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Velocity distribution in non-uniform/unsteady flows and the validity of log law

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VELOCITY DISTRIBUTION IN NON-UNIFORM/UNSTEADY FLOWS AND THE VALIDITY OF LOG LAW

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ABSTRACT
This study investigates the longitudinal velocity profiles in steady and unsteady non-uniform open channel flows by analyzing the data available in the literature. It was found that for steady/unsteady flow the Log law is applicable only in the inner region where $y/h < 0.2$ when compared with the measured longitudinal velocity. However, in the main flow, the measured velocity deviates significantly from the Log law's prediction. The detail assessment shows that the deviation is negative when a flow is accelerating, and positive when decelerating. The reason for this deviation is explained by linking these deviations with the presence of wall–normal velocity. A new equation was developed to express the velocity in non-uniform flow which is valid for uniform flows (Log law) and accelerating and decelerating steady/unsteady flows, in which Log law, Cole's Wake law and Dip law have been combined together to reflect the influence of acceleration on the velocity profile. This modification has been verified with experimental data sets available in the literature, and a reasonable agreement between the measured and predicted mean velocity profile is obtained.

Keywords: uniform/non-uniform flows; velocity distribution; Log-Wake–Dip law; steady/unsteady flows; accelerating/decelerating flows.

INTRODUCTION

Accurate knowledge of the velocity distribution in an open channel is of crucial importance to practicing engineers as its helps in the estimation of erosion and sediment transport. Extensive and intensive research has been conducted over the past century. [10] proposed the "universal wall function" or Log law of the wall as:

$$\frac{\bar{u}}{u_*} = \frac{1}{k} \ln \frac{y u_*}{\nu} + B \qquad (1)$$

where the integral constant B =5.29; the universal von Kármán constant k = 0.41; u_* is the shear velocity; ν is the kinematic viscosity; y is the vertical distance measured from the reference level; and $\bar{u}$ is the point mean horizontal velocity at y. For a rough bed, Equation 1 can be rewritten as follows:

$$\frac{\bar{u}}{u_*} = \frac{1}{k} \ln \frac{y + y_0}{k_s} + B \qquad (2)$$

where k_s is the roughness height $= d_{50}$; y_0 is the reference bed level = $0.2 k_s$ and B is the constant of integration. In Equations 1 and 2, there are two constant (k and B) and these can be calculated from the experimental results. However, the experimental data from flat boundary layer flows [5], [8] and pipe flows [14] suggested that the Log-law may be not always be correct in describing the velocity distribution. This was also observed in open channel flows over 100 years ago by river engineers, such as [6], who discovered from their measurement in rivers that maximum velocity does not appear at the free surface as the Log law predicts, but occurs below the free surface. This effect, also called “dip-phenomenon”, remains an open question for researchers who still debate is mechanism. Consequently, an article in Science [3] commented that: “the law of the wall was viewed as one of the few certainties in the difficult field of turbulence, and now it should be dethroned. Generations of engineers who learned the law will have to abandon it”. In open channel flows, the same phenomenon has been known for a long time, but the mechanism is still unclear. [5] introduced an additional term (i.e. Coles Wake term) to express the deviation of measured velocity from the prediction of Log law, which has the following formula:

$$\frac{\bar{u}}{u_*} = \frac{1}{k}\ln\left(\frac{y}{y_0}\right) + B + \frac{2\Pi}{k}\sin^2\left(\frac{\pi}{2}\frac{y}{h}\right) \qquad (3)$$

where Π is the wake strength parameter. Different values have been found for this parameter based on the type of flow and bed configuration [1], [4], [7], [9], [11]. Thus, the value of Π is not universal. From the brief review, the Log law is applicable only in the inner region with $(y/h < 0.2)$. Therefore, the Log law cannot express the velocity distribution accurately in the outer region. Whilst significant advances have been made by using Cole’s Wake law, the mechanism of Cole’s Wake law and the associated wake strength parameter are not fully understood. [12] discussed the dip phenomenon, in which the maximum longitudinal velocity occurs below the water surface. They suggested that the Cole’s Wake law is not able to describe the entire velocity profile when the dip-phenomenon exists. Therefore, [12] modified Log law by adding a term to express the dip phenomenon instead of the Cole’s Wake law based on Reynolds equations:

$$\frac{\bar{u}}{u_*} = \frac{1}{k}\ln\left(\frac{y}{y_0}\right) + \frac{\alpha}{k}\ln\left(1-\frac{y}{h}\right) \qquad (4)$$

where α is the dip correction factor and can be determined by:

$$\alpha = 1.3\exp\left(-\frac{z}{h}\right) \qquad (5)$$

where z is the distance from the sidewall in z direction. It is clearly seen from Equation 4 that a dip model consists of two logarithmic distances, one from the bed (i.e. Log law) and the other from the free surface i.e. $\ln(1-y/h)$. Similarly, the [12] model is unable to fit the cases where the measured local velocity is higher than the prediction by Log law. In the literature however, there is no universal model to express the velocity in the complex flow conditions, thus more research is needed to clarify why the Log law cannot predict the measured longitudinal velocity well in non-uniform flows. This leads to the present research aims to develop a universal model to express the velocity profile in uniform and non-uniform as well as steady and unsteady open channel flows.

THE RELATIONSHIP BETWEEN THE FLOW ACCELERATION AND VELOCITY DISTRIBUTION

In the literature, it has already been discussed that the consistency of flow velocity and water depth in open channel flows from upstream to downstream generate the uniform flow. However, in steady and unsteady non-uniform flows, the flow velocity and water depth are different upstream to downstream. These differences relate to flow acceleration. Generally, the flow acceleration i.e. a means that there is a difference in velocities in two adjacent measuring stations or different time at the same location. Based on this definition, the flow acceleration is equal to zero when the flow is uniform while in steady and unsteady non-uniform flows, the flow acceleration is different. When the flow velocity increases along the open channel the flow acceleration is increased or has a positive value and this type of flow is accelerating steady/unsteady non-uniform flows. In contrast, the flow acceleration is less than zero or has a negative value when the flow is decelerating steady/unsteady non-uniform flows due to the decrease of flow velocity along the channel. According to this, flow acceleration is the most important parameter to distinguish the flows, thus it is possible to develop empirical formulas to predict the mean horizontal velocity using the flow acceleration. The distinction of velocity profiles in a uniform flow with those of accelerating and decelerating flows is shown in Figures 1, where the velocity profiles were measured by [7], in the form of $\bar{u}/u_*$ against $y+y_0/k_s$, in which $y_0 = 0.2*k_s$ is the reference bed level; and $k_s = d_{50}$. Figure 1 shows that the measured velocity profiles match well in the inner region with the Log law as no influence for the flow acceleration (a = 0). While the measured data points bend down from the straight line of Log law prediction as the flow acceleration (a >0) is positive; and they bend up over the Log law prediction when the flow acceleration (a <0) is negative.

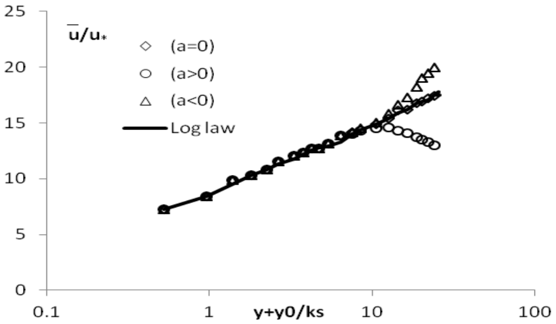

Figure 1: Comparison between measured velocity profile with Log law's prediction based on [7], where a is the depth averaged flow acceleration.

The reason for this deviation was explained by [13] based on the Reynolds equation, and they concluded that the vertical velocity or wall-normal velocity (v) is responsible for the invalidity of Log law, they found that the Log law is valid if v=0 (uniform flow); and the [5] model becomes valid only when v>0 (decelerating flows), and the maximum velocity is submerged below the water surface when and only when v<0 (accelerating flows). But in practice, the formulae proposed by [13] are very difficult to use because the wall-normal velocity is generally too small in quantity to measure. Therefore, to help river engineers to solve their practical problems easily, it is necessary to develop a formula that is valid for all flow conditions and only the mean streamwise velocity, not the wall-normal velocity is used. To simplify such as expression, the Log law together with Cole's Wake law and Dip law are combined to express the longitudinal velocity across the whole water depth from the bed to the water surface for both steady and unsteady flows.

