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Distribution of reynolds shear stress in steady and unsteady flows

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Abstract

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Keywords

steady, distribution, flows, reynolds, unsteady, shear, stress

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DISTRIBUTION OF REYNOLDS SHEAR STRESS IN STEADY AND UNSTEADY FLOWS

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ABSTRACT
This study investigates the Reynolds shear stress distribution in steady and unsteady non-uniform flows. Specifically, it deals with how to express the deviation of this turbulence characteristic from that of uniform flow line; it is found that flow acceleration can well represent the deviation of Reynolds shear stress from its standard linear distribution. By connecting the flow acceleration with Reynolds shear stress, the study demonstrates empirically that the linear distribution of Reynolds shear stress can be observed when the flow acceleration is zero; the concave distribution of Reynolds shear stress can be observed when the flow acceleration is negative or when the flow velocity is decreased along the channel; the convex distribution of Reynolds shear stress can be observed when the flow acceleration is positive or the flow velocity is increased along the channel. These empirical results have been verified using published experimental data and good agreement between the predicted and observed profiles has been achieved.

Keywords: Uniform/non-uniform flow, unsteady flow, Reynolds shear stress, flow acceleration.

INTRODUCTION

The distribution of Reynolds shear stress can determine the distribution of sediment concentration and pollution dispersion. Therefore, its distribution is crucial for predicting sediment transport in river systems where the influence of steadiness or unsteadiness is more significant. Because all river flows are unsteady or non-uniform, uniform flow is very rare to occur, thus it is necessary to investigate the influence of non-uniformity and unsteadiness on Reynolds shear stress distribution, the knowledge of the distribution of Reynolds shear stress in non-uniform open channel flows is of essential in hydraulic engineering. But its distribution in steady/unsteady flows is not fully understood and there exist no widely accepted equations available in the literature, thus it requires more investigations. Many researchers have studied the distribution of Reynolds shear stress in uniform flow [2], [3], [4], [6], [7], [9]; however, few researchers have studied the influence of non-uniform flow on the distribution of Reynolds shear stress. [5], [8] measured Reynolds shear stress in accelerating and decelerating flows. They observed that an accelerating flow generally hampers the local Reynolds shear stress, vice versa; these observations clearly demonstrate that the influence of unsteadiness/non-uniformity on the Reynolds shear stress is not negligible. Unfortunately, the mechanism for this phenomenon has not been well revealed, and the quantitative description for the Reynolds shear stress is not available in the literature.

[8] measured Reynolds shear stress in steady/unsteady non-uniform flows using an Acoustic Doppler Velocity Profiler (ADVP). The measured data show that the concave distributions are observed in accelerating flow where the bed slope was negative; while convex distributions are observed for decelerating flow with positive bed slope. However, the

reason for this deviation was not discussed in his study. [8] also established equations to predict the Reynolds shear stress profiles in steady and unsteady flows i.e.

$$\tau(y)/\tau_0 = 1+\beta * y/h - \rho/\tau_0 * U^2 dh/dx[(m+1)^2/(m^2+2m)]*(y/h)^{(m+2)/m} \quad (1)$$

$$\tau(y)/\tau_0 = 1+\beta_u * y/h + \rho h/\tau_0 * \partial U/\partial t * [y/h^{1/m} - 1]* y/h - \rho U^2/\tau_0 * \partial h/\partial x$$
$$*[(m+1)^2/(m^2+2m)]* y/h^{(2+m)/m} - \rho U/\tau_0 * \partial h/\partial t$$
$$*[1/m * y/h^{(m+1)/m} + (m+1)/(m+2)*(y/h)^{(m+2)/m}] \quad (2)$$

where Equations 1 and 2 developed for this prediction in steady and unsteady flows, respectively; β and β_u are the pressure gradient parameter in steady and unsteady flows respectively; h is the water depth; m =5.5; τ_0 is the bed shear stress which can be determined using the following equation:

