrasonic Doppler-effect velocity meter as tested was found to have definite potential as a low cost, non-contact velocity detector. It was found extremely useful in measuring grain stream velocities at orifice exit, at chute entry and chute exit as part of the testing programme for the uniform flow of grain down chutes.

10.2 Flowmeter Filter

In order to record step responses using the strain gauge flowmeter, and recognising that this meter was extremely sensitive to grain impingement, a suitable high frequency cut-off filter was required between the meter and the ultraviolet recorder. As none was available a second order maximally flat (Butterworth) filter was
Frame 1  Experimental rig

Frame 2  Frequency monitoring equipment

Frame 3  Transducers without tubes

Frame 4  Transducers with tubes

Plate A1  Ultrasonic velocity detector prototype
**Frame 1**  Portable meter in position

**Frame 2**  Internal view of meter

**Plate A2**  Portable digital readout ultrasonic velocity transducer
designed using a readily available type 741 operational amplifier. Figure A1.12 shows the circuit of the filter with $R_1$, $R_2$, $C_1$ and $C_2$ being selectable by a single rotary switch so as to give design cut-off frequencies of 5, 10, 25 and 50Hz. Using readily available components, the actual cut-off frequencies of 5.2, 10.1, 29 and 52 were achieved. These were considered adequate for the range of tests to be conducted. This unit proved quite useful in a number of applications related to the experimental work that required some degree of filtering.

<table>
<thead>
<tr>
<th>FREQ</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$C_1$</th>
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<tbody>
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<td>5</td>
<td>750 k</td>
<td></td>
<td>150 k</td>
<td>.27</td>
<td>.056</td>
</tr>
<tr>
<td>10</td>
<td>768 k</td>
<td></td>
<td>159 k</td>
<td>.1</td>
<td>.0208</td>
</tr>
<tr>
<td>25</td>
<td>768 k</td>
<td></td>
<td>159 k</td>
<td>.047</td>
<td>.000828</td>
</tr>
<tr>
<td>50</td>
<td>768 k</td>
<td></td>
<td>159 k</td>
<td>.022</td>
<td>.000414</td>
</tr>
</tbody>
</table>

Figure A1.12 Flowmeter Filter
10.3 Camera Timing Lights

The two neon timing lights fitted on either side of the film transport of the Hycam high speed camera were used to mark, or expose, the edges of the film during a filming run and thus provide a timing reference. Since no timing light generator was available the simple circuit in Figure A1.13 was used to provide this triggering function. For the step response testing, particularly, it was useful to have the motion of the flow control gate, in this case the perturbing gate, actually switch one of the timing lights on and off to correspond to the opening and closing of the gate. With a 500Hz square wave applied to the other timing light the film is marked into one millisecond sections. From these timing marks the portion of the film correctly exposed after the camera has reached operating or set speed is easily seen as is the time interval between frames. As this unit provides the timing reference for all subsequent film analysis its accuracy and reliability are of paramount importance and frequent calibration checks are called for.

![Figure A1.13 Camera timing light generator](image-url)
10.4 Camera Technique

The Hycam is a high speed rotating prism camera accommodating film spools up to 120 metres in length. The optical head used was suitable for exposing 16mm full frame positive or negative movie film and would accept any standard 'C' mount lens. The quoted resolution at the centre line was 68 lines/mm dropping to 56 lines/mm at the edges.

The procedure adopted to photograph the front lighted experiments was as follows. Firstly, an estimate is made of the framing rate required to 'stop' the motion to be photographed minimising smear and a light meter reading of the average illumination is taken. By using the camera performance curves of Figure 34 a compromise between length of film and recording time at the required framing rate is reached. The maximum aperture allowed by the prism lens shutter system is \( f \) 3.3 so there is no point trying to expose at an \( f \) stop lower than this. The exposure time for this camera is calculated as follows:

\[
t_e = \frac{1}{2.5 \times \text{pictures per second}}
\]  
(Reference 76)

For a given light level and film speed this exposure time would correspond to a certain \( f \) stop.

The random motion of the grains necessitates a reasonable depth of field so as to track grain motion. This, in turn, requires a relatively high \( f \) stop setting which can only be obtained, all other parameters held constant, by increasing the film speed. The Ilford Mark V negative film rated at 400 ASA was forced developed to double that ASA, an improvement of one stop. The graininess of the emulsion hampered the analysis of the projected image when the film was forced beyond this value. It should be noted that for framing rates above about 1000
frames/second, the reciprocity characteristics of the film need to be accounted for in determining the effective exposure. The use of a spot type exposure light meter becomes essential when the field to be photographed is too small for an averaging type meter. The alternative of trial-and-error estimation using an averaging type meter is time consuming for satisfactory results.

10.5 Computer Programmes

10.5.1 System Identification Package

10.5.1.1 Algorithms

The algorithms used were obtained by digitising the respective signal analysis formulae derived in Section 5.2.

Correlation

The correlation algorithm below performs all the correlation calculations

\[ R(j\Delta T) = \frac{1}{N-j} \left[ \sum_{i=0}^{N-j+1} x(i)x(i+j) \right] \quad (A5.1) \]

for \( j = 0, 1, \ldots, m-1 \)

(note that \( R(0) = \bar{x}^2 = \text{mean square value} \))

Power Spectral Density

To analyse the power versus frequency relationships in the signals under test, use is made of the fact that the power spectral density function of an ergodic* random process is given by the Fourier transform of its autocorrelation function. To provide smooth estimates of the power spectrum use is made of Hanning's window

*See Section 7.1
function as outlined by Bendat et al (Reference 40, p.318).

The discrete unique frequencies calculated are given by

\[ f = \frac{k f_c}{m} \]  \hspace{1cm} (A5.2)

for \( k = 1, 2 \ldots m-1 \)

where \( f_c \) is the Nyquist folding frequency.

Leakage reduction resulting from the calculation of power spectra is achieved using the Hanning lag weighting function,

\[
D(r) = \frac{1}{2}(1 + \cos \frac{\pi r}{m}) \quad \text{for} \quad r = 1, 2 \ldots m-1 \\
= 0 \quad \text{for} \quad r \geq m \]  \hspace{1cm} (A5.3)

The final digitised formula for the power spectral density becomes

\[
G(k) = 2\Delta T \left[ R(0) + 2 \sum_{r=1}^{m-1} D(r)R(r) \cos \left( \frac{\pi r k}{m} \right) \right] \]  \hspace{1cm} (A5.4)

for \( k = 1, 2 \ldots m-1 \)

The cross spectral density function enables the system's magnitude and phase versus frequency relationships to be determined. These indicate the frequency response of the system, the presence of any time delays and any phase shift. In a similar manner to the single record spectrum, the following equations are derived by Bendat et al (Reference 40, p.334) at the discrete frequencies

\[ f = \frac{k f_c}{m} \]

for \( k = 1, 2 \ldots m-1 \)

The magnitude is

\[ |C_{xy}(f)| = \sqrt{C(k)^2 + Q(k)^2} \]  \hspace{1cm} (A5.5)

and the phase is

\[ \theta_{xy}(f) = \tan^{-1} \left( \frac{Q(k)}{C(k)} \right) \]  \hspace{1cm} (A5.6)

for \( k = 1, 2, \ldots m-1 \),
where
\[ C(k) = 2\Delta T \left[ A(0) + \sum_{r=1}^{m-1} A(r) \cos \frac{\pi r K}{m} + (-1)^k \zeta(r) \right] \]  
\[ (A5.7) \]

and
\[ Q(k) = 4\Delta T \left[ \sum_{r=1}^{m-1} B(r) \sin \frac{\pi r K}{m} \right] \]  
\[ (A5.3) \]

The odd and even parts of the cross-correlation function \( A(r) \) and \( B(r) \) are given by
\[ A(r) = \frac{1}{2} \left[ R_{xy}(r) + R_{yx}(r) \right] \]  
\[ (A5.9) \]
and
\[ B(r) = \frac{1}{2} \left[ R_{xy}(r) - R_{yx}(r) \right] \]  
\[ (A5.10) \]
for \( r = 1, 2, \ldots, m-1 \)

**System Computation**

For a linear time invariant system the cross-correlation function for the input and output signals may be expressed in terms of the autocorrelation function of the input signal and the system weighting function. Equation 5.5 is re-arranged and integration is approximated by summation to yield the discrete impulse function,
\[ H(0) = \frac{R_{xy}(0)}{R_x(0)\Delta \lambda} \]  
\[ (A5.11) \]
\[ H(j) = \frac{1}{R_x(0)} \left[ \frac{R_{xy}(j)}{\Delta \lambda} - \sum_{k=0}^{j-1} R_x(j-k)H(k) \right] \]
for \( j = 1, 2, \ldots, m-1 \).

Alternatively, in matrix form it is
\[ \bar{H} = \frac{1}{\Delta \lambda} \left[ R_x \right]^{-1} [R_{xy}] \]  
\[ (A5.12) \]
where \( R_x \) is the lower triangular autocorrelation matrix.

An indication of the accuracy of the model estimation can be gauged by comparison of the actual system output to that predicted by
convolution when the system is subjected to some prescribed input. This corresponds to the problem of controlling the flow rate so as to produce a prescribed flow output.

The output signal is given by

\[ y(t) = \int_0^t x(\lambda) H(t-\lambda) d\lambda \quad (A5.13) \]

Limiting the number of computed points to the number of discrete impulse function values previously calculated, and using the midordinate method of calculating the common convoluted areas, the digital form of the convolution routine becomes,

\[ y(c) = (x(0)H(0)) \Delta \lambda \]
\[ y(1) = \left[ \frac{x(0)H(1) + x(1)H(0)}{2} \right] \Delta \lambda \quad (A5.14) \]
\[ y(j) = \left[ \frac{x(c)H(j) + x(j)H(0)}{2} + \sum_{i=1}^{j-1} x(i)H(j-i) \right] \Delta \lambda \]

for \( j = 2, 3, \ldots, m-1 \)

The corollary for predicting the required input to yield a desired output, inverse convolution, has proved to be more difficult to implement. By rewriting Equation A5.13 and using the end value method of calculating the common convoluted area, the following expression, under certain conditions has yielded satisfactory results.

\[ x(j) = \frac{1}{H(0)} \left[ \frac{y(j)}{\Delta \lambda} - \sum_{k=0}^{j-1} H(j-k) x(k) \right] \quad (A5.15) \]

for \( j = 0, 1, \ldots, m-1 \)

The frequency response of the system under test can be found by taking the Fourier transform of the impulse function. Davies (Reference 41, p.184) outlines the simplifications consonant with pseudo-random binary signal testing, yielding a digital expression for the Fourier transform of the impulse function.
\[ G(j\omega) = \Delta T \sum_{i=0}^{N-1} H(i\Delta T)\cos(i\omega\Delta T) - j\Delta T \sum_{i=0}^{N-1} H(i\Delta T)\sin(i\omega\Delta T) \]  

(A5.16)

In the programme package \( \omega \) is varied from 0.1 to 1000 radians/second in four logarithmically spaced decades.

10.5.1.2 **User Experience**

Once operational experience is gained with the system identification package, some of the obvious advantages of the assembler language level of programming make the package a powerful data processing tool. The Data General User Manual (Reference 77) was found useful here. The intimate interaction between operator and hardware enables him, for example, to stop the process, examine or change counters or programme variables, skip over or back to some other part of the programme and then to continue processing by the press of a switch. This facility of checking the recorded data prior to lengthy data calculations can save considerable computer time. This is particularly so when working with a system involving a number of transducers, any one of which, should they become faulty, would render all the results meaningless.

On the other hand, the very considerable disadvantage of programming effort required to write programmes at this basic hardware level needs careful consideration. To attempt to write software of this nature on a newly installed system having only slow speed input-output reading and writing facilities is, in retrospect, very time consuming.

The utility of the package programme is evidenced by the fact that it has been used on many occasions for student demonstrations, data logging and data reduction for student projects within the Department of Mechanical Engineering.
10.5.1.3 Programme Flowcharts and Listing

The following is a brief description of the main subroutines used by the system identification programme. Flowcharts for the more complex subroutines are included with their listings.

SIGAN  - generate the operating system and by asking a series of hierachial questions schedule the subroutine path.

SERV  - contains all the device drivers; character input, output and text writing routines.

ATODC  - enables data logging using the analogue to digital converter operating under programme control.

PLOT  - an autoranging autoscaling plotting routine to present results in graphical form on a standard teletypewriter.

DATAIN  - general data entry via any of the devices available. Data is counted establishing N and M the correlation counter set to N/10.

LIST  - normalises the data; prints the mean; enables the listing or the punching of paper tape (with parity); or the plotting of data.

CORREL  - general correlation routine for computing RXX, RYY, RXY, etc.

SPECT  - computes the power spectral density of the input and output signals using the autocorrelation functions and a Hanning window for leakage reduction.

XSPECT  - computes the cross-spectral density function between input and output signals using the correlation functions and a Hanning window for leakage reduction.
IMPUL - computes the system's impulse function.

CONVOL - predicts an output based on the system model described by the impulse function.

INVERCON - predicts an input based on the system model described by the impulse function.

FRESP - computes a Bode diagram by taking the Fourier transform of the impulse function.

MESSR - start-up message, all running messages and storage arrays.
Figure A5.1 SIGAN(1)-Generation of operating runstream
Figure A5.2  SIGAN(2) - OPERATING SYSTEM SUBROUTINE HIERARCHY
SIGNAL/ON LINE SIGNAL ANALYSIS...G J MONTAGNER
LAST UPDATE 9/8/75

DATA ACCEPTED FROM TELETYPE, HIGH SPEED PAPER
TAPE READER, CARTRIDGE, A TO D CONVERTER.
NORMALISED DATA MAY BE PLOTTED OR LISTED.

SIGNAL COMPUTATIONS INCLUDE AUTOCORRELATION OF
INPUT OR OUTPUT, CROSSCORRELATION, POWER SPECTRAL
FUNCTIONS.

SYSTEM COMPUTATIONS INCLUDE IMPULSE FUNCTION
CONVOLUTION (TO PREDICT THE OUTPUT)
INVERSE CONVOLUTION (TO PREDICT THE INPUT)
FREQUENCY RESPONSE (BODE DIAGRAM)

PRBS PARAMETERS:
DT = TIME DISPLACEMENT (DIGIT INTERVAL)
SELECT NOISE BANDWIDTH TO SUIT SYSTEM
USUALLY TEN TIMES THE SYSTEM'S BANDWIDTH.
PRBS PERIOD TO BE APPROX. 1.5*SETTLING TIME.
T(PRBS)=1.5*T(SETTLING)
T(SETTLING)=T(DEAD TIME)+4*T(LONGEST
TIME CONST. OF INTEREST)
SELECT DT <0.5*T(SHORTEST TIME CONST.
OF INTEREST)

CORRELATION PARAMETERS:
N = NO OF DATA POINTS (SELF COUNTING)
M = NO OF CORRELATION LAGS (SET TO N/10)
DELAM = TIME INTERVAL BETWEEN LAGS
(SET EQUAL TO DT)

-TITL SIGAN ; GENERATE OPERATING SYSTEM -

-ENT WSA,GETC,PUTC,CTLF,TXTT,NEXTP,RESLT
-ENT TEMPC,TEMP1,C12,C15,NUM,NUMPL,J,M1,N
-ENT SUM,SUMP1,RXXC,DELAM,ONE,TWO,START
-ENT CSP1,C9P1,MSK,NLP1,N1,ICHN,SRATE,SCNTR
-ENT PLOT,NL,C9,C5,W,WW,D,DD,K,M,TWNTY
-ENT AMESC,AMESL,GTCT,GTCP,GTCC,PI
-ENT DEG,LNLOG,TEN

-EXTN USAR,TTIN,TTWR,RR,H,G,PLOT,GXY,CON,FRESP
-EXTN RAY1,RAY2,RAY3,RAY4,RAY5,RAY6,RAY7,RAY8
-EXTN DEL,DATAIN,INPUT,OUTPUT,AUTO1,AUTO2,CROSS
-EXTN MESC,MES1,MES2,MES3,MES4,MES5,MES6
-EXTN MEX,MES8,MEX9,MEX10,MEX11,MEX12,MEX13
-EXTN TTY,RSP,TTH,MPH,CTAF,A2DI,CRP,TXT
-EXTN NEXTP,RESLT,NPAGE,HEAD,FIN,T,FIN
-EXTN GTCT,GTCP,GTCC,INPOW,OUTPOW,XPWD
-EXTN CONV,INCP,ICN,FREQ,STRM
00003-177777 WSA: USAR ; POINTER TO USER AREA
00001-177777 GETC: TTIN ; POINTER TO GET A CHAR ROUTINE
00002-177777 PUTC: TTWR ; POINTER TO WRITE A CHAR ROUTINE
00000-177777 TTIN: TTIN ; POINTERS IN PAGE ZERO TO ROUTINES

