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Bunker Consumption Optimization Methods in Shipping: A Critical Review and Extensions

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Abstract

It is crucial nowadays for shipping companies to reduce bunker consumption while maintaining a certain level of shipping service in view of the high bunker price and concerned shipping emissions. After introducing the three bunker consumption optimization contexts: minimization of total operating cost, minimization of emission and collaborative mechanisms between port operators and shipping companies, this paper presents a critical and timely literature review on mathematical solution methods for bunker consumption optimization problems. Several novel bunker consumption optimization methods are subsequently proposed. The applicability, optimality, and efficiency of the existing and newly proposed methods are also analyzed. This paper provides technical guidelines and insights for researchers and practitioners dealing with the bunker consumption issues.

Key Words: Shipping; Sailing speed; Bunker consumption optimization; Mixed-integer nonlinear programming

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1 Introduction

Maritime transportation is the backbone of world trade, and world seaborne trade was estimated at 8.4 billion tons in terms of the total goods loaded in 2011 (UNCTAD, 2011). In recent years, increased competition and global shipping downturn have been putting downward pressure on the revenues of shipping companies; at the same time, increased security regulations and fuel prices continued to increase their operating costs. The bunker cost constitutes a large proportion of the operating cost of a shipping company (Notteboom, 2006). For example, Ronen (2011) estimated that when bunker fuel price is around 500 USD per ton the bunker cost constitutes about three quarters of the operating cost of a large containership.

The amount of bunker consumed by ships also determines the amount of gas emission, including Green House Gas (GHG) such as carbon dioxide (CO_2), methane (CH_4), and nitrous oxide (N_2O), Non-Green House Gases such as sulphur oxides (SO_x) and nitrogen oxides (NO_x), and various other pollutants, such as particulate matter, volatile organic compounds, and black carbon (Psaraftis and Kontovas, 2013). The above gases have negative effect on global climate. For example, GHGs contribute to global warming, SO_x causes acid rain and deforestation, and NO_x causes undesirable health effects. According to the 2009 GHG study by the International Maritime Organization (IMO, 2009), international shipping contributes 2.7% of the CO_2 emitted globally. IMO is currently considering many measures to reduce GHGs (Psaraftis, 2012). For instance, the IMO Marpol 73/78 Annex VI regulations aim to reduce nitrogen oxide (NO_x) emissions and prevent sulphur oxide (SO_x) and particulate matter emissions from ships. In view of strict regulations on CO_2 emission, tradable CO_2 emission schemes have been developed and applied, and the current average contract price is about 8 Euros per ton of CO_2 emitted (ICE-ECX, 2012). To meet future regulation on emission, shipping companies must either reduce bunker consumption or use cleaner but more expensive bunker fuel, or purchase emission quota from other companies.

1.1 Impact of sailing speed on shipping capacity, inventory cost and bunker consumption

The bunker consumption of a ship on one hand depends on the design and structure of the ship, and it is on the other hand very sensitive to the sailing speed. This study focuses on the impact analysis of sailing speed on bunker consumption.

Fig. 1 plots the relations between sailing speed and bunker consumption for 4 types of ships: ships with a capacity of 3000 twenty-foot equivalent units (3000-TEU ships for short), 5000-TEU ships, 8000-TEU ships and 10000-TEU ships. Clearly, when the speed increases, the bunker consumption increases more than linearly. Ronen (1982) mentioned that daily bunker consumption is approximately proportional to the sailing speed cubed, and Wang and Meng (2012a) further calibrated the relation using historical operating data of containerships and found that the exponent is between 2.7 and 3.3, which supports the third power approximation. Du et al. (2011) used the exponent of 3.5 for feeder containerships, 4 for medium-sized containerships, and 4.5 for jumbo containerships according to suggestions of a ship engine manufacturing company. Kontovas and Psaraftis (2011) suggested using an exponent of 4 or greater when the speed is greater than 20 knots.

<*Fig 1 is inserted here*>

In general, a higher sailing speed has both advantages and disadvantages. The first advantage is that the amount of cargo that can be shipped annually is larger. For example, consider a ship with a capacity of 10,000 tons that sails between two ports (A and B) whose distance is 10000 n miles, and suppose that the total time for discharging and then loading a full ship load is 3 days at each port, as shown in **Fig. 2**. If the ship sails at 15 knots, it needs 3+10,000/(24×15)≈30.8 days to transport 10,000 tons of cargo from port A to port B (or from port B to port A). Therefore in one year it can transport 365/30.8×10,000 = 1.19×10^6 tons of cargo. If the ship sails at 20 knots, it needs only 23.8 days to ship cargo from A to B and hence would be able to transport 1.53×10^6 tons of cargo annually. The second advantage is that the inventory cost associated with shipping is lower. In the above example, the cargo needs a total of 30.8 days for maritime transportation and handling if the ship sails at 15 knots, and needs only 23.8 days at the speed of 20 knots. The inventory cost of containerized cargos is high because of the high value of the cargos. For instance, Notteboom (2006) estimated that one day delay of a 4, 000-TEU ship implies a total cost of 57, 000 Euros associated with the cargos in the containers; Bakshi and Gans (2010) estimated the inventory cost of containerized cargo at 0.5 per cent the value of a container per day.

The disadvantage of a higher sailing speed is that the amount of bunker burned is much higher. Suppose that the daily bunker consumption is proportional to the sailing speed cubed. As a result, the bunker consumption for accomplishing a trip from port A to port B in **Fig. 2** is proportional to the sailing speed squared (the daily bunker consumption is proportional to the sailing speed cubed, but the number of days required is inversely proportional to the sailing speed). Therefore, the amount of bunker consumed annually at the speed of 20 knots (proportional to 20^2×(365/23.8) ≈6134) is 130% higher than that at the speed of 15 knots (proportional to 15^2×(365/30.8) ≈2666), and the amount of cargo carried is only (1.53-1.19)/1.19≈29% higher. Consequently, the optimal sailing speed is desirable to balance the tradeoffs between cargo shipping capability, inventory cost, and bunker cost.

<Fig 2 is inserted here>

1.2 Contexts of bunker consumption optimization

In literature, bunker consumption optimization is cast into three application contexts. The first one is minimizing the operating cost of a shipping company by optimizing the sailing speed. For example, in shipping network design (Alvarez, 2009), ship fleet deployment (Gelareh and Meng, 2010), ship schedule construction (Qi and Song, 2012; Wang and Meng, 2012b), sailing speed optimization (Norstad et al., 2011; Ronen, 2011; Wang and Meng, 2012a), and selection of bunkering port and volume (Yao et al., 2012). As aforementioned, a lower speed means larger inventory cost. However, the inventory cost is borne by shippers and hence is not directly related to the shipping companies. Therefore, inventory cost is not considered in most of the studies in this category. Some studies explicitly incorporate the inventory cost (e.g., Wang and Meng, 2011 for schedule design), or impose a certain level of service in terms of the maximum allowable origin-to-destination (OD) transit time (Meng and Wang, 2011).

In the second category, the amount of emission (usually converted to CO_2 equivalent) is formulated in the model (Corbett, 2009; Kontovas and Psaraftis, 2011). From the government's viewpoint, imposing a fuel tax would effectively lower down the sailing speed of ships, thereby reducing the emissions at least in the short term (Corbett, 2009). From the shipping company's viewpoint, taking the minimization of bunker consumption (which is proportional to emission) as an objective has two implications: one is to fulfill the international or local regulations on ship emission; the other is to build an image of social responsibility. To account for emission in modeling, one approach is to minimize the weighted sum of operating cost and emission. Mathematically, this approach is equivalent to an increase of bunker price. Another possible approach aims to minimize the operating cost while ensuring that the emission cannot exceed a certain upper limit. This approach can be adopted to find Pareto-optimal solutions that minimize the operating cost and emission, as shown in Fig. 3.

