Moment tensor analysis of transversely isotropic shale based on the discrete element method

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Abstract
In this study, the moment tensor of transversely isotropic shale was analyzed using a discrete element method-acoustic emission model (DEM-AE model). Firstly, the failure modes of the shale obtained from the acoustic emission (AE) events and physical experiments were compared. Secondly, the relationships between AE events and seismic magnitudes, and AE events and the resulting cracks were analyzed. Finally, a moment tensor T-k chart describing the seismic source was introduced to demonstrate the differences in the transversely isotropic shale. The results showed that, for different anisotropy angles, a linear logarithmic relationship existed between the cumulative AE events and the seismic magnitude in the concentration area of the AE events. A normal distribution was observed for the number of AE events as the seismic magnitude changed from small to large. The moment tensor T-k chart indicated that the number and proportion of linear tension cracks in the shale were highest. When $\theta = 30^\circ$, the peak seismic magnitude was at a minimum. The average seismic magnitude in the concentration area of the AE events was also relatively small. Points close to the $U = -1/3 \ V$ line and the number of cracks included in a single AE event were at a minimum, and the corresponding peak stress also reached its lowest level. In contrast, when $\theta = 90^\circ$, all related parameters were contrary to the above $\theta = 30^\circ$ case. The DEM-AE model and the moment tensor T-k chart are suitable for analyzing the distribution of shale cracks appearing during the loading process. This study can provide constructive references for future research on the fracturing treatment of shale.

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Moment tensor analysis of transversely isotropic shale based on the discrete element method

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Keywords: Bedding plane of shale AE events Failure mode Moment tensor T-k chart Crack type

1. Introduction

The bedding plane of rock has a significant effect on shale gas exploration, hydraulic fracture, fracturing treatment, crack propagation, and stability of gas wells, as well as other constructions [1–5]. The tensile strength of shale can be influenced easily and significantly by the bedding plane, which often results in anisotropy [6]. Therefore, more insights and reference data can be provided for construction projects by researching transversely isotropic shale.

At the laboratory scale, acoustic emission (AE) technology is often used to study the initiation, propagation, and coalescence of cracks in a rock mass [7]. Goodman first pointed out that the Kaiser effect exists in rock materials and proposed the concept of the moment tensor to research the seismic source [8]. Gilbert used the moment tensor to detect seismicity for the first time [9]. Subsequently, fracture mechanisms in a seismic source have been analyzed using the moment tensor theory. The moment tensor is divided into three main parts, namely the isotropy part $M^{ISO}$, the pure shear crack part $M^{PSR}$, and the compensated linear vector dipole part $M^{CVD}$ [10]. Liu et al. concluded that the processes of deformation or fracture in a rock mass, as well as seismicity, were caused by the quick release of elastic strain energy [11]. Therefore, a similar approach was used to investigate both processes. The characteristics of an AE source of granite were back-analyzed with an AE moment tensor. Similarly, back analysis was also conducted on the formation of mine water inrush using an AE moment tensor [12].

For the most part, researchers have used two methods to determine the mechanical properties of shale using a Brazilian test. First, physical experiments were conducted to investigate the bedding plane effect and the AE properties [13,14]. In addition, the mechanical behavior of the hydraulic fracture was analyzed further. Second, numerical simulation analysis was used to investigate the

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failure modes and dynamic responses [15–17]. Using AE technology in numerical simulations to investigate the anisotropy of shale can facilitate the understanding of rock mechanics, but AE data cannot be derived from numerical simulations directly. Therefore, an independent sub-program was needed for the discrete element method (DEM). Additionally, some AE models have been developed in numerical simulations [18,19]. However, no studies have been conducted to date using the DEM-AE model to research transversely isotropic shale.

This study is based on the results of our previous physical experiments and uses a DEM-AE model with an independent sub-program to analyze the moment tensor of transversely isotropic shale. The results of the failure modes of shale represented by the AE events in the DEM and in physical experiments were compared based on the moment tensor. The seismic magnitude of the AE events and their relationship with the numbers of cracks were analyzed. Finally, a moment tensor T-k chart describing the seismic magnitude of the transversely isotropic shale. This model can improve our understanding of the anisotropy of shale.

### 2. Method of moment tensor

The source related to the deformation and cracks of rocks is known as the AE source [20]. In order to detect and determine the signal magnitude of the AE source in rocks, AE instruments or micro-seismographs are often used for the inspection and quantitative analysis in the laboratory and the field. The basic method for measuring the seismic magnitude is the seismic moment. In addition, the seismic moment can also be analyzed using the discrete element method.

