Simultaneous Estimation of Primary and Cross-Channel Gains for Underlay Cognitive Radios

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Abstract
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Simultaneous Estimation of Primary and Cross-Channel Gains for Underlay Cognitive Radios

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ABSTRACT In underlay cognitive radios, the primary-channel gain from a primary transmitter to a primary receiver (PU-Rx) and the cross-channel gain from a cognitive transmitter (SU-Tx) to the PU-Rx are crucial for the SU-Tx in controlling its transmit power when sharing spectrum with primary users (PUs). However, the non-cooperation between PUs and secondary users (SUs) makes it difficult for the SU-Tx to acquire the primary-channel and cross-channel gains, in particular when the channel reciprocity is not applicable. Most existing works focus on cross-channel gain estimation and are often based on the channel reciprocity. In this paper, we propose a scheme where the SU-Tx relays primary signals and transmits its own signals to the secondary receiver during the phase of channel gain estimation. Then, an estimation method is developed, which enables the SU-Tx to acquire the primary-channel and cross-channel gains simultaneously. The performance of the proposed estimator is analyzed. Numerical simulations are provided to validate the theoretical analysis and demonstrate the superior performance of the proposed method compared with state-of-the-art methods.

INDEX TERMS Cognitive radio, channel gain estimation, spectrum sharing.

I. INTRODUCTION Cognitive radio (CR) has been recognized as one of the promising technologies to tackle the issue of spectrum scarcity in wireless communications [1], [2]. In CR, secondary users (SUs) share the licensed spectrum of primary users (PUs), leading to improved spectral efficiency. There are mainly three spectrum sharing modes: interweave, overlay and underlay [3], [4]. In interweave mode, SUs are allowed to access the spectrum only when PUs are inactive, and SUs should vacate the spectrum when PUs return [5]. Hence, the spectral efficiency will not be improved when PUs are active. In overlay and underlay modes, SUs are allowed to transmit with PUs simultaneously. In overlay mode, SUs employ sophisticated signal processing and coding techniques to mitigate the interference due to simultaneous transmission with PUs [6]. However, in this mode, SUs require additional information of primary signals which are transmitted by PUs. In underlay mode, SUs access the licensed spectrum without any information of primary signals, if the signal to interference plus noise ratio (SINR) at the PUs is kept above a given level to guarantee the performance of PU communications [7].

Underlay CRs have attracted much attention due to its high spectral efficiency and requiring no information of primary signals [8]–[11]. In the existing works, the primary-channel gain from a primary transmitter (PU-Tx) to a primary receiver (PU-Rx) and the cross-channel gain from a cognitive transmitter (SU-Tx) to the PU-Rx are usually assumed to be known by the SU-Tx [8]–[11]. It is reasonable to assume the cross-channel gain to be known, when PUs employ the time division duplex (TDD) technique [12]. In this case, cross-channel gain can be estimated at SU-Tx based on the channel reciprocity [13]. However, when PUs employ the frequency division duplex (FDD) technique, a backhaul link from PUs to SUs is required to feedback the cross-channel gain [14]. As for acquiring the primary-channel gain at
SU-Tx, a backhaul link from PUs to SUs is often required no matter whether TDD or FDD is employed. However, PUs and SUs usually do not work cooperatively, and a backhaul link can be unavailable. This renders a very difficult problem for the SU-Tx to acquire the primary-channel and cross-channel gains.

To solve the above problem, several methods were proposed for estimating either the primary-channel gain or the cross-channel gain. In [15], assuming an constant phase difference between the small-scale fading channels, the cross-channel gain is estimated by probing the transmit power levels of the PU-Tx before and after SU relay, where the SU-Tx does not transmit its own signals during the estimation process. However, estimating the phase difference between the small-scale fading channels with a bisector searching algorithm requires several time slots, which limits the application because its actual phase difference may change. In [16], given the primary-channel gain, an SU-Tx estimates the cross-channel gain using its signal-to-noise ratios (SNRs) before and after SU relay. Rather than relaying the primary signals, in [17], an SU-Tx transmits probing signals with different power levels and observes the SNR variation of PU-Tx. Then, using the observed SNRs and the characteristic of channel reciprocity, the cross-channel gain is estimated at the SU-Tx. Without the channel reciprocity but having the knowledge of the primary-channel gain, an active probing scheme was proposed for the SU-Tx to estimate the cross-channel gain in [18]. To avoid performance degradation due to deep shadowing between PU-Tx and SU-Tx, in [19], multiple cooperative SU-Rxs are deployed to assist the SU-Tx in estimating the cross-channel gain. However, this estimation also requires the knowledge of primary-channel gain. It is noticed that the primary-channel gain is required by most cross-channel gain estimators. In [12], a primary-channel gain estimator was proposed by assuming that the small-scale channels from PU-Tx to PU-Rx and SU-Tx have an identical probability density function. To the best of our knowledge, only the work in [12] has dealt with the estimation of the primary-channel gain.