<Fig 3 is inserted here>

In the third category, port operators take into account the bunker cost of the shipping companies (Golias et al., 2010; Lang and Veenstra, 2010; Du et al., 2011; Wang et al., 2013), which contrasts conventional planning approaches where port operators maximize their own efficiency in berth allocation. In such a setting, port operators prioritize the berthing of incoming ships while accounting for the bunker cost of incoming ships. After that, port operators inform each ship captain a suggested arrival time, and as a result the ship could slow down to save bunker if the port is already very congested. For example, suppose that the ship is 200 n miles away from the port, and it has to wait for 5 hours for a berth if it sails at its current speed 20 knots. If the port operator informs the ship captain that a berth is available only 200/20+5=15 hours later, then the captain could slow down to a speed of

200/15=13.3 knots, resulting in a significant reduction in bunker consumption. We give another example with more than one ship. Suppose that there are two identical ships approaching one port. One ship is sailing at the speed of 20 knots from 1000 n miles away, and the other is sailing at 25 knots from 1250 n miles away. Both ships need 10 hours' time for container handling at berth and both ships desire to be berthed in 60 hours. Only one berth is available for these two ships. If all other conditions are the same (identical ships sailing under the same condition and requiring the same container handing operations at the port), the port operators should let the ship at 20 knots be berthed first, and inform the ship at 25 knots to slow down to the speed of 1250/(1000/20+10)=20.8 knots. Note that if the ship at 25 knots is berthed first, the resulting bunker cost reduction is smaller, because the bunker consumption is more sensitive to speed when the speed is higher.

1.3 Objectives and contributions

Investigations on the solution methods are of considerable difficulty/significance for the bunker consumption optimization problems, due to the nonlinearity of bunker consumption relation with sailing speed and existence of discrete decision variables (the number of ships to deploy or berth allocation decisions). As a consequence, the objective of this paper is to critically review the solution methods proposed in the literature and then design efficient solution methods that supplement the existing methods. Contributions of this paper are threefold. First, we provide a complete framework on tailored ε-optimal solution methods, and this framework enables us to design six new tailored ε-optimal solution methods. Second, based on Du et al. (2011), we introduce an auto-conduction second-order cone programming

(SOCP)-transformation procedure that provides the optimal solution. Third, we review the existing methods in the literature and methods proposed by this paper and then analyze the advantages and disadvantages of each method. Hopefully, this review could provide guidelines for researchers and practitioners for optimizing bunker consumption to minimize operating cost and emission from the viewpoints of both shipping companies and port operators. Moreover, the approaches may also be applied to optimize sailing speed in settings with fixed speed (Christiansen et al., 2004; Shintani et al., 2007; Karlaftis et al., 2009; Gelareh et al., 2010; Bell et al., 2011; Brouer et al., 2011; Reinhardt and Pisinger, 2012).

The remainder of this paper is organized as follows. Section 2 gives a simple bunker consumption example for us to demonstrate the solution approaches. Section 3 presents two basic solution methods: enumeration and dynamic programming. Section 4 introduces a discretization approach. Section 5 proposes a complete framework on tailored ε-optimal solution methods. Section 6 is dedicated to an exact SOCP approach. A summary of these methods are provided in Section 7.

2 A simple bunker consumption optimization example

We present a simple speed optimization example that belongs to the first category of bunker consumption optimization context. Using this simple example we analyze the steps and properties of each solution method. It should be mentioned that these methods are also applicable to other bunker consumption optimization contexts.

Consider the Central China Express (CCX) container liner shipping service operated by OOCL (2012), as shown in Fig. 4. The port rotation of CCX can be coded by its port calling sequence - $1 \rightarrow 2, \rightarrow \cdots \rightarrow N \rightarrow 1$ - where the numbers 1 and N denote its first and last ports of call, respectively. Define $\mathcal{I} := \{1, 2 \cdots N\}$. Any port on the service can be coded as the first port of call because the itinerary is a directed loop. Two different ports of call may denote the same port with different port calling sequences. The voyage between two consecutive ports of call on the service is referred to as a *leg*. The i^{th} leg is defined as the voyage from the i^{th} port of call to the $(i+1)^{\text{th}}$ port of call when $i = 1, 2, \cdots, N-1$ and the N^{th} leg is from the N^{th} port of call to the 1^{st} port of call. For example, after choosing Qingdao as the first ports of call in Fig. 4, the CCX service can be coded as follows: 1 (Qingdao) → 2 (Ningbo) → 3 (Shanghai(WGQ)) → 4 (Shanghai(YAN)) → 5 (Pusan) → 6 (Los Angeles) → 7 (Oakland) → 8 (Pusan) → 1 (Qingdao).

<Fig 4 is inserted here>

A string of homogeneous ships are deployed on CCX to provide a weekly service frequency. For example, if the round-trip time is 42 days, then six ships are deployed to ensure that each port of call is visited once every week. Every port of call is visited on the same day of each week. Note that a port of call is different from a port in this study. For example, the port of Pusan in Fig. 4 is visited twice a week, and hence it corresponds to two ports of call, one of which is after Shanghai(YAN), and the other of which is after Oakland.

The round-trip time consists of port time and sea time. We assume that the time spent at each port of call $i \in \mathcal{I}$ in a round-trip is fixed and denoted by t_i^{port} (h). The time at sea depends on the distance of each voyage leg and the sailing speed. Let L_i be the oceanic distance (n mile) and v_i (knot) be the speed on leg i. Then the sailing time on leg i is L_i / v_i (h). We assume that ships have a maximum speed $V^{\max}$ that is subject to the mechanical properties of the ships. Assuming that a total of m ships are deployed on CCX, to maintain a weekly service, we have

$$\sum_{i \in I} L_i / v_i + \sum_{i \in I} t_i^{\text{port}} = 168m \tag{1}$$

where 168 is the number of hours in a week.

Providing a weekly service alone is not sufficient for customer satisfaction. As a consequence of competition, a maximum allowable transit time from port of call $i \in I$ to port of call $j \in I, j \neq i$ would be set when designing the service. In fact, the sailing speed has a significant impact on the level of service. For example, if the distance between two ports is 5000 n miles, then the difference in transit time when sailing at 25 knots and 20 knots is 50 hours, which translates to a total cost of 119, 000 Euros associated with the cargos on a 4, 000-TEU ship (Notteboom, 2006). Let T_{ij} (h) represent this maximum allowable transit time. If there is no container shipped from port of call i to port of call j, then we could simply set T_{ij} at a very large number. The transit time from port of call i to port of call j, including the container handling time at these two ports of call, should not exceed T_{ij}.

Eq. (1) further indicates that generally when the sailing speed is higher (higher bunker consumption), fewer ships are required to maintain a weekly service (lower ship cost), and vice versa. Therefore an optimal trade-off between bunker cost and ship cost is desirable. As the bunker consumption function is different on different voyage legs, we denote by $g_i(v_i)$ (tons/n mile) the bunker consumption per nautical mile at the speed v_i on leg i. If the daily bunker consumption is proportional to the speed to the power of ω_i, then $g_i(v_i)$ is proportional to the speed to the power of $\omega_i - 1$. It is reasonable to assume that $g_i(v_i)$ is a strictly convex and non-decreasing function. It should be mentioned that in reality the relation between speed and bunker consumption $g_i(v_i)$ may change in different trips because of the uncertain currents, wind, tides and seasonal storms. In fact, the function $g_i(v_i)$ is calibrated from historical data of different trips. Therefore, when modeling the function $g_i(v_i)$ can be considered as an average bunker consumption at the speed v_i.

