Modeling the internal erosion behavior of lignosulfonate treated soil

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Abstract
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Keywords
lignosulfonate, behavior, treated, soil, internal, erosion, modeling

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Modeling the Internal Erosion Behavior of Lignosulfonate Treated Soil

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ABSTRACT: This paper presents the development of an analytical model to predict the erosion rate of stabilised soil based on the law of conservation of energy. The model incorporates the strain energy intensity until failure, obtained from the stress-strain curves of lignosulfonate treated soil, to account for the stabilisation. The novel approach of this model is to correlate the shear strength characteristics with the erosion resistance. The proposed equation was used to predict the erosion rates of a non-plastic silty sand treated with lignosulfonate. The erosion resistance was interpreted in terms of the coefficient of soil erosion and the critical shear stress initiating erosion. It was observed that with the increasing amount of lignosulfonate, the critical shear stress increases and the coefficient of soil erosion decreases. The proposed model predicts the erosion resistant behaviour of lignosulfonate treated soil similar to the laboratory experiments.

INTRODUCTION

Traditional chemical admixtures such as cement, lime, fly ash were often used for stabilising erodible soil commonly found in the world. These chemical admixtures will enhance the inherent properties such as strength and stiffness by altering the mineralogy of the soil resulting in a highly stable soil substance. In the recent past, numerous studies were conducted to investigate the applicability of traditional stabilisers on erodible and problematic soils such as soft clay and erodible soils (e.g. Balasubramaniam et al. 1989; Indraratna et al. 1995; Rajasekaran et al. 1997; Uddin et al. 1997; Chew et al. 2004). However, such traditional admixtures (i.e. cement, lime, fly ash etc.) are not frequently used because of the various threats to the environment attributed to inadvertent increase in soil and ground water alkalinity. Moreover, traditionally stabilised soil has a pH of 9 (Rollings et al. 1999), which often affects the longevity of concrete reinforcements and steel frame structures (e.g. Biggs and Mahony 2004; Perry 1977). On the other hand, the traditionally treated soil exhibits excessive brittle performance (e.g. Sariosseiri and Muhunthan 2009) that affects the stability of structures, especially during cyclic and impact loading from high speed rail and aircraft runways. In this context, it is necessary to find out an alternative soil stabilizer, which could provide sustainable soil improvement without harming the environment. Recently, lignosulfonate, a lignin based chemical, has shown promising aspects in stabilising the erodible and problematic soils (Puppala and Hanchanloet 1999; Pengelly et al. 1997; Tingle and Santori 2003;
Indraratna et al. 2008, Vinod et al. 2010). A number of research studies have been conducted on low volume road construction to investigate the performance of lignosulfonate for improving the strength behaviour of sub-grade and also for effective control of dust emission (e.g. Chemstab 2003; Tingle and Santori 2003; Lohnes and Coree 2002). Lignosulfonate belongs to a family of lignin based organic polymers derived as a waste by-product from wood and paper processing industry. Compared to highly alkaline and sometimes corrosive chemical admixtures, lignosulfonate is an environmental friendly, non-corrosive and non-toxic chemical that does not alter the soil pH upon treatment. Indraratna et al. (2008) conducted research on internal erosion behaviour of lignosulfonate treated dispersive soils using a novel Process Simulation Apparatus for Internal Crack Erosion (PSAICE). They concluded that the lignosulfonate treatment improves the erosion resistance of the treated soil similar to traditional admixtures such as lime and cement. In addition, Indraratna et al. (2009) developed an erosion model to calculate the mass erosion rate taking the tensile strength characteristics in to account. However, since tensile strength tests are suitable only for cohesive soils, it is essential to correlate the widely used shear strength parameters with erosion resistance. Therefore, the main focus of this paper is to explain an analytical model developed to capture the erosion behaviour of chemically treated erodible soil incorporating well-known shear strength characteristics.

**EROSION MODEL FOR LIGNOSULFONATE STABILIZED SOIL**

The formulation of the proposed erosion model is based on the principle of conservation of energy. It is assumed that rolling of particles, caused by the hydrodynamic forces, is the predominant mechanism by which the particles are detached from the soil bed. When the hydrodynamic forces acting on the soil particles exceed a critical value, inter-particle bonds are broken and the particles begin to move. These particles are then suspended, and transported by the eroding fluid flow. The energy required by soil particles to erode, sum of the energy required to break the inter-particle bonds and make particles suspended, should be taken as to be equal to the energy dissipated by the excess hydraulic shear stress during erosion.