00004-177777 * RR: RR
00005-177777 * H: H
00006-177777 * G: G
00007-177777 * GXY: GXY
00008-177777 * QUES: QUES
00009-177777 * GTCT: GTCT
00010-177777 * GTCP: GTCP
00011-177777 * GTCC: GTCC
00012-177777 * TTYR: TTYR
00013-177777 * HSPTR: HSPTR
00014-177777 * CARTF: CARTF
00015-177777 * A2DI: A2DI
00016-177777 * A2DC: A2DC
00017-177777 * PLOT: PLOT
00018-177777 * CON: CON
00019-177777 * ICON: ICON
00020-177777 * FRESP: FRESP
00021-177777 * DATAIN: DATAIN
00022-177777 * LST1: LST1
00023-177777 * LST2: LST2
00024-177777 * CRLF: CRLF
00025-177777 * TXTT: TXTT
00026-177777 * NEXTP: NEXTP
00027-177777 * RESLT: RESLT
00028-177777 AMESC: -1 ; DATA SOURCE
00029-177777 AMES1: -1 ; DATA LISTING
00030-177777 AMES2: -1 ; FREQUENCY RESPONSE
00031-177777 AMES3: -1 ; CONVOLUTION
00032-177777 AMES4: -1 ; AUTOCOR. OF INPUT
00033-177777 AMES5: -1 ; AUTOCOR. OF OUTPUT
00034-177777 AMES6: -1 ; CROSSCORRELATION
00035-177777 AMES7: -1 ; IMPULSE FUNCTION
00036-177777 AMES8: -1 ; POWER SPECTRAL ROUTINES
00037-177777 NL: -10 ; NL
00038-177777 NLP1: -74 ; NLP1
00039-177777 C9: -12 ; C9
00040-177777 C9P1: -12 ; C9P1
00041-177777 C5: -5 ; C5
00042-177777 C5P1: -5 ; C5P1
00043-177777 TEMPC: 3 ; TEMPC
00044-177777 TEMP1: 3 ; TEMP1
00045-177777 C12: 12 ; C12
00046-177777 C15: 15 ; C15
00047-177777 NUM: 3 ; NUM
00048-177777 NUMP1: 3 ; NUMP1
00049-177777 J: 3 ; J
00050-177777 M1: 3 ; M1
00051-177777 M: 3 ; M
00052-177777 N: 3 ; N
00053-177777 W: 7 ; W
00054-177777 WW: 10 ; WW
00055-177777 D: 2 ; D
00056-177777 DD: 3 ; DD
00057-177777 K: 3 ; K
0003 SIGAN

00072-000177 MSK: 177 ; MSK
00073-000000 SUM: C ; SUM
00074-000000 SUMPR: C
00075-000000 ICHN: C
00076-000000 SRATE: C
00077-000000 SCNTR: C
00103-000015-PFT: H5PTR
00101-000000 RXXG: C ; RXXC
00102-000000 CN100-000000 DELM: C ; DELM
00104-000000 AFIN: C ; ARITHMETIC CONSTANTS
00105-041371 DEG: 57.2957795 ; RADS. TO DEGS.
00106-0045673
00107-040462 PI: 3.14159265
00110-041462 LNLOG: 2.30258509 ; LN TO LOG
00112-041542 ONE: 1.C ; ONE
00114-000000 TWO: 2.C ; TWO
00116-000000 TEN: 10.C
00120-000000 TWNY: 20.C
00122-000000 

00000*000000 START: 100ST ; RESET START
00001*000377 HALT ; WAIT , PRESS CONTINUE
00002*000632- JSR 0.NEXTP
00003*000633- JSR 0.TXTT ; STARTUP MESSAGE
00004*177777 STRTM
00005*000403 JMP +3
00006*000301- JSR 0.TXTT ; RESTART HEADING
00007*177777 HEAD
00010*034333- LDA 3,WSA ; SET FORMAT FOR
00011*002366- LDA 0,WW ; FLOATING POINT
00012*041521 STA 0,121,3
00013*02070- LDA 0,DD
00014*041522 STA 0,122,3
00015*177777 FINT ; INITIALISE FLT. PT.
00016*036312- JSR 0.QUES ; SET THE RUN STREAM BY
00017*177777 MESS ; ASKING ALL RELEVENT
00020*034133- LDA 3,PFT ; QUESTIONS.
00021*177777 ADD 0,3
00022*04034- STA 0,AMES2 ; DATA SOURCE
00023*007400- JSR 0,3
00024*006331- JSR 0,TXTT ; ANS. FOLL. QUES.
00025*177777 MESS2
00026*006012- JSR 0,QUES
00027*177777 MESS1
00029*04035- STA 0,AMES1 ; DATA LISTING
00031*006012- JSR 0,QUES
00032*177777 MESS9
00033*04036- STA 0,AMES2 ; FREQUENCY RESPONSE
00034*180015 COM# 0,SNR
00035*000404 JMP +4
00036*04034- STA 0,AMES4 ; AUTOCOR. OF INPUT
00037*04034- STA 0,AMES6 ; CROSSCORRELATION
00039*04034- STA 0,AMES7 ; IMPULSE FUNCTION
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00134*040133- FSTA 3,DELA
00135*120000- FEXT
00136*040043- STA 3,GETC
00137*120612- ADCZL 1,1
00140*000403- LDA 3,AMES5 ; DATA SOURCE
00142*000443- JMP +3 ; NO
00143*026028- JSR @A2DC ; YES, JUMP TO SUB.
00144*000443- JMP +13
00145*024035- LDA 1AMES1
00146*024040- LDA 3AMES4
00147*030042- LDA 2AMES6
00149*150015- COM# 2,2,SNR
00152*024035- LDA 1AMES1
00153*124014- COM# 1,1,SNR
00154*000414- JMP NC ; NO
00155*006025- JSR @DATAIN; INPUT DATA CALL
00156*177777- RAY1
00157*026032- JSR @NEXTP
00158*026031- JSR @TXTT ; INPUT HEADING
00160*006031- JSR @TXTT ; INPUT HEADING
00161*177777- INPUT
00162*026026- JSR @LST1
00163*020015- RAY1
00164*020035- LDA 3AMES1
00165*130015- COM# 3,3,SNR ; LIST. AND OR PLOT. ?
00166*013043- JMP NC
00167*026027- JSR @LST2
00168*020016- RAY1
00171*126120- ADCZL 1,1 ; START TO EXECUTE THE
00172*020034- LDA 3AMES5 ; RUNSTREAM
00173*127015- ADD# 3,1,SNR ; DATA SOURCE ?
00174*013041- JMP +13
00175*024035- LDA 1AMES1
00176*020041- LDA 3AMES5
00177*030042- LDA 2AMES6
00180*150015- COM# 2,2,SNR
00183*024014- COM# 1,1,SNR
00184*000403- JMP +3
00185*024035- COM# 3,3,SNR ; AUTOCOR. OF OUTPUT ?
00187*030415- JMP N1
00188*026025- JSR @DATAIN; OUTPUT DATA CALL
00190*177777- RAY2
00191*026032- JSR @NEXTP
00192*026031- JSR @TXTT ; OUTPUT HEADING
00194*177777- OUTPUT
00195*026026- JSR @LST1
00196*020016- RAY2
00197*020035- LDA 3AMES1
00198*130015- COM# 3,3,SNR ; LIST. AND OR PLOT. ?
00199*000413- JMP N1
00201*026027- JSR @LST2
00202*020013- RAY2
00204*020035- LDA 3AMES1
00205*030043- JMP N1
00207*026025- JSR @DATAIN; OUTPUT DATA CALL
00208*177777- RAY2
00209*026032- JSR @NEXTP
00211*026031- JSR @TXTT ; OUTPUT HEADING
00212*026026- JSR @LST1
00213*020016- RAY2
00214*020035- LDA 3AMES1
00215*130015- COM# 3,3,SNR ; LIST. AND OR PLOT. ?
00216*020043- JMP N1
00217*026027- JSR @LST2
00218*020013- RAY2
00221*020035-N1: LDA 3,M ; SET PLOTTING COUNTER
00222*020032- STA 3,M1
00223*020043- LDA 3AMES4 ; AUTOCORRELATION OF
00224*130005- COM 3,3,SNR ; INPUT DATA CALL
00225*020046- JMP N2
00226*020032- JSR @NEXTP
00321'100004
COM C,CSZ; PLOTTING ?
00323'100451
JMP N5
00324'100021-
JSR @.PLOT
00325'100316-
RAY6
00326'100032-
JSR @.NEXTP
00327'100031-
JSR @.TXTT
00330'177777
OUTPW
00331'100006-
JSR @.G ; OUTPUT SPECTRAL
00332'177777
RAY7 ; DENSITY
00333'100262-
LDA C,AMES8
00334'100044-
NEG C,C
00336'100044-
COM C,CSZ; PLOTTING ?
00337'100435
JMP N5
00340'100021-
JSR @.PLOT
00341'100332-
RAY7
00342'100032-
JSR @.NEXTP
00343'100024-
JSR @.RR
00344'100273-
RAY1
00345'1000272-
RAY2
00346'100333-
RAY4
00347'100031-
JSR @.TXTT
00350'177777
XP0W
00351'100037-
JSR @.GXY ; CROSS-SPECTRAL
00352'100032-
RAY5 ; DENSITY
00353'1000346-
RAY4
00354'1000325-
RAY6
00355'1000341-
RAY7
00356'1000353-
RAY4
00357'177777
RAY8
00360'100044-
LDA C,AMES8
00361'100400
NEG C,C
00362'100034-
COM C,CSZ; PLOTTING ?
00363'100041
JMP N5
00364'100032-
JSR @.NEXTP
00365'100021-
JSR @.PLOT ; PLOT MAGNITUDE
00366'1000356-
RAY4
00367'100032-
JSR @.NEXTP
00370'100021-
JSR @.PLOT ; PLOT PHASE
00371'100357-
RAY8
00372'100063-
LDA C,M ; RESET M COUNTER
00373'100062-
STA C,M1
00374'100043-N5:
LDA C,AMES7 ; IMPULSE FUNCTION CALL
00375'100005-
COM C,SNR ; REQUIRES RXR AND RXY
00376'1000413
JMP N6
00377'100005-
JSR @.H
00400'100317-
RAY3
00402'1000352-
RAY5
00403'100034-
RAY6
00404'100043-
LDA C,AMES7
00404'100040
NEG C,C
00405'100024-
COM C,CSZ; PLOTTING ?
00406'100043-
JMP N6
00407'100021-
JSR @.PLOT
00410'100042-
RAY6
00411'100037-N6:
LDA C,AMES3 ; CONVOLUTION AND
00412'100005-
COM C,SNR ; INVERSE CONVOL. CALL
00413'100043
JMP N7
00414'100032-
JSR @.NEXTP
CONVOLUTION ROUTINE

INVERSE CON. ROUTINE

FREQUENCY RESPONSE

RESET FOR TELETYPewriter

START

END START
Input a character (teletype) → produce parity
Output a character (teletype) → perform pagination
Write a text string

CRLF:
- generate a carriage return and a line feed

NEXTP:
- advance paper to next page

RESLT:
- format the results
- print into columns and pages

QUES:
- routine to ask questions

GTCT:
GTCC:
- teletype non-echoing data entry
- cartrifile

GTCO:
- high speed paper tape reader

TTYR:
HSPTR:
CARTF:
A2DI:
- initialise teletype
- initialise high speed paper tape reader
- initialise cartrifile mag. tape unit
- initialise analogue to digital converter

Figure A5.3 SERV- Device drivers
Figure A5.4    A2DI- Analogue to Digital Converter Driver
TITL SERVICE

- ENT TTIN, TTWR, TXTT, CRLF, NEXTP, RESLT, QUES
  - ENT TTYR, CARTF, A2DI, GTCC, GTCP, GTCT, HSPTR

- EXTD • TXTT, • NEXTP, • RESLT, NL, NLP1, MSK
- EXTD GETC, PUTC, C5, C5P1, C12, C15, SCNTR
- EXTD AMESC, ICHN, SRAE, GTCT, GTCP, GTCC
- EXTN NPAGE, SPCE3, RAY3, FENT, START, MES1G, MES1L
- EXTN MES12

NREL

<table>
<thead>
<tr>
<th>TTIN:</th>
<th>STA 3*R13</th>
<th>; INPUT A CHARACTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>STA 3*R13</td>
<td>; INPUT A CHARACTER</td>
<td></td>
</tr>
<tr>
<td>LDA 3*MSK</td>
<td>; IDLE T•TYPE READER</td>
<td></td>
</tr>
<tr>
<td>SKPDN</td>
<td>TTI</td>
<td>; GET CHARAC. AND</td>
</tr>
<tr>
<td>JMP -1</td>
<td>; CLEAR READER</td>
<td></td>
</tr>
<tr>
<td>DIA3</td>
<td>TTI</td>
<td></td>
</tr>
<tr>
<td>AND 3*SKP</td>
<td>; CLEAR READER</td>
<td></td>
</tr>
<tr>
<td>STA 3*R13</td>
<td>; OUTPUT A CHARACTER</td>
<td></td>
</tr>
<tr>
<td>STA 3*SAVC</td>
<td>; SAVE ACC</td>
<td></td>
</tr>
<tr>
<td>SUB 3.3</td>
<td>; ZERO AC3</td>
<td></td>
</tr>
<tr>
<td>MOVHR 3*G, SZC</td>
<td>; MOVE IST BIT INTO CARRY</td>
<td></td>
</tr>
<tr>
<td>INC 3.3</td>
<td>; IDLE T•TYPE READER</td>
<td></td>
</tr>
<tr>
<td>MOVZ 3*G, SZR</td>
<td>; ALL BITA CHECKED</td>
<td></td>
</tr>
<tr>
<td>JMP -3</td>
<td>; NO, GO BACK</td>
<td></td>
</tr>
<tr>
<td>LDA 3*SAVC</td>
<td>; YES, LOAD THE NO.</td>
<td></td>
</tr>
<tr>
<td>MOV3 3G</td>
<td>; SWAP IT TO UPPER HALF</td>
<td></td>
</tr>
<tr>
<td>MOVL 3C</td>
<td>; MOVE PARITY INTO CARRY</td>
<td></td>
</tr>
<tr>
<td>MOVR 3C</td>
<td>; ADD PARITY TO WORD</td>
<td></td>
</tr>
<tr>
<td>MOV3 3C</td>
<td>; SWAP TO LOWER HALF</td>
<td></td>
</tr>
<tr>
<td>SKPBZ</td>
<td>TTO</td>
<td>; IDLE T•TYPE PRINTER</td>
</tr>
<tr>
<td>JMP -1</td>
<td>; GET CHARAC. AND</td>
<td></td>
</tr>
<tr>
<td>DOAS 3*TTO</td>
<td>; START TYPING IT</td>
<td></td>
</tr>
<tr>
<td>LDA 3*C12</td>
<td>; START TYPING IT</td>
<td></td>
</tr>
<tr>
<td>SUB# 3,3*SNR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ISZ</td>
<td>NL</td>
<td>; KEEP TRACK OF</td>
</tr>
<tr>
<td>JMP 3*R13</td>
<td>; NUMBER OF LINES</td>
<td></td>
</tr>
<tr>
<td>LDA 3*NLP1</td>
<td>; PRINTED</td>
<td></td>
</tr>
<tr>
<td>STA 3*NL</td>
<td>; PRINTED</td>
<td></td>
</tr>
<tr>
<td>LDA 3*R13</td>
<td>; SAVE REENTRANT</td>
<td></td>
</tr>
<tr>
<td>STA 3*R14</td>
<td>; ADDRESS</td>
<td></td>
</tr>
<tr>
<td>LDA 3*R15</td>
<td>; ADDRESS</td>
<td></td>
</tr>
<tr>
<td>STA 3*R17</td>
<td>; ADDRESS</td>
<td></td>
</tr>
<tr>
<td>STA 2*SAV2</td>
<td>; SAVE AC 2</td>
<td></td>
</tr>
<tr>
<td>JSR TXTT</td>
<td>; DELINATE TOP</td>
<td></td>
</tr>
<tr>
<td>NPAGE</td>
<td>; OF PAGE</td>
<td></td>
</tr>
<tr>
<td>LDA 2*SAV2</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>LDA 3*R17</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>LDA 3*R15</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>STA 3*R15</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>JMP @R14</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>; RESTORE AC 2</td>
<td></td>
</tr>
</tbody>
</table>
...
SUB 1, P ; DEPENDS ON
LDA 3, R1 ; RESPONSE
ADC 1, 1
ADD 1, P
JMP 1, 3

LDA 3, R16 ; TELETEYPE
JMP 1, 3

AND 3, C ; MASK LEFT BYTE
0005 SERV I
00334*120000 FDEC C ; SAMPLES
00335*046410 FSTA C,TEM
00336*074407 FFIX TEM
00337*100000 FEXT
00342*022426 LDA C,TEM+1
00341*101400 INC C,C
00342*046815$ STA C,SCNTR ; STORE IN SAMPLE
00343*022401 JMP @R7 ; COUNTER
00344*000000 R7: 0
00345*000000 TEM: 0
00346*000000 0
00347*000020 C20: 20 ; 16 CHANNELS
* END
Enter Subroutine

Save return address
Get result format

Set plotting counter

Load plot pointer

Load first No.