When the channel reciprocity is not applicable for a primary system with FDD, and primary-channel gain is unknown, the existing cross-channel gain estimators will fail. Moreover, the existing primary-channel gain estimator will not work when the small-scale channels from PU-Tx to PU-Rx and SU-Tx have different probability density functions. To deal with this issue, in this work, we propose a new estimation method to simultaneously estimate the primary channel gain and the cross-channel gain. Moreover, the theoretical performance of the proposed estimator is analyzied. Different from existing methods, in our proposed method, the SU-Tx relays the primary signals and transmits its own signals to SU-Rx as well during the phase of channel gain estimation. This can further improve the spectrum efficiency. Numerical simulations are provided to validate the theoretical analysis and demonstrate the superior performance of the proposed estimator.

The remainder of this paper is organized as follows. The signal model is introduced in Section II. In Section III, we propose a new estimation method to simultaneously estimate the primary channel gain and the cross-channel gain, and the theoretical performance for the proposed estimator is derived. Simulation results are presented in Section IV and concluding remarks are made in Section V.

II. SYSTEM MODEL

Consider a scenario as shown in Fig. 1 where an SU link consisting of an SU transmitter (SU-Tx) and an SU receiver (SU-Rx) shares a narrow-band channel with a PU link consisting of a PU transmitter (PU-Tx) and a PU receiver (PU-Rx). Besides transmitting signals to SU-Rx, the SU-Tx relays primary signals from the PU-Tx to the PU-Rx by employing the amplify-and-forward (AF) scheme. To enable the SU-Tx to serve as an AF relay, a full-duplex antenna or two antennas (with one for reception and the other for transmission) are adopted. Both the small-scale fading and the large-scale fading are considered for the channels between the transmitters (PU-Tx and SU-Tx) and receivers (PU-Rx and SU-Rx). We assume that the small-scale fading is time-invariant, while the large-scale fading is time-invariant. Denote by \( h_0(t) \) the channel coefficients between the transmitters and the receivers, where \( h_0(t) \) and \( g_k \) represent the small-scale and large-scale fading, respectively. Specifically, as shown in Fig. 1, \( h_0(t) \sqrt{g_0} \) is the channel from PU-Tx to PU-Rx, \( h_1(t) \sqrt{g_1} \) from PU-Tx to SU-Tx, \( h_2(t) \sqrt{g_2} \) from SU-Tx to PU-Rx, \( h_3(t) \sqrt{g_3} \) from PU-Tx to SU-Rx and \( h_4(t) \sqrt{g_4} \) from SU-Tx to SU-Rx. We assume that \( h_k(t) \) follows complex a Gaussian distribution, i.e., the small-scale fading is Rayleigh. Without loss of generality, we let \( \mathbb{E}[|h_k^2(t)|] = 1 \) by absorbing its gain into \( g_k \). The log-normal shadowing model is adopted to denote the large-scale fading channels \( \{g_k, \forall k\} \). By considering that PU and SU operate at a carrier frequency over 2 GHz, and adopting the path loss model in [16] and [23], \( g_k \) in dB is given by

\[
g_k(d_k)[\text{dB}] = -128 - 37.6 \log_{10}(d_k) + X_0, \quad d_k \geq 0.035 \text{ km}
\]

FIGURE 1. The system model which consists of a PU transmitter (PU-Tx), a PU receiver (PU-Rx), an SU transmitter (SU-Tx) and an SU receiver (SU-Rx).
where $d_k$ denotes the distance from a transmitter to a receiver and $X_k$ follows a real Gaussian distribution with mean zero and standard deviation $\sigma$. In the right-hand side of (1), the first two terms represent the log-distance path loss, and the last term represents the log-normal shadowing. We also assume that the channel coefficients $h(t,k), g_k, \forall k$ are independent of each other. The knowledge of the gains $\{g_k, \forall k\}$ are important for SU-Tx to improve the performance of the SU link without degrading the performance of the PU link, by appropriately allocating its power between relay amplifier and self-transmission. The gain $g_1$ can be estimated at the SU-Tx by receiving the signal from PU-Tx, and the gains $g_2$ and $g_4$ can be estimated at the SU-Rx and these two estimates can be forwarded to the SU-Tx through a backhaul link from the SU-Rx to the SU-Tx. However, it may be challenging for the SU-Tx to obtain the gains $g_0$ and $g_2$ from the PU-Rx, as a backhaul link from the PU-Rx to the SU-Tx is usually unavailable. In this work, we aim to design a scheme to enable the SU-Tx acquire $g_0$ and $g_2$ simultaneously.

Let $\sqrt{p_0}s_0(t)$ be the transmitted signal from the PU-Tx to the PU-Rx where $p_0$ represents the transmission power of the PU-Tx. Let $\sqrt{p_1}s_1(t)$ be the transmitted signal from the SU-Tx to the SU-Rx where $p_1$ represents the transmission power. Assume that $s_0(t)$ is independent of $s_1(t)$ if a full-duplex AF relay technique is employed at the SU-Tx, some residual self-interference exists because of imperfect cancelation [20]–[22]. Let $w_r(t)$ be the residual self-interference at the SU-Tx, and it can be modelled to be complex Gaussian distributed with mean zero and variance $\sigma_w^2$ [20]–[22]. As shown in [20]–[22], the self-interference at the SU-Tx can almost be cancelled to the noise floor, and a fixed power $\sigma_w^2$ of $w_r(t)$ is often assumed, as long as the transmit power of SU-Tx does not exceed 20 dBm. Thus, the received signal at SU-Tx is given by