Represent by α^{bun} (USD/ton) the bunker fuel price and let c^{ship} (USD/week) be the fixed operating cost of a ship on CCX. The bunker consumption optimization (BCO) problem aims to determine the number of ships m to deploy and the sailing speed v_i on each leg, in order to minimize the total operating cost while fulfilling the weekly service and transit time constraints. The BCO problem can be formulated as:

[BCO]
$$\min_{v_i,m} \alpha^{\text{bun}} \sum_{i\in I} L_i g_i(v_i) + c^{\text{ship}} m \tag{2}$$

subject to:

$$\sum_{i\in I} L_i / v_i + \sum_{i\in I} t_i^{\text{port}} = 168m \tag{3}$$

$$\sum_{i\le k\le j-1} L_k / v_k + \sum_{i\le k\le j} t_k^{\text{port}} \le T_{ij}, i, j \in I, i < j \tag{4}$$

$$\sum_{i\le k\le N, 1\le k\le j-1} L_k / v_k + \sum_{i\le k\le N, 1\le k\le j} t_k^{\text{port}} \le T_{ij}, i, j \in I, i > j \tag{5}$$

$$0 \le v_i \le V^{\max}, \forall i \in I \tag{6}$$

$$m \text{ is a positive integer} \tag{7}$$

The objective function (2) minimizes the sum of bunker cost and ship cost. Constraint (3) imposes the weekly service frequency. Constraints (4)-(5) enforce the transit time requirement. Note that we assume that T_{ij} is greater than the second term on the left-hand side of Eqs. (4)-(5) as otherwise there is no solution. Constraint (6) defines the speed range. We may also impose a minimum speed as in Ronen (2011) to account for engine wear, then we need to incorporate some slack time at port or at sea to ensure that "=" holds in constraint

(3). Whether the minimum speed is equal to 0 or greater than 0 does not affect the solution method. Constraint (7) defines the number of ships to be a positive integer.

3 Basic optimization methods

[BCO] is a mixed-integer nonlinear programming model with nonlinear terms in its objective function (2) and constraints (3)-(5). Moreover, constraints (3)-(5) are non-convex. Therefore it is very difficult to solve [BCO] directly. There are some basic optimization methods in literature that address special cases of the BCO problem. One method addresses the problem by assuming that bunker consumption function $g_i(v_i)$ does not change over different voyage legs and that there is no transit time constraints shown in Eqs. (4)-(5). The other is a dynamic programming approach which extends the analytical method by relaxing the assumption of uniform bunker consumption function $g_i(v_i)$. Other approaches, such as linear programming by assuming the bunker consumption is linear with speed (Lang and Veenstra, 2010) and genetic-algorithm (Golias et al., 2010) cannot guarantee optimality; the gradient descent method (Qi and Song, 2012) is a general solution method. Therefore, these methods are not elaborated.

3.1 Enumeration method

Corbett et al. (2009) and Ronen (2011) have implicitly made two assumptions about the BCO problem. First, they assume that that bunker consumption function $g_i(v_i)$ does not change over different voyage legs. Second, T_{ij} is assumed to be infinite, or in other words, constraints (4)-(5) are not incorporated. Under these two assumptions, we prove the following two theorems, which are used by Corbett et al. (2009) and Ronen (2011) without a rigorous proof.

Theorem 1: The optimal sailing speed v_i^* is uniform over all voyage legs.

Proof: Suppose that there exist $i, j \in I, i \neq j$ such that $v_i^* \neq v_j^*$. Define

$$u_i^* = 1/v_i^*, \forall i \in I \tag{8}$$

and

$$G(u_i) = G(1/v_i) = g_i(v_i), \forall i \in I \tag{9}$$

The total bunker consumption on legs i and j is:

$$L_i g_i(v_i^*) + L_j g_j(v_j^*) = (L_i + L_j)\left[\frac{L_i}{L_i + L_j} G(u_i^*) + \frac{L_j}{L_i + L_j} G(u_j^*)\right] \tag{10}$$

If the ship sails at the same speed on legs i and j and the total travel time does not change, then the common speed would be

$$v_{ij} = \frac{L_i + L_j}{L_i / v_i^* + L_j / v_j^*} \tag{11}$$

Its reciprocal is

$$u_{ij} = 1/v_{ij} = \frac{L_i}{L_i + L_j} u_i^* + \frac{L_j}{L_i + L_j} u_j^* \tag{12}$$

The total bunker consumption is:

$$(L_i + L_j)G(u_{ij}) = (L_i + L_j)G\left(\frac{L_i}{L_i + L_j} u_i^* + \frac{L_j}{L_i + L_j} u_j^*\right) \tag{13}$$

Since $g_i(v_i)$ is strictly convex and non-decreasing and $u_i = 1/v_i$ is also strictly convex when $v_i > 0$, $G(u_i)$ is strictly convex. Therefore the bunker consumption (13) is less than (10). Hence, the optimal sailing speed v_i^* is uniform over all voyage legs. □

Similarly, we have

Theorem 2: The optimal sailing speed is constant on each voyage leg. □

In view of these two theorems, the BCO problem can be solved easily as it only has two decision variables: the number of ships m and the common speed denoted by v. The number of ships m is a positive integer and smaller than e.g. 20 from practical point of view. Therefore we could enumerate all the possible values of m. For each m we determine the speed according to Eq. (3) and subsequently calculate the total cost function (2). The optimal number of ships and the optimal common sailing speed could be determined. Ronen (2011) employed exactly this procedure and plotted a figure of the change of total operating cost with the common speed, as shown in Fig. 5. If $g_i(v_i)$ changes over different voyage legs or T_{ij} is finite, then the optimal speeds on different voyage legs may be different and hence the above enumeration procedure is no longer applicable.

<Fig 5 is inserted here>

3.2 Dynamic programming method

The dynamic programming (DP) method was applied by Norstad et al. (2011) for solving a tramp ship routing and scheduling problem. This method is also applicable to the BCO problem excluding constraints (4)-(5). To implement the dynamic programming method, we first construct a space-time network where the horizontal axis corresponds to time (time is discretized into units of e.g. days, 12 hours, 4 hours, or 1 hour, depending on the precision) and the vertical axis corresponds to the ports of call, as shown in Fig. 6. Note that the discretization of time corresponds to the discretization of ship speed. For clarity, in Fig. 6 we assume that the port time $t_i^{\text{port}} = 0$, $i \in \mathcal{I}$. Without loss of generality, the ship visits the first port of call on day 0. An arc from the node (Port 1, Day 0) to a node corresponding to the 2nd port of call determines the sailing time, speed, and bunker consumption (note that $g_i(v_i)$ changes over different voyage legs). For example, the arc from (Port 1, Day 0) to (Port 2, Day 1) corresponds to a much higher bunker cost than the arc from (Port 1, Day 0) to (Port 2, Day 3) as the former has a much larger sailing speed. Path 1 and Path 2 converge at (Port 3, Day 6). These two paths have different bunker costs and the same ship cost (or more exactly, the same trip time from the 1st port of call to the 3rd one). Evidently, the optimal path in the space-time network starting from (Port 3, Day 6) relies exclusively on the state (Port 3, Day 6, and the optimal total bunker cost on leg 1 and leg 2). As a result, only the best path from (Port 1, Day 0) to (Port 3, Day 6) needs to be recorded when we extend the path to the 4th, 5th ports of call, etc. Therefore, a dynamic programming approach is suitable for finding the optimal number of ships to deploy and the optimal speed v_i^* on each voyage leg $i \in \mathcal{I}$.

If T_{ij} is finite, then at each node more information must be recorded. For example, at (Port 3, Day 6) we also need to record the arrival time at the 2nd port of call because it affects the feasibility of the transit time constraint from the 2nd port of call to other ports of call. As a consequence, the state of a node contains information on the arrival time at all the previous ports of call. Therefore, the BCO problem is no longer tractable due to the curse of dimensionality.

<Fig 6 is inserted here>

4 Discretization Methods

To overcome the deficiencies of the basic optimization methods, Gelareh and Meng (2010) and Yao et al. (2012) have proposed a discretization method. The method works as follows. First, similar to Eqs. (8)-(9), the reciprocal of speed is used as the decision variable, and [BCO] is reformulated as follows:

[P]
$$\min_{u_i, m} \alpha^{\text{bun}} \sum_{i \in I} L_i G_i(u_i) + c^{\text{ship}} m \tag{14}$$

subject to:

$$\sum_{i \in I} L_i u_i + \sum_{i \in I} t_i^{\text{port}} = 168m \tag{15}$$

$$\sum_{i \le k \le j-1} L_k u_k + \sum_{i \le k \le j} t_k^{\text{port}} \le T_{ij}, i, j \in I, i < j \tag{16}$$

$$\sum_{i \le k \le N, 1 \le k \le j-1} L_k u_k + \sum_{i \le k \le N, 1 \le k \le j} t_k^{\text{port}} \le T_{ij}, i, j \in I, i > j \tag{17}$$

$$u_i \ge 1/V^{\max}, \forall i \in I \tag{18}$$

$$m \text{ is a positive integer} \tag{19}$$

[P] is a mixed-integer nonlinear programming model where only the objective function (14) has nonlinear terms.