**Energy required to break inter-particle bonds**

Figure 1 illustrates the applied and resistance forces on a single inter-particle bond under plane shear condition. $F_{ni}$ (N) and $F_{si}$ (N) are the applied normal and shear forces due to shearing, $S_i$ (N) and $N_i$ (N) are the mobilized shearing and normal resistances at the contact point. $\delta_i$ (m) and $\delta_i$ (m) are the displacements of $i^{th}$ contact along and normal to the average failure surface respectively and $\delta l_i$ (m) is the displacement along the failure surface of $i^{th}$ contact.
The total work done by the applied normal and shear forces should be equal to the energy required to break the inter-particle bonds and for overcoming the frictional resistance. Hence,  

\[ F_i \delta x_i - F_{ni} \delta y_i = \delta E_{bi} + F_i \delta d_i \]  

(1)

where \( E_{bi} \) (J) is the energy required to break a single inter-particle bond and \( F_i \) (N) is the frictional resistance between the soil grains which can be written as \( \mu_i N_i \) considering no cohesion between grains after inter-particle bond is broken. \( \mu_i \) is the coefficient of friction between particles. Now substituting for the relevant terms in Equation (1) from the relationships shown in Figure 1, \( E_{bi} \) is given by,

\[
E_{bi} = \frac{1}{n} \left\{ \int_0^{d_s} \tau' (d_s) \left[ (1 - \mu_i \tan \beta) - \sigma_{n}'(\tan \beta + \mu_i) \right] \right\} 
\]

(2)

where \( n \) is the number of inter-particle bonds in a unit surface area, \( d_s \) (m) is the measured shear displacement at failure along the average shear surface, \( \tau' \) (Pa) is the measured shear resistance \( \sigma_{n}' \) (Pa) is the applied effective normal stress and \( \beta \) (deg) is the angle of reference particle to the vertical with respect to the contact particle (Figure 1).

To calculate the energy required for a single soil grain, \( E_b \) (J), to break the inter-particle bonds is calculated by multiplying \( E_{bi} \) by the average number of contacts per particle which can be expressed as \( nqD^2 \). \( q \) is a constant that depends on the packing arrangement of particles and \( D \) (m) is the mean particle diameter. The shear
displacements were converted into shear strains and zero effective normal stress ($\sigma_{n}'=0$) state was considered to ensure the real field conditions for erosion. Subsequently, $E_b$ can be expressed by (Indraratna et al. 2012):

$$E_b^* = qD^2 \frac{h_0}{100} \left[ \int_0^{\gamma_f} \tau d(\gamma) \right]^{*} \left( 1 - \frac{2}{3} \sin \phi' \tan \beta \right)$$

(3)

where $h_0$ (m) is the initial height of the soil specimen used in the shear tests, $\gamma$ (%) is the shear strain and $\gamma_f$ (%) is the shear strain at failure. The superscript * was used to indicate the terms influenced by lignosulfonate stabilization. $\mu_i$ was replaced by a correlation between $\mu_i$ and $\phi'$ developed by Bishop (1954) where $\phi'$ (deg) is the angle of internal friction of soil. The term $\int \tau d\gamma$ in Equation (3) is the strain energy density (J/m$^3$) due to shearing (area under the shear stress-strain curve) up to the failure at $\sigma_{n}'=0$.

**Energy Required to Bring the Particles into Suspension**

The energy required for a particle to come into suspension is formulated by considering its movement as two distinct processes; (i) it will roll on the contact particle until the contact is lost, (ii) it will be lifted while moving with the fluid until it attains the same velocity as the fluid flow.