Find max. and min.

Increment pointer
Decrement counter

Set range to 6

Multiply by 10

Cal. scale factor

$\text{Cal.} |\text{Int1}| = |\text{M1} / .F.|$

$\text{Cal.} |\text{INT2}| = |\text{MAX/S.F.+.95}|$

Fig. A5.5 PLOT- Autoranging/Autoscaling plotter
*TITL PLOT

*ENT PLOT

*EXTN YAXIS, SPCE3, FENT

*EXTD W, D M, ONE, TWO, CRLF, TXTIT, C9, C9P1

*EXTD WSA, NL, WW, DD, PUTC, TEN, M1

```
NREL

054533 PLOT: STA 3, R7 ; plotting routine
03412$ LDA 3, WSA ; change format
03423$ LDA C, W ; specification
04452$ STA 3, 121, 3
04462$ LDA C, D
04472$ STA 3, 122, 3
04482$ LDA C, M1 ; set plotting counter
04492$ STA C, C2
044A2$ LDA 2, OR7 ; pointer to plot
044B2$ FENT ; source
044C2$ FLDA C, C2
044D2$ FMOV C, 2
044E2$ FLDA C, C2
044F2$ FMOV C, 2
04502$ FLDA C, C2
04512$ FMOV C, 2
04522$ FSTA 1, MAX ; save max.
04532$ FSTA 2, MIN ; save min.
04542$ FSUB 2, 1, FSNR ; max. > min. ?
04552$ FMOV 2, 1 ; no
04562$ FSUB 2, 2, FSNR ; is range = 3 ?
04572$ FLDA 1, SIX ; yes, set it = to 6
04582$ FLDA C, ONE ; calculate scale factor
04592$ FLDA 2, SIX
045A2$ FSUB# 1, 2, FSLT ; range < 6 ?
045B2$ FJMP PL3 ; yes
045C2$ FLDA 3, TWO ; no
045D2$ FJSR PL4 ; S.F.*2
045E2$ FLDA 3, TWO5 ; S.F.*2.5
045F2$ FJSR PL4
04602$ FLDA 3, TWO ; S.F.*2
04612$ FJSR PL4
04622$ FJMP PL2 ; return
04632$ FLDA 3, TEN ; load ten
04642$ FDIV 3, 3 ; S.F./10
04652$ FMPS 3, 1 ; range*10
04662$ FJMP PL2-2 ; return
04672$ FMPS 3, 3 ; S.F.*AC3
04682$ FDIV 3, 1 ; range/AC3
04692$ FSUB# 1, 2, FSGE ; range <= 6 ?
046A2$ FJMP C, 3 ; no, return
046B2$ FRND C, C ; yes, round S.F.
```
CC152*36326$ JR 0.CRLF
CC153*12612C ADCZL 1,1 ; GENERATE -2
CC154*12712C ADDZL 1,1 ; -2 4
CC155*35911* FENT
CC156*24733 FLDA 1,INT1
CC157*2273C FLDA 3, SF
CC158*1541C FMPY 5,1
CC161*33574C PL6: FLDA 2,PCC5 ; ROUND OFF THE
CC162*127405 FRND 1,1,FSEG ; PRINTED SCALE
CC163*15040C FNEG 2,2
CC164*14730C FADD 2,1
CC165*144801 FFDCF 1 ; PRINT Y AXIS
CC166*14640C FSUB 2,1 ; SCALE
CC167*13000C FEXT
CC170*336027$ JR 0.CRLF
CC171*17777 SPCE3
CC172*125405 INC 1,1,SNR
CC173*338406 JMP *+6
CC174*338155* FENT
CC175*22712C FLDA 3, SF
CC176*10700C FADD 2,1
CC177*33762C FJMP PL6
CC201*36006$: JSR 0.CRLF
CC202*36007$: JSR 0.CRLF ; PRINT Y AXIS
CC203*177777 YAXIS
CC204*332727 LDA 2, @ R7 ; POINTER TO PLOT
CC205*32302$: LDA 3, M1 ; SOURCE
CC206*34726C STA 3, C2
CC207*12600C ADC 1,1
CC208*344011$ STA 1, C9P1
CC211*30174*PL7: FENT
CC212*32100C FLDA 3,C,2 ; LOAD FIRST POINT
CC213*10400C FIC2
CC214*324673 FLDA 1,SF ; SCALE IT
CC215*33317$: FLDA 2,TEN
CC216*11510C FMPY 5,2
CC217*13200C FDIV 1,2
CC220*324703 FLDA 1,P5 ; ADD C.5 TO IT
CC221*13300C FADD 1,2
CC222*550675 FSTA 2, DDATA
CC223*74674 FFIX DDATA ; TRUNCATE IT
CC224*660673 FFLO DDATA
CC225*22670C FLDA 3, ISTRT ; CALCULATE SPACES TO
CC226*33671C FLDA 2, DDATA ; PLOTTING POINT
CC227*12400C FSUB 5,2
CC230*550667 FSTA 2, DDATA
CC231*74666 FFIX DDATA
CC232*12580C FEXT
CC233*14520C MOV 2,1
CC234*336027$ JSR 0.CRLF ; LEAVE 3 SPACES
CC235*33171* SPCE3
CC236*13100C MOV 1,2
CC237*22700C LDA 5,C53
CC240*110011$ ISZ C9P1
CC241*22767 LDA 5,C55 ; TYPE EITHER A
CC242*336016$ JSR 0.PUTC ; OR A -
CC243*224655 LDA 1, DDATA+1
CC244*125112 MOV 1,1,SZC
00245*12646C  SUBC  1,1
00246*12400C  COM  1,1
00247*022666  LDA  #C4C
00250*125405  INC  #1,SNR
00251*000403  JMP  +3
00252*006016$  JSR  @PUTC  ; SHIFT CARRIAGE TO
00253*005774  JMP  -4  ; PLOTTING POINT
00254*020662  LDA  #C52
00255*006016$  JSR  @PUTC  ; TYPE AN ASTERISK
00256*006306$  JSR  @CRLF  ; CARR. RETURN, LINE FEED
00257*024311$  LDA  #C9P1  ; KEEP TRACK OF
00258*022313$  LDA  #C9  ; X AXIS SPACING
00261*125005  MOV  #1,SNR
00262*0400115  STA  #C9P1
00263*020013$  LDA  #NL
00264*101400  INC  #3
00265*101415  INC#  #3,SNR  ; SUPPRESS NEW PAGE
00266*014013$  DSZ  NL  ; ROUTINE
00267*014645  DSZ  #C2
00270*00721  JMP  PL7
00271*034012$  LDA  #WSA  ; RESET FORMAT
00272*052214$  LDA  #WW  ; SPECIFICATION
00273*041521  STA  #121,3
00274*020315$  LDA  #DD
00275*041522  STA  #122,3
00276*006007$  JSR  @TXTT  ; PRINT Y AXIS
00277*022203*  YAXIS
00300*034633  LDA  #3,R7
00301*031401  JMP  #1,3  ; RETURN
Figure A5.6  ATODC- Analogue to Digital Converter
• TITL  ATOPCONVERTER
• ENT A2DC,INTP
• EXTD ICHN,SRATE,TXTT,NUM,NUMP1
• EXTD SCNTR,TEN
• EXTN FENT,RAY1,RAY2,RAY3

00000-000000
00000-000000 WCNT:  C  ; PAGE ZERO POINTERS
00001-000000 SCNT:  C  ; NO. OF WORDS IN BUFFER
00002-000000 BUFP:  C  ; ADDRESS OF FIRST WORD IN BUFFER
00003-000000 C  ; ADDRESS OF LAST WORD IN BUFFER
00005-177777 RAY3:  C  ; MIN. ADDRESS
00006-000000 C  ; MAX. ADDRESS
00007-000044 42C.:  C  ; MAX. NO. OF WORDS
00010-000000 CHNF:  C  ;

00010-000522 A2DC:  STA 3# SAV3
00010-000252 SUBZL 2# C  ; SET WAIT COUNTER
00010-000000 STA 2# WCNT ; TO 1 (SYNC.)
00010-000114 MOVOL 2# C  ; SET CLOCK TO
00010-00014 C  ; RTC 1 kHz
00010-00126 STA 1# SCNTR ; SET CHANNEL COUNTER
00010-000144 STA 1# SCNT  ;
00010-000265 LDA 3# PRAY1  ; POINTER TO X
00010-000471 STA 2# INAD  ;
00010-000264 LDA 3# PRAY2  ; POINTER TO Y
00010-000473 STA 3# OUTAD  ;
00010-000335 LDA 3# BUFP+3  ; RESET DATA
00010-000337 LDA 2# BUFP+5  ; BUFFER
00010-00133 C  ;
00010-00026 ADD 2#  ;
00010-000266 STA 2# BUFP+4  ;
00010-001436 DSZ BUFP+4  ;
00010-001212 MOVRE 0# C  ; EVEN BYTE?
00010-00140C INC 2# C  ;
00010-001433 STA 3# BUFP+1  ;
00010-001434 STA 3# BUFP+2  ;
00010-001243 SUB 3# C  ;
00010-001245 STA 3# BUFP  ;
00010-001245 LDA 3# TMSK  ;
00010-001277 MSKO 3# ; MASK OUT ALL INTERRUPTS
00010-001277 JSR 0# TXTT  ; EXCEPT A TO D
00010-001277 MESS 3# ; AND CLOCK
00010-001277 NIOTC TTI  ; CLEAR T-TYPE READER
00010-001277 SKPDN TTI  ; IDLE T-TYPE READER
00010-001277 JMP -1  ;
00010-001277 NIOTC TTI  ; CLEAR IT
00010-001277 INTEN 3# ; ENABLE INTERRUPTS
00010-001277 NIOS RTC  ; START CLOCK
00010-001277 LDA 1# BUFP  ; LOAD NO. OF WORDS
MOV 1\*1, SNR ; LAST ONE ?
JMP EMPTY ; YES, BUFFER EMPTY
DSZ BUFP ; NO, DEC. NO. OF
JMP +1 ; WORDS IN BUFFER
LDA 2, BUFP+1
LDA 2\*2, 2 ; LOAD WORD
INC 2\*3 ; INC. ADDRESS
LDA 1, BUFP+4
ADC 1\*3, SNR ; AT MAX. ADDRESS ?
LDA 3, BUFP+3
STA 3, BUFP+1 ; NEXT WORD ADDRESS
LDA 3, INAD
MOVEZ 2\*2, SZC ; EVEN ADDRESS ?
LDA 3, OUTAD ; NO, OUTPUT
MOV 2\*2, SNC ; YES, INPUT NEGATIVE ?
SUBC 1, 1; SKP ; NO, SET UP MOST
ADC 1, 1 ; SIGNIFICANT 16 BITS
STA 1, OS ; STORE DOUBLE
STA 3, OS ; PRECISION NUMBER
INC 3, OS ; NEXT ADDRESS
LDA 3, 3
MOV 3\*2, SZC ; EVEN ADDRESS ?
JMP +3 ; NO, OUTPUT
STA 3, INAD ; YES, INPUT
JMP +2
STA 3, OUTAD
JMP GET
POINTER TO INPUT ARRAYS
LDA 0, SCNT ; SAMPLE COUNTER
MOV 0, SZR ; ALL SAMPLES TAKEN
JMP GET ; NO
STA 0, NUM ; YES
LDA 2, PRAY1 ; POINTER TO X
LDA 0, SCNTR
NEG 0, 3
COM 0, 3
STA 0, N ; INPUT ARRAY
STA 0, INAD ; COUNTER
STA 0, NUMP1
FENT
FDIV 1, 2 ; SET M TO N/10
FSTA 2, TEM
FFIX TEM ; TRUNCATE M
FLEC PRAY2 ; POINTER TO Y
FDLA 0, C2C48 ; BINARY TO VOLTS FACTOR
FDLA 0, 2 ; CONVERSION ROUTINE
FDLA 2, 2
FDIV 0, 2 ; STORE IN INPUT ARRAY
```
# TIMING ROUTINE
LDA SRATE ; RESUME WAIT COUNTER
JMP DISM+3 ; NOT READY TO START

# SAMPLING FINISHED?
DSZ SCNT ; SAMPLING FINISHED?
JMP DISM+2 ; NO

# CLEAR A TO D CONVERTER
NIOC ADCV ; YES, CLEAR A TO D
NIOC RTC ; CLEAR CLOCK

# CLEAR CLOCK
LDA SAVC
JMP AC ; RETURN

MESS: *TXT "<15><12><12> TO START, STRIKE

```

* END
Figure A5.7  DATAIN- GENERAL DATA ENTRY ROUTINE
- TITL DATAIN
- ENT DATAIN
- EXT D CRLF, TXTT, N, NUM, NUMP, TEMPC
- EXTD TEMP1, C15, WSA, AMES1, TEN
- EXTN EXT, MEAN

- NREL

DATAIN: STA 3#R5 ; DATA READ IN,
SUB 3#3 ; AND STORED
STA 3#NUM ;
LDA 2#R5 ; STORAGE POINTER
STA 2#TEMP1 ; LOADED HERE
SUB 1#1
STA 1#N ; START COUNTING
STA 1#NUMP1 ; NO. OF DATA POINTS
LDA 1#C115
LDA 2#WSA
FENT
FDAFC 1 ; GET THE CHARAC.
FEXT
LDA 3#2#2
SUP# 3#1#SNR ; IS IT AN M ?
JMP D4 ; YES, LAST DATA
LDA 3#1#2 ; NO, IS IT A
MOV 3#3#SRZ ; BREAK CHARAC. ?
JMP +#3 ; YES, STORE NO-
FENT
FDMP D2 ; NO, GET NEXT CHARAC.
FENT
LDA 3#24#C
FMP 3#3#US
INC 1#1
FEXT
LDA 1#N ; INCREMENT DATA
INC 1#1 ; COUNTER
JMP D1
FENT
LDA 1#N ; CALCULATE NO. OF
FFLO NUM ; CORRELATIONS REQUIRED
FLDA 2#NUM ;
FLDA 1#TEM
FDIV 1#2 ; N/10
FSTA 2#TEM
FFIX TEM
FEXT
LDA 1#TEM+1
STA 1#M ; STORE IN M
LDA 3#R5
JMP 1#3 ; RETURN
R5:
C
TEM:
C
C
ASCII M
Enter Subroutine

Save return address

Load No. of data points

Load pointer to array

Load No., add to sum

No

additions

Yes

Calc. mean

Print heading & mean

Normalise and store data

Exit

Figure A5.8  LIST(1) - NORMALISE DATA
Figure A5.9  LIST(2)- DATA LISTING AND PUNCHING
**TITL LIST**

**ENT LIST1, LIST2**

**EXTD - CRLF, TXTT, N, NUM, TEMPC, TEMPL, AMES1**

**EXTD M, M1, RESLT, PLOT, C5, C5P1**

**EXTN FENT, MEAN**

Calculate mean and normalise pointer to array

Load number and add to sum

Inc. pointer

Dec. counter

Calculate mean

Print heading

Print mean

Pointer to array

Reset result format

Data listing

List it
LOC02 LIST
LOC057'100000
LOC060'050423
LOC061'066012$ STA 2,SAV2 ; SAVE AC 2
LOC062'030421 JSR 0,RESL
LOC063'04006$ LDA 2,SAV2 ; RESTORE AC2
LOC064'000767 DSZ TEMP1
LOC065'034415 JMP D3
LOC066'124513 D4: LDA 3,R5
LOC067'071401 NEGL# 1,1,SNC ; DATA PLOTTING ?
LOC068'02010$ JMP 1,3 ; NO, EXIT
LOC069'04011$ LDA 3,M ; YES, SET COUNTER
LOC070'02641C STA 3,M1 ; TO PLOT M OR N VALUES
LOC071'044402 LDA 1,0R5
LOC072'044402 STA 1,0+2
LOC073'06013$ JSR 0,PLOT ; PLOT THE ARRAY
LOC074'000000 C
LOC075'034404 LDA 3,R5
LOC076'034404 LDA 3,M
LOC077'02010$ STA 3,M1 ; RESET M COUNTER
LOC078'021401 JMP 1,3 ; RETURN
LOC079'02010$ R5: C
LOC080'033000 SAV2: C
LOC081'033000 C
LOC082'033000 C
LOC083'033000 C
LOC084'033000 C
- END
Figure A5.10  CORREL- GENERAL CORRELATING ROUTINE
*TITL CORRL