The range of u_i can be uniformly or non-uniformly divided into K_i segments, see Fig. 7. The larger K_i is, the more accurate the solution is. Note that although u_i does not have an upper bound, it is not difficult to impose a reasonable upper bound $u_i^{\max}$ considering that in practice ships will not sail at a speed lower than e.g. 1 knot. After division, we obtain $K_i + 1$ speed values (strictly speaking, values of the reciprocal of speed), denoted by u_i^0, u_i^1 … $u_i^{K_i}$. To indicate which speed to adopt, we define binary variable b_i^κ, $\kappa \in \{0, 1 \cdots K_i\}$, which equals 1 if and only if speed u_i^κ is adopted on leg $i \in I$, and 0 otherwise. As a result, [P] can be approximated by an integer programming model:

[P1] $$\min_{b_i^\kappa, m} \alpha^{\text{bun}} \sum_{i \in I} L_i \sum_{\kappa \in \{0,1 \cdots K_i\}} b_i^\kappa G_i(u_i^\kappa) + c^{\text{ship}} m \tag{20}$$

subject to:

$$\sum_{i \in I} L_i \sum_{\kappa \in \{0,1 \cdots K_i\}} b_i^\kappa u_i^\kappa + \sum_{i \in I} t_i^{\text{port}} \le 168m \tag{21}$$

$$\sum_{i \le k \le j-1} L_k \sum_{\kappa \in \{0,1 \cdots K_k\}} b_k^\kappa u_k^\kappa + \sum_{i \le k \le j} t_k^{\text{port}} \le T_{ij}, i, j \in I, i < j \tag{22}$$

$$\sum_{i \le k \le N, 1 \le k \le j-1} L_k \sum_{\kappa \in \{0,1 \cdots K_k\}} b_k^\kappa u_k^\kappa + \sum_{i \le k \le N, 1 \le k \le j} t_k^{\text{port}} \le T_{ij}, i, j \in I, i > j \tag{23}$$

$$\sum_{\kappa \in \{0,1 \cdots K_i\}} b_i^\kappa = 1, \forall i \in I \tag{24}$$

$$b_i^\kappa \in \{0,1\}, \forall i \in I, \forall \kappa \in \{0,1 \cdots K_i\} \tag{25}$$

$$m \text{ is a positive integer} \tag{26}$$

Note that in Eq. (21) we use "≤" rather than "=" as in Eq. (15) because of the discretization. Note further that the dynamic programming method in Section 3.2 is also based on the discretization of speed.

[P1] is an integer linear programming model and may be solved by optimization solvers such as CPLEX. The discretization method is capable of handling all the necessary constraints. The precision depends on the number of discretization intervals and how the speed range is discretized. Nevertheless, the disadvantage is that there are a large number of integer decision variables in [P1], thereby posing considerable computational difficulties.

<Fig 7 is inserted here>

5 Tailored methods

Although the objective function (20) in [P1] is nonlinear, it is convex. In view of this sound property, a number of tailored methods are proposed, as summarized in Table 1. We elaborate on a few representative methods, and other methods follow in a similar manner.

<*Table 1 is inserted here*>

5.1 Linear static outer-approximation method

In contrast to the discretization method that adds more binary decision variables, the linear static outer-approximation method adds linear constraints to model [P]. As shown in Fig. 8 (a), a number of tangent lines are generated, for example, by uniformly dividing u_i, or uniformly dividing $G_i(u_i)$, or using as few lines as possible while guaranteeing a maximum approximation tolerance. The slopes and intercepts of these tangent lines are recorded in a set Ω_i. After introducing auxiliary variables G_i, [P] can be linearized as follows:

[P2] $$\min_{u_i, m, G_i \ge 0} \alpha^{\text{bun}} \sum_{i \in \mathcal{I}} L_i G_i + c^{\text{ship}} m \quad (27)$$

subject to:

$$G_i \ge \text{slope}_i^{\kappa} \times u_i + \text{intercept}_i^{\kappa}, \forall i \in \mathcal{I}, \forall (\text{slope}_i^{\kappa}, \text{intercept}_i^{\kappa}) \in \Omega_i \quad (28)$$

$$\sum_{i \in \mathcal{I}} L_i u_i + \sum_{i \in \mathcal{I}} t_i^{\text{port}} = 168m \quad (29)$$

$$\sum_{i \le k \le j-1} L_k u_k + \sum_{i \le k \le j} t_k^{\text{port}} \le T_{ij}, i, j \in \mathcal{I}, i < j \quad (30)$$

$$\sum_{i \le k \le N, 1 \le k \le j-1} L_k u_k + \sum_{i \le k \le N, 1 \le k \le j} t_k^{\text{port}} \le T_{ij}, i, j \in \mathcal{I}, i > j \quad (31)$$

$$u_i \ge 1 / V^{\max}, \forall i \in \mathcal{I} \quad (32)$$

$$m \text{ is a positive integer} \quad (33)$$

[P2] is a mixed-integer linear programming model. Compared with [P1], the number of integer decision variables does not increase. Therefore, the computational efficiency of [P2] is much higher than [P1].

<*Fig 8 is inserted here*>

5.2 Linear dynamic outer-approximation method

The tangent lines can also be generated dynamically whenever necessary, as shown in Fig. 9. Wang and Meng (2012c) applied the linear dynamic outer-approximation approach in a slightly different context. The procedure is as follows. First, solve [P2] without constraints (28). The optimal solution is denoted by $(m_i^*, u_i^*, G_i^*, i \in \mathcal{I})$. For each leg $i \in \mathcal{I}$, check the gap between $G_i(u_i^*)$ and G_i^*. If this gap is too large, generate a new tangent line at point $(u_i^*, G_i(u_i^*))$, add such a constraint to [P2], and resolve. Otherwise, the approximation gap is acceptable, and the solution is ε-optimal, where ε>0 is a pre-specified tolerance level.

In general, linear dynamic outer-approximation method needs fewer tangent lines than its static counterpart. However, [P2] has to be solved more than once. Therefore, there is no straightforward answer whether the dynamic or the static method is preferable.

The static and dynamic methods could also be combined. First, some tangent lines are generated a priori. Then model [P2] is solved subject to the constraints (28) of these tangent lines. If the approximation gap is large, generate more tangent lines and resolve. Otherwise the solution is good enough for practical applications.

<Fig 9 is inserted here>

5.3 Linear branch-and-bound outer-approximation method

The approximation gap can also be narrowed by a branch-and-bound (B&B) scheme. This method works as follows. First, a few (e.g. 2) tangent lines are generated for each leg, and model [P2] is solved. The optimal solution is denoted by $(m_i^*, u_i^*, G_i^*, i \in \mathcal{I})$. If the gap between $G_i(u_i^*)$ and G_i^* is large, then we branch the feasible range of u_i, which is

$[1/V^{\max}, u_i^{\max}]$, into two ranges: $[1/V^{\max}, u_i^*]$ and $[u_i^*, u_i^{\max}]$. Therefore in one branch, $u_i \in [1/V^{\max}, u_i^*]$ and in the other branch $u_i \in [u_i^*, u_i^{\max}]$. The tangent lines for the original range $[1/V^{\max}, u_i^{\max}]$ are removed and two new tangent lines for the feasible range of u_i in each branch are generated. Since the width of the ranges of the two new branches is narrower than the original range, the approximation error on the two new branches should be smaller. This process is repeated combined with a bounding process, and finally an ε-optimal solution is obtained.

Meng and Wang (2011) compared the efficiency of the linear B&B outer-approximation method and the discretization method. Results demonstrate that the former is several orders more efficient than the latter.

<Fig 10 is inserted here>

5.4 Linear static secant-approximation method

Instead of using tangent lines, we could also use secant lines to approximate the nonlinear function $G_i(u_i)$, as shown in Fig. 8 (b). The tangent lines always underestimate bunker consumption and secant lines may underestimate or overestimate bunker consumption. Tangent lines seem to be the natural choice of approximation. However, to achieve the same accuracy, fewer secant lines are needed than tangent lines.