$$\tau_0 = g\rho h*[S_0 - \{1-(m+1)^2/(m^2+2m)*F_r{}^2\}*dh/dx] \quad (3)$$

where F_r is the Froude number. Based on his theoretical distribution, Song investigated that a good agreement between the measured and predicted Reynolds shear stress can be observed when the bed slope is not very large, for example, the theoretical distributions match well with his measurements when the bed slope =-0.15% and 0.30%, which is better than when the slope =-0.60% . [10] investigated theoretically the deviation of Reynolds shear stress distribution in accelerating/decelerating non-uniform flow. They reported that this deviation is related to the presence of upward/downward vertical velocity. Based on this reason, they developed formula that can be predicted the full profiles of Reynolds shear stress in uniform and non-uniform flows:

$$-\frac{\overline{u'v'}}{u_*^2} = (1-\frac{y}{h}) + b\frac{y}{h} + \frac{\overline{u}\overline{v}}{u_*^2} \quad (4)$$

where $-\overline{u'v'}/u_*^2$ is the normalized Reynolds shear stress; h is the water depth; y is the vertical distance measured from the reference level; $\overline{u}\ \overline{v}$ are the momentum flux caused by the vertical velocity; and b is a parameter and can be expressed as follows:

$$b = -(\frac{\overline{u}_h}{u_*})^2 dh/dx \quad (4a)$$

It can be seen from Equation 4a that b is always positive in an accelerating flow where $dh/dx < 0$ and becomes negative in a decelerating flow where $dh/dx > 0$. While it becomes zero when the flow is uniform and thus the first term on the right hand side of Equation 4 can be used to estimate the Reynolds shear stress in uniform flow i.e.

$$-\frac{\overline{u'v'}}{u_*^2} = (1-\frac{y}{h}) \quad (5)$$

Based on the review outlined, the Reynolds shear stress in non-uniform flows has little investigation in hydraulic engineering, the conclusions from different authors are different, and thus more research is needed to clarify why the measured Reynolds shear in non-uniform flow deviates from that in uniform flow. From the practical point of view, almost all flows in rivers are unsteady or non-uniform flows, and in the literature there is no a universal model to express the Reynolds shear stress in the complex flow conditions, this leads to the present research aims to develop a universal model to express the Reynolds shear stress profile in uniform/non-uniform and steady/unsteady flows.

THE RELATIONSHIP BETWEEN THE FLOW ACCELERATION AND THE DISTRIBUTION OF RYNOLDS SHEAR STEESS

In the literature, it has already been discussed that the consistency of flow velocity and water depth in open channel flows from upstream to downstream generate the uniform flow. However, in steady/unsteady non-uniform flows, the flow velocity and water depth are different upstream to downstream. These differences relate to flow acceleration. Generally, the flow acceleration i.e. a means that there is a difference in velocities in two adjacent measuring stations or different time at the same location. Based on this definition, the flow acceleration is equal to zero when the flow is uniform while in non-uniform flows, the flow acceleration is different. When the flow velocity increases along the open channel the flow acceleration is increased or has a positive value and this type of flow is accelerating steady/unsteady non-uniform flows. In contrast, the flow acceleration is less than zero or has a negative value when the flow is decelerating steady/unsteady non-uniform flows due to the decrease of flow velocity along the channel. According to this, flow acceleration is the most important parameter to distinguish the flows, thus it is possible to develop empirical formulas to predict the mean horizontal velocity using the flow acceleration. This explanation can be seen from experimental runs of [5] as shown in Figure 1, in which the measured data in uniform and accelerating/decelerating non-uniform flows are included for comparison legends. It is clearly seen from Figure 1 that the measured data points of Reynolds shear stress in non-uniform flow deviate from that in uniform flow (Equation 5). When the flow is accelerating or positive flow acceleration the value of Reynolds shear stress is lower than the uniform flow line and vice versa in decelerating flow. Therefore, Equation 5 needs to be modified to extend its applicability in both uniform and non-uniform flows, it is reasonable to express the deviation with the flow acceleration. But the difficulty is that the flow acceleration is a variable in spatial and temporal domain, it is needed to develop a proper way to express the flow acceleration, and then investigate the relationship between the deviations shown in Figure 1 and the characteristic acceleration.