$$\frac{\bar{u}}{u_*} = \frac{1}{k}\ln\left(\frac{y+y_0}{k_s}\right) + k_1 \sin^2\left(\frac{\pi}{2}\frac{y}{h}\right) + k_2 \ln\left(1-\frac{y}{h}\right) \quad (6)$$

where k_1 and k_2 are coefficients to be determined empirically. The first term in Equation 6 refers to the Log law while the second and third terms are Cole's Wake law and Dip law respectively. Obviously, k_1 and k_2 are a function of flow acceleration, and they become zero if the acceleration is zero (uniform flow), terms 2 and 3 on the right hand side of Equation 6 are negligible in the inner region, but they are noticeable in the main flow region. Due to $\ln(1-y/h) \to -\infty$ as $y/h \to 1$, therefore, Equation 6 is invalid in a very thin layer near the free surface just like the classical Log law that becomes invalid at the layer closer to the boundary, i.e. $y = 0$. In Equation 6, k_1 and k_2 are empirical coefficients and the relationships will be evaluated with the flow acceleration instead of the wall-normal velocity. As mentioned before, the wall-normal velocity is equivalent to predict the longitudinal velocity but the flow acceleration is more direct and simpler in the mathematical treatment. For this purpose a simple expression of flow acceleration should be established.

DETERMINATION OF FLOW ACCELERATION FOR BOTH STEADY AND UNSTEADY FLOW

The flow acceleration in 2D flows can be written as follows:

$$\bar{a} = \frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \bar{u}\bar{v}}{\partial y} \quad (7)$$

where $\bar{a}$ is the flow acceleration in each point in a flow field, the first term on the right–hand side of Equation 7 becomes zero in a steady flow. The depth averaged flow acceleration (a) for both steady and unsteady non-uniform flow is defined as:

$$a = \frac{1}{h}\int_0^h \bar{a}dy \quad (8)$$

Then, insert Equations 7 into Equation 8 to obtain:

$$a = \frac{1}{h}\int_0^h \frac{\partial \bar{u}}{\partial t}dy + \frac{1}{h}\int_0^h \frac{\partial \bar{u}^2}{\partial x}dy + \frac{1}{h}\int_0^h \frac{\partial \bar{u}\bar{v}}{\partial y}dy \quad (9)$$

Using the Leibnitz theorem, Equations 9 can be integrated with respect to water depth y from the channel bed ($y = 0$) to the free water surface ($y = h$), one has:

$$a = \left(\frac{dU}{dt} - \frac{\bar{u}_h}{h}\frac{dh}{dt}\right) + \left(\beta\frac{dU^2}{dx} - \frac{\bar{u}_h^2}{h}\frac{dh}{dx}\right) + \left(\frac{1}{h}\bar{u}_h\bar{v}_h\right) \quad (10)$$

where U = depth average longitudinal velocity; h = water depth; v = wall-normal velocity or vertical velocity; the subscript h denotes the free surface where $y = h$; β is the momentum flux correction factor that takes into the non-uniform of flow velocity across the inlet and outlet. The value of β ranges between 1.01 and 1.04 [2] and in this study, the value of β is assumed to be 1.03. The depth averaged flow acceleration can be obtained without the wall–normal velocity, thus it avoids the shortcomings of [13]'s method as only these parameters are required: longitudinal velocity at the water surface, $\bar{u}_h$; vertical velocity at the water surface, $\bar{v}_h$; the variation of water depth with time, dh/dt; the variation of water depth along the longitudinal direction, dh/dx; water depth, h; the variation of depth averaged longitudinal velocity with time, dU/dt; and the variation of depth averaged longitudinal velocity squared along the channel, dU^2/dx. The vertical velocity at the free surface can be determined from the continuity equation, i.e.

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \quad (11)$$

The wall-normal velocity at the free surface can be expressed as follows:

$$\bar{v}_h = -\int_0^h \frac{\partial \bar{u}}{\partial x}dy = -\frac{d}{dx}\int_0^h \bar{u}dy + \bar{u}_h\frac{dh}{dx} \quad (12)$$

Equation 12 can be alternatively expressed as:

$$\bar{v}_h = -\frac{d(Uh)}{dx} + \bar{u}_h\frac{dh}{dx} \quad (13)$$

Equation 13 relates the wall-normal velocity with streamwise velocity and water depth only. Thus, using Equations 13 and 10, one is able to express the flow acceleration without the wall-normal velocity that is difficult for an ordinary engineer to determine. The first term of Equation 13 on the right-hand-side is zero in a steady flow, but it becomes non-zero in unsteady flows.