Given an approximation tolerance ε>0 , the secant lines can be generated as follows, as shown in Fig. 11:

Function Generate Secant Lines $(u_i, G_i(u_i), u_i^{\min} = 1/V^{\max}, u_i^{\max})$ {

Define points $A_1 = (u_i^{\min}, G_i(u_i^{\min}))$, $D_1 = (u_i^{\max}, G_i(u_i^{\max}))$, $A_2 = (u_i^{\min}, G_i(u_i^{\min}) - \varepsilon)$, $D_2 = (u_i^{\max}, G_i(u_i^{\max}) + \varepsilon)$.

(a) If the maximum gap between line A_2D_2 and the curve $G_i(u_i)$ over the interval $[u_i^{\min}, u_i^{\max}]$ does not exceed ε, as shown in Fig. 11 (a)-(b), then only the line A_2D_2 is generated. Return.

(b) Generate a line that passes point A_2 with slope k (the value of k is to be determined). The line is:

$$G_i = k(u_i - u_i^{\min}) + G_i(u_i^{\min}) - \varepsilon \tag{34}$$

Let u_i^* be the point corresponding to the maximum difference of the line and the curve $G_i(u_i)$ over the interval $[u_i^{\min}, u_i^{\max}]$, that is,

$$u_i^* := \arg\max\{u_i \in [u_i^{\min}, u_i^{\max}] \mid k(u_i - u_i^{\min}) + G_i(u_i^{\min}) - \varepsilon - G_i(u_i)\} \tag{35}$$

The value of k is chosen such that the maximum gap is equal to ε, that is,

$$k(u_i^* - u_i^{\min}) + G_i(u_i^{\min}) - \varepsilon - G_i(u_i^*) = \varepsilon \tag{36}$$

as shown in Fig. 11 (c), where the point $B_1 = (u_i^*, G_i(u_i^*))$, and point $B_2 = (u_i^*, G_i(u_i^*) + \varepsilon)$. Apparently,

$$u_i^{\min} < u_i^* < u_i^{\max} \tag{37}$$

(b.1) If the gap between the curve $G_i(u_i)$ and the line at $u_i = u_i^{\max}$ is not greater than ε, as shown in Fig. 11 (c). The line is sufficient to ensure ε-optimality. Return.

(b.2) Record the generated line and find the value of $u_i = u_i^{\text{min-new}}$ such that the difference of the curve $G_i(u_i)$ and the line at $u_i = u_i^{\text{min-new}}$ is equal to ε, as shown in Fig. 11 (d). Call the function Generate Secant Lines $(u_i, G_i(u_i), u_i^{\text{min-new}}, u_i^{\max})$.

Return.

}□

<Fig 11 is inserted here>

5.5 Quadratic static outer-approximation method

Instead of using straight lines, we could also use parabolic curves to approximate the nonlinear function $G_i(u_i)$, as shown in Fig. 8 (c). A parabola can be defined by three parameters $\mathrm{a}_i^\kappa, \mathrm{b}_i^\kappa, \mathrm{c}_i^\kappa$. Let $\Omega_i^{\text{parabola}}$ be a set representing the parameters $\mathrm{a}_i^\kappa, \mathrm{b}_i^\kappa, \mathrm{c}_i^\kappa$ of the parabolas). Eq. (28) can be replaced with:

$$G_i \geq \mathrm{a}_i^\kappa \times (u_i)^2 + \mathrm{b}_i^\kappa \times u_i + \mathrm{c}_i^\kappa, \forall i \in \boldsymbol{I}, \forall (\mathrm{a}_i^\kappa, \mathrm{b}_i^\kappa, \mathrm{c}_i^\kappa) \in \Omega_i^{\text{parabola}} \tag{38}$$

Evidently, $\mathrm{a}_i^\kappa > 0$ and therefore Eq. (38) can be transformed to second-order cone programming (SOCP) constraints. A simple SOCP constraint has the form

$$\| x, y \|_2 \leq z, \text{ that is, } \sqrt{x^2 + y^2} \leq z \tag{39}$$

A frequently encountered form $x^2 \leq yz$, $x, y, z \geq 0$ can also be transformed to SOCP constraint

$$\| x, (y-z)/2 \|_2 \leq (y+z)/2 \tag{40}$$

Optimization solvers such as CPLEX could solve mixed-integer SOCP models. How to generate the parabolic curves and how to transform Eq. (38) to SOCP constraints are elaborated in Wang et al. (2013).

A parabola outperforms a straight line in approximating the nonlinear function $G_i(u_i)$ because a straight line can be considered an extreme case of a parabola when $\mathrm{a}_i^\kappa = 0$. However, solving a model with an SOCP constraint is more time-consuming than a linear constraint. Therefore it is not easy to say whether straight lines or parabolic curves are preferable.

6 An exact second-order cone programming approach

The function $G_i(u_i)$ is generally assumed or calibrated to be a power function in most studies. If the daily bunker consumption is proportional to the ω_ith power of the speed, defining $\rho_i = 1 - \omega_i$, then $G_i(u_i)$ can be represented by:

$$G_i(u_i) = \beta_i (u_i)^{1-\omega_i} = \beta_i (u_i)^{\rho_i}, \forall i \in I \tag{41}$$

where β_i is a parameter calibrated from historical data. After introducing intermediate variables h_i, the objective function (27) and constraint (28) in [P2] can be replaced by

[P3] $$\min_{u_i, m, h_i \geq 0} \alpha^{\text{bun}} \sum_{i \in I} L_i \beta_i h_i + c^{\text{ship}} m \tag{42}$$

$$h_i \geq (u_i)^{\rho_i}, \forall i \in I \tag{43}$$

To simplify the notation, we suppress the subscript i in Eq. (43) in the sequel. Du et al. (2011) showed that when $\omega \in \{3.5, 4.0, 4.5\}$, constraint (43) can be transformed (*NOT approximated*) to SOCP constraints. For example, when $\omega = 3$, the constraint $h \geq u^{-2}$ is equivalent to two SOCP constraints by introducing an intermediate variable s:

$$1 \leq su,\ s^2 \leq h,\ \text{that is, } 1^2 \leq su,\ s^2 \leq h \tag{44}$$

As a result, [P3] can be transformed to a mixed-integer SOCP model and solved by CPLEX.

The seminal work by Du et al. (2011) pointed out that a more general constraint $h \geq u^{\rho}$ can be transformed to SOCP constraints. However, this work did not mention how to implement such a transformation. In this paper we introduce an auto-conduction SOCP-transformation procedure. To this end, we rewrite it as:

$$h \geq u^{\rho} \Leftrightarrow h \geq u^{\rho=1\text{-}\omega} \Leftrightarrow 1 \leq h \cdot u^{\omega-1} \tag{45}$$

We can state $\omega-1$ as the quotient of two positive integers n_1 and n_2:

$$\omega-1=\frac{n_2}{n_1} \tag{46}$$

Hence, Eq. (45) is:

$$1 \leq h \cdot u^{\frac{n_2}{n_1}} \tag{47}$$

or

$$1 \leq h^{n_1} u^{n_2} \tag{48}$$

Define

$$\kappa := \arg\min\{\tilde{\kappa} \text{ is an integer} \mid 2^{\tilde{\kappa}} \geq n_1 + n_2\} \tag{49}$$

Eq. (47) can be transformed to:

$$1^{2^{\kappa}} \leq h^{n_1} u^{n_2} 1^{2^{\kappa}-n_1-n_2} \tag{50}$$

We examine a general case of Eq. (50) and transform it to SOCP constraints. The general case we consider is:

$$s_1^{2^{\kappa}} \leq h^{\theta_1} u^{\theta_2} s_2^{\theta_3} \tag{51}$$

where s_1, h, u, s_2 are nonnegative variables, $\theta_1, \theta_2, \theta_3, \kappa$ are nonnegative integers, and

$$\theta_1 + \theta_2 + \theta_3 = 2^{\kappa} \tag{52}$$

Evidently, if we can transform Eq. (51) to SOCP constraints, we can also transform Eq. (50) to SOCP constraints by adding linear constraints $s_1 = 1$ and $s_2 = 1$. Without loss of generality, we define that $\theta_1 \geq \theta_2 \geq \theta_3$. In other words, whenever we call the repetitive SOCP Transformation function below, we should ensure that:

$$\theta_1 \geq \theta_2 \geq \theta_3 \tag{53}$$

Function SOCP Transformation $(s_1, h, u, s_2, \kappa, \theta_1, \theta_2, \theta_3)$ {

(a) If $\kappa = 1$, then there are only two possible scenarios: $\theta_1 = 2, \theta_2 = \theta_3 = 0$ or $\theta_1 = \theta_2 = 1, \theta_3 = 0$. (a.1) If $\theta_1 = 2, \theta_2 = \theta_3 = 0$, constraint (51) is equivalent to:

$$s_1 \le h \tag{54}$$

(a.2) Else we have $\theta_1 = \theta_2 = 1, \theta_3 = 0$ and thus constraint (51) is equivalent to:

$$s_1^2 \le hu \tag{55}$$

Return.