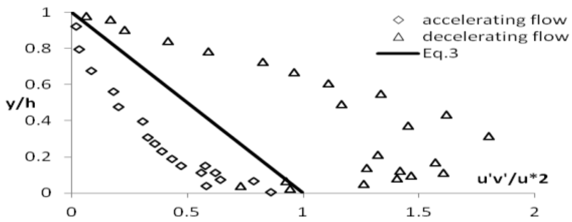

Figure 1: The measured Reynolds shear stress in uniform and non-uniform flows.

DETERMINATION OF FLOW ACCELERATION FOR BOTH STEADY AND UNSTEADY FLOWS

This paper makes an attempt to express the Reynolds shear stress in steady non-uniform flow or unsteady flow, to achieve this, appropriate expression of flow acceleration is needed; the point and instantaneous acceleration can be written as follows:

$$\bar{a} = \frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \overline{uv}}{\partial y} \quad (6)$$

where $\bar{a}$ is the flow acceleration in each point in a flow field, the first term on the right–hand side of Equation 6 becomes zero in a steady flow. In this study, we use the depth averaged flow acceleration as the characteristic acceleration for both steady/unsteady non-uniform flow, it is defined as:

$$a = \frac{1}{h}\int_0^h \bar{a}\,dy \quad (7)$$

where a is the depth averaged flow acceleration in steady and unsteady flows. Then, inserting Equations 6 into Equation 7 one has:

$$a = \frac{1}{h}\int_0^h \frac{\partial \bar{u}}{\partial t} dy + \frac{1}{h}\int_0^h \frac{\partial \bar{u}^2}{\partial x} dy + \frac{1}{h}\int_0^h \frac{\partial \bar{u}\bar{v}}{\partial y} dy \tag{8}$$

Using the Leibnitz theorem to integrate Equations 8 with respect to water depth y from the channel bed ($y = 0$) to the free water surface ($y = h$), one has:

$$a = \left(\frac{dU}{dt} - \frac{\bar{u}_h}{h}\frac{dh}{dt}\right) + \left(\beta \frac{dU^2}{dx} - \frac{\bar{u}_h^2}{h}\frac{dh}{dx}\right) + \left(\frac{1}{h}\bar{u}_h \bar{v}_h\right) \tag{9}$$

where U = depth average velocity; h = water depth; v = wall-normal velocity or vertical velocity; the subscript h denotes the free surface where $y = h$; β is the momentum flux correction factor that takes into the non-uniform of flow velocity across the inlet and outlet. The value of β ranges between 1.01 and 1.04 [1] and in this study, the value of β is assumed to be 1.03. In Equation 9, one needs to determine the wall-normal velocity at the free surface; it is a difficult task for measurement in practice. The vertical velocity at the free surface can be determined from the continuity equation, i.e.

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{10}$$

The wall-normal velocity at the free surface can be expressed as follows:

$$\bar{v}_h = -\int_0^h \frac{\partial \bar{u}}{\partial x} dy = -\frac{d}{dx}\int_0^h \bar{u} dy + \bar{u}_h \frac{dh}{dx} \tag{11}$$

Equation 11 can be alternatively expressed as:

$$\bar{v}_h = -\frac{d(Uh)}{dx} + \bar{u}_h \frac{dh}{dx} \tag{12}$$

Equation 12 relates the wall-normal velocity with streamwise velocity and water depth only. Thus, using Equation 9 and 12 one is able to express the flow acceleration without the wall-normal velocity that is difficult for an ordinary engineer to determine. The first term of Equation 12 on the right-hand-side is zero in a steady flow, but it becomes non-zero in an unsteady flows. The depth averaged flow acceleration can be obtained without the wall–normal velocity, thus it avoids the shortcomings of [10]'s method as only these parameters are required: longitudinal velocity at the water surface, $\bar{u}_h$; vertical velocity at the water surface, $\bar{v}_h$; the variation of water depth with time, dh/dt; the variation of water depth along the longitudinal direction, dh/dx; water depth, h; the variation of depth averaged longitudinal velocity with time, dU/dt;and the variation of depth averaged longitudinal velocity squared along the channel, dU^2/dx.