THE INFLUENCE OF FLOW ACCELERATION ON VELOCITY DISTRIBUTION IN STEADY AND UNSTEADY FLOW

In the literature, a comprehensive measurement was conducted by [11] who measured mean horizontal velocity ($\bar{u}$) in steady and unsteady non-uniform flows using an Acoustic Doppler Velocity profiler (ADVP). Based on [11] experimental datasets, Equation 10 is used to determine the flow acceleration in steady and unsteady flows. In order to prove the influence of flow acceleration on the deviation of these velocity distribution in non-uniform flow from Log law, the measured velocity profiles are plotted in Figure 2 in the form of dimensionless velocity versus the relative distance i.e., $y + y_0 / k_s$ as $'x'$, where the acceleration is normalized by u_*^2 / h. The typical velocity profiles in accelerating and decelerating flows are shown in Figure 2, in which “A” means accelerating steady; “D” denotes decelerating steady; “AU” refers to accelerating unsteady; “S” represents the bed slope; “Q” is flow discharge; “t” means time; and “931 or 936” is the hydrograph’s number in Song’s experimental datasets. In Figure 2, the open and solid symbols are the measured point velocities and the solid line is the Log law from Equation 2. The calculated values of $a \ /(u_*^2 / h)$ are shown in legend of Figure 2. It can be seen clearly that the measured velocity from the inner region in steady and unsteady flows matches well with Log law’s prediction but in the main flow region ($y / h \geq 0.2$), the measured velocity becomes larger than Log’s formula when the flow is decelerating or it has lower value when the flow is accelerating, this is why in Equation 6 the dip-term and wake function are kept in the expression. Therefore, it seems that the flow acceleration is a controlling parameter which can affect the shape of the measured longitudinal velocity profile.

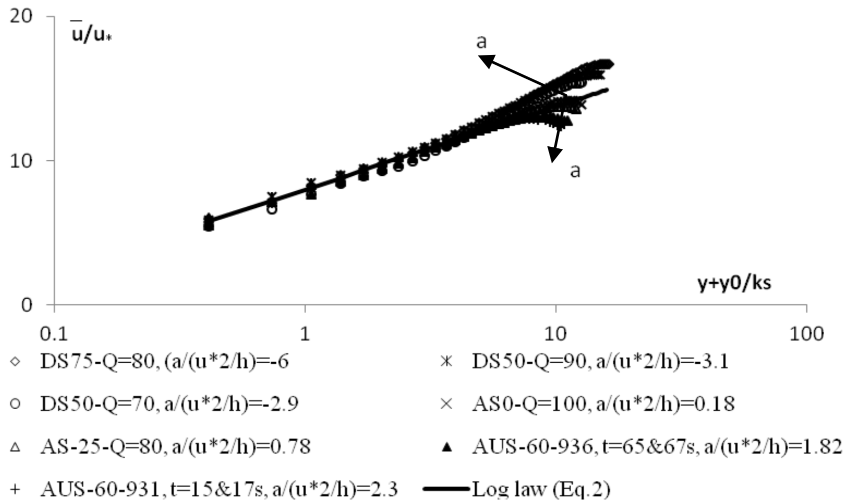

Figure 2: The influence of dimensionless flow acceleration on the deviation of measured longitudinal velocity in accelerating and decelerating steady and unsteady flows from Log law based on [11]’s experimental data sets.