$$x_{SU-Tx}(t) = h_1(t)\sqrt{g_1p_0}s_0(t) + w_1(t) + w_r(t)$$

(2)

where $w_1(t)$ denotes the noise at the SU-Tx. We assume that $w_1(t)$ is independent and identically Gaussian distributed with mean zero and variance $\sigma_w^2$. The noise power model in dBm is given by [24]

$$\sigma_w^2[\text{dBm}] = N_0 + 10\log_{10} B + F$$

(3)

where $N_0$ denotes the noise power spectral density, $B$ denotes the system bandwidth in Hz and $F$ denotes the noise figure in dB. For notation simplification, we let $\sigma_w^2 = r\sigma_w^2$. The SU-Tx forwards the received signal $x_{SU-Tx}(t)$ to PU-Rx with an amplitude gain $G$, and transmits $s_1(t)$ to SU-Rx simultaneously. Hence, the received signal at the PU-Rx is given by

$$x_{PU-Rx}(t) = h_0(t)\sqrt{g_0p_0}s_0(t) + Gx_{SU-Tx}(t) + w_0(t) + h_3(t)\sqrt{g_2p_1}s_1(t)$$

(4)

$$= h_0(t)\sqrt{g_0p_0}s_0(t) + Gh_1(t)h_2(t)\sqrt{g_1g_2p_0}s_0(t) - \tau + Gh_2(t)\sqrt{g_2}(w_1(t) + w_r(t)) + w_0(t)$$

(5)

Hence, the SINR of $x_{PU-Rx}(t)$ is given by

$$\gamma_{PU-Rx} = \frac{E[h_0(t)\sqrt{g_0p_0} + Gh_1(t)h_2(t)\sqrt{g_1g_2p_0}]}{G^2g_2(1 + r) + 1}\sigma_w^2 + g_2p_1$$

(6)

Similarly, the received signal at the SU-Rx is given by

$$x_{SU-Rx}(t) = h_3(t)\sqrt{g_3p_0}s_0(t) + Gx_{SU-Tx}(t) + w_3(t) + h_4(t)\sqrt{g_5}s_1(t)$$

(7)

where $w_3(t)$ denotes the noise at SU-Rx and it is assumed to be Gaussian distributed with zero and variance $\sigma_w^2$. The SINR of $x_{SU-Rx}(t)$ is given by

$$\gamma_{SU-Rx} = \frac{g_4p_1}{G^2g_4(1 + r) + 1}\sigma_w^2 + g_3p_0 + G^2g_1g_4p_0$$

(8)

As in [15]–[19], we assume that the close loop power control (CLCP) strategy is employed by the PU link, so that the SINR at the PU-Rx is maintained at a target level $\gamma_t$ by adjusting the transmit power $p_0$. In other words, the PU link maintains

$$\gamma_{PU-Rx} = \gamma_t$$

(9)

by automatically adjusting $p_0$ according to the values of $\gamma_{PU-Rx}$ and $\gamma_t$. Specifically, the PU-Tx increases the transmit power $p_0$ when $\gamma_{PU-Rx}$ is smaller than $\gamma_t$, otherwise, the PU-Tx decreases the transmit power $p_0$.

### III. GAIN ESTIMATION AND PERFORMANCE ANALYSIS

In this section, we propose to estimate the channel gains $g_0$ and $g_2$ by exploiting the CLCP strategy of the PU link. The proposed estimator is expressed with closed-form expressions. We also analyze the performances of the estimators by deriving the statistical distributions of the estimates.

#### A. CHANNEL GAIN ESTIMATION

Substituting (9) into (6) gives

$$p_0 = \frac{\gamma_t}{g_0 + G^2g_1g_2} + \frac{\gamma_t}{g_0 + G^2g_1g_2}p_1.$$  

(10)

By letting

$$a = \frac{\gamma_t}{g_0 + G^2g_1g_2}$$

(11)

where $\tau$ denotes the time-delay difference between the direct and the relay links, and $w_1(t)$ denotes the noise at the PU-Rx.
and

\[ b = \frac{\gamma_1 g_2}{g_0 + G^2 g_1 g_2}, \]  

(12)

Eq. (10) can be rewritten as

\[ p_0 = a + bp_1 \]  

(13)

which shows that \( p_0 \) is a linear function of \( p_1 \) with unknown coefficients \( a \) and \( b \). The equation in (13) holds because of the CLCP strategy at the PU link for different \( p_1 \), as long as \( a + bp_1 \) does not exceed the maximum transmit power of PU-Tx. In other words, the transmit power \( p_0 \) at PU-Tx varies with the power \( p_1 \) of \( s_1(t) \) at the SU-Tx. Then, we will obtain the estimates of the coefficients \( a \) and \( b \), if several pairs of \((p_0, p_1)\) are obtained by changing \( p_1 \). Here, the power \( p_1 \) is under control of SU-Tx and it can be changed easily. Assume that the \( j^{th} \) \((j = 1, \ldots, J)\) transmitted power of \( s_1(t) \) is given by \( p_1(j) \), and the \( j^{th} \) transmit power \( p_0(j) \) of PU-Tx can be estimated at the SU-Tx as

\[ \hat{p}_0(j) = a + bp_1(j) + e(j), \quad j = 1, \ldots, J \]  