(b) Else if all $\theta_1, \theta_2, \theta_3$ are even and $\kappa \ge 2$, we can divide each of $\theta_1, \theta_2, \theta_3$ by 2, and set $\kappa \leftarrow \kappa - 1$. Call SOCP Transformation $(s_1, h, u, s_2, \kappa - 1, \theta_1 / 2, \theta_2 / 2, \theta_3 / 2)$, return.

(c) Else there are two possible scenarios: (c.1) $\theta_1 \ge 2^{\kappa-1}$ and (c.2) $\theta_1 = \max(\theta_1, \theta_2, \theta_3) < 2^{\kappa-1}$.

(c.1) If $\theta_1 \ge 2^{\kappa-1}$, after introducing intermediate nonnegative variable s_3, Eq. (51) is transformed to:

$$s_1^{2^\kappa} \le s_3^{2^{\kappa-1}} h^{2^{\kappa-1}} \text{ and } s_3^{2^{\kappa-1}} h^{2^{\kappa-1}} \le h^{2^{\kappa-1}} h^{\theta_1 - 2^{\kappa-1}} u^{\theta_2} s_2^{\theta_3} \tag{56}$$

or,

$$s_1^2 \le s_3 h \text{ and } s_3^{2^{\kappa-1}} \le h^{\theta_1 - 2^{\kappa-1}} u^{\theta_2} s_2^{\theta_3} \tag{57}$$

The first constraint in Eq. (57) is already an SOCP constraint. To transform the second constraint in Eq. (57) and impose the condition in Eq. (53), there are three scenarios. (c.1.1) If $\theta_1 - 2^{\kappa-1} \ge \theta_2$, call SOCP Transformation $(s_3, h, u, s_2, \kappa - 1, \theta_1 - 2^{\kappa-1}, \theta_2, \theta_3)$; (c.1.2) else if $\theta_1 - 2^{\kappa-1} < \theta_3$, call SOCP Transformation $(s_3, u, s_2, h, \kappa - 1, \theta_2, \theta_3, \theta_1 - 2^{\kappa-1})$; (c.1.3) else call SOCP Transformation $(s_3, u, h, s_2, \kappa - 1, \theta_2, \theta_1 - 2^{\kappa-1}, \theta_3)$. Return.

(c.2) Else we have $\theta_1 = \max(\theta_1, \theta_2, \theta_3) < 2^{\kappa-1}$. Hence $\theta_3 < 2^{\kappa-1}$. As $\theta_1 + \theta_2 + \theta_3 = 2^{\kappa}$, we have $\theta_1 + \theta_2 > 2^{\kappa-1}$. After introducing intermediate nonnegative variables s_4 and s_5, Eq. (51) is transformed to:

$$s_1^{2^\kappa} \le s_3^{2^{\kappa-1}} s_4^{2^{\kappa-1}} \text{ and } s_3^{2^{\kappa-1}} s_4^{2^{\kappa-1}} \le h^{\theta_1} u^{2^{\kappa-1}-\theta_1} u^{\theta_1+\theta_2-2^{\kappa-1}} s_2^{\theta_3} \tag{58}$$

or,

$$s_1^2 \le s_3 s_4,\, s_3^{2^{\kappa-1}} \le h^{\theta_1} u^{2^{\kappa-1}-\theta_1} \text{ and } s_4^{2^{\kappa-1}} \le u^{\theta_1+\theta_2-2^{\kappa-1}} s_2^{\theta_3} \tag{59}$$

The first constraint in Eq. (59) is already an SOCP constraint. The second constraint can be written as:

$$s_3^{2^{\kappa-1}} \le h^{\theta_1} u^{2^{\kappa-1}-\theta_1} s_3^0 \tag{60}$$

To transform the second constraint in Eq. (59), noting that we have $\theta_1 > 2^{\kappa-1} - \theta_1$, we call SOCP Transformation $(s_3, h, u, s_3, \kappa-1, \theta_1, 2^{\kappa-1} - \theta_1, 0)$. The third constraint can be written as:

$$s_4^{2^{\kappa-1}} \le u^{\theta_1+\theta_2-2^{\kappa-1}} s_2^{\theta_3} s_4^0 \tag{61}$$

To transform Eq. (61) and impose the condition in Eq. (53), there are two scenarios. (c.2.1) If $\theta_1 + \theta_2 - 2^{\kappa-1} \ge \theta_3$, call SOCP Transformation $(s_4, u, s_2, s_4, \kappa-1, \theta_1 + \theta_2 - 2^{\kappa-1}, \theta_3, 0)$; (c.2.2) else call SOCP Transformation $(s_4, s_2, u, s_4, \kappa-1, \theta_3, \theta_1 + \theta_2 - 2^{\kappa-1}, 0)$. Return.

}□

Note that the above function of SOCP transformation terminates in a finite number of iterations because after one iteration the value of κ decreases by 1.

In theory, the SOCP approach is exact. However, in reality the coefficient β_i and the exponent ω_i are obtained from regression of historical data, and therefore there will be errors associated with the estimation of β_i and ω_i.

7 Conclusions

This study has reviewed and extended a number of bunker consumption optimization methods. The enumeration method is supplemented by proving that the sailing speed is constant in a round-trip. The dynamic programming method is borrowed from tramp shipping speed optimization to solve the liner ship speed optimization problem. The scheme of the discretization method is introduced in detail. A complete framework on tailored ε-optimal solution methods that take advantage of the convexity of the problem is proposed based on the existing studies. This framework enables us to design six new tailored ε-optimal solution methods. Finally, an auto-conduction second-order cone programming (SOCP)-transformation procedure is introduced. These methods could be used to optimize the sailing speed of ships, minimize emissions, and plan jointly for port operations and shipping operations. The properties of these approaches are summarized in Table 2.

<Table 2 is inserted here>

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List of Figures and Tables

(All figures are in color on the web only. Please use black and white figures in the printed version)