Since the stress exerted on shale particles and their movement can be obtained directly using this numerical simulation, it is feasible to calculate the scalar of the seismic moment based on the change of the contact force between occurring particles while new cracks are appearing. As shown in Fig. 1, the moment tensor in the DEM can be calculated by summing the moment values of the contact forces between the crack particles:

\[
M_j = \sum_s (\Delta F_i l_j)
\]
where $M_j$ is the scalar seismic moment in the calculation; $\Delta F_i$ and $L_j$ the $n$th component of the contact force and the corresponding arm of force respectively.

According to the moment tensor matrix, the maximum scalar moment $M_0$ can be calculated as following:

$$ M_0 = \sqrt{\frac{\sum_{j=1}^{3} M_j^2}{2}} $$

The signal magnitude of the AE event $M$ can be calculated with the following empirical equation [21]:

\[
M = \sqrt{\sum_{j=1}^{3} M_j^2}
\]

### Table 2
Micro-properties used in SJ contact for simulated specimen after calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Angle range (°)</th>
<th>Normal stiffness $sj_{kn}$ (GPa/m)</th>
<th>Shear stiffness $sj_{ks}$ (GPa/m)</th>
<th>Tensile strength $\delta_s$ (MPa)</th>
<th>Cohesion $c_s$ (MPa)</th>
<th>Friction angle $\phi_s$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>±4</td>
<td>54,000</td>
<td>68,000</td>
<td>12.5 ± 2</td>
<td>14.5 ± 1</td>
<td>0</td>
</tr>
</tbody>
</table>

![Image of micro-properties used in SJ contact for simulated specimen after calibration.](image)
The principle of the AE event simulated with the DEM is to calculate the maximum scalar change of the moment tensor during the movement of shale particles, which has been verified by previous related works [18]. Since the moment tensor in the DEM is a symmetry two-tensor, the three feature values are all real numbers and there are three orthogonal feature vectors. If the principal feature values of the three feature vectors are $M_1$, $M_2$, and $M_3$ ($M_1 \geq M_2 \geq M_3$) with corresponding feature vector values of $t_1$, $t_2$, and $t_3$, the moment tensor can be diagonalized in a principal axes system as [22]:

$$M = \frac{2}{3} \lg(M_0) - 6$$

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$$M = \begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix} = \begin{bmatrix}
M_1 & 0 & 0 \\
0 & M_2 & 0 \\
0 & 0 & M_3
\end{bmatrix}$$

$$= \frac{1}{3}(M_1 + M_2 + M_3) \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} + \frac{1}{2}(M_1 - M_3) \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{bmatrix}$$

$$+ \frac{1}{6}(2M_2 - M_1 - M_3) \begin{bmatrix}
-1 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & -1
\end{bmatrix} = M^{ISO} + M^{PSC} + M^{CLVD}$$

where $M^{ISO}$ is the isotropy part of the moment tensor (ISO); $M^{PSC}$ the pure shear crack part (PSC); and $M^{CLVD}$ the compensated linear vector dipole part (CLVD).

The pure shear crack part $M^{PSC}$ of the moment tensor can be calculated as follows:
Hudson put forward a moment tensor $T$-$k$ chart to analyze and better understand the mechanisms of rock fractures, in which the moment tensor is divided into two parameters, $T$ and $k$. The equation for these two parameters is presented below:

$$T = \frac{2M}{\max(|M_1|, |M_3|)}$$  \hspace{1cm} (6)

$$k = \frac{M_{\text{ISO}}}{M_{\text{ISO}} + \max(|M_1|, |M_3|)}$$  \hspace{1cm} (7)

As shown in Fig. 2, $T$ refers to the parameter controlling the character of the constant-volume component and $k$ refers to the parameter that evaluates the dilatational component of the seismic source, with the value range from -1 to 1 for both parameters. The smaller the $T$ value, the closer to -1 the CLVD is; and the bigger the $T$ value, the closer to 1 the CLVD is. In addition, the smaller the $k$ value, the more even the compression is; and the bigger the $k$ value, the more even the tension is. The proposed method in Fig. 2 was adopted to demonstrate and analyze the crack types and the magnitudes of the AE moment tensor.

### 3. Anisotropic Brazilian test model

In this study, a DEM-AE model was used to simulate the AE properties of a transversely isotropic shale based on a Brazilian test. Black shale selected from the Longmaxi Formation in the Pengshui shale gas area in China was used as the specimen for the physical experiment. In order to determine the anisotropic characteristics of the shale bedding plane in the Brazilian test in an effective and scientific manner, the angle between the bedding plane and the vertical direction was chosen as the anisotropy angle ($\phi$), see Fig. 3a and b. Seven different values for $\phi$ were selected to conduct the tests, including $0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$, and $90^\circ$. The testing procedures and instruments used in this study are based on our previous research [24].