*ENT RR

*EXTD RESLT,N,M,J,C5,C5P1,TEMPO,TEMPP

*EXTN FENT

*WREL

CODE 054461 RR:
STA  3,R9  ; CORRELATION ROUTINE
LDA  1,C5  ; RESET RESULT FORMAT
STA  1,C5P1
LDA  3,2,3  ; POINTER FOR STORAGE
STA  3,TEMPP
SUB  2,2,SKP  ; J=0
INC  2,2  ; SET COUNTERS
STA  2,J
SUB  3,C
STA  3,NJF
LDA  3,N
SUR  2,3  ; N-J
STA  3,NJ  ; STORE IN FLOAT. LIMIT
STA  3,NJF+1
MOVZL 2,C  ; 2*J
LDA  1,R9
ADD  3,1  ; SET POINTER
STA  1,TEMPP  ; TO X(I+J)
LDA  2,M
LDA  3,R9
SUB# 0,2,SNR  ; J=M ?
JMP  3,3  ; YES, EXIT
LDA  2,1,3  ; POINTER TO DELAYED
FENT  ; SIGNAL LOADED HERE
FSUB  3,C
FLD3  TEMPP  ; POINTER TO RR(J)
FLD3  2,C,2  ; LOAD X(I)
FLD3  3,C,3  ; LOAD X(I+J)
FMPY  3,2  ; X(I)*X(I+J)
FADD  2,C  ; ADD TO SUM
FIC2  ; INC. POINTERS
FIC3
FIC3
FD3Z  NJ  ; DEC. COUNTER
FJMP  RR2
FFLO  NJF
FLDA  1,NJF
FDIV  1,C  ; CAL. RR(J)
FLD3  TEMPP  ; LOAD STORAGE POINTER
FSTA  3,C,3  ; STORE IN RR(J)
FFDCF  3  ; PRINT RR(J)
FIC3  ; INC. POINTER
FST3  TEMPP  ; STORE IT
FE3T
JSR  @RESLT  ; FORMAT THE RESULT
LDA  2,J  ; LOAD COUNTER
JMP  RR1
VJ:  0
VJF:  0
Figure A5.11  IMPUL- SYSTEM IMPULSE FUNCTION
; IMPUL
; 00057'020005$ FLDA C RXXC
; 00060'064013$ FLDA TEMPC
; 00061'025400 FLDA 1, G, 3 ; LOAD RXY(J)
; 00062'030016$ FLDA 2, DELAM ; RXY(J)/DT
; 00063'144200 FDIV 2, 1
; 00064'110000 FIC3 ; INC, AND STORE
; 00065'070013$ FST3 TEMPC ; POINTER
; 00066'030011$ FLDA 2, SUM
; 00067'146400 FDIV 0, 1 ; CALCULATE H(J)
; 00070'104200 FSTAI 1, C, 2 ; STORE IT
; 00071'345000 FSTCF 1 ; PRINT IT
; 00072'144201 FEXT
; 00073'100000 JSR @RESLT
; 00074'06015$ LDA 3, M ; LOAD M COUNTER
; 00075'034010$ ISZ J ; INC. J
; 00076'010006$ LDA 1, J
; 00077'024006$ SUB# 3, 1, SZR ; J=M ? FINISHED ?
; 00100'166414 JMP H1 ; NO, CONTINUE
; 00101'000780 LDA 3, RIC ; YES, EXIT
; 00102'034402 JMP 3, 3
; 00103'001403 RIC ; C
; 00104'000000 END
Figure A5.12 CONVOL–CONVOLUTION
*TITL CONVOLUTION

*ENT CON

*EXTN FNT

*EXTL CS,CSPI,J,K,M,SUM
*EXTL TEMPO,TEMPO1,DELAM,RESLT

WHEL

ST3 3,R12 ; CONVOLUTION ROUTINE
LDA 1,C5
STA 1,CSPI
LDA 0,0,3 ; POINTER TO H
LDA 1,1,3 ; POINTER TO X
LDA 2,2,3 ; POINTER TO Y
STA 0,TP0 ; (TEMP. POINTERS TO )
STA 1,TP1
STA 2,TP2
MOV 3,2

FELI3 TP1 ; POINTER TO X
FLDA 0,0,2 ; LOAD H(C)
FLDA 1,0,3 ; LOAD X(C)
FSTA 0,H3
FSTA 1,X0
FLDA 2,DELAM
FMPY 3,1 ; H(C)*X(C)
FMPY 1,2
FHLV 2,2
FIC3 ; INC. POINTERS
FIC3
FLDA 0,0,2 ; LOAD H(1)
FLDA 1,X0
FMPY 1,3 ; H(1)*X(C)
FLDA 1,0,3 ; LOAD X(1)
FLDA 3,H3
FMPY 1,3 ; H(C)*X(1)
FALD 3,3
FHLV 3,0
FLDA 1,DELAM
FMPY 1,0
FIC3 ; INC. ANL STORE
FST3 TP1 ; POINTER
FLL3 TP2 ; LOAD PI. TO Y
FSTA 2,0,3 ; STORE IN Y(C)
FIC3
FSTA 0,0,3 ; STORE IN Y(1)
FDECF 2
FIC3
FST3 TP2
FXT
STA 2,TPC
JSH @RESLT
FENT
FICF 3
FXT
JSR @RESULT

LOAD EXIT ADDRESS

J=M?

YES, EXIT

LOAD H(J)

LOAD X(J)

SET POINTER TO X(J)

H(J)*X(J)

H(C)*X(J)

H(J)*X(K)

LOAD H(J-K)

LOAD X(K)

LOAD X(K)

X(K)*H(J-K)

INC K

PRINT IT
0003    CONVO
00152'110000  FIC3
00153'070415  FST3  TP2
00154'100000  FEXT
00155'0360125  JSR  @.RESLT
00156'0100035  ISZ  J  ; INC. J
00157'0200035  LFA  C,J
00160'000702  JMP  C1
00161'000000 R12:  C
00162'000000 XO:  C
00163'000000  C
00164'000000 HC:  C
00165'000000  C
00166'000000 TP0:  C  ; H
00167'000000 TP1:  C  ; X
00170'000000 TP2:  C  ; Y
                      * END.
Figure A5.13  INVERCON- INVERSE CONVOLUTION


```assembly
VNFL

00000'054003 ICON: STA 3,R12 ; INVERSE CONVOLUTION
00001'024001$ LDA 1,C5 ; ROUTINE
00002'044002$ STA 1,C5P1
00003'177777 FEXT
00004'066477 FLD3 2,C5,3 ; FUNCTION H
00005'040020 STA 2,HC
00006'030004$ FST3 TEMPI
00007'100000 FEXT
00011'126400 SUP 1,1 ; SET J=0
00012'040006$ STA 1,J
00013'034473 STA 3,R12
00014'025401 L1A 1,1,3 ; POINTER TO OUTPUT
00015'040003$ STA 1,TEMPC ; SIGNAL Y
00016'126400 C01: SUP 0,SUM ; SET SUM TO C
00017'030004$ STA 3,TEMPI
00018'040011$ STA 2,TEMP1
00019'034461 L1A 3,R12
00020'031403 L1A 2,2,3 ; POINTER TO INPUT
00021'034004$ L1A 3,TEMPI ; SIGNAL X
00022'025401 L1A 1,J ; SET POINTER TO
00023'127000 ALL 1,1 ; H(J-K) WITH K=0
00024'137000 ALL 1,3
00025'054004$ STA 3,TEMPI
00026'024006$ STA 3,TEMP1
00027'034415 SUP# 3,1,SNR ; K=0 ?
00028'000003$ JMP C03 ; YES
00029'030003$ FEXT ; K<0=1
00030'066434$ FLL3 TEMPI
00031'025401 FLDA 2,C5,3 ; LOAD H(J-K)
00032'025401 FLDA 1,C5,2 ; LOAD X(K)
00033'025401 FLC2 TEMPI ; INC* X POINTER
00034'025401 FLSZ TEMPI ; DEC* H POINTER
00035'030003$ FDSZ TEMPI
00036'030003$ FMPY 2,1
00037'020011$ FLDA 0,SUM
00038'133000 FALD 1,0
00039'040011$ FST3 0,SUM
00040'100000 FEX1
00041'010007$ LST 2,3 ; INC* K
00042'020007$ LDA 2,K
00043'000003$ JMP C02
00044'0000034$ FFWT TEMPC
00045'000003$ FLD3 TEMPO
00046'025400 FLDA 1,2,3 ; LOAD Y(J)
```
```assembly
0002  INVER
00056'110000 FIC3
00057'070000 FST3 TEMPC
00060'034000 FLDA 3,DELAM
00061'164000 FDIV 3,1 ; Y(J)/DT
00062'330011 FLDA 2,SUM
00063'146400 FSUB 2,1
00064'320415 FLDA 3,HC
00065'104200 FDIV 3,1 ; CALCULATE X(J)
00066'045000 FSTA 1,3,2 ; STORE IT
00067'144001 FFDCF 1 ; PRINT IT
00070'000000 FEXT
00071'066135 JSR @.RESLT
00072'034010 LDA 3,M
00073'010061 ISZ J ; INC. J
00074'024066 LDA 1,J
00075'166414 SUB# 3,1,SZR ; J=M ? FINISHED ?
00076'003720 JMP C01 ; NO, CONTINUE
00077'034404 LDA 3,R12 ; YES, EXIT
00100'021403 JMP 3,3
00101'000000 HC: C
00102'000000 C
00103'000000 R12: C
00104'000000 END
```
Figure A5.14  SPECT- Power Spectral Density
**TITL SPECTRAL**

**ENT G**

**EXTN FENT, SPCE3**

- **EXTD C5, C5P1, TEMPI, M, K, DELAM, RXXC**
- **EXTD ONE, TWO, RESLT, SUM**
- **EXTD CRLF, TXTT, PI**

- **NREL**

```assembly
C001: C54536 G: STA 3, R11 ; POWER SPECTRAL DENSITY
C000: C04831$ LDA 1, C5 ; ROUTINE
C002: C04402$ STA 1, C5P1
C003: C021403 LDA 3, C3, 3 ; POINTER FOR STORAGE
C004: C04333$ STA 3, TEMPI
C005: C06614$ JSR @ CRLF
C006: C06615$ JSR @ TXTT
C007: 177777 SPCE3
C013: 122400 SUB 3, C ; SET COUNTERS
C011: 043533 STA 3, MF ; M FLOATING
C012: 044044 LDA 1, M
C013: 045532 STA 1, MF + 1
C014: 031401 LDA 2, 1, 3 ; POINTER TO AUTOCOR-
C015: 177777 FENT ; RELATION ARRAY
C016: 095536 FFLO MF
C017: 08336$ FLDA 3, DELAM
C022: 040511 FLDA 1, TWO
C021: 122100 FMPY 1, C ; 2 * DT
C022: 040515 FSTA 3, DT2
C023: 045521 FLDA 1, MF
C024: 134103 FMPY 3, 1 ; 2 * DT * M
C025: 022013$ FLDA 3, ONE
C026: 122000 FDIV 1, C ; FREQ. STEP HZ.
C027: 140331 FFDCF 3 ; PRINT FIRST STEP
C030: 021000 FLDA 3, C, 2 ; LOAD FIRST AUTO-
C031: 104000 FIC2 ; CORRELATION VALUE
C032: 040607$ FSTA 3, RXXC
C033: 103000 FEXT
C034: 026014$ JSR @ CRLF
C035: 026014$ JSR @ CRLF
C036: 026014$ JSR @ CRLF
C037: 122400 SUB 3, C
C040: 101400 G1: INC 3, C ; SET TO 1ST. FREQ.
C041: 040005$ STA 3, K
C042: 040535 STA 3, KF + 1
C043: 024044 LDA 1, M
C044: 034472 LDA 3, R11 ; LOAD EXIT ADDRESS
C045: 122015 ADC# 1, C, SNR ; K > M ?
C046: 021402 JMP 2, 3 ; YES, EXIT
C047: 122400 SUR 3, C
C053: 040466 STA 3, R ; STORE IN FLOAT. K
C051: 040471 STA 3, RF
C055: 131400 INC 3, C
C053: 040466 STA 3, R ; SET R = 1
C054: 033315' FENT
C055: 020037$ FLDA 3, RXXC
```
SPECT

FSTA C SUM ; SET SUM = RXX(C)

FFLO C RF ; FLOAT R

FLDA C RF ; LOAD R

FLDA 1 RF ; LOAD M

FLDA 2 PI ; LOAD PI

FADD 1 ; PI*R

FDIV 1 ; PI*R/M

FCOS C ; COS(PI*R/M)

FADD 3 ONE ; (COS(PI*R/M)+1)

FLDA 1 ; PI*R*K/M

FCOS C C ; COS(PI*R*K/M)

FADD 3 ; (COS(COS+1)

FLDA 1 ; LOAD NEXT AUTO-

FIC2 ; CORRELATION VALUE

FLDA 2 , SUM ; ADD TO SUM

FSTA 2 SUM

FEXT

SUB 3 C

STA C RF

ISZ R

LDA C R

JMP G2

FENT

FLDA C DT2 ; DIGIT INTERVAL

FMPY C 2 ; CAL. G(K)

FLD3 TEMPL

FSTA 2 C 3 ; STORE IN ARRAY

FIC3 ; INC. AND STORE

FST3 TEMPL ; POINTER

FFDCF 2 ; PRINT G(K)

FEXT

JSR 0 RESLT ; FORMAT IT

LDA 2 R11 ; RESET POINTER TO

LDA 2 1 2 ; SECOND AUTOCOR-

INC 2 2 ; RELATION VALUE

INC 2 2 ; INC. FRE0. COUNTER

LDA 3 K ; AND STORE IN K

JMP G1

R11: C

DT2: C

RF: C

MF: C

KF: C

* END
Enter Subroutine

- Save return address
- Set result format

- Set gain and phase pointers

Calc. freq. step

Set up temp. pointers

Set counters, load M

K=M?

No

Calc. A(K), B(K)

Increment K

Yes

Set K = 1
Calc. A(0), A(M)

K=M?

No

Calc. sum 1, sum 2

Yes

R=M?

No

Calc. and store gain and phase
Load K

Calc. and store C(K), Q(K)

Increment R load M

Print and format gain

Print and format phase

Exit

Figure A5.15  XSPECT- CROSS-SPECTRAL DENSITY
TITL XSPEC

ENT GXY,P180

EXTN FENT,SPCE3

EXTD C5,C5P1,TEMPC,TEMPI,M,K

EXTD DELAM,ONE,CRLF,TXTT,RESLT

EXTD TWO,TWNTY,PI,DEG,LNLOG

NREL

GXY: STA 3,R20 ; CROSS-SPECTRAL
LDA 1,C5 ; DENSITY FUNCTION
CSP1 LDA 1,4,3 ; STORAGE PT. FOR GXY
CSP1 LDA 2,5,3 ; STORAGE PT. FOR QXY
TEMPC STA 1,TEMPC ; MAGNITUDE
TEMPI STA 2,TEMPI ; PHASE
M LDA 1,M ; SET HIGH M=C
MF LDA 1,MF+1 ; SET LOW M
CRLF JSR 0,CRLF ; FORMAT
CRLF JSR 0,TXTT
SPCE3 SPCE3
FENT

GXY1 LDA 3,DELAM ; CAL. 1ST FREQ.
LDA 1,TW0 ; AND FREQ. STEP
FMPY 1,C ; 2*DT
DT2 FSTA 3,DT2
MF FFLO MF
MF FMPY 3,1 ; 2*DT*M
ONE FLDA 3,ONE
V FDIV 1,C ; FREQ. STEP HZ.
C FDFCF 3 ; PRINT FIRST STEP
R20 LDA 3,R20
R20 LDA 1,2,3 ; POINTER TO RXY
R20 LDA 2,3,3 ; POINTER TO B(R)
TP0 LDA 3,C,3 ; POINTER TO HXY
TP0 STA 3,TP0 ; RXY
TP1 STA 5,TP1 ; RXY
TP2 STA 1,TP2 ; A(R)
TP3 STA 2,TP3 ; B(R)
TP0 LDA 2,TP0 ; LOAD POINTER TO RXY
TP0 SUB 3,C
R20 LDA 1,M
RX1 ADC# 3,1,SNR ; K=M?
RX2 JMP RX2
FENT
ADC# 1*0*SNR ; K=M? (FINISHED)
JMP GX6 ; YES, GO TO PRINTOUT
FENT ; NO, CONTINUE
FFLO KF
FSTA 1*SUM2 ; SET Q(K) TO C
FLDA C*AC ; LOAD A(C)
FLDA 3*AM ; LOAD A(M)
FEXT
MOVR C*CSNC ; ODD OR EVEN?
JMP +4 ; EVEN
FENT ; ODD
FSUB 3*3 ; A(C)-A(M)
FJMP +3
FADD 3*3 ; A(C)+A(M)
FSTA C*SUM1
FEXT
LDA 3*R2C
LDA 2,2,3 ; RESET POINTER TO A(C)
LDA 3,3,3 ; RESET POINTER TO B(C)
STA 3*TP3
SUBZL C*G ; GENERATE 1
STA C*R ; SET R=1
JMP GX5 ; YES
STA C*RF+1 ; NO
FENT
FFLO RF
FLDA C*RF
FLDA 1*MF
FLDA 2*PI
FMXY 2*3 ; PI*R
FDIV 1*3 ; PI*R/M
FLDA 1*KF
FMXY 1*3 ; PI*R*K/M
FCOS C*1
FSIN C*2
FLD3 TP3
FIC2
FIC3
FLDA C*3*2 ; LOAD 2*A(R)
FLDA 3,3,3 ; LOAD 2*B(R)
FMXY 2,1 ; 2*A(R)*COS
FMXY 3,2 ; 2*B(R)*SIN
FLDA C*SUM1
FLDA 3*SUM2
FADD 3*1
FADD 3,2
FSTA 1*SUM1
FSTA 2*SUM2
FSTA 2,SUM5
FST3 TP3
FEXT
SUB 3*3 ; RESET RF TO C
STA C*RF
ISZ R
LDA C*R
LDA 1*M
JMP GX4
273