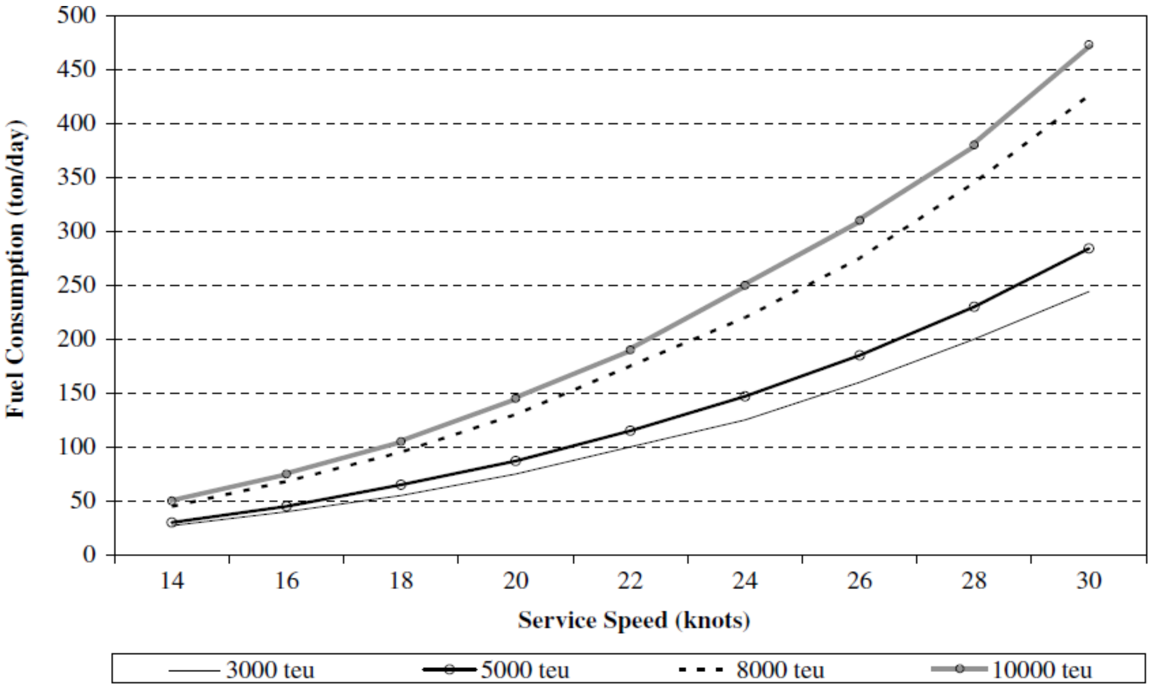
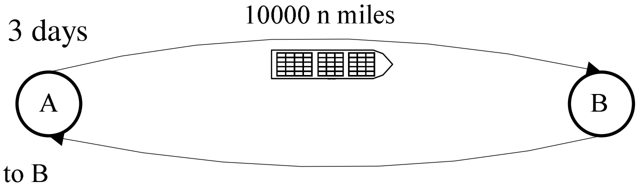


Fig. 1 Sensitivity of bunker consumption with regard to speed (Notteboom, 2006)



From A to B

Speed (knots)	Voyage time (days)	Total time (days)	#Voyage per year	Transported cargo (tons)	Bunker consumption (proportional to)
15	10000/(24×15) ≈ 27.8	3+27.8=30.8	365/30.8≈11.9	1.19×10^6	$15^2\times11.9$ ≈2666
20	10000/(24×20) ≈ 20.8	3+20.8=23.8	365/23.8≈15.3	1.53×10^6	$20^2\times15.3$ ≈6134

Fig. 2 Impact of different sailing speeds

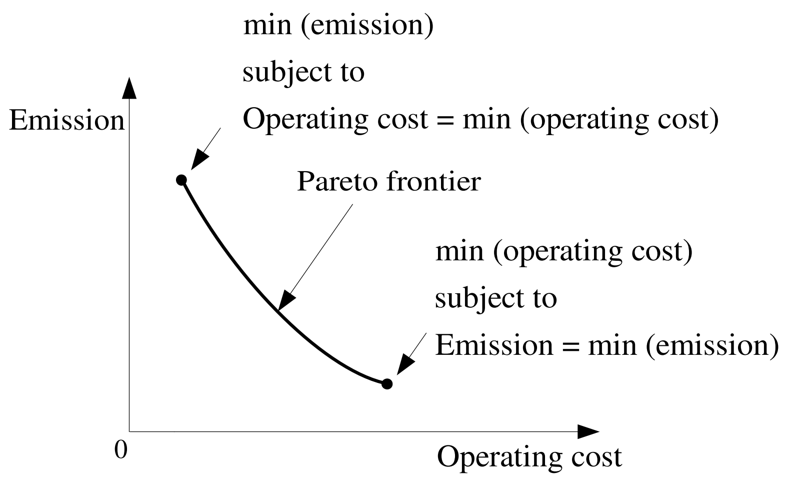


Fig. 3 Pareto frontier that minimizes the operating cost and emission

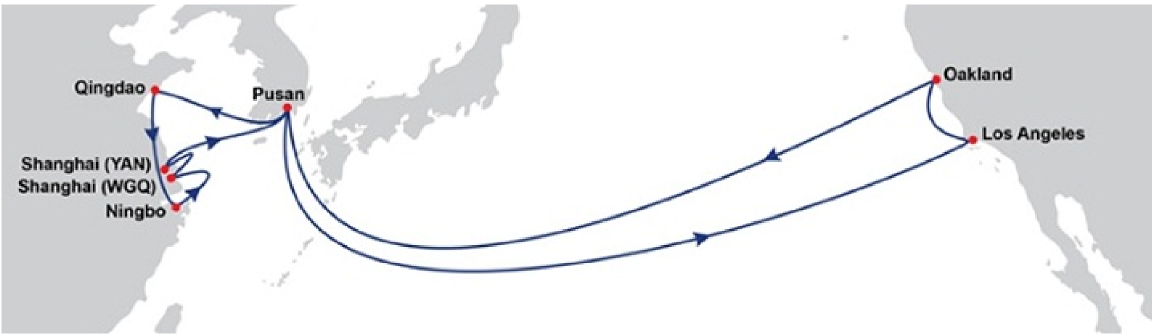
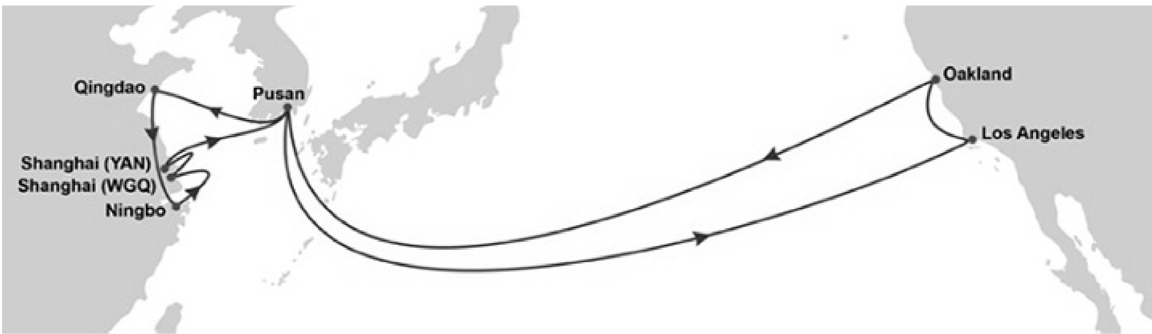


Fig. 4 CCX liner service route operated by OOCL (2012)



(Black and white version of Fig. 4 for the printed version)

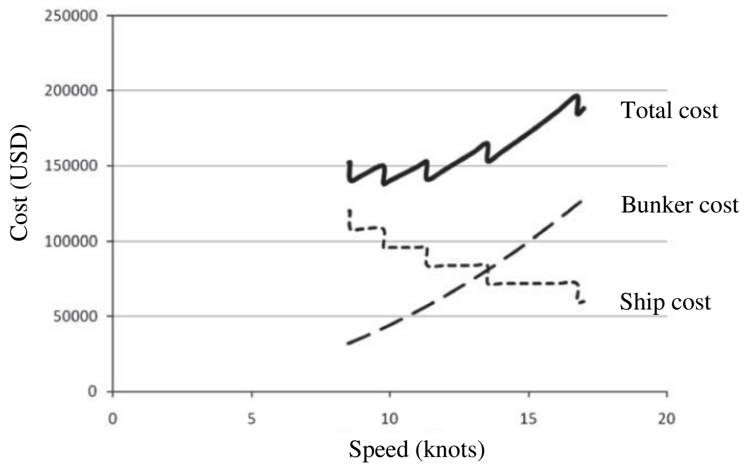


Fig. 5 Enumeration method (Ronen, 2011)