To yield the best agreement between Equation 6 and the measured velocity profiles, one can obtain k_1 and k_2 from experimental data in steady and unsteady flows. For example, k_1 and k_2 can be evaluated from the velocity defect between the measured and Log law predicted velocities $\Delta\bar{u}$ (i.e. $\Delta\bar{u} = (\bar{u}_{measure} - \bar{u}_{Loglaw}) / u_*$). This difference indicates the amount of deviation from the measured mean velocity and the Log law. By fitting the velocity difference, one may obtain the expressions of k_1 and k_2. For accelerating flow, k_1 and k_2 were obtained from [11] experimental data as shown in Figure 3, in which only the data from accelerating flows in both steady and unsteady flows were selected, and the following empirical expressions are obtained:

$$k_1 = 1.4[a/(u_*^2/h)]^{0.7} \quad (14)$$

$$k_2 = [a/(u_*^2/h)]^{0.7} \quad (15)$$

A clear dependence of these coefficients k_1 and k_2 on the dimensionless acceleration can be observed and solid lines can be drawn based on the data points presented in each figure. If a=0, then k_1 and k_2=0, Equations 14 and 15 were obtained. In Figure 3, significant similarity is observed between dimensionless flow acceleration in steady and unsteady flows and these similar values of flow acceleration give similar values for k_1 and k_2. Thus, this is the reason why the acceleration symbol (a) is used in Equations 15 and 16 without any subscript relating to the steady or unsteady flow. From these figures, it is clear that the values of k_1 and k_2 increase with the flow acceleration.

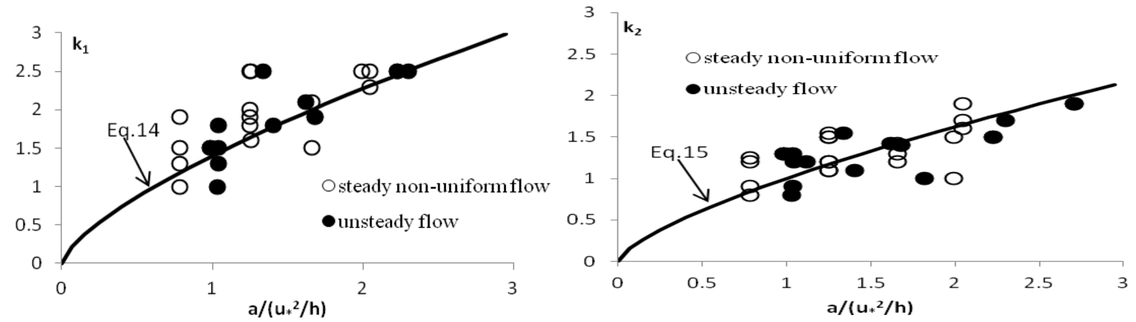

Figure 3: Relationship between k_1, k_2 and dimensionless flow acceleration $a/(u_*^2/h)$ in steady and unsteady flow based on [11]experimental data sets.

For decelerating flows, the empirical equations for k_1 and k_2 can be obtained with the similar method as shown in Figure 4, and the k_1 can be evaluated as an exponential function of negative flow acceleration:

$$k_1 = \exp\{-0.26 * [a/(u_*^2/h)]\} - 1 \quad (16)$$

It was found by analyzing [17] data that k_2 is very small and $k_2 \approx 0$.

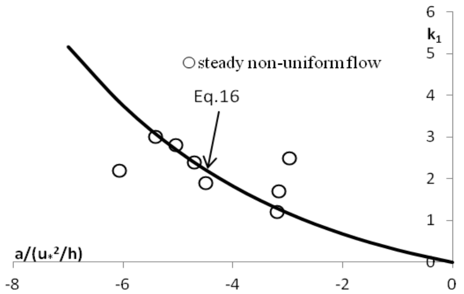

Figure 4: Relationship between k_1 and dimensionless flow acceleration a/(u*2/h) in decelerating steady flow based on [11] experimental data sets.

In order to check the validity of Equation 6 with these empirical values for k_1 and k_2, the remaining datasets in [11] experiments expect those shown in Figures ¾ are plotted in Figures 5-7. In order to demonstrate the performance of Equation 6 and the two empirical values of k_1 and k_2 obtained from their relationship with the flow acceleration, in each figure the relative error between the measured and predicted longitudinal velocity profile is determined as $E = \left| \bar{u}_m - \bar{u}_c \right| / \bar{u}_m * 100$, where the subscript m and c are the measured and calculated velocities respectively. As can be seen from Figures 5-7, the average values of E are less than 4% error band between the measurement and calculations velocity profiles in accelerating unsteady flow. Therefore, it can be seen from this comparison that Equation 6 is able to capture the velocity distribution in the entire profile that includes the inner and outer regions depending on the value of flow acceleration.