THE INFLUENCE OF FLOW ACCELERATION ON REYNOLDS SHEAR STESS IN STEADY AND UNSTEADY FLOWS

In order to verify that flow acceleration is responsible for the deviation of the measured Reynolds shear stress in non-uniform from that in uniform flow, [8]experimental data is used as it may be one of the most comprehensive dataset in the literatures, all parameters needed for Equation 9 were measured and documented. The full profiles for ($-\overline{u'v'}/u_*^2$) are plotted in Figure 2 for varying flow conditions using dimensionless velocity with respect to the shear velocity (u_*) against y/h. In Figure 2, the open and solid symbols represent the measured Reynolds shear stress in non-uniform flow i.e. $-\overline{u'v'}/u_*^2$ while the straight solid line is the predicted Reynolds shear stress in uniform flow using Equation 5. It can be seen that the measured Reynolds shear stress in non-uniform flow becomes zero as y/h approaches to the water surface, and its value becomes 1 as y/h approaches to the bed surface $(y=0)$, these indicate that at $y=0$ and $y=h$, the measured Reynolds shear stress in non-uniform flows becomes identical as a uniform flow. However, between these two extremes, the measurement of data points in accelerating and decelerating steady/unsteady flows locate on both sides of the solid line or Equation 5. From this comparison, the Reynolds shear stress profiles in unsteady flow are similar to those in steady flow, thus it is possible to establish a unify formula to express the Reynolds shear stress distribution of steady/unsteady flows based on the effect of flow acceleration on the measured Reynolds shear stress.

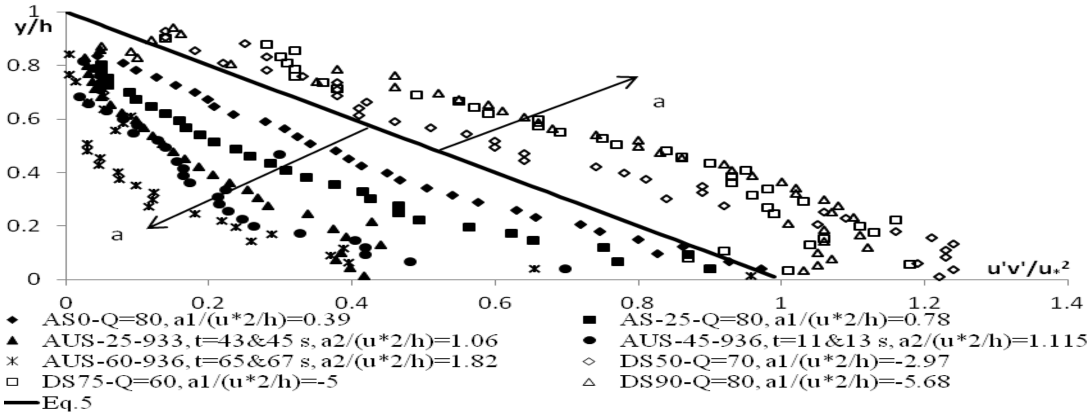

Figure 2: The influence of dimensionless flow acceleration on the deviation of measured Reynolds shear stress in accelerating and decelerating steady and unsteady flows from uniform flow based on [8] experimental data sets.