(15)

where \( e(j) \) denotes the estimation error of \( p_0(j) \). According to the central limit theorem with a sufficiently large \( M \), it can be obtained from (14) that the error \( e(j) \) follows Gaussian distribution with mean zero. So the maximum likelihood estimates of \( a \) and \( b \) can be obtained by minimizing the square sum of \( e(j) \). The square sum of \( e(j) \) is given by

\[ Q = \sum_{j=1}^{J} e^2(j) = \sum_{j=1}^{J} (\hat{p}_0(j) - a - bp_1(j))^2. \]  

(16)

Letting the partial derivatives of \( Q \) with respect to \( a \) and \( b \) be zero, we get

\[
\begin{align*}
\frac{\partial Q}{\partial a} &= -2 \sum_{j=1}^{J} (\hat{p}_0(j) - a - bp_1(j)) = 0, \\
\frac{\partial Q}{\partial b} &= -2 \sum_{j=1}^{J} (\hat{p}_0(j) - a - bp_1(j)p_1(j) = 0.
\end{align*}
\]

(17)

Solving the equations in (17) gives

\[
\begin{align*}
\hat{a} &= \frac{\sum_{j=1}^{J} p_1^2(j) \sum_{j=1}^{J} \hat{p}_0(j) - \sum_{j=1}^{J} p_1(j) \sum_{j=1}^{J} p_1(j) \hat{p}_0(j)}{J \sum_{j=1}^{J} p_1^2(j) - \left( \sum_{j=1}^{J} p_1(j) \right)^2}, \\
\hat{b} &= \frac{J \sum_{j=1}^{J} p_1(j) \hat{p}_0(j) - \sum_{j=1}^{J} p_1(j) \sum_{j=1}^{J} \hat{p}_0(j)}{J \sum_{j=1}^{J} p_1^2(j) - \left( \sum_{j=1}^{J} p_1(j) \right)^2}.
\end{align*}
\]

(18)

Note that multiple estimate pairs \((\hat{a}, \hat{b})\) will be obtained if multiple values of the amplifier \( G \) are employed. Under this case, it can be easily obtained from (11) and (12) that the target SINR \( \gamma_1 \) can be estimated, if it is unknown at SU-Tx. However, for simplicity in this paper, we assume that \( \gamma_1 \) is known at SU-Tx. This is because the value of \( \gamma_1 \) is usually publicly available knowledge [16]. Combining (11), (12) and (18) gives

\[
\begin{align*}
\hat{g}_0 &= \frac{\gamma_1 g_1^2 - \hat{b} G^2 g_1 g_2}{\hat{a} - \hat{b} G^2 (1 + r) a^2}, \\
\hat{g}_2 &= \frac{\hat{g}_0^2}{\hat{a} - \hat{b} G^2 (1 + r) a^2}.
\end{align*}
\]

(19)

B. PERFORMANCE ANALYSIS

In this subsection, we analyze the probability density functions (PDFs) of the estimates \( \hat{g}_0 \) and \( \hat{g}_2 \). According to the central limit theorem with a sufficiently large \( M \), we obtain that \( \hat{p}_0(j) \) follows Gaussian distribution with mean and variance being, respectively, given by

\[
\begin{align*}
\mathbb{E}[\hat{p}_0(j)] &= p_0(j), \\
\mathbb{V}[\hat{p}_0(j)] &= \frac{3 g_1^2 p_0^2(j) + 4 g_1 p_0(j)(1 + r) \sigma_w^2 + (1 + r) \sigma_r^2}{M g_1^2}.
\end{align*}
\]

(20)

i.e.,

\[ \hat{p}_0(j) \sim \mathcal{N} \left( \mathbb{E}[\hat{p}_0(j)], \mathbb{V}[\hat{p}_0(j)] \right). \]

(21)

By considering that the estimates \( \{\hat{p}_0(j), \forall j\} \) are independent of each other, it can be obtained that \( \hat{a} \) and \( \hat{b} \) are also Gaussian distributed. The expectation of \( \hat{a} \) is given by

\[
\mathbb{E}[\hat{a}] = \frac{\sum_{j=1}^{J} p_1^2(j) \sum_{j=1}^{J} \hat{p}_0(j) - \sum_{j=1}^{J} p_1(j) \sum_{j=1}^{J} p_1(j) \hat{p}_0(j)}{J \sum_{j=1}^{J} p_1^2(j) - \left( \sum_{j=1}^{J} p_1(j) \right)^2},
\]

(22)

and the variance of \( \hat{a} \) is given in (23) at the top of the next page. The expectation of \( \hat{b} \) is given by

\[
\mathbb{E}[\hat{b}] = \frac{J \sum_{j=1}^{J} p_1(j) \hat{p}_0(j) - \sum_{j=1}^{J} p_1(j) \sum_{j=1}^{J} \hat{p}_0(j)}{J \sum_{j=1}^{J} p_1^2(j) - \left( \sum_{j=1}^{J} p_1(j) \right)^2},
\]

(24)

and the variance of \( \hat{b} \) is given in (25) at the top of the next page.