The geometries of the experimental and simulated specimens are described in Fig. 3a and b. The simulation model was conducted using DEM, i.e. particle flow code (PFC) software. A parallel bond model was employed for modeling the bonding of particles in the shale matrix. A smooth joint model was embedded into the parallel bond model to simulate the bedding plane. The microstructure of the shale revealed that its bedding plane was not consistent (Fig. 3c). Therefore, a random smooth joint model was chosen as the bedding plane for the anisotropic simulation model. Moreover, an anisotropic mineral brittleness-based model (AMBBM) was developed, which divided the minerals in the shale
into two categories, brittle minerals and non-brittle minerals. Model development and parameter calibration used in this study have been described in our previous research [25]. The micro-parameters obtained after the calibration procedure are shown in Tables 1 and 2.

4. Results and analysis

Based on the results of the physical Brazilian test and the numerical simulations, the seismic magnitude of the AE events in the transversely isotropic shale was analyzed. Crack types were also investigated according to the moment tensor $T-k$ chart.

Fig. 4 depicts the comparison between the results from physical experiments and numerical simulations in terms of failure mode and macro-cracks; there was good agreement between the results of the numerical simulation and the physical experiments for the failure mode and the macro-cracks represented by the AE events.

4.1. Magnitude analysis of AE events

The cumulative AE events for seven different anisotropic angles ($\theta$) are shown in Fig. 5. When $\theta$ was relatively small ($\theta \leq 45^\circ$) and $M = -7.50$ to $-5.75$, there was a linear logarithmic relationship between the seismic magnitude and the cumulative number of AE events. The average seismic magnitude in the concentration area of the AE events increased with an increase in $\theta$. Specifically, when $\theta = 30^\circ$, the average seismic magnitude in the concentration area of the AE events reached the minimum level. According to our previous research, when $\theta = 30^\circ$, the bearing capacity of the shale was reduced to the lowest level and the probability of emerging cracks and specimen failure was the greatest during the loading process [25].

When $M < -7.25$, the number of AE events decreased with the drop in the seismic magnitude. The slope of the fitting curve of the cumulative AE events was close to zero. However, when $\theta = 90^\circ$ and $M < -6.25$, the slope of the curve began to decrease gradually, which meant that both the seismic magnitude of the AE events and the bearing capacity of the specimen at $\theta = 90^\circ$ were the largest.

Fig. 6 shows the number of AE events corresponding to each seismic magnitude. In general, the numbers represent a normal distribution along with the change of seismic magnitudes from small to large. For the specimens with a smaller $\theta$ ($\theta \leq 45^\circ$), the AE events were mainly concentrated in the range of $-7.5$ to $-7.0$ and the number of AE events reached its peak at $-7.25$. With an increase in $\theta$, the seismic magnitude in the peak area of the number of AE events also increased. When $\theta$ is relatively large ($\theta \geq 60^\circ$ or $75^\circ$), most AE events were mainly concentrated in the range of $-7.0$ to $-6.5$. In particular, when $\theta$ reached its largest value ($\theta = 90^\circ$), most AE events were mainly concentrated in the range of $-6.5$ to $-6.0$. 

Fig. 7. Relationship between number of AE events and crack number included in a single AE event.
For specimens with different anisotropy angles, the peak seismic magnitudes of the AE events were quite different. For example, the largest magnitude for the specimen at $\theta = 30^\circ$ was $-5.75$, and for values of $\theta$ at $0^\circ$, $15^\circ$, $45^\circ$, and $60^\circ$, the magnitude of the specimen was close to $-5.25$. The specimens at $\theta = 75^\circ$ and $90^\circ$ had the largest magnitude of $-4.75$. This trend in the change in the magnitude was directly related to the bearing capacities of the specimens with different anisotropy angles.

The relationship between the numbers of AE events and the crack numbers of each AE event is shown in Fig. 7. Fitting curves and 95% confidence interval envelopes were developed based on the distribution of numbers of different crack types corresponding to each AE event. The curves demonstrated that, for all different specimens with different anisotropy angles, there was a negative exponential relationship between the number of AE events and the crack number of each AE event. 60%–70% percent of the AE events only exhibited a single crack, and about 10% of the AE events included two cracks. There was an inverse relationship between the crack number per AE event and the number of the AE events. When $\theta = 30^\circ$, there was no AE event even when more
than ten cracks occurred. When $\theta = 0^\circ$, only three AE events occurred, including the occurrence of more than ten cracks. With the increase of the anisotropy angle ($\theta$), the number of AE events including ten cracks also increased, but these events represented a small proportion of all AE event numbers (less than 10%).