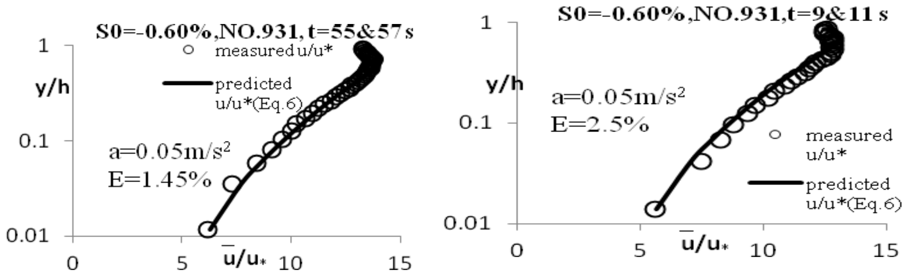

Figure 5: Comparison of measured and predicted mean horizontal velocity profile in accelerating unsteady flow based on [11] experimental data.

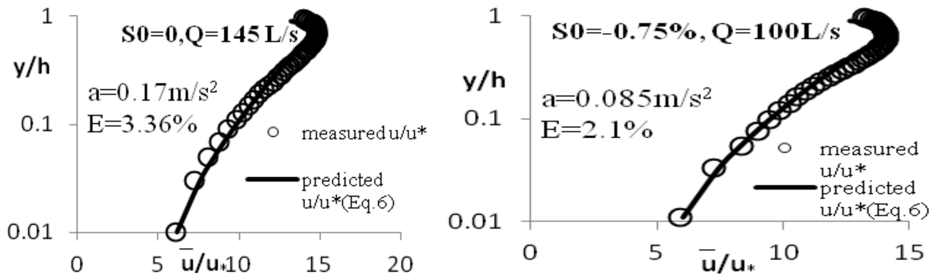

Figure 6: Comparison of measured and predicted mean horizontal velocity profile in accelerating steady flow based on [11] experimental data.

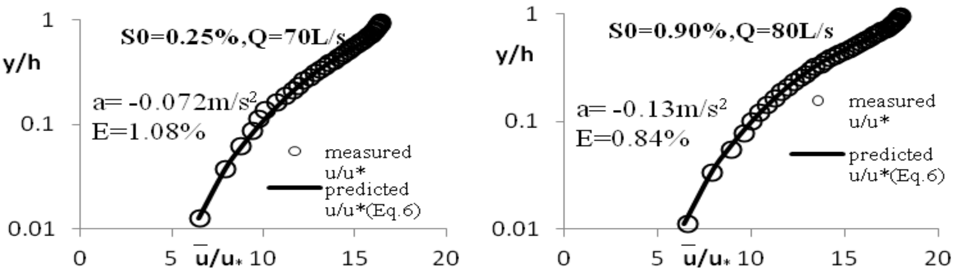

Figure 7: Comparison of measured and predicted mean horizontal velocity profile in decelerating steady flow based on [11] experimental data.

CONCLUTIONS

It has been widely reported that the longitudinal velocity distribution deviates from the classical Log law in steady and unsteady open channel flows. In accelerating flow, the measured data fall below the Log law while it has higher value when the flow is decelerating. In this paper, we attribute this deviation to the existence of vertical velocity generated from non-uniform flows. Therefore, a new empirical equation was developed to predict the longitudinal velocity in non-uniform flow which is valid for uniform flows (Log law) and accelerating/ decelerating steady and unsteady flows, in which Log law, Cole's Wake law and Dip law have been combined together to reflect the influence of acceleration on the velocity profile. When the longitudinal velocity increases along the channel the flow acceleration is positive; and the acceleration becomes negative when this velocity decreases. This flow acceleration or deceleration can be predicted using Equation 10 which is valid for both steady and unsteady flows. The relationship between these three laws with the influence of flow acceleration has been proposed, in which two k factors that depend on dimensionless acceleration are determined. The one factor of k is introduced for the Cole's Wake law and the other for the Dip law. Song's experimental data was used to verify the relationship between the flow acceleration and the values of k. It is found that for both positive and negative flow accelerations, the values of k_1, k_2 are positive. Comparing Equation 6 with experimental data by [11], a good agreement was obtained between the measured and predicted value of velocity profiles depending on the existence of flow acceleration.

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