Let

\[
\begin{align*}
\hat{g}_0 &= \frac{\hat{g}_{0w}}{\hat{g}_d}, \\
\hat{g}_2 &= \frac{\hat{g}_{2w}}{\hat{g}_d},
\end{align*}
\]

(26)
where the numerators (\(\hat{g}_{0,n}\) and \(\hat{g}_{2,n}\)) and the denominator \(\hat{g}_d\) are given by
\[
\hat{g}_{0,n} = \gamma_n \sigma_w^2 - \hat{b} G_1 \sigma_w^2, \quad \text{(27)}
\]
\[
\hat{g}_{2,n} = \hat{b} \sigma_w^2, \quad \text{(28)}
\]
and
\[
\hat{g}_d = \hat{a} - \hat{b} G_2 (1 + r) \sigma_w^2. \quad \text{(29)}
\]

Because \(\hat{a}\) and \(\hat{b}\) are Gaussian distributed, the numerators and the denominator are also Gaussian distributed. The expectation and the variance of \(\hat{g}_{0,n}\) are given by
\[
\begin{align*}
\mathbb{E}[\hat{g}_{0,n}] &= \gamma_n \sigma_w^2 - G_2 \hat{g}_1 \sigma_w^2 \mathbb{E}[\hat{b}], \\
\mathbb{V}[\hat{g}_{0,n}] &= G^2 \mathbb{V}[\hat{g}_{w}].
\end{align*}
\quad \text{(30)}
\]

The expectation and the variance of the numerator \(\hat{g}_{2,n}\) are given by
\[
\begin{align*}
\mathbb{E}[\hat{g}_{2,n}] &= \sigma_w^2 \mathbb{E}[\hat{b}], \\
\mathbb{V}[\hat{g}_{2,n}] &= \sigma_w^2 \mathbb{V}[\hat{b}].
\end{align*}
\quad \text{(31)}
\]

Moreover, we can obtain that the expectation and the variance of the denominator of \(\hat{g}_d\) are
\[
\begin{align*}
\mathbb{E}[\hat{g}_d] &= \mathbb{E}[\hat{a}] - G^2 (1 + r) \sigma_w^2 \mathbb{E}[\hat{b}], \\
\mathbb{V}[\hat{g}_d] &= \mathbb{V}[\hat{a}] + G^2 (1 + r)^2 \sigma_w^2 \mathbb{V}[\hat{b}] - 2 G^2 (1 + r) \sigma_w^2 \mathbb{C}(\hat{a}, \hat{b})
\end{align*}
\quad \text{(32)}
\]

where \(\mathbb{C}(\hat{a}, \hat{b})\) denotes the covariance between \(\hat{a}\) and \(\hat{b}\), and it can be obtained by straightforward but tedious derivations as in (33) at the top of the next page. Further, with some mathematical manipulations, the correlation coefficient between the numerator \(\hat{g}_{0,n}\) and the denominator \(\hat{g}_d\) can be derived as
\[
\rho_{\hat{g}_0, \hat{g}_d} = \frac{G^4 \sigma_w^2 \mathbb{V}[\hat{b}] - G^2 \sigma_w^2 \mathbb{C}(\hat{a}, \hat{b})}{\sqrt{\mathbb{V}[\hat{g}_{0,n}] \mathbb{V}[\hat{g}_d]}}. \quad \text{(34)}
\]

and the correlation coefficient of the numerator \(\hat{g}_{2,n}\) and the denominator \(\hat{g}_d\) can be derived as
\[
\rho_{\hat{g}_2, \hat{g}_d} = \frac{\sigma_w^2 \mathbb{C}(\hat{a}, \hat{b}) - G^2 (1 + r) \sigma_w^2 \mathbb{V}[\hat{b}]}{\sqrt{\mathbb{V}[\hat{g}_{2,n}] \mathbb{V}[\hat{g}_d]}}. \quad \text{(35)}
\]

**Proposition 1:** The cumulative distribution function (CDF) of \(\hat{g}_0\) and \(\hat{g}_2\) are, respectively, given by
\[
F_{\hat{g}_0}(u) = Q(\gamma_0, -\infty, \rho_0) + Q(-\infty, \eta_0, \rho_0) - 2Q(\gamma_0, \eta_0, \rho_0) \quad \text{(36)}
\]
and
\[
F_{\hat{g}_2}(u) = Q(\gamma_2, -\infty, \rho_2) + Q(-\infty, \eta_2, \rho_2) - 2Q(\gamma_2, \eta_2, \rho_2) \quad \text{(37)}
\]