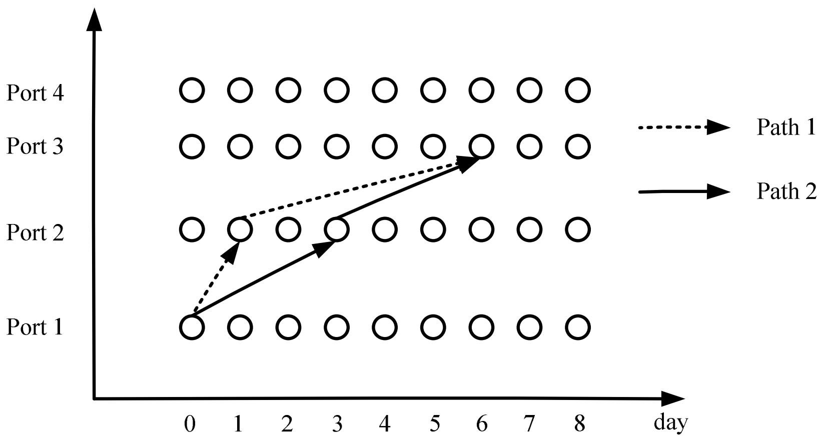


Fig. 6 Dynamic programming

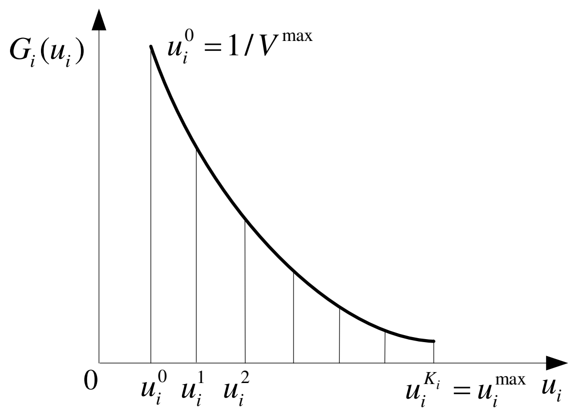
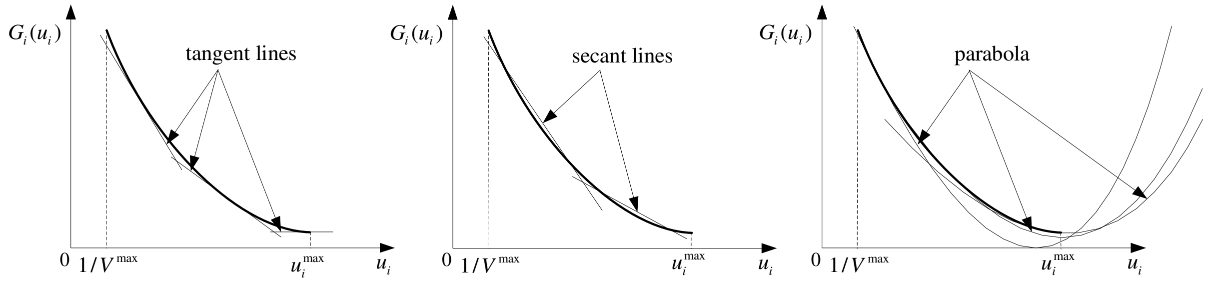


Fig. 7 Discretization



(a) Linear static outer-approximation (b) Linear static secant-approximation (c) Quadratic static outer-approximation

Fig. 8 Three static tailored methods

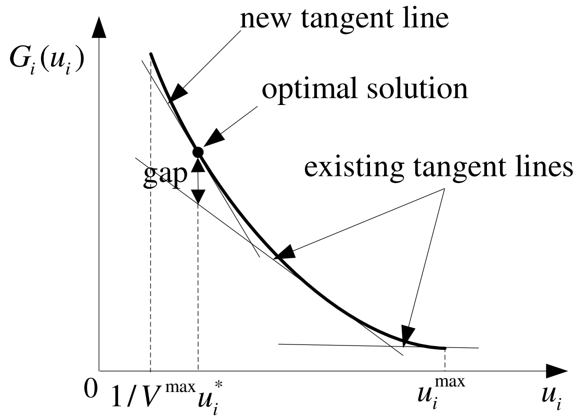


Fig. 9 Linear dynamic outer-approximation method

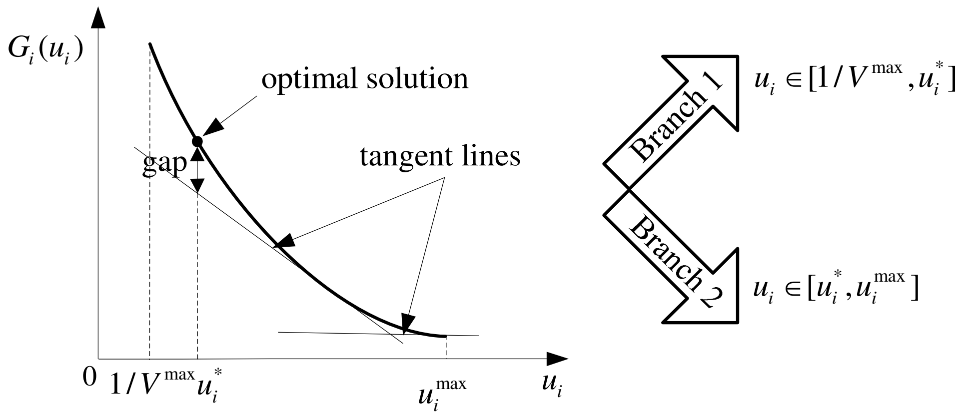


Fig. 10 Linear B&B outer-approximation method

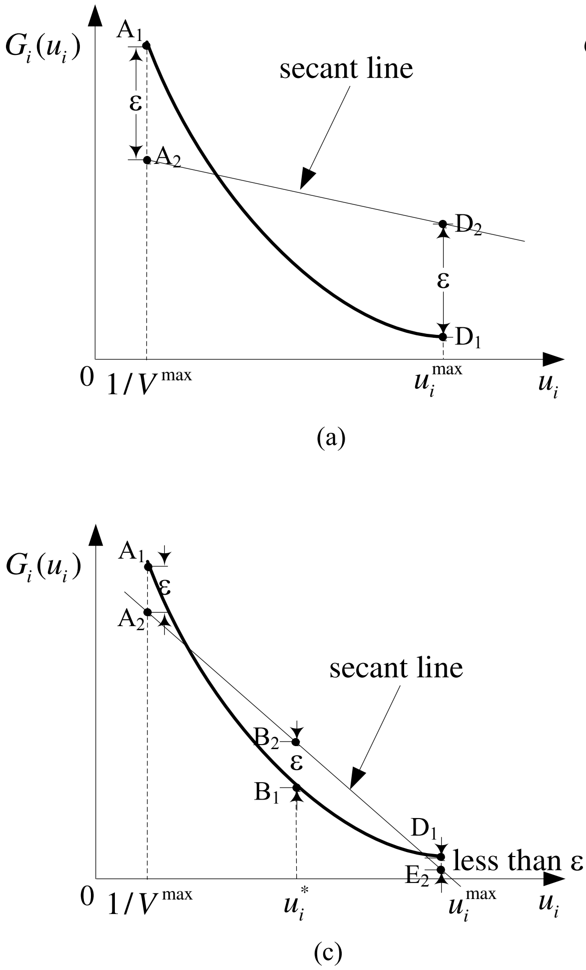


Fig. 11 Linear static secant-approximation method

Table 1 Tailored methods

Literature	Outer-approximation			Secant-approximation		
	Static	Dynamic	B&B	Static	Dynamic	B&B
Linear	Wang and Meng (2012a)	Wang and Meng (2012c)	Meng and Wang (2011)	Wang and Meng (2012b)	*	*
Quadratic	Wang et al. (2013)	Wang et al. (2013)	*	*	*	*

*Proposed by this paper

Table 2 Comparison of the solution methods

Methods	Application capabilities		Optimality		Efficiency	Implement in software tools
	Varying $g_i(v_i)$	Transit time constraint	ε-optimal	Optimal		
Enumeration	N	N	Y	Y	High	Easy
Dynamic programming	Y	N	Y	N	High	Easy
Discretization	Y	Y	Y	N	Low	MIP
Linear methods[a]	Y	Y	Y	N	High	MIP
Quadratic methods[b]	Y	Y	Y	N	High	MISOCP
SOCP	Y	Y	Y	Y	Depend on input parameters	MISOCP

Note: Y: Yes, N: No; MIP: Require mixed-integer linear programming (MIP) solvers; MISOCP: Require mixed-integer SOCP (MISOCP) solvers

[a] Include 6 methods – linear outer/secant static/dynamic/B&B approximation methods;

[b] Include 6 methods – quadratic outer/secant static/dynamic/B&B approximation methods;