CGC5 XSPEC

00337*000762
00340*006011SGX8:
00341*006011$
00342*006011$
00343*020001$
00344*040002$
00345*102400

00346*040006$GX9:
00347*024005$
00348*036416

00350*036416
00351*122415
00352*000761
00353*000114 PR20:

00354*000326*
00355*021000
00356*140001
00357*124000
00358*100000

00360*100000
00361*050004$
00362*000326$
00363*000326$
00364*100003
00365*000761
00366*000114'PR2C:

00369*ZREL

00370-141264 P18C: -180.0
00371-000000

*END
Figure A5.16  FRESP- FREQUENCY RESPONSE
• TITL FRESP
• ENT FRESP
• EXTN FENT

• EXTD C5, C5P1, TEMPO, TEMPO, DELAM, RESLT, M
• EXTD DEG, LNLOG, TWNTY, M1, P18G

```assembly
C000'054564 FRESP: STA 3*R18 ; FREQUENCY RESPONSE
C000'024554$ LDA 1*C5 ; SAVE RESULT FORMAT
C000'044564 STA 1*CSAV
C000'024562 LDA 1*C3
C000'044631$ STA 1*C5
C000'044632$ STA 1*C5P1
C000'024561 LDA 1*C5G ; SET PLOTTING COUNTER
C000'044613$ STA 1*M1
C000'028401 LDA 1,1,3 ; OUTPUT POINTER G
C000'031402 LDA 2,2,3 ; OUTPUT POINTER PHI
C000'044633$ STA 1*TEMPO
C000'0500043$ STA 2*TEMPO1
C000'102400 SUR G+G
C000'040554 STA G*X ; SET FREQ. COUNTER
C000'044554 STA G*X+1
C000'177777 TR1: FLNOT ; CAL. FREQ. RADS./SEC
C000'060551 FFL0 X ; IN LOG. DECADES
C000'020550 FLDA G*X ; IE. OW=C*1EXP(230259X)
C000'024555 FLDA 1*P23
C000'104500 FMPY G*1
C000'124200 FEXP 1*1
C000'020550 FLDA G*P1
C000'104100 FMPY G*1
C000'044552 PIA 1*OW
C000'020543 FLIA G*PI2
C000'164500 FLT.V G*1 ; FREQ. IN HZ.
C000'144501 FFDCT 1
C000'100000 FEXT
C000'060666$ JSR G*RESLT
C000'028257 LIA G*R18 ; POINTER TO H
C000'040553 STA G*TPC
C000'102400 SUE G*C
C000'040543 STA G*J ; SET J=C
C000'040543 STA G*J+1
C000'040543 STA G*SUM1
C000'040543 STA G*SUM1+1
C000'040543 STA G*SUM2
C000'040543 STA G*SUM2+1
C000'040543 STA G*SUM2+1
C000'060017'TR2: FLNOT
C000'060534 FLLO J
C000'020534 FLTA 1*OW
C000'020532 FLTA G*J
C000'030055 FLDA 2*DELAM
C000'104100 FMPY G*1 ; J*W
C000'130100 FMPY 1*2 ; J*W*LT
C000'060453 FLLO 1*PC ; POINTER TO H(J)
C000'021400 FLIA G*G*3 ; LOAD H(J)
```
COS(J*W*DT)  

H(J)*COS(J*W*DT)  

SINC J*W*DT)  

H(J)*SIN(J*W*DT)  

J=M ?  

C*C+Q*Q  

CAL. GC(J)  

20*LN(MAG.)  

20+LOGCMAG.)  

STORE IN ARRAY  

PRINT IT  

Q/C  

ARCTAN(Q/C)  

LAG NETWORK  

STORE IN ARRAY  

PRINT IT  

PHASE IN DEGREES  

180 DEG. SHIFT  

QUAD. TERM NEG.?  

NO  

YES  

POINTER TO PHI  

STORE IN ARRAY  

PRINT IT  

X
2303 FRESB
23152.006006$ JSR @RESLT
23153.010417 ISZ X+1 ; INC. FREQ. COUNTER
23154.024416 LDA 0,X+1
23155.024412 LDA 1,C5C ; 4 DECADES ?
23156.010417 SUPL 0,1,SNC
23157.030000 JMP TR1
23158.024406 LDA 1,CSAV ; RESET RESULT FORMAT
23159.010406 STA 1,C5
23160.000600 LDA 3,R18
23161.0440015 JMP 3,3 ; RETURN
23162.034403 R18: 0
23163.034402 C3: -3
23164.0440015 CSAV: 0
23165.040031 C5C: 50 ; OCTAL 50= DEC. 40
23166.040031 MM: 0
23167.040031 X: 0
23168.040031 R18: 0
23169.040031 P12: 6.283185 ; TWO PI
23170.040031 P1: 0,1
23171.040031 P23: 0.230259
23172.040031 P20: 0.171100
23173.040031 NW: 0
23174.040031 J: 0
23175.040031 SUMP: 0
23176.040031 TP0: 0.03064
23177.040031 SUM1: 0
23178.040031 SUM2: 0
23179.040031 TPC: 0.0306
23180.040031 ENI
10.5.2 Optimisation Programme

10.5.2.1 Algorithm

The principal algorithm used was that of Fletcher and Powell and was outlined in Section 3.4. The minimum of a multivariable non-linear function subject to non-linear equality constraints is formulated as follows:

Minimise subject \( F(x_1, x_2, \ldots, x_N) \)

to \( G_k(x_1, x_2, \ldots, x_N) = 0 \)

where \( k = 1, 2, 3 \ldots M \)

By incorporating the constraints into a modified unconstrained objective function, the method of Fletcher and Powell for the unconstrained minimisation is then used. Inequality constraints are treated by the use of slack variables with appropriate transformations. The algorithm proceeds to define a new unconstrained objective function.

\[
\Phi = F - \sum_{k=1}^{M} \lambda_k G_k + B \sum_{k=1}^{M} G_k^2
\]

where \( \lambda_k \) and \( B \) are constants. Having selected initial estimates for the unknown variables. The numerical value of the function \( F \), its derivatives and the value of \( G_k \) are calculated. The \( \lambda_k \) values above are determined from

\[
\sum_{i=1}^{N} \sum_{j=1}^{M} \lambda_j \frac{\partial G_j}{\partial x_i} \frac{\partial x_i}{\partial x_i} = \sum_{i=1}^{N} \frac{\partial G_k}{\partial x_i} \frac{\partial F}{\partial x_i}
\]

for \( k = 1, 2, 3 \ldots M \)

With an estimate of \( B \) a series of search directions and one dimensional search steps are determined, and after the necessary iteration to obtain the required convergence criteria \( G_k = 0 \) and the function to be optimised is equal to \( \Phi \). \( (F = \Phi \) at convergence.)
10.5.2.2 User Experience

Some effort is required to formulate the polynomial approximation algorithm mentioned in Section 3.3.2 into a form amenable to the Fletcher-Powell algorithm. However once this was achieved and the programme package was tested on some problems whose solution was known, confidence in the package was gained. The resulting final programme package has proved computationally fast, efficient, very stable numerically and with the added plotting routines very useful for this investigation. The Univac-Fortran 5 User Manual (Reference 78) proved useful.

10.5.2.3 Programme Listing

The conclusions for the design of chute geometries under uniform flow conditions suggest the use of this package programme. Accordingly a listing of the optimisation programme follows.
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
REAL MU
DIMENSION X(25), GRADF(25), G(25), CPADG(25), H(25), FLAM(25), S(25)
DIMENSION GO(25), GN(25), W(50), A(25)
COMMON IREAD, IWRITE, ISWITCH, NN, MM, XSC(100), FSC, GSC(100), DELDI(100)
COMMON /CHUTE/ NV, N, NSTEP, MC, XCOR, YCOR, TAU, VINIT, STEP, MU, C1, C2, YR, 19, X(25), TF
EXTERNAL LINK

C SETUP INITIAL VALUES AND PROGRAM PARAMETERS
C NV = NO OF VARIABLES ( = DEPEND. * INDEPEND.)
C N = NO OF INDEPENDENT VARIABLES
C M = NO OF CONSTRAINT EQUATIONS
C MC = NO OF OUTPUT PARAMETERS (YR PARAMETERS)
C X = COEFFS. OF POLYNOMIAL TO BE OPTIMISED
C F = FUNCTION TO BE OPTIMISED (DESCENT TIME)
C XCOR = END X COORDINATE
C YCOR = END Y COORDINATE
C DELX = X VARIABLE CRITERION
C EPS = MAGN. OF AUX. FUNCTION PHI NEAR THE MIN.
C DEFX = X VARIABLE CRITERION DUE AN INCREASE OF EPS IN PHI
C DLEG = GRADIENT OF F AND G CRITERION
C MU = RESISTANCE COEFFICIENT (TANG. DRAG FORCE, VEL. DEPENDENT)
C TAU = COULOMB FRICTION COEFFICIENT (VEL. DEPENDENT)
C VINIT = INITIAL VELOCITY
C HB = HEIGHT TO BREADTH RATIO
C T - LIMITING THETA FOR FAST FLOW

TREAD=5
IWRITE=6
10 WRITE (IWRITE,40)
READ (IREAD,50,END=30) NV, M, MC, HB, VINIT, MU, TAU, XCOR, YCOR, TF
C1=0.346*VINIT*HB
C2=0.00009096*VINIT*HB
DELX=1.0-12
EPS=1.0-12
DEFX=1.0-4
DLEG=1.0-6
N=NV-1
WRITE (IWRITE,70)
WRITE (IWRITE,90) NV, M, MC, HB, VINIT, MU, TAU, C1, C2, XCOR, YCOR, TF
NSTEP=40
STEP=YCORDER NSTEP

C SET INITIAL ESTIMATES FOR POLYNOMIAL COEFFS.
C WRITE (IWRITE,60)
READ (IREAD,50) (X(I),I=1,N)
C SET THE LAST COEFFICIENT
SLM=0.
Z= YCOR*Z
DO 20 I=1,N
SUM=SUM+X(I)*Z/(I+1)
20 Z=Z*YCOR
TRUC=(XCOR-SUM)*(NV+1)/Z
XINV=1/TRUC
WRITE (IWRITE,120) (I*X(I),I=1,NV)
ISWICH=0
CALL SCALER (N,M,X,1.D-12,1.D6,5.D-6,1.D2,60.GN)
WRITE (IWRITE,120) (I*X(I),I=1,NV)
CALL LINK (1*X*GRADF*G*GRADG)
CALL CONKIN (LINK,N,M,X,GRADF,G,GRADG,1.D2,1.D1*H,FLAM,DELX,EPS,
1*DEFX,DELG,20,G10,IERS*G0,G0*G0,G0*W,A1)
WRITE (IWRITE,801)
ISWICH=1
WRITE (IWRITE,110)
CALL LINK (1*X*GRADF*G*GRADG)
VAL=YR(1)
WRITE (IWRITE,1201) (I*X(I),I=1,NV)
WRITE (IWRITE,100) VINIT,VAL,TF
ISWICH=0
GC TC 10
30 STOP
40 FORMAT (*)"TYPE IN THE FOLLOWING DATA SEPARATED BY A COMMA"/* NV".
1*MC,H,..VINIT,MU,TAU,XCOR, YCOR"/
50 FORMAT (*)
60 FORMAT (*)" TYPE IN THE ESTIMATES OF THE N COEFFICIENTS"/* X(I)",".
IX(2)....ETC"/
70 FORMAT (*)"//4X""INITIAL VALUES"
80 FORMAT (*)""/4X""FINAL VALUES"
90 FORMAT "/""/ NV= "/I2.5X,""M= "/I2.5X,""MC= "/I2.5X,""4B= "/F8.5,""VI
INIT= "/F8.5,""MU= "/F8.5,""TAU= "/F8.5,""C1= "/F11.8"/"E="/*F10.3"
 /""// THE TA"F="/*F10.3"
100 FORMAT "/""// INITIAL VELOCITY = "/F10.5"/" MIN. DESCENT TIME = "*/1P
1E15.5/"" TH""A= "*/0F10.21
*PAR,X*,8X,""DIFF. IN X"
120 FORMAT "/""// COEFFICIENTS ARE "*/4(2X,""X(*)="*/1PE15.8"

END
SUBROUTINE CONMIN (FNS*NM*X*F*GRADF*GGRADG*B*R*H*FLAM*DELEXEPS*O
1EFX*DELG*IT*IPR*IND*IER*S*GO*GN*W*A)

MAIN OPTIMISATION SUBROUTINE: PERFORMS OR COORDINATES ALL CALCULATIONS TO DETERMINE THE SCALED COEFFICIENTS OF THE INDEPENDENT VARIABLES.

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DIMENSION X(25), GRADF(25), G(25), GRADG(25), H(50), FLAM(25), S(25), GO(25), GN(25), W(50), A(25)

COMMON IREAD*WRITE*ISWICH*NNMM*XSC(100)*FSC*GSC(100)*DELDIF(100)

INITIALIZE MATRIX H

IF (IND-1) 10,20,20
10 CALL MATUP (H*1*N*S*GO*GN*ALFA*DELX*DELG*W)

PRINT OUT INITIAL CONDITIONS

20 IP=IPR/10
30 WRITE (IWRITE,500) N*M*IT*IPR*IND*B*R*DELEX*EPS*DEFX*DELG
WRITE (IWRITE,570) (I*X(I),I=1,N)

FUNCTIONS AND GRADIENTS AT INITIAL POINT

40 CALL FNS (2*X*F*GRADF*G*GRADG)
50 IET=0
60 ICV=-1
70 IEL=0
80 INL=1

OBTAIN NEW SET OF LAMBDA VALUES

IF (1.GT.M) GO TO GO
DO 50 I=1,M
50 FLAM(I)=0.0
GO CONTINUE
70 CALL LAM3 (N*M*GRADF*GRADG*G*S*FLAM*R*DELG*W*ALFA*ICV)
90 IEL=IEL+1

OBTAIN THE AUXILIARY FUNCTION (PHI), ITS GRADIENT(GN), THE MAGNITUDE (GNA) OF GN AND THE SUM(ES) OF 3*G(I)**2 FOR I=1 TO M