where \(Q(\gamma, \eta, \rho)\) is a two-dimensional Gaussian Q-function as
\[
Q(\gamma, \eta, \rho) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-\frac{\gamma^2 - 2uv + \eta^2}{2(1 - \rho^2)}} dudv,
\]
\[
\gamma_0 = \frac{u \mathbb{E}[\hat{g}_d] - \mathbb{E}[\hat{g}_{0,n}]}{\sqrt{(\alpha_{11} - \beta_0 \alpha_{21})^2 + (u - \beta_0)^2 \mathbb{V}[\hat{g}_d]}}, \quad \text{(39)}
\]
\[
\eta_0 = -\frac{\mathbb{E}[\hat{g}_d]}{\sqrt{\mathbb{V}[\hat{g}_d]}}, \quad \text{(40)}
\]
\[
\rho_0 = \frac{- (u - \beta_0) \mathbb{V}[\hat{g}_d]}{\sqrt{(\alpha_{11} - \beta_0 \alpha_{21})^2 + (u - \beta_0)^2 \mathbb{V}[\hat{g}_d]}}, \quad \text{(41)}
\]
\[
\alpha_{11}^0 = \frac{\mathbb{V}[\hat{g}_{0,n}]}{\mathbb{V}[\hat{g}_d]}, \quad \text{(43)}
\]
\[
\alpha_{21}^0 = \frac{\mathbb{V}[\hat{g}_d]}{\mathbb{V}[\hat{g}_{0,n}]} \frac{1 + \rho_{\hat{g}_0}}{1 + \rho_{\hat{g}_0}}, \quad \text{(44)}
\]
\[
\gamma_2 = \frac{\hat{g}_2 \mathbb{E}[\hat{g}_d] - \mathbb{E}[\hat{g}_{2,n}]}{\sqrt{(\alpha_{11} - \beta_0 \alpha_{21})^2 + (\hat{g}_2 - \beta_2)^2 \mathbb{V}[\hat{g}_d]} - \mathbb{V}[\hat{g}_d]}, \quad \text{(45)}
\]
\[
\eta_2 = -\frac{\mathbb{E}[\hat{g}_d]}{\sqrt{\mathbb{V}[\hat{g}_d]}}, \quad \text{(46)}
\]
\[ C(\hat{a}, \hat{b}) = \left( \frac{1}{\sqrt{2\pi}} \right)^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du \]

The PDF of \( \hat{g}_2 \) is given by

\[
 f_{\hat{g}_2}(u) = \frac{b_2(u) d_2(u)}{\sqrt{2\pi(1 - \mu_2^2) V[\hat{g}_2] V[\hat{g}_d] a_2^2(z)}} \times \left[ Q \left( \frac{b_2(u)}{a_2(u)} \right) - Q \left( \frac{b_2(u)}{a_2(u)} \right) \right] + \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{u^2}{2}}}{\sqrt{1 - \mu_2^2} V[\hat{g}_d] a_2^2(u)}
\]

with

\[
 a_2(u) = \sqrt{\frac{(1 - \mu_2^2) V[\hat{g}_2] V[\hat{g}_d] a_2^2(u)}{1 - \mu_2^2}}
\]

and

\[
 b_2(u) = \frac{E[\hat{g}_2] - \mu_2 E[\hat{g}_d]}{V[\hat{g}_2]} (u - \mu_2) + \frac{E[\hat{g}_d]}{\sqrt{V[\hat{g}_d]}}
\]

\[
 c_2 = \frac{E[\hat{g}_2] - \mu_2 E[\hat{g}_d]}{V[\hat{g}_2]} \frac{E[\hat{g}_d]}{\sqrt{V[\hat{g}_d]}}
\]

\[
 d_2(u) = e^{-\frac{u^2}{2a_2^2(u)}}
\]

\[
 a_0(u) = \sqrt{\frac{(1 - \mu_0^2) V[\hat{g}_0] V[\hat{g}_d] a_0^2(u)}{1 - \mu_2^2}}
\]

Finally, means and variances of \( \hat{g}_0 \) and \( \hat{g}_2 \) can be obtained with \( f_{\hat{g}_0}(u) \) and \( f_{\hat{g}_2}(u) \), i.e.,

\[
 E[\hat{g}_0] = \int_0^{+\infty} u f_{\hat{g}_0}(u) du,
\]

\[
 V[\hat{g}_0] = \int_0^{+\infty} (u - E[\hat{g}_0])^2 f_{\hat{g}_0}(u) du,
\]

\[
 E[\hat{g}_2] = \int_0^{+\infty} u f_{\hat{g}_2}(u) du,
\]

\[
 V[\hat{g}_2] = \int_0^{+\infty} (u - E[\hat{g}_2])^2 f_{\hat{g}_2}(u) du.
\]

IV. SIMULATION RESULTS

In this section, we validate the theoretical analysis and evaluate the proposed estimators by numerical simulations. To verify the theoretical analysis about the distributions of the estimates, we only consider the path loss for each link in Monte Carlo simulations, i.e., the random shadowing effect...
is not considered. While in the other simulations, the random shadowing effect is taken into account. The cross-channel gain estimator which does not requires the primary-channel gain in [15] and the primary-channel gain estimator in [12] are compared with the proposed estimator.