DISTRIBUTION OF REYNOLDS SHEAR STRESS

After demonstrating the impact of flow acceleration on the deviation of Reynolds shear stress in steady/unsteady flows from that in the uniform flow, one may conclude that the difference of Reynolds shear stress in uniform form/unsteady or non-uniform flows is proportional to $(1-y/h)$ and y/h as the difference between these two types of flow must become zero at these boundary conditions, where $y/h \approx 0$. Thus the proportionality $k_{-\rho\overline{u'v'}}$ should depend on a dimensionless flow acceleration i.e. $a/(u_*^2/h)$. Therefore, these empirical formulas are proposed as follows:

$$(-\frac{\overline{u'v'}}{u_*^2})_{nonuf.} = (-\frac{\overline{u'v'}}{u_*^2})_{uf.} + k_{-\rho\overline{u'v'}} * (1-y/h)^{1.5} * y/h \tag{13}$$

where $(-\overline{u'v'}/u_*^2)$ is the normalized Reynolds shear stress, and the subscript "$uf.$" and "$nonuf.$" refer to the Reynolds shear stress in uniform and non-uniform flows, respectively. In Equation 13, the value of $k_{-\rho\overline{u'v'}}$ has two signs, one positive and the other negative. The positive one can be used when the flow is decelerating while the negative sign will be used when the flow is accelerating for both steady and unsteady flows. To yield the best agreement between Equation 13 and the measured Reynolds shear stress, one can determine $k_{-\rho\overline{u'v'}}$ from experimental data in steady and unsteady flows. For example, the value of $k_{-\rho\overline{u'v'}}$ can be evaluated from Reynolds shear stress defect between the measured Reynolds shear stress in uniform and non-uniform flows i.e. $\Delta(-\overline{u'v'}/u_*^2) = (\overline{u'v'}_{nonuf.} - \overline{u'v'}_{unf.})/u_*^2$. By fitting the Reynolds shear stress difference, one may obtain the expressions of $k_{-\rho\overline{u'v'}}$. The values of $k_{-\rho\overline{u'v'}}$ are obtained from [8]'s experimental data as shown in Figure 3, thus, the empirical expressions of are obtained:

$$k_{-\rho\overline{u'v'}} = -\ln\{1 + 5.8 * [a/(u_*^2/h)]\} \qquad \text{(Accelerating flow)} \qquad (14)$$

$$k_{-\rho\overline{u'v'}} = \exp\{-0.2 * [a/(u_*^2/h)]\} - 1 \qquad \text{(Decelerating flow)} \qquad (15)$$

From Equations 14 and 15, the observed value of $k_{-\rho\overline{u'v'}}$ is negative when the flow is accelerating because the distribution of Reynolds shear stress in accelerating non-uniform flow is lower than those in uniform flow (see Figure 3a). While this empirical value of $k_{-\rho\overline{u'v'}}$ is positive (see Figure 3b) when the flow is decelerating because the data points for Reynolds shear stress in this flow are higher than those in uniform flow.

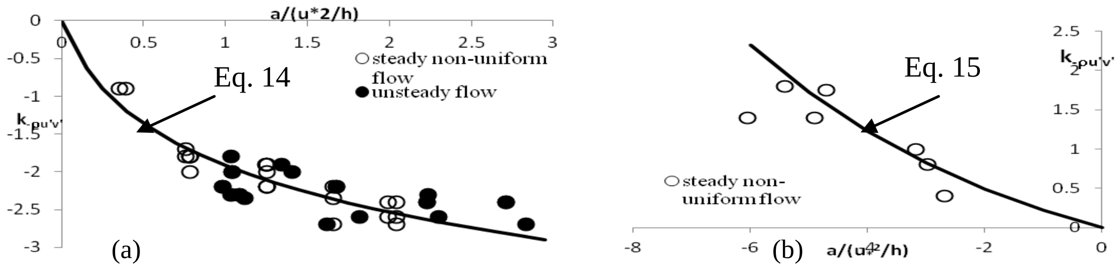

Figure 3: Relationship between dimensionless flow acceleration and the value of $k_{-\rho\overline{u'v'}}$ for Reynolds shear stress $(-\overline{u'v'}/u_*^2)$ in (a) accelerating and (b) decelerating steady and unsteady flow based on selected data sets from [8] experimental data.