L=1
80 PHI=F
IF (1.GT.M) GO TO 100
DO 90 I=1,M
90 PHI=PHI+G(I)**2*FLAM(I))
100 ICKCGT=L
IF (ICKCGT.LE.1) GO TO 110
IF (ICKCGT.GE.2) GO TO 310
GO TO (110,310), ICKCGT
110 GNA=0.
IF (I.GT.N) GO TO 150
DO 140 I=1,N
T=GRADF(I)
K=I
IF (I.GT.M) GO TO 130
DO 120 J=1,M
T=T+GRADF(K)*((2.*B*C(J)-FLAM(J))
120 K=K+N
130 GNII=T
140 GNA=GNA*T*T
150 GNA=SGRT(GNA)
GS=0.
IF (I.GT.M) GO TO 170
DO 160 I=1,M
160 G S=GS+B*G(I)*G(I)
170 ICCKCCT=L
IF (ICKCGT.LE.1) GO TO 180
IF (ICKCGT.GE.2) GO TO 350
GO TO (180,350), ICKCGT
C C PRINT OUT DATA FOR LAMBDA DETERMINATION
C 180 IF (IF) 200,200,190
190 WRITE (IWRITE,520) IEL,IET,ICV
WRITE (IWRITE,570) (I,X(I),I=1,N)
WRITE (IWRITE,510) (FLAM(I),I=1,M)
WRITE (IWRITE,530) (G(I),I=1,M)
WRITE (IWRITE,540) PHI,F,GS,GNA
WRITE (IWRITE,570) (I,XSC(I),I=1,N)
C C DETERMINE WHETHER CONVERGENCE HAS BEEN OBTAINED
C 200 IF (ICV-1) 210,220,230
210 IF (INL) 230,220,230
220 IER=0
GO TO 450
C C LINEAR ITERATION
C 230 L=2
INL=1
ITN=0
IF (I.GT.N) GO TO 250
DO 240 I=1,N
240 GSII=GN(I)
250 CONTINUE
260 IF (IET-IT) 280,270,270
270 IER=1
GO TO 450
290 IF (I.GT.N) GO TO 300
DC 290 I=1,N
290 W(I)=X(I)
300 IET=IET+1
ITN=ITN+1
INLP=INL
INL=0
ALFA=1.0
CALL LINPIN (INL,X,KF,W,S,ALFA,X,PHI,DELX,DEFX,EPS)
    I=1 (INL-1) 340,330,320
320 IER=2
    GO TO 450
330 CALL FNS (1,X,F,GRADF,G,GRADG)
    GO TO 80
C
C    OBTAIN GN,GNA, AND GS
C
340 CALL FNS (2,X,F,GRADF,G,GRADG)
    GO TO 110
C
    FIND INDEX INL FOR LINEAR ITERATIONS
C
350 INL=INL+2
    IF (ALFA) 370,370,360
360 IF (ICPA-DELG) 370,370,390
370 INL=0
C
    PRINT OUT DATA FOR LINEAR ITERATION
C
380 IF (IP*IR) 410,410,390
390 IF (IPD(IEIT,IR)) 410,400,410
400 WRITE (IWRITE,550) IET,INL,ALFA,NF
    WRITE (IWRITE,570) (I,X(I),I=1,N)
    WRITE (IWRITE,540) PHI,F,GS,GNA
C
    UPDATE H AND OBTAIN NEW S
C
410 CALL MATUP (H,3,N,S,GN,ALFA,DELX,DELG,W)
C
    CONVERGENCE OBTAINED IF INLP=0 AND INL=0 (ITN=2)
C
    IF (INLP*INL) 420,420,430
420 IER=0
    GO TO 450
C
    OBTAIN NEW SET OF LAMDA VALUES OR CARRY OUT A FURTHER ITERATION
C
430 IF (ITN=1) 260,260,440
440 IF (INL*(N-M+1-ITN)) 70,70,260
C
    CONCLUSION
C
450 IF (IP) 490,490,460
460 WRITE (IWRITE,560) IER
    K=1
    I=1,GT,N) GO TO 480
    DC 470 I=1,N
    W(I)=H(K)
470 K=K+N+1-I
480 CONTINUE
    WRITE (IWRITE,580) (W(I),I=1,N)
490 RETURN
C
500 FORMAT (/* INITIAL DATA FOR CONSTRAINED MINIMIZATION*/,
    N=*13, 13X,*P=*13,3X,*IT=*14,3X,*IPR=*12,3X,*IND=*11/* D=*1PE10.3,5X)
2R = \text{E}10.3 \times 5X, *DELX = \text{E}10.3 \times 5X, *EPS = \text{E}10.3/1X, *DEFX = \text{E}10.3 \times 5X, *DELG = \text{E}10.3

510 FORMAT (12X, *LAMBDA = * (1P5E16.7))
520 FORMAT (/// LAMBDA DETERMINATION = I4, * AFTER ITERATION = I5, 7X, *ICV 1 = * I1)
530 FORMAT (17X, *C = * (1P5E16.7))
550 FORMAT (/// ITERATION = I5, 7X, *INL = * I1, 6X, *ALFA = *1PE10.3, 6X, *NO OF 1 POINTS = * I3)
560 FORMAT (/// CONstrained MINIMIZATION COMPLETED = 7X, *IER = * I1)
570 FORMAT (1X, *COEFFICIENTS ARE = 3(2X, *X(*, I1, *)) = *1P5E15.8))
580 FORMAT (/// DIAGONAL ELEMENTS OF H = * (1P5E12.3))

C  END
FUNCTION FUNF (X)
EVALUATES PARAMETERS TO BE OPTIMISED INCLUDING
YCOR, W, XCOR, TMIN, TIME, THETA, VEL

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON IRREAD, IWRITE, ISWITCH, NN, MM, XSC(100), FSC, GSC(100), DELDIF(100)
COMMON /CHUTE/ NV, N, NSTEP, MC, XCOR, YCOR, TAU, VINIT, STEP, MU, C1, C2, YR(19), X(25), TF
DIMENSION X(25), XARRAY(50), YARRAY(50)
RAD = 57.29577951
IF (ISWICH.EQ.1) GO TO 30

UPDATE INITIAL COEFFICIENT ESTIMATES

DO 10 I=1,N
10 XX(I) = X(I)

SET THE LAST COEFFICIENT
SUM = 0.
Z = YCOR ** 2
DO 20 I=1,N
SUM = SUM + XX(I) ** 2 / (I+1)
20 Z = Z * YCOR
TRUC = (XCOR - SUM) ** 2 / (NV ** 2) / Z
XX(NV) = TRUC

SET INITIAL VALUES OF YCOR, W, XCOR, T TO ZERO

30 DO 40 I=1,MC
40 YRI(I) = 0.
IF (ISWICH.EQ.1) WRITE (IWRITE,60) (YRI(J), J=1, MC)

SOLVE EQUATIONS FOR NSTEP INCREMENTS

DO 50 I=1,NSTEP
CALL RUNGE (MC, STEP, YP)
THET = ATAN(THET)

V = VINIT ** 2 + 2 * (GY + W) ** 0.5
VEL = VINIT ** 2 + 2 * (9.81 * YR(1) + YR(2))
THET1 = THET * RAD

COMPARE CURVE TO PAPABOLA
C = XCOR / (YCOR ** 2)
PX = C * YR(1) ** 2
DIFF = YR(3) - PX
IF (ISWICH.EQ.1) WRITE (IWRITE,60) (YR(J), J=1, MC), THET1, VEL, PX, DIFF
IF XARRAY(I+1) = YR(3)
YARRAY(I+1) = -YR(1)
50 CONTINUE
IF (ISWICH.EQ.0) CALL PLT (XARRAY, YARRAY)
FUNF = YR(4)
RETURN
C 60 FORMAT (1X,1P8.8)
C
END
SUBROUTINE FUNC (X, F, G)

DEFINE FUNCTION TO BE OPTIMISED AND CONSTRAINTS

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION X(25), G(25)
COMMON /CHUTE/ NV, NSTEP, MC, XCOR, YCOR, TAU, VINIT, STEP, MU, C1, C2, YR(19), XX(25), TF

T^ = LIMITING THETA FOR FAST FLOW (IN RADIANS)
TFA = TF * 0.0174532925

TEST FOR BOUNDARY PENETRATION
U = MIN1(F, 0.0 D0)

EVALUATE FUNCTION VALUE. IF BOUNDARY PENETRATED APPLY PENALTY.
F = FUNF(X) - 1.0 D0 * U

CONSTRAINT EQUATIONS
THET = FT4ETAP(YR(1))
THET = ATAN(THET)
G(1) = THET - TFA
RETURN

END
FUNCTION FTHETA(Y)
EVALUATES THE FUNCTION FOR THETA AS A FUNCTION OF Y

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON /:UFE/ NV,NSTEP,MC,XCOR,YCOR,TAU,VINIT,STEP,MU,C1,C2,YR
19,XX(25),TF

S=0.
AX=Y
DO 10 I=1,NV
S=S+XX(I)*AX
10 AX=AX*Y
FTHETA=S
RETURN

END
SUBROUTINE DSI MQ (A, B1, B2, N, KS)

SOLVES A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION A(25), B1(50), B2(50)

FORWARD SOLUTION

TCL=1.0-20
KS=0
JY=-1
IF (1.GT.N) GO TO 120
DC 110 J=1,N
JY=J+1
JY=J*(N+1)
BIGA=0
IT=J-J
IF (J.GT.N) GO TO 30
DC 20 I=J*N

SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN

IF (ABS(BIGA)-ABS(A(I,J))) 10.20.20
10 BIGA=A(I,J)
IMAX=I
20 CONTINUE
30 CONTINUE

TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)

IF (ABS(BIGA)-TOL) 40.40.50
40 KS=1
RETURN

INTERCHANGE ROWS IF NECESSARY

50 I1=J*N*(J-2)
IT=IMAX-J
IF (J.GT.N) GO TO 70
DC 60 K=J*N
I1=I1*N
I2=I1+IT
SAVE=A(I1)
A(I1)=I2
A(I2)=SAVE

DIVIDE EQUATION BY LEADING COEFFICIENT

60 A(I1)=A(I1)/BIGA
70 SAVE=B1(IMAX)
B1(IMAX)=B1(J)
B1(J)=SAVE/BIGA
SAVE=B2(IMAX)
ELIMINATE NEXT VARIABLE

IF (J-N) 80*130*80
80 IQS=N*(J-1)
   DO 110 IX=JY*N
   IXJ=IQS*IX
   IT=J-IX
   IF (JY.GT.N) GO TO 100
   DO 90 JX=JY*N
   IXJX=N*{(JX-1)*IX
   JJX=IXJX+IT
90   A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
100  B1(IXI)=B1(IXI)-B1(JJ)*A(IXJ))
110  B2(IXI)=B2(IXI)-(B2(J)*A(IXJ))
120  CONTINUE

BACK SOLUTION

130 IF (N-1) 170*170*140
140 NY=N-1
   IT=N*N
   IF (I.GT.NY) GO TO 160
   DO 150 J=1*NY
   IA=I-T-J
   IT=N-J
   IC=N
   DO 150 K=1*J
   B1(IB1)=B1(IB1)-A(IA)*B1(TC)
   B2(IB1)=B2(IB1)-A(IA)*B2(IC)
   IA=IA-N
150  IC=IC-1
160  CONTINUE
170  RETURN

END
SUBROUTINE MDW (XP, DY, Y)
C
C EVALUATES DW/DY*TAN(THETA)*COS(THETA)*VEL**-1
C REQUIRED FOR FINDING YCOR, XCOR, T
C I.E. DY(1), DY(2), DY(3)
C
REAL MU
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON IREAD, IWRIT, ISWITCH, NN, MM, XSC(100), FSC, GSC(100), DELDIF(100)
COMMON /CHUTE/ NV, NSTEP, MC, XCOR, YCOR, TAU, VINIT, STEP, MU, C1, C2, YR(19), XX(25), TF
DIMENSION XP(MC), DY(NV)
THET=FTHETA(Y)
THET=ATAN(THET)
CS=COS(THET)
SS=SIN(THET)
VEL2=VINIT**2+2.0*(9.81*Y*XP(1))
C
V=VINIT**2*2*G(Y+1)**0.5
C
VEL=SQRT(ABS(VEL2))
S=0.
Z=1.
DO 10 I=1,NV
S=S+XX(I)*Z
10 Z=Z*Y
TERM=9.81*SS-VEL2*(CS**3)*S
TERM=ABS(TERM)*TAU*(1.0+C1/VEL+C2*VEL)
C
CALCULATE DW/DY
C
TERM = TAU*(1+C1/V+C2*V)*(CS*SIN(THETA))-V**2*COS**3(THETA))
C
DW/DY = -(MU*V*TERM)/COS(THETA)
C
DY(1) = (TERM*MU*VEL)/CS
DY(2) = TAN(THET)
DY(3) = 1./(CS*VEL)
RETURN
C
END
SUBROUTINE CRADFG (IF, X, F, GRADF, G, GRADG)
C
CALCULATES NUMERICAL DERIVATIVES
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION X(25), GRADF(25), G(25), GRADG(25), GD(100)
COMMON /READWRITE/ISWICH,NN,MM,XSC(100),FSC,GSC(100),DELDIF(100)
N=NN
M=MM
C
FIND VALUES FOR F AND G (FUNCTION AND CONSTRAINTS)
C
IF (M) 2C,20,10
10 CALL FUNC (X,F,G)
GO TO 30
20 CALL FUNC (X,F,G)
30 IF (IF-1) 40,40,50
40 RETURN
C
FIND GRADIENTS OF F AND G
C
50 IF (1.GT.N) GO TO 120
 DO 110 I=1,N
 DT=DELDIF(I)
 X(I+1)=X(I)+DT
 IF (M) 90,90,60
60 CALL FUNC (X,FD,GD)
 K=I
 IF (1.GT.M) GO TO 80
 DO 70 J=1,M
 GRADG(K)=(GD(J)-G(J))/DT
70 K=K+N
80 CONTINUE
 GO TO 100
90 CALL FUNC (X,FD,G)
100 X(I+1)=X(I)-DT
110 GRADF(I)=(FD-F)/DT
120 CONTINUE
 RETURN
C
END
SUBROUTINE LAMB (NM, GM, GV, SL, FLAM, R, DELG, W, A, ICV)

COMPUTES LAMBDA VALUES

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FLAM(25), W(50), A(25), SL(50), GV(50), H(125), F(50),
GM(50)

EVALUATE W(I) = F * G(I)

AS = 0.0
 Q = 0.0
 T = 0.0

10 K = 1
 IF (I1 GT M) GO TO 50
 DC 40 I = 1 + M
 T = 0.0
 IF (I1 GT N) GO TO 30
 DO 20 J = 1 + N
 T = T * F(J) * GM(K)
 20 K = K + 1
 30 CONTINUE
 40 W(I) = T

EVALUATE A(M*(I-1) + J) = G(I) * G(J)

50 K = 1 - N
 IF (I1 GT W) GO TO 80
 DC 70 I = 1 + M
 K = K + N
 L = K
 DC 70 J = 1 + M
 T = 0.0
 DO 60 IN = 1 + N
 T = T * GM(K) * GM(L)
 L = L + 1
 60 K = K + 1
 K = K - N
 LL = M*(I-1) + J
 A(LL) = T
 LL = M*(J-1) + I
 70 A(LL) = T

EVALUATE RD AND Q
 - S = SUM(I = 1 + M) OF FLAM(I) * G(I)
 - AGI = MAGNITUDE OF G(I)
 - SGI = ABS(S.GI)
 - RD = MAXIMUM VALUE OF SGI/AGI/DELG WITH RESPECT TO I
 - Q = SUM OF DIAGONAL ELEMENTS OF A

80 R3 = 0.0
 K = 1
 L = 1
 IF (I1 GT M) GO TO 120
 DC 110 I = 1 + M
SGI=W(I)
IF (I.GT.M) GO TO 100
DC 90 J=1*M
SGJ=SGI-FLAM(J)*A(K)
90 K=K+1
100 SGI=ABS(SGI)
AGI=SQRT(A(I)*I)
Q=A*I
L=L*M*1
T=AGI*DELG
T=SGI/T/DELG
110 RD=AMAX1(RD,T)
C
COMPUTE SL(I) AND NEW VALUES FOR FLAM(I)
C
-D=MAGNITUDE OF CHANGE IN FLAM
C
120 K=1
Q=Q*Z
IF (I.GT.M) GO TO 140
DO 130 I=1,M
A(K)=A(K)*Q
SL(I)=G(V(I))
130 K=K*M+1
140 CONTINUE
CALL DSIPO (A,W,SL,M,IER)
IF (IER) 160,160,150
150 Z=10.*Z+1.D-12
GO TO 10
160 D=0.0
IF (I.GT.M) GO TO 180
DC 170 I=1,M
Z=W(I)-FLAM(I)
FLAM(I)=W(I)
W(I)=SL(I)
170 D=D*Z*Z
180 D=SQRT(D)
IF (I.GT.M) GO TO 200
DO 190 I=1,N
K=I
SL(I)=0.0
DC 190 J=1,M
SL(I)=SL(I)*W(J)*GM(K)
190 K=K+N
C
MODIFY SL(I) IF NECESSARY
C
W=NORMALISED GRADIENT OF SUM OF GV(I)*SL(I) FROM I=1 TO M
C
-SW=W*SL
C
-SP=MAGNITUDE OF COMPONENT OF SL PERPENDICULAR TO W
C
-R=MAXIMUM ALLOWED VALUE OF SP/ABS(SW)
C
200 T=0.0
IF (I.GT.N) GO TO 240
DC 230 I=1,N
W(I)=0.0
K=I
IF (I.GT.M) GO TO 220
DC 210 J=1,M
W(I) = W(I) * G(V(J)) * GM(K)

210 K = K + 1

220 Z = W(I) * SIGN(1.0 - 20 * W(I))

225 W(I) = Z

230 T = T + Z

240 T = SQRT(T)

SW = 0.0
IF (1. GT. N) GO TO 260
DO 250 I = 1, N

250 W(I) = W(I) / T

260 SW = SW + W(I) * SL(I)

270 SP = 0.0
IF (1. GT. N) GO TO 280
DO 270 I = 1, N

280 Z = SL(I) - SW * W(I)

Sl(I) = Z

290 SP = SP + Z * Z

300 T = SQRT(SP) * 1.0 - 20
IF (1. GT. N) GO TO 310
DO 300 I = 1, N

310 SL(I) = T * SL(I) * SW * W(I)

320 CONTINUE

CONVERGE parameter

330 ICV = ICV + 330
DO 340 I = 1, 4

350 ICV = ICV + 1

360 RETURN

370 ICV = 0
RETURN

380 IF (I100 EQ 5) 410, 390, 390
390 IF (I100 EQ 5) 410, 400, 400

400 ICV = 1
410 RETURN

END
SUBROUTINE LINMIN (IND, ND, NPT, BEGIN, ALFA, END, F, NLX, DFX, EPS)