First, we verify the theoretical results through Monte Carlo simulations. Let the positions of PU-Tx, PU-Rx, SU-Tx, SU-Rx be (0 km, 0 km), (0 km, 2.5 km), (0.25 km, 0 km), (0.25 km, 0.25 km) and (0.175 km, 0.2165 km), respectively. Let $N_0 = -174$ dBm/Hz, $B = 20$ MHz, $F = 1$ dB and $r = 1$. The relay amplitude gain $G$ is given by 40 dB, and the target SINR at PU-Rx is set to be $\gamma_T = 10$ dB. The power $p_1$ varies from 0 dBm to 6 dBm in step of 2 dBm, i.e., $p_1$ has four power levels for estimating the gains $g_0$ and $g_2$. We take $M = 1000$ samples for estimating the power $p_0$. At each power level of $p_0$, $p_1$ is independently estimated for 25 times, i.e., we have $J = 4 \times 25 = 100$. Fig. 2 shows the theoretical analysis and Monte Carlo simulation results for the CDFs of estimates $\hat{g}_0$ and $\hat{g}_2$. It shows that the theoretical results match the simulation results well. Fig. 3 shows the theoretical analysis and Monte Carlo simulation results for the PDFs of estimates $\hat{g}_0$ and $\hat{g}_2$. It can be observed that the theoretical results for the PDFs of estimates $\hat{g}_0$ and $\hat{g}_2$ match their corresponding simulation results.

Fig. 4 shows the theoretical and Monte Carlo results for the means of $\hat{g}_0$ and $\hat{g}_2$. It is observed that the difference between the theoretical and Monte Carlo results is smaller than 0.2%, and this validates the theoretical analyses. Moreover, the means of $\hat{g}_0$ and $\hat{g}_2$ approach the true values of $g_0$ and $g_2$ as $M$ increases, and this implies the proposed estimator is asymptotically unbiased. Fig. 5 shows the theoretical and Monte Carlo results for the variances of $\hat{g}_0$ and $\hat{g}_2$. It can be seen that the theoretical results match the Monte Carlo results. Moreover, the variances of $\hat{g}_0$ and $\hat{g}_2$ approach zero as $M$ increases, which implies that the proposed estimator is consistent.

In the following, we evaluate the performance of the proposed estimator by comparing it with existing estimators using normalized estimation errors. As in [12] and [15], the normalized estimation error is defined as

$$
\epsilon_i = \left| \frac{10 \log_{10}(\hat{g}_i) - 10 \log_{10}(g_i)}{10 \log_{10}(g_i)} \right| \quad (68)
$$

for $i = 0, 2$. Let the positions of PU-Tx and PU-Rx be (0 km, 0 km) and (2 km, 0 km), respectively. Two location regions of SU-Tx are considered. As shown in Fig. 6, one region is around the PU-Tx with a radius of 1 km, and the SU-Tx is randomly deployed in the region in each trial.
Fig. 6. Locations of PU-Tx and PU-Rx and the location region of SU-Tx in the simulations.

Fig. 7. Normalized estimation errors of $\hat{g}_0$ and $\hat{g}_2$ with SU-Tx located around PU-Tx for different $G$ when $M = 100$ and $J = 100$.

The other region of the SU-Tx is around the PU-Rx. The other parameters are kept the same as aforementioned.

Fig. 7 shows the normalized estimation errors of $\hat{g}_0$ and $\hat{g}_2$ for different $G$ when SU-Tx is located around PU-Tx. It can be observed from Fig. 7 that the proposed estimator has normalized estimation errors when the relay amplitude power gain $G$ is smaller than 70 dB. Fig. 8 shows the normalized estimation errors of $\hat{g}_0$ and $\hat{g}_2$ for different $G$ when SU-Tx is located around PU-Rx. It can be seen that the proposed estimator outperforms the others. This is because the proposed estimator has used multiple power levels of the probe signal $\sqrt{P_{\text{Tx}}}(t)$ of SU-Tx. It should be noted that our proposed estimator can estimate $g_0$ and $g_2$ simultaneously, while other estimators can only estimate either $g_0$ or $g_2$.

Fig. 9 shows the normalized estimation errors of $\hat{g}_0$ and $\hat{g}_2$ with SU-Tx located around PU-Tx for different $M$ when $G = 60$ dB and $J = 100$. It can be seen that the errors of all estimators decreases as the number of samples $M$ increases, and the proposed estimator achieves a better performance than other estimators. For the cross gain $g_2$, the estimation error of the proposed estimator is smaller than half of the estimation error of the estimator in [15].

Fig. 10 shows the normalized estimation errors of $\hat{g}_0$ and $\hat{g}_2$ with SU-Tx located around PU-Rx for different $M$ when $G = 60$ dB and $J = 100$. It demonstrates the superior performance of the proposed estimator.
Fig. 11 shows the normalized estimation errors of $\hat{g}_0$ and $\hat{g}_2$ with SU-Tx located around PU-Tx for different $J$ when $G = 60$ dB and $M = 100$. It can be seen that the errors of all estimators decreases as the number of samples $J$ increases, and the proposed estimator achieves a better performance than other estimators. For the cross gain $g_2$, the estimation error of the proposed estimator is smaller than half of the estimation error of the estimator in [15]. Fig. 12 shows the normalized estimation errors of $\hat{g}_0$ and $\hat{g}_2$ with SU-Tx located around PU-Rx, and it demonstrates the superior performance of the proposed estimator again.

**FIGURE 12.** NRMSE of the estimates $\hat{g}_0$ and $\hat{g}_2$ with SU-Tx located around PU-Rx for different $J$ when $G = 60$ dB and $M = 100$.