The solid symbols in Figure 3 denote the data obtained from unsteady flows, and the open symbols represent the same but in steady flow. In Figure 3b, there is no decelerating unsteady data available to compare with decelerating steady flows because [8] reported that the flow in his experiments was accelerating in unsteady flows. A clear dependence of the coefficient of $k_{-\rho\overline{u'v'}}$ on the dimensionless acceleration can be observed and solid lines can be drawn based on the data points and the condition that if a=0, then $k_{-\rho\overline{u'v'}}$=0 in both accelerating and decelerating flows, Equations 14 and 15 were obtained. In Figure 3, significant similarity is observed between dimensionless flow acceleration in steady and unsteady flows and these similar values of flow acceleration give similar values for $k_{-\rho\overline{u'v'}}$. Thus, this is the reason why the acceleration symbol (a) is used in Equations 14 and 15 without any subscript relating to the steady or unsteady flow. When the flow is accelerating for both steady/unsteady cases, a positive a gives a negative value of $k_{-\rho\overline{u'v'}}$ and the vice versa for decelerating flow. While if a=0, then $k_{-\rho\overline{u'v'}}$=0 and as a result the second term of Equation 13 is negligible and this is uniform flow. Consequently, if the flow acceleration in steady or unsteady flows is known and then the value of $k_{-\rho\overline{u'v'}}$ can be estimated based on Equations 14 and 15 for both accelerating and decelerating flows. In order to check the validity of Equation 13 with these empirical values for $k_{-\rho\overline{u'v'}}$, the remaining datasets in [8] experiments except that show in Figure 3 are plotted in Figures 4-6. To demonstrate the performance of Equation 13, [8]'s equations for the prediction of Reynolds shear stress in steady and unsteady flows are compared in each figure based on the determination of the relative error (E) between the measured and predicted values as $E = \left| \overline{u'v'}_m - \overline{u'v'}_c \right| / \overline{u'v'}_c$, where the subscript m and c are the measured and calculated values. In each figure, the symbol E is introduced with subscripts i.e. $(Eq.13)$ and $([8]'sEq.)$, where the first subscript refers to the relative error between the measured and predicted values based on the developed model in the present study while the second one is determined based on the predicted value using Songs' equation.

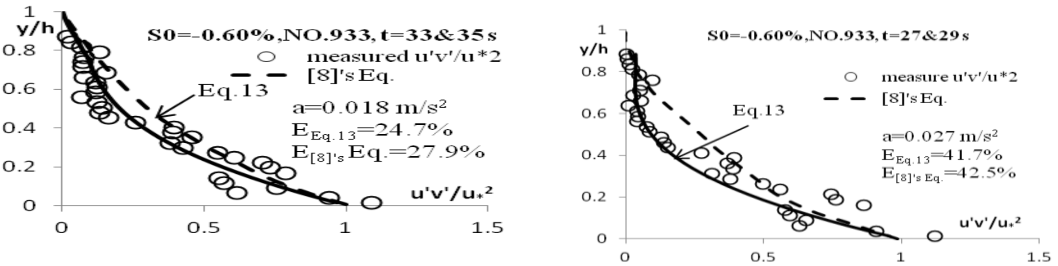

Figure 4: Comparison of measured and predicted Reynolds shear stress profile in accelerating unsteady flow based on [8] experimental data.