PERFORMS ONE DIMENSIONAL SEARCH

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION S(25), F(50), T(50), BEGIN(50), END(50)

IF (IND) 50, 50, 10
10 F(KK)=F(N)
NPT=NPT+1
ICKCGT=N30T0
IF (ICKCGT.LE.1) GO TO 80
IF (ICKCGT.GE.6) GO TO 380
GO TO (80, 90, 130, 170, 310, 380), ICKCGT
20 IF (1.GT.ND) GO TO 40

GET FUNCTION VALUES

DO 30 I=1, ND
30 E KDCI I-BEGINI I MTI KK)*SII)

CONTINUE

RETURN

CHANGE IN ALFA CORRESPONDING TO DELX AND DFX

50 IND=1
NPT=0
Z=0.0
IF (1.GT.IND) GO TO 70
GO TO 60 I=1, ND
60 Z=Z*S(I), S(I)
70 Z=SQT(2)
Z=MAX1(Z*1.0, 20)
DEL=NLX/Z
DFX=DFX/Z

OBTAIN THREE POINTS, L, M, AND N, WITH L AND N ON OPPOSITE SIDES OF
M, F(L) AND F(N) NOT LESS THAN F(M), AND THE DISTANCES (L TO M)
AND (M TO N) AT LEAST DFX

-MINIMAL AND RECOMMENDED POINTS

T(1)=0.0
KK=1
NGOT=1
GO TO 20
80 FKEEP=F(1)
T(2)=ALFA
KK=2
NGOT=2
GO TO 20

-POINTS P AND L

90 L=1
M=2
IF \( | F(1) - F(2) | > 1000 \)  
100 \( M = 1 \)  
\( L = 2 \)  
110 IF \( | ABS(T(M) - T(L)) - DEF | > 1200 \)  
120 \( T(L) = T(M) \cdot \text{SIGN(DEF \cdot T(L) - T(M))} \)  
\( KK = L \)  
\( \text{GO TO 20} \)  
130 IF \( | F(L) - F(M) | > 1900 \)  
140 \( I = L \)  
L = M  
\( N = I \)  
C - POINT N (DOUBLE STEPLENGTH EACH TIME)  
C  
150 \( Z = 1.0 \)  
\( N = 3 \)  
160 \( T(N) = T(M) + Z \cdot (T(M) - T(L)) \)  
\( KK = N \)  
\( \text{GO TO 20} \)  
170 IF \( | F(N) - F(M) | > 1800 \)  
180 \( I = L \)  
L = M  
M = N  
N = I  
\( Z = 2.0 \)  
190 \( \text{IND} = 2 \)  
RETURN  
C  
DECREASE THE DISTANCE (L TO N) TO LESS THAN 4 \( \cdot \) DEFX, KEEPING THE  
DISTANCES (L TO M) AND (M TO N) AT LEAST DEFX  
C  
200 NEW = 4  
\( N \cdot A D = 0 \)  
C - LET L BE CLOSER TO M THAN IS N  
C  
210 IF \( | ABS(T(M) - T(L)) - ABS(T(M) - T(N)) | > 2200 \)  
220 \( I = L \)  
L = N  
N = I  
C - ESTIMATE THE POSITION OF THE MINIMUM POINT (NEW) FROM A PARABOLIC  
FIT \( F = A \cdot 3 \cdot T + C \cdot T \cdot T \cdot 2 \)  
C  
230 \( T1 = T(L) - T(M) \)  
\( T2 = T(N) - T(M) \)  
\( H1 = \frac{ABS(F(L) - F(M))}{T1} \)  
\( H2 = \frac{ABS(F(N) - F(M))}{T2} \)  
\( C = \frac{H2 - H1}{(T2 - T1)} \)  
\( B = H1 \cdot T2 - H2 \cdot T1 \cdot (T2 - T1) \)  
\( T(\text{NEW}) = T(M) - B / 2 \cdot / \text{SIGN}(1.0 - 30.0 \cdot C) \)  
C - END CYCLE WHEN DISTANCE (L TO N) IS LESS THAN 4 \( \cdot \) DEFX  
C
IF (ABS(T1(T1)-T1(T2))-DEF) 370.370.340

GEOMETRIC AVERAGE OF THE DISTANCES (L TO M) AND (L TO N)

240 IF (NBAD<2) 260.250.250
250 T1(NEW)=SORT(T1(T1(T1-T2)))
   T1(NEW)=T1(L)*SIGN(T1(NEW)*T2)
   NBAD=0

-NEW WILL BE CLOSER TO M THAN TO N OR L. IF NEW LIES WITHIN
DEFX OF M, CHANGE T1(NEW) SO THAT NEW LIES BETWEEN M AND N, AT A
DISTANCE DEFX FROM M

260 IF (ABS(T1(NEW)-T1(M))-DEF) 270.280.280
270 T1(NEW)=T1(M)*SIGN(DEF*T2)

-LET NEW LIE BETWEEN M AND N

280 IF (T1(T1(NEW)-T1(M))<T2) 290.300.300
290 I=L
   L=N
   N=I

-IMPROVE L, M AND N

300 K=NEW
   NGOTO=5
   GO TO 20
310 Z=ABS(T1(T1)-T1(L))
   IF (F(T1(NEW)-F(M)) 320.330.330
320 I=L
   L=M
   M=NEW
   NEW=I
   GO TO 340
330 I=N
   N=NEW
   NEW=I

-TEST THAT DISTANCE (L TO N) DECREASED BY AT LEAST TEN PER CENT

340 IF (ABS(T1(T1)-T1(L))/Z<.9) 350.360.360
350 NBAD=0
   GO TO 210
360 NBAD=NBAD+1
   GO TO 210

OBTAIN THE FUNCTION VALUE AT THE ESTIMATED MINIMUM POINT

370 K=NEW
   NGOTO=6
   GO TO 20
380 I=IF (NEW)-F(M)) 400.400.390
390 NEW=Y
400 ALFA=T1(NEW)
   FN=F(NEW)
   IF (1.0T.90) GO TO 420
C TEST WHETHER IMPROVEMENT IS SIGNIFICANT
C
IF (FKEEP-FN-EPS) .GE. 1.E-10 .AND. 1.E-10 .GE. IMP
430   IND=-2
      RETURN
440   IND= IABS(ALFA)-DELT
450   IND=-1
      RETURN
C
END
SUBROUTINE LINK (IF, X, F, GRADF, G, GRADG)

APPLIES SCALING FACTORS TO FUNCTION AND INDEPENDENT VARIABLES

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DIMENSION X(25), GRADF(25), G(25), GRADG(25), XRAW(100)

COMMON IREAD, IWRITE, ISWICH, NN, MM, XSC(100), FSC, GSC(100), DELDIFF(100)

N=NN
M=MM

IF (I.GT.N) GO TO 20
10 XRAW(I)=X(I)*XSC(I)
20 CONTINUE

IF (M).LT.60.30
30 CALL GRADFG (IF, XRAW, F, GRADF, G, GRADG)

IF (I.GT.M) GO TO 50
40 J=1
40 DO 30 J=1, M
G(J)=G(J)/GSC(J)
30 CONTINUE

GO TO 70

60 CALL GRADFG (IF, XRAW, F, GRADF, G, GRADG)
70 F=F/FSC

IF (I-1).LT.140.140.80
80 IF (I.GT.M) GO TO 130
90 K=I

IF (I.GT.M) GO TO 110
100 J=1

GRADG(K)=GRADG(K)/GSC(J)*XSC(I)
100 K=K+N

110 CONTINUE
120 CONTINUE
130 CONTINUE
140 RETURN

END
SUBROUTINE MATUP (H,IND,N,S,GO,GN,ALFA,DELX,DELY)

COMPUTES SEARCH DIRECTIONS

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION H(50), S(25), GO(25), GN(25), W(50)

GENERATE IDENTITY MATRIX
DIAGONAL ELEMENTS ARE H(1), H(N+1), H(2*N), H(3*N-2), ..., H(N*N)

IF (IND-2) 10,80,120
10 K=1
IF (1.GT.N) GO TO 60
DO 50 I=1,N
H(I,K)=1.0
NJ=N-I
IF (NJ) 70,70,20
20 IF (1.GT.NJ) GO TO 40
DO 30 J=1,NJ
KJ=K+J
30 H(I,KJ)=0.0
40 CONTINUE
50 K=K+1
60 CONTINUE
70 IF (IND) 80,80,110

DIRECTION OF STEEPEST DESCENT

80 IF (1.GT.N) GO TO 100
DO 90 I=1,N
90 S(I)=H(I,I)
100 CONTINUE
110 RETURN

UPDATE MATRIX H

- COMPUTE GRADIENT CHANGE Y AND STORE IN GN

120 IF (IND-4) 130,130,300
130 IF (1.GT.N) GO TO 150
DO 140 I=1,N
T=GN(I)
GN(I)=T-GC(I)
140 CONTINUE

- COMPUTE Y AND STORE IN G(1) TO G(N)

150 CONTINUE
IF (1.GT.N) GO TO 210
DO 200 I=1,N
T=0.0
K=I
IF (1.GT.N) GO TO 190
DO 190 J=1,N
T=T+GN(J)*H(K)
I=I-1
160,170,170
160 K=K+K-J
   GO TO 130
170 K=K+1
180 CONTINUE
190 CONTINUE
200 W(I1)=T
C  C  COMPUTE POSITION CHANGE Z AND STORE IN G(N+1) TO G(2*N)
C
210 K=N
   IF (I.GT.N) GO TO 230
   DO 220 I=1,N
      K=K+1
   220 W(I)=ALPHA*S(I)
C  C  COMPUTE Q=Y*H*Y, P=Y*Z AND R=SQRT(Z*Z)
C
230 R=0.0
   Q=0.0
   P=0.0
   K=N
   IF (I.GT.N) GO TO 250
   DO 240 I=1,N
      K=K+1
      R=R*W(K)*W(K)
      Q=Q*GN(I)*W(I)
   240 P=P*GN(I)*W(K)
250 R=SQRT(R)
C  C  LEAVE H UNCHANGED IF P OR R ARE VERY SMALL
C
260 IF (R-DELM) 300,300,260
   270 IF (P-R*DELM) 300,300,270
C  C  COMPUTE NEW H
C
270 K=1
   IF (I.GT.N) GO TO 290
   DO 280 I=1,N
      LI=N+I
   DC 280 J=I,N
      LJ=N+J
      H(K)=H(K)*W(LI)*W(LJ)/P-W(I)*W(J)/Q
   290 CONTINUE
C  C  COMPUTE NEW DIRECTION
C
300 IF (IND-4) 310,380,310
310 IF (I.GT.N) GO TO 370
   DC 360 T=0.0
   K=I
   IF (I.GT.N) GO TO 350
   DC 340 J=I,N
   T=T*GO(JI)*H(K)
   IF (IJ-I) 320,330,330
320 K=K+N-J
   GC TO 340
330 K=K+1
340 CONTINUE
350 CONTINUE
360 SIJ=-T
370 CONTINUE
380 RETURN
C END
SUBROUTINE PLT (XARRAY, YARRAY)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
REAL XXRAY, YYRAY

C C

SUBROUTINE TO PLOT RESULTS
C

COMMON IREAD, IWRITE, ISWICH, NN, MM, XSC(100), FSC, GSC(100), DELD1(100)
COMMON /CHUTE/ NV, N, NSTEP, MC, XCOR, YCOR, TAU, VINIT, STEP, MU, C1, C2, YR(19), XX1251, TF
DIMENSION XARRAY(50), YARRAY(50), XXRAY(50), YYRAY(50)
1 IF (INTEQ .GT. 0) GO TO 10
CALL START
NSTEP1 = NSTEP + 1
XXRAY(NSTEP1 + 1) = 0
XXRAY(NSTEP1 + 2) = .8466
YYRAY(NSTEP1 + 1) = 0
YYRAY(NSTEP1 + 2) = .8466
10 INTEQ = INTEQ + 1
DO 20 I = 1, NSTEP1
XXRAY(I) = SNGL(XARRAY(I))
20 YYRAY(I) = SNGL(YARRAY(I) + 55)
CALL LINE (XXRAY, YYRAY, NSTEP1, INTEQ)
IF (INTEQ .EQ. 1) CALL AXEN (5.9055, 5906, 1.1812, 2, 0.1, 1.1812, 2, -5, 1, 15, 55, 5906, 1.1812, 2, -5, 1, 15, 55, 0.05, -1)
RETURN
C

END
SUBROUTINE RUNGE (MC,STEP,YR)

4TH ORDER RUNGE KUTTA ROUTINE
EVALUATES NEXT INCREMENT FOR YCOR, W, XCOR, T, THETA, VEL

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION DY(3), YR(MC), XT(100), YM(100), YK(100)
DIMENSION XP(100), ZK(100), WK(100), XK(100)
NG=MC-1
TW=YR(1)
STEPH=0.5*STEP
TZ=TW*STEPH

EVALUATE RUNGE KUTTA COEFFICIENTS ZK*WK*XK*YK

DC 10 I=1,NG
10 XP(I)=YT(I)
DC 20 I=1,NG
20 XP(I)=XT(I)
   CALL F4D(WP,DP,TW)
   DO 30 IM=1,NG
30 ZK(IM)=DY(IM)
   DC 40 I=1,NG
40 XP(I)=XT(I)+STEPH*ZK(I)
   CALL F4D(WP,DP,TZ)
   DC 50 I=1,NG
50 WK(I)=DY(I)
   DC 60 I=1,NG
60 XP(I)=XT(I)+STEPH*WK(I)
   CALL F4D(WP,DP,TZ)
   DO 70 I=1,NG
70 XK(I)=DY(I)
   DC 80 I=1,NG
80 XP(I)=XT(I)+STEP*XK(I)
   CALL F4D(WP,DP,TW*STEP)
   DC 90 I=1,NG
90 YK(I)=DY(I)

SET NEW VALUES FOR W, XCOR, T, THEN YCOR

DO 100 I=1,NG
   TERM=ZK(I)+2.*WK(I)+2.*XK(I)+YK(I)
100 YM(I)=XT(I)*TERM/6.
   YR(I)=TW*STEP
   DO 110 I=1,NG
110 YRI(I)=YM(I)
RETURN

END
SUBROUTINE SCALER (N,M,X,DEL,TOP,BOT,B,GRADG1,GRADG2)

CALCULATES SCALING FACTORS FOR FUNCTION AND INDEPENDENT VARIABLES

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION X(25), GRADG1(25), GRADG2(25), G(100), GP(100), GM(100), 
1 GRADF1(100), GRADF2(100), ABD(100)
COMMON IREAD, IWRITE, ISWHICH, NN, MM, XSC(100), FSC, CSC(100), DELDIFF(100)

COMMON IREAD, IWRITE, ISWHICH, NN, MM, XSC(100), FSC, CSC(100), DELDIFF(100)

C

FIND FIRST AND SECOND DERIVATIVES

DEL1=SQRT(ABS(DEL))
DEL3=DEL*(*1.0/3.)
FSC=1.0
IF (I.GT.N) GO TO 20
DO 10 I=1,N
10 XSC(I)=1.0
20 CONTINUE
IF (M) 60,30
30 IF (I.GT.M) GO TO 50
DO 40 J=1,M
40 GSC(J)=1.0
50 CONTINUE
60 CALL LINK (1,0,F,GRAD=1,G,GRADG1)
IF (I.GT.N) GO TO 110
DO 100 I=1,N
X(I)=X(I)+DEL3
CALL LINK (1,0,F,GRAD=1,G,GRADG1)
X(I)=X(I)-2.*DEL3
CALL LINK (1,0,F,GRAD=1,G,GRADG1)
X(I)=X(I)+DEL3
GRADF1(I)=(FP-FM)/2./DEL3
GRADF2(I)=(FP-FM-2.*F)/DEL3
100 CONTINUE
110 CONTINUE

C

FIND FSC AND FIRST APPROXIMATION TO XSC (SCALE FACTORS)

FSC=ABS(F)
IF (M) 260,260,120
120 K=1
IF (I.GT.M) GO TO 160
DO 150 J=1,M
5=0.0
IF (1. GT. N) GO TO 140
DJ 130 I=1*N
S=S*GRADG1(K)*2
130 K=K+1
140 CONTINUE
150 AGRD(IJ)=SQRT(S)
160 AN=N
RN=10.*SQRT(AN)
IF (1. GT. N) GO TO 210
DO 200 I=1*N
FS1=ABS(GRADF1(I)/F)
FS2=SQRT(ABS(GRADF2(I)/F))
S=0.*0.
K=1
IF (1. GT. M) GO TO 190
DO 180 J=1*M
GRAT=ABS(GRADG1(K)/ABGD(J)*TOP*RN/BOT
IF (GRAT-1.1 180 180,170
170 T=ABS(GRADG2(K)/GRADG1(K))
S=AMAX(S,T)
180 K=K+N
190 S=AMAX(S1*FS1*FS2*S1/TOP)
S=AMIN(S1/1./BOT)
200 XSC(I)=1./S
C
C FIND GSC (G SCALE)
C
210 K=1
IF (1. GT. M) GO TO 250
DO 240 J=1*M
S=0.0
IF (1. GT. N) GO TO 230
DO 220 I=1*N
T=GRADG1(K)*XSC(I)
S=S*T*T
220 K=K+1
230 CONTINUE
240 GSC(J)=SQRT(S)
250 CONTINUE
260 IF (1. GT. N) GO TO 320
C
C FIND FINAL VALUES OF XSC AND DELDIF
C
DO 310 I=1*N
S=0.0
IF (IMP) 300,300,270
270 K=1
IF (1. GT. M) GO TO 290
DO 280 J=1*M
T=GRADG1(K)/GSC(J)
S=S*T*T
280 K=K+N
290 S=2.*S*TS
300 S=S*GRADF2(I)/GSC
S=AMAX(S1/1./TOP/TOP)
S=AMAX(S1/SQRT(1.*/S))
XSC(I)=S
DELDIF(I) = XSC(I) * DEL1
310 X(I) = X(I) / XSC(I)
320 CONTINUE
   RETURN

C

END
10.5.3 Step Responses

Often in system identification studies, as was the case here, there arises the need to compute the step response of the system. The hopper-chute system's response to the stepped opening and shutting of the flow control valve was investigated. The interpretation of the cross-correlation and impulse function responses for various chute geometries was greatly facilitated by integrating these to find their equivalent step response. Transient characteristics such as rise-time, overshoot and settling time becoming clearly evident and readily interpretable from the equivalent step response.