![Figure 2. (a) Reference particle and (b) velocities of the reference particle](image-url)

Figure 2. (a) Reference particle and (b) velocities of the reference particle

A single reference particle was considered at an angle $\theta$ (deg) [Figure 2 (a)] and the velocities [angular velocity $\omega_p$ (rad/s), tangential velocity $V_t$ (m/s) and radial velocity $V_r$]
of the reference particle due to hydrodynamic forces are shown in Figure 2 (b). Considering the equations of motion, $V_t$ was obtained as:

$$V_t^2 = \frac{5D}{7m} \left\{ F_D [\psi + \sin \beta - \sin(\beta - \psi)] + (W_S \cos \theta - F_L) [\cos \beta - \cos(\beta - \psi)] \right\}$$

(4)

where $m$ (kg) is the mass of the soil grain, $F_D$ (N) and $F_L$ (N) are drag and lift forces acting on the grain respectively, $W_S$ (N) is the weight of the grain and $\psi$ (deg) is the angle by which the reference particle has rolled. At the instant $\psi = \psi_{max}$, the moving particle leaves the contact surface of the other particle and then it will be suspended in the water flow and will attain the same flow velocity, $V_f$ (m/s). The energy ($E_1$) required for a single particle to roll on the contact particle was found to be:

$$E_1 = \frac{7}{10} m \left[ V_t^2 \right]_{\psi = \psi_{max}}$$

(5)

by applying the equations of motion for tangential movement and for angular movement. The energy ($E_2$) required in the suspension stage was calculated to be:

$$E_2 = \frac{1}{2} m \left\{ V_f^2 - \left[ V_t^2 \right]_{\psi = \psi_{max}} \right\}$$

(6)

by applying the equations of motion in tangential and normal directions to the soil bed. Sum of Equation (5) and (6) is the energy required by a single soil particle, $E_S$ (J) for this whole process expressed as:

$$E_S = m \left( \frac{\left( V_t^2 \right)_{\psi = \psi_{max}}}{5} + \frac{V_f^2}{2} \right)$$

(7)

Now, the total energy required by a single soil grain, at an angle $\theta$, to be eroded can be calculated by summing up the Equation (3) and (7).

**Rate of Erosion**

Figure 3 shows the change in the radius of the crack of a specimen subjected to erosion test by $dr_t$ at a time interval of $dt$. It is assumed that the radius of the crack is changing linearly over time and that erosion is uniform over the crack surface.
Figure 3. Change in crack size caused by erosion

Now, considering an infinitesimal volume \((dv)\) of soil, the total energy \((E_T)\) required by soil particles in a volume of \(2\pi r_t \cdot dr_t\) can be expressed as (Indraratna et al. 2012):

\[
E_T = \frac{12 \rho_d}{G_s \rho_w \pi D^3} r_t \int_0^\pi \left( E'_b + E_S \right) d\theta
\]  \(8\)

where \(\rho_d (\text{kg/m}^3)\) and \(G_s\) are the dry density and the specific gravity of soil respectively, \(\rho_w (\text{kg/m}^3)\) is the density of water \(r_t (\text{m})\) is the radius of the crack at time \(t (\text{s})\) and \(l (\text{m})\) is the length of the crack. The energy dissipated by excess hydraulic shear stress, \(\Delta E'\) (J) during erosion in time duration \(dt\), is given by Muttuvel (2008) as:

\[
\Delta E' = \omega (\tau_a - \tau_c) V_j 2\pi r_t l dt
\]  \(9\)

where \(\omega\) is the efficiency factor used to capture the energy loss due to heat and noise, \(\tau_a\) (Pa) is the hydraulic shear stress applied and \(\tau_c\) (Pa) is the critical shear stress that initiates erosion. According to the conservation of energy, the energy dissipated by excess hydraulic shear stress during erosion \((\Delta E')\) should be equal to the energy taken by the particles for erosion, \(E_T\) (J). Therefore, Equation (6) and (7) were evaluated for \(dr_t/dt\) and multiplied by \(\rho_d\) to obtain the rate of erosion \([\varepsilon (\text{kg/s/m}^2)]\), i.e., the amount of soil eroded in unit time over a unit surface area (Muttuvel 2008). Then, the term \((E'_b + E_S)\) was substituted from Equations (3), (4) and (7), and simplified to calculate the erosion rate of stabilized soil, \((\varepsilon)\), as:
\[ (\dot{e})^* = \frac{G_s \rho_w \omega V_f}{6q} \left( 1 - \frac{2}{3} \sin\phi^* \tan\beta \right) \frac{G_s \rho_w V_f^2}{12} \left( \tau_c - \tau_c^* \right) \]  

where \( A^* \) replaces the term \( \frac{h_0}{100} \left[ \int_0^{r_c} \tau d(y) \right] \) in Equation (3). \( A^* \) is a property of soil that consists of the area under the shear stress-strain curve up to the failure (at \( \sigma_n^* = 0 \)) and the initial height of the specimen used for the shear test. It is influenced by the lignosulfonate treatment and expects to be increased with the increasing amount of lignosulfonate providing higher resistance to applied hydraulic stresses.