**V. CONCLUSIONS**

In this paper, we have proposed a scheme and an estimator for the SU-Tx to acquire the primary-link and cross-link gains simultaneously. The theoretical performance of the proposed estimator has been also analyzed. Numerical simulations have been provided to validate the theoretical analysis and demonstrate the superior performance of the proposed estimator.

**APPENDIX**

**PROOF OF PROPOSITION 1**

Let a random variable $Z$ be

\[ Z = \frac{X}{Y}, \quad (A.1) \]

where $X$ and $Y$ are real Gaussian distributed with expectations $\mu_x$ and $\mu_y$, and variances $\sigma_x^2$ and $\sigma_y^2$, respectively. Assume that the cross-correlation coefficient between $X$ and $Y$ is given by $\rho_{xy}$. Let $V_1$ and $V_2$ be independent standard Gaussian variables, i.e.,

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right) \quad (A.2) \]

Then the vector $[X, Y]^T$ can be expressed as

\[ [X, Y]^T = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} \quad (A.3) \]

where

\[ \alpha_{11} = \frac{\sigma_x}{\sqrt{1 + \rho_{xy}^2}}, \quad (A.4) \]

\[ \alpha_{12} = \frac{\rho_{xy} \sigma_y}{\sqrt{1 + \rho_{xy}^2}}, \quad (A.5) \]

\[ \alpha_{21} = \frac{\rho_{xy} \sigma_x}{\sqrt{1 + \rho_{xy}^2}}, \quad (A.6) \]

\[ \alpha_{22} = \frac{\sigma_y}{\sqrt{1 + \rho_{xy}^2}}. \quad (A.7) \]

Let $\xi = \rho_{xy} \frac{\sigma_x}{\sigma_y}$, it can be easily verified that

\[ \mathbb{E} [(X - \xi Y)Y] = \mathbb{E} [X - \xi Y] \mathbb{E} [Y]. \quad (A.8) \]

Considering that $X - \xi Y$ and $Y$ are Gaussian distributed, together with (A.8), we can obtain that $X - \xi Y$ and $Y$ are independent of each other. Thus, it can be derived that

\[ \frac{X - \xi Y}{Y} = \frac{(\alpha_{11} - \xi \alpha_{21}) V_1 + \mu_x - \xi \mu_y}{\sigma_y V_2 + \mu_y} \quad (A.9) \]

and therefore, $Z$ in (A.1) can be rewritten as

\[ Z = \xi + \frac{(\alpha_{11} - \xi \alpha_{21}) V_1 + \mu_x - \xi \mu_y}{\sigma_y V_2 + \mu_y}. \quad (A.10) \]

Then, we can obtain that

\[ \text{Prob}(Z \leq z) = \text{Prob} \left( \xi + \frac{(\alpha_{11} - \xi \alpha_{21}) V_1 + \mu_x - \xi \mu_y}{\sigma_y V_2 + \mu_y} \leq z \right) \]

\[ = \text{Prob} \left( (\alpha_{11} - \xi \alpha_{21}) V_1 + \mu_x - \xi \mu_y \leq (\sigma_y V_2 + \mu_y) \times (z - \xi), \sigma_y V_2 + \mu_y > 0 \right) \]

\[ + \text{Prob} \left( (\alpha_{11} - \xi \alpha_{21}) V_1 + \mu_x - \xi \mu_y \geq (\sigma_y V_2 + \mu_y) \times (z - \xi), \sigma_y V_2 + \mu_y < 0 \right) \]

\[ = \text{Prob} \left( (\alpha_{11} - \xi \alpha_{21}) V_1 - (z - \xi) \mu_y V_2 \leq z \mu_y - \mu_x, \sigma_y V_2 > -\mu_y \right) \]

\[ + \text{Prob} \left( (\alpha_{11} - \xi \alpha_{21}) V_1 - (z - \xi) \mu_y V_2 \geq z \mu_y - \mu_x, \sigma_y V_2 < -\mu_y \right) \quad (A.11) \]

It can be easily obtained that the cross-correlation coefficient of $(\alpha_{11} - \xi \alpha_{21}) V_1 - (z - \xi) \mu_y V_2$ and $\sigma_y V_2$ is

\[ \rho = \frac{-(z - \xi) \sigma_y}{\sqrt{\alpha_{11} - \xi \alpha_{21}}^2 + (z - \xi)^2 \sigma_y^2}. \quad (A.12) \]

Then Prob($Z \leq z$) in (A.11) can be rewritten as

\[ \text{Prob}(Z \leq z) = Q(y, -\infty, \rho) + Q(-\infty, \eta, \rho) - 2Q(y, \eta, \rho) \quad (A.13) \]

where $Q(y, \eta, \rho)$ represents a two-dimensional Gaussian Q-function as

\[ Q(y, \eta, \rho) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \int_{y}^{+\infty} \int_{\eta}^{+\infty} e^{-\frac{u^2 - 2\rho uv + \rho^2 v^2}{2(1 - \rho^2)}} dudv \quad (A.14) \]
with

\[ \gamma = 2 \mu_y - \mu_x \sqrt{(\alpha_{11} - \xi \alpha_{21})^2 + (z - \xi)^2 \sigma_y^2} \]  
(A.15)

and

\[ \eta = -\frac{\mu_y}{\sigma_y}. \]  
(A.16)

REFERENCES


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