A short programme using Simpson's one-third rule was written to calculate these step responses and it incorporated plotting routines to present the results in graphical form. Since this technique is well known and easily programmed, a listing is not included here.

10.6 Check of Value of Viscous Drag

10.6.1 Viscous Drag Based on $a = \frac{dv}{dt}$

Continuing from Equation 4.4

$$
\int_1^2 \frac{dv}{(E - \frac{C}{v} - Fv)} = \int_1^2 dt \\
(4.4)
$$

L.H.S. = \int \frac{dv}{(E - \frac{C}{v} - Fv)} = \int \frac{v dv}{(E_v - c - Fv^2)}

= \frac{1}{-F} \int \frac{v dv}{(v^2 - \frac{E v}{F} + \frac{C}{F})}

= -\frac{1}{2F} \int \frac{(2v - \frac{E}{F} + \frac{E}{F}) dv}{(v^2 - \frac{E v}{F} + \frac{C}{F})}$
\[ v^2 - \frac{2E}{2F} \frac{v}{F} + C = \left( v - \frac{E}{2F} \right)^2 + \frac{C}{F} - \frac{E^2}{4F^2} \quad \text{(completing the square)} \]

\[ = - \frac{1}{2F} \int \frac{2v - \frac{E}{F}}{v^2 - \frac{E}{F} v + \frac{C}{F}} + \frac{E}{F} \int \frac{1}{\frac{E}{F} v + \frac{C}{F}} \]

\[ = - \frac{1}{2F} \ln \left( v^2 - \frac{E}{F} v + \frac{C}{F} \right) - \frac{E}{2F^2} \int \frac{dv}{\frac{E}{F} v + \frac{C}{F}} \quad (4.5) \]

Now the second term in 4.5 becomes:

\[ - \frac{E}{2F^2} \int \frac{dv}{v^2 - 2E v F + C F} \]

\[ = - \frac{E}{2F^2} \int \frac{dv}{\left( v - \frac{E}{2F} \right)^2 + \left( \frac{C}{F} - \frac{E^2}{4F^2} \right)} \quad \text{(completing the square)} \]

Let \( \omega = v - \frac{E}{2F} \)

\[ a^2 = \frac{E^2}{4F^2} - \frac{C}{F} \]

\[ - \frac{E}{2F^2} \int \frac{dv}{\left( v - \frac{E}{2F} \right)^2 - \left( \frac{E^2}{4F^2} - \frac{C}{F} \right)} \]

Upon substitution

\[ = - \frac{E}{2F^2} \int \frac{dv}{\omega^2 - a^2} \]

\[ = - \frac{E}{2F^2} \int \frac{1}{2a} \left( \frac{1}{\omega - a} - \frac{1}{\omega + a} \right) \]

\[ = - \frac{E}{4F^2 a} \left[ \ln(\omega - a) - \ln(\omega + a) \right] \]
Back substitution into 4.4 yields

\[= - \frac{1}{2F} \ln \left( v^2 - \frac{Ev}{F} + \frac{c}{F} \right) - \frac{E}{4F^2a} \left[ \ln(\omega - a) - \ln(\omega + a) \right]^{v_2}_{v_1} \]

\[= - \frac{1}{2F} \ln \left( v^2 - \frac{Ev}{F} + \frac{c}{F} \right) - \frac{E}{4F^2a} \left[ \ln(\omega/a) - \ln(\omega/a) \right]^{v_2}_{v_1} \]

\[= - \frac{1}{2F} \ln \left( v^2 - \frac{Ev}{F} + \frac{c}{F} \right) + \frac{E}{4F^2a} \left[ \ln(\omega/a) - \ln(\omega/a) \right]^{v_2}_{v_1} \]

\[= - \frac{1}{2F} \ln \left( v^2 - \frac{Ev}{F} + \frac{c}{F} \right) + \frac{E}{4F^2a} \left[ \ln(\omega/a) - \ln(\omega/a) \right]^{v_2}_{v_1} \]

\[= \left[ \ln \left( \frac{v^2 - \frac{Ev}{F} + \frac{c}{F}}{\omega/a} \right) \right]^{v_2}_{v_1} \]

\[= \left[ \ln \left( \frac{v_2^2 - \frac{Ev_1}{F} + \frac{c}{F}}{v_2^2 - \frac{Ev_2}{F} + \frac{c}{F}} \right) \right]^{v_2}_{v_1} = \Delta T \]

Simplifying by collecting terms

\[= \ln \left( \frac{(\omega_2 + a)(\omega_1 - a)}{(\omega_2 - a)(\omega_1 + a)} \right) \frac{E}{4F^2a} \left( \frac{Fv_1^2 - Ev_1 + c}{Fv_2^2 - Ev_2 + c} \right) \frac{1}{2F} = \Delta T \]

Taking the inverse or anti-log

\[= \ln \left( \frac{\omega_1 \omega_2 + a(\omega_1 - \omega_2) - a^2}{\omega_1 \omega_2 + a(\omega_2 - \omega_1) - a^2} \right) \frac{E}{4F^2a} \left( \frac{Fv_1^2 - Ev_1 + c}{Fv_2^2 - Ev_2 + c} \right) \frac{1}{2F} = e \Delta T \]

(4.6)
Now substituting back the variable values.

\[ \omega_1 = v_1 - \frac{E}{2F} \]

\[ \omega_2 = v_2 - \frac{E}{2F} \]

\[ a^2 = \frac{E^2}{4F^2} - \frac{C}{F} = \frac{E^2}{4F^2} - \frac{4F}{4F^2} C \]

\[ \therefore \quad a = \frac{1}{2F} \sqrt{E^2 - 4FC} \]

Consider

\[ \omega_1 \omega_2 = v_1 v_2 - \frac{E}{2F} (v_1 + v_2) + \frac{E^2}{4F^2} \]

and

\[ \omega_2 - \omega_1 = v_2 - \frac{E}{2F} - v_1 - \frac{E}{2F} = v_2 - v_1 \]

substituting into 4.6 yields

\[ v_1 v_2 - \frac{E}{2F} (v_1 + v_2) + \frac{E^2}{4F^2} - a (v_2 - v_1) - \frac{E^2}{4F^2} + \frac{C}{F} \]

\[ v_1 v_2 - \frac{E}{2F} (v_1 + v_2) + \frac{E^2}{4F^2} + a (v_2 - v_1) - \frac{E^2}{4F^2} + \frac{C}{F} \]

\[ v_1 v_2 - \frac{E}{2F} (v_1 + v_2) - a (v_2 - v_1) + \frac{C}{F} \]

\[ v_1 v_2 - \frac{E}{2F} (v_1 + v_2) + a (v_2 - v_1) + \frac{C}{F} \]

\[ \frac{2Fv_1 v_2 - E (v_1 + v_2) - 2Fa (v_2 - v_1) + 2c}{2Fv_1 v_2 - E (v_1 + v_2) + 2Fa (v_2 - v_1) + 2c} \]

And finally

\[ \left( \frac{2Fv_1 v_2 - E (v_1 + v_2) - 2Fa (v_2 - v_1) + 2c}{2Fv_1 v_2 - E (v_1 + v_2) - 2Fa (v_2 - v_1) + 2c} \right) \left( \frac{Fv_1^2 - Ev_1 + C}{Fv_1^2 - Ev_2 + C} \right) \]

\[ 2F \left( \frac{E}{4F^2a} \right) \]

\[ = e^{(t_2 - t_1)} \]

(4.7)
10.6.2 Viscous Drag Based on $a = \frac{v \, dv}{ds}$

Continuing from Equation 4.9

$$\frac{v^2 \, dv}{(v^2 + \frac{C}{F} - \frac{Ev}{F})} = -F \, ds \quad (4.9)$$

Now the L.H.S. of Equation 4.9 yields

$$\frac{(v^2 + \frac{C}{F} - \frac{Ev}{F} + \frac{Ev}{F} - \frac{C}{F}) \, dv}{(v^2 + \frac{C}{F} - \frac{Ev}{F})}$$

$$= \left(1 + \frac{Ev}{F} - \frac{C}{F} \right) \frac{dv}{(v^2 + \frac{C}{F} - \frac{Ev}{F})}$$

$$= \left(1 + \frac{E}{2F} (2v - \frac{E}{F}) - \frac{C}{F} + \frac{E^2}{2F^2} \right) \frac{dv}{(v^2 + \frac{C}{F} - \frac{Ev}{F})}$$

Substituting into Equation 4.9 yields

$$\left(1 + \frac{E}{2F} (2v - \frac{E}{F}) - \frac{C}{F} + \frac{E^2}{2F^2} \right) \frac{dv}{(v^2 + \frac{C}{F} - \frac{Ev}{F})} = -F \, ds \quad (4.10)$$

Integrating

$$\left[ v + \frac{E}{2F} \ln \left(\frac{v^2 + \frac{C}{F} - \frac{Ev}{F}}{v^2 + \frac{C}{F} - \frac{Ev}{F}} \right) \right]_{v_1}^{v_2} + \left(\frac{E^2}{2F^2} - \frac{C}{F} \right) \int \frac{dv}{(v^2 + \frac{C}{F} - \frac{Ev}{F})}$$

$$= -F(s_2 - s_1) \quad (4.11)$$

Right hand term of Equation 4.11 becomes

$$\left(\frac{E^2}{2F^2} - \frac{C}{F}\right) \int \frac{dv}{(v^2 + \frac{C}{F} - \frac{Ev}{F})}$$

which is similar to R.H.S. of Equation 4.5.

Letting

$$a^2 = \frac{E^2}{4F^2} - \frac{C}{F}$$
and
\[ \omega = v - \frac{E}{2F} \]

the right hand term of Equation 4.11 becomes
\[
\left( \frac{E^2}{2F^2} - \frac{C}{F} \right) \int \frac{dw}{(\omega^2 - a^2)}
= \left( \frac{E^2}{2F^2} - \frac{C}{F} \right) \frac{1}{2a} \ln \left( \frac{1}{\omega - a} \right)
= \frac{1}{2a} \left( \frac{E^2}{2F^2} - \frac{C}{F} \right) \ln \left( \frac{\omega - a}{\omega + a} \right)
\]

Equation 4.11 becomes on substitution
\[
\left[ v + \frac{E}{2F} \ln(v^2 + \frac{C - Ev}{F}) \right]^v_2 + \frac{1}{2a} \left( \frac{E^2}{2F^2} - \frac{C}{F} \right) \ln \left( \frac{\omega - a}{\omega + a} \right) \]^{v_2}_{v_1}
= -F(s_2 - s_1)
\]
i.e.,
\[
\left[ v + \frac{E}{2F} \ln(v^2 + \frac{C - Ev}{F}) + \frac{1}{2a} \left( \frac{E^2}{2F^2} - \frac{C}{F} \right) \ln \left( \frac{\omega - a}{\omega + a} \right) \right]^{v_2}_{v_1}
= -F(s_2 - s_1)
\]

(4.12)

Applying the limits of integration 4.11 becomes
\[
\ln \left[ \frac{(\nu_2^2 + \frac{C}{F} - \frac{Ev_2}{F})}{(\nu_1^2 + \frac{C}{F} - \frac{Ev_1}{F})} \right]^{E}{2F} \left( \frac{(\nu_2-a)(\nu_1+a)}{(\nu_2+a)(\nu_1-a)} \right) \frac{1}{2a} \left( \frac{E^2}{2F^2} - \frac{C}{F} \right)
= -F(s_2 - s_1) - (v_2 - v_1)
\]

(4.13)
Taking the antilog of Equation 4.13 yields
\[
\frac{Fv_2^2 + c - Ev_2}{Fv_1^2 + c - Ev_1} \left( \frac{\omega_2-a}{\omega_2+a} \right) \left( \frac{\omega_1+a}{\omega_1-a} \right) \left( \frac{E}{2F} \right) \left( \frac{1}{2a} \right) \left( \frac{E^2}{2F^2} - \frac{c}{F} \right)
\]

\[= e^{-F(s_2 - s_1) - (v_2 - v_1)}
\]

\[= \frac{e^{-F(s_2 - s_1)}}{e(v_2 - v_1)}
\]

i.e.
\[e(v_2 - v_1) \left( \frac{Fv_2^2 + c - Ev_2}{Fv_1^2 + c - Ev_1} \right) \left( \frac{\omega_2-a}{\omega_2+a} \right) \left( \frac{\omega_1+a}{\omega_1-a} \right) \left( \frac{E}{2F} \right) \left( \frac{1}{2a} \right) \left( \frac{E^2}{2F^2} - \frac{c}{F} \right)
\]

\[= e^{-F(s_2 - s_1)}
\]

(4.14)

Now
\[
\frac{(\omega_2-a)(\omega_1+a)}{(\omega_2+a)(\omega_1-a)} = \frac{w_1\omega_2-a(w_1-w_2)-a^2}{w_1\omega_2+a(w_1-w_2)-a^2}
\]

Similarity of expression between Equations 4.7 to 4.14

\[\omega_1 - \omega_2 = v_1 - v_2
\]

\[\omega_1\omega_2 = v_1v_2 - \frac{E}{2F} (v_1+v_2) + \frac{E^2}{4F^2}
\]

:\. \omega_1\omega_2-a(w_1-w_2)-a^2 becomes
\[
v_1v_2 - \frac{E}{2F} (v_1+v_2) + \frac{E^2}{4F^2} - a(v_1-v_2) - \frac{E^2}{4F^2} + \frac{c}{F}
\]

\[= \frac{2Fv_1v_2 - E(v_1+v_2) - 2Fa(v_1-v_2) + 2c}{2Fv_1v_2 - E(v_1+v_2) + 2Fa(v_1-v_2) + 2c}
\]

Substitution into Equation 4.14 yields
\[
e(v_2-v_1) \left( \frac{Fv_2^2 + c - Ev_2}{Fv_1^2 + c - Ev_1} \right) \left( \frac{2Fv_1v_2 - E(v_1+v_2) - 2Fa(v_1-v_2) + 2c}{2Fv_1v_2 - E(v_1+v_2) + 2Fa(v_1-v_2) + 2c} \right) \left( \frac{E}{2F} \right) \left( \frac{1}{2a} \right) \left( \frac{E^2}{2F^2} - \frac{c}{F} \right)
\]

\[= e^{-F(s_2 - s_1)}
\]

(4.15)
10.7 Publications

During the course of this research programme the opportunity of presenting the progressive results at conferences, etc., was considered appropriate and the following co-authored papers were written.

MONTAGNER, G.J. and TROTT, G.W. "A General Programme for System Identification and Analysis Using a Desk-Top Mini Computer".

ROBERTS, A.W. and MONTAGNER, G.J. "Solids in a Hopper-Discharge Chute System".


ROBERTS, A.W. and MONTAGNER, G.J. "Flow in a Hopper-Discharge Chute System".
The author's involvement in the Creative Design Competition conducted as part of the subject Design I offered by the Department of Mechanical Engineering at the University of Wollongong resulted in the following co-authored papers.

MONTAGNER, G.J. et al. "Student Design Contest".

MONTAGNER, G.J. et al. "Closing University-Industry-Community Gap Through Conceptual Design in First Year Engineering Courses".