**MODEL ASSESSMENT**

The predictions of the proposed model (Equation 8) were assessed for different model parameters before validating it with erosion test results. Figure 4 illustrates the model prediction results for different \( A^* \) and \( \tau_c \) values corresponding to different amounts (percentage by dry soil weight) of lignosulfonate. Due to lignosulfonate treatment, the strain energy intensity is expected to be increased and hence increases \( A^* \). It was observed by Indraratna et al. (2008) that \( \tau_c \) is increased with the increased amount of chemical admixture. An expression for efficiency factor (\( \omega \)) was found (Indraratna et al. 2012) as:

\[ \omega = \frac{9.5}{(\tau_c^* A^*) V_f^{0.8}} \]  

The properties of the soil and lignosulfonate used in this study are given in Table 1. Other model parameters used in this assessment are; \( \rho_w = 1000 \text{kg/m}^3, D = 150 \mu \text{m}, \phi^* = 58^\circ, q = 0.886 \) and \( \beta = 30^\circ \). Values of \( q \) and \( \beta \) are corresponding to a dense packing arrangement which has a porosity of 31% (Gray 1968).
Table 1 Properties of materials used in the study

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>Lignosulfonate</td>
<td>Soil</td>
<td>Lignosulfonate</td>
</tr>
<tr>
<td>Liquid limit</td>
<td>22.5%</td>
<td>Appearance</td>
<td>Dark brown liquid</td>
</tr>
<tr>
<td>Plasticity index</td>
<td>Non-plastic</td>
<td>Gs</td>
<td>1.2 approx.</td>
</tr>
<tr>
<td>Gs</td>
<td>2.67</td>
<td>pH</td>
<td>3.8 approx.</td>
</tr>
<tr>
<td>MDD</td>
<td></td>
<td>Solubility in water</td>
<td>Completely soluble</td>
</tr>
<tr>
<td>OMC</td>
<td>11.6%</td>
<td>Other</td>
<td>Non-flammable</td>
</tr>
<tr>
<td>Percent dispersion (percent dispersion test)</td>
<td>44%</td>
<td></td>
<td>Non-toxic</td>
</tr>
<tr>
<td>Erodibility (standard pinhole test)</td>
<td>D1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classification (USCS)</td>
<td>SM</td>
<td></td>
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</tr>
</tbody>
</table>

It is clear from Figure 4 that the coefficient of soil erosion, i.e. the slope of the erosion rate versus hydraulic shear stress, is decreased due to increase of the amount of lignosulfonate. The same behavior was observed by Indraratna et al. (2008) for chemically treated soils. Therefore, this assessment confirms that the model is capable of capturing the improvement in erosion resistance due to lignosulfonate treatment.

![Figure 4. Predicted erosion rates for lignosulfonate treated silty sand](image-url)
CONCLUSIONS

A theoretical model was proposed to predict the erosion rate of lignosulfonate treated soil correlating the shear strength characteristics with erosion parameters. The formulation was based on the law of conservation of energy. The parameter $A^*$ derived from the area under the stress-strain curve was introduced to account for the lignosulfonate treatment. Erosion rates for different values of $A^*$ and $\tau^*$ corresponding to different amounts of lignosulfonate were predicted from the proposed equation and plotted against the hydraulic shear stress to assess the erosional behavior of lignosulfonate treated soil. The graphs clearly illustrated that the coefficient of soil erosion is decreasing with the increasing amount of lignosulfonate. Therefore, the assessment of model predictions verified that the model was capable of simulating the erosional behavior of lignosulfonate treated soils as observed from experimental studies.

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