In addition, for different anisotropy angles, the largest crack numbers included in a single AE event were also different. For example, when $\theta = 0^\circ$, the largest crack number included in a single AE event was sixteen, but there was only one such AE event. When $\theta = 30^\circ$, a single AE event included the least crack numbers (8), and there were two such AE events. When $\theta$ was relatively large ($\theta = 75^\circ$ and $90^\circ$), a single AE event included eighteen cracks and had the largest seismic magnitude and influence area.

4.2. Moment tensor $T-k$ chart analysis

Fig. 8 shows the moment tensor $T-k$ charts for seven different anisotropy angles ($\theta$) where each $\theta$ has four different loading points, namely 50% peak stress, 80% peak stress, peak stress, and post-peak stress.

When $\theta$ was relatively small ($\theta = 0^\circ$, $15^\circ$, and $30^\circ$), the points in the moment tensor $T-k$ chart were almost evenly distributed on the $U = -1/3 V$ line when specimens were loaded to 50% and 80% peak stress, which meant that the crack types increased evenly during this period. The rate of increase in the crack types varied with the increase in loading. The moment tensor increased in the coordinate range of $(-1, 1/3)$ and increased rapidly when the shale was loaded at peak stress, which represented most linear tension cracks in this range. The situation was similar for the post-peak stress period and the peak stress period. Overall, all points for the AE events were distributed around the $U = -1/3 V$ line with few points far away from the line, therefore, the seismic magnitude was also small. Especially when $\theta = 30^\circ$, all points were distributed close to the $U = -1/3 V$ line and both the seismic magnitude of the AE events and the bearing capacity were at a minimum.

A transition stage occurred at $\theta = 45^\circ$. When the shale was loaded to 50% and 80% peak stress, a few points were far away from the $U = -1/3 V$ line. In the post-peak stage, i.e. the shale failure stage, linear tension cracks still represented the main type of crack.

When $\theta$ was large ($\theta = 60^\circ$, $75^\circ$ and $90^\circ$) and the shale was loaded at 50% and 80% peak stress, the number of points that were far away from and close to the $U = -1/3 V$ line was roughly equal. In the peak stress and post-peak stress stages, more points were concentrated in the coordinate range of $(-1, 1/3)$ when $\theta$ had a
large compared to a small value. There were still many points far away from the $U = -1/3 \ V$ line, especially when $\theta = 90^\circ$, which meant the seismic magnitude of the AE events and the bearing capacity were at a maximum when $\theta = 90^\circ$.

Fig. 9 depicts the number and proportion of shale crack types. Both the number of linear tension cracks and their proportion increased along with an increase in $\theta$. The numbers and proportions reached a minimum level when $\theta = 15^\circ$ and $30^\circ$ respectively and a maximum level when $\theta = 90^\circ$. The change trend in double couple cracks was contrary to that of the linear tension cracks, and both number and proportion of cracks were smaller for double couple cracks than for the linear tension cracks, even when $\theta = 30^\circ$. The proportion of linear tension cracks remained almost steady regardless of the value of $\theta$.

5. Conclusions

A DEM-AE model was employed in this study to analyze the moment tensor of transversely isotropic shale based on the results of our previous physical experiment. Based on the simulated results, the following conclusions were drawn:

1. As for the failure mode of the transversely isotropic shale, the numerical simulation results matched the physical experiment results well, which means the DEM-AE model is suitable for describing the anisotropies of rock materials.

2. As for different anisotropic angles, there was a linear logarithmic relationship between cumulative AE events and seismic magnitudes in the concentration area of the AE events. The number of different AE events had a normal distribution as the seismic magnitude changed from small to large. There was a negative exponential decreasing relationship between the number of AE events and the number of cracks included in a single AE event. Particularly, the peak seismic magnitude was at a minimum and the average seismic magnitude in the concentration area of the AE events was relatively small when $\theta = 30^\circ$.

3. An analysis and comparison of all moment tensor T-k charts of the transversely isotropic shale indicated that the linear tension cracks represented the largest number and the highest proportion of all crack types, and the increasing trend was relatively immune to a change in $\theta$. However, the change trend of the number and proportion of double couple cracks was contrary to that of the linear tension cracks. Specifically, when $\theta = 90^\circ$, the largest number of points were close to the $U = -1/3 \ V$ line, the peak seismic magnitude was at a maximum, and the bearing capacity was strongest.

In summary, this study serves as a basis for future research on the fracturing treatment of shale, because it investigates the moment tensor of transversely isotropic shale and provides further insights into the understanding of the distribution, temporal-spatial location, and types of shale cracks appearing in the loading process.

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