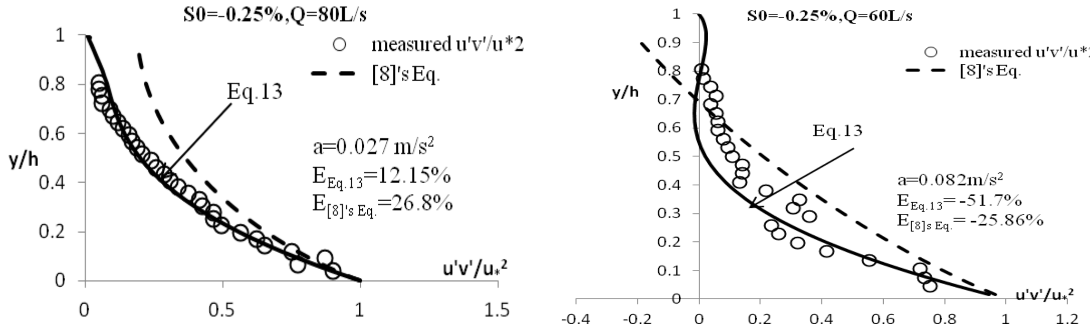

Figure 5: Comparison of measured and predicted Reynolds shear stress profile in accelerating steady flow based on [8] experimental data.

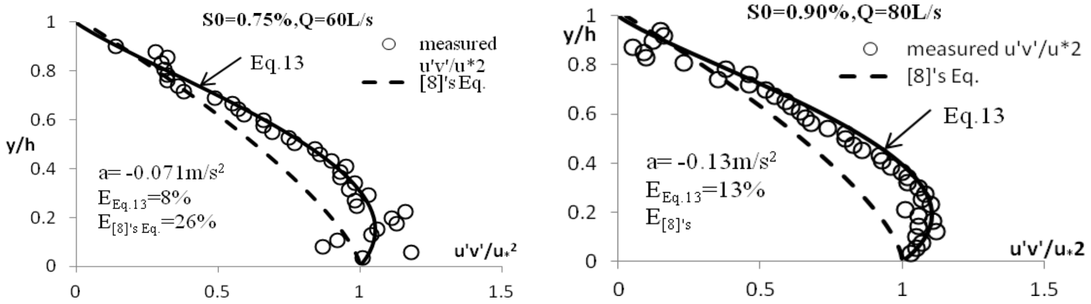

Figure 6: Comparison of measured and predicted Reynolds shear stress profile in decelerating steady flow based on Song's (1994) experimental data.

In Figures 4-6, the calculated Reynolds shear stress profiles using Equation 13 are plotted as a solid line, the dashed line is the calculate Reynolds shear stress using [8]'s equation and the open circles denote to the measured data points in non-uniform steady/ unsteady. The average values of determined E in Figures 4/6 using Equation 13 are less than that determined using [8]'s equations for all types of flow, indicating that the proposed model agreed the existing datasets in steady/unsteady flows, especially when the value of flow acceleration is existed. Overall, it can be found that the predicted formulas can give similar profiles to the measurements of Reynolds shear stress, concave distribution when the applying negative value of $k_{-\rho\overline{u'v'}}$ and convex distribution when the value of $k_{-\rho\overline{u'v'}}$ is positive.

CONCLUSIONS

The distributions of Reynolds shear stress in non-uniform steady and unsteady flows have been predicted. As observed from the literature, the distribution of Reynolds shear stress in non-uniform flow deviate from the linear distribution of uniform flow. Song's (1994) experimental results were used to develop the relationships between the dimensionless flow acceleration and values of $k_{-\rho\overline{u'v'}}$ for the prediction of Reynolds shear stress. These empirical formulas were developed dependent on the impact of flow acceleration on the deviation of the Reynolds shear stress in non-uniform steady and unsteady flows from that in uniform flow. In Reynolds shear stress, the positive acceleration (accelerating flow) generates negative value of $k_{-\rho\overline{u'v'}}$ while the negative value of flow acceleration (decelerating flow) generates positive value of $k_{-\rho\overline{u'v'}}$. Using these values of $k_{-\rho\overline{u'v'}}$ the distribution of Reynolds shear stress across the whole water depth was then predicted. The experimental data from [8] support these predictions. When the effect of flow acceleration is considered, the agreement between the measured and estimated Reynolds shear stress profiles in steady and unsteady flows is found to be good.

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