Shape formation and ultimate load behaviour of post-tensioned space trusses

Gholamreza Dehdashti

University of Wollongong

1994

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Shape Formation and Ultimate Load Behaviour of Post-Tensioned Space Trusses

A thesis submitted in fulfilment of the requirements for the award of the degree

Doctor of Philosophy

from

UNIVERSITY OF WOLLONGONG

by

Gholamreza Dehdashti, MSc.

Department of Civil and Mining Engineering

1994
To the defenceless and innocent peoples of Bosnia and East Timor
Declaration

This is to declare that the research work contained herein has been carried out at the Civil and Mining Engineering Department of the University of Wollongong and has not been presented elsewhere for the fulfilment of the requirements of an academic degree.

The material presented in this thesis covers the following two conference papers co-authored by the writer as well as additional work.


In the Name of God, the Beneficent, the Merciful

Praise and thanks to God, the Lord of the heavens and the earth Who created and guided everything in the best possible way.

Acknowledgments

I would like to pay tribute to the sublime souls of all the prophets of God and all those believers who have sacrificed their lives, in the course of history, in defence of humanity and faith, providing us with the opportunity to live, learn, and prosper. Without them, the forces of greed and injustice would have destroyed the earth.

In memory of my late father who was my best support, I would like to thank my best friends, my mother and wife, for their incredible encouragement and patience throughout the preparation of this work.

I am very grateful to Professor L.C. Schmidt for his very efficient supervision and support. It was my pleasure and one of my best lifetime experiences to know him and benefit from his in-depth knowledge.

I would also like to acknowledge the kind assistance of the secretaries and the technical staff of the Department of Civil and Mining Engineering, University of Wollongong. Special thanks go to Messrs. Joe Abbott, Richard Webb, Charles Allport and Scot Dunster.
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Notation

The following is a list of the notation used in this thesis in alphabetical order.

$A$ = cross-sectional area
$b$ = total number of bars or elements
$\beta$ = edge chord subtended angle
$c$ = distance from neutral axis to the extreme fibre
$c$ = total number of kinematical constraints
$d$ = depth of truss
$\delta_s$ = the out-of-straightness of a member
$\Delta U$ = the incremental displacement vector
$e$ = eccentricity
$E$ = Young's modulus
$e$ = strain
$F$ = force
$F^A$ = vector of applied forces
$F^{NR}$ = vector of the Newton-Raphson restoring load
$F_d$ = member axial force due to vertical load
$F_p$ = member axial force due to post-tensioning
$F_y$ = yield stress
$F_{at}$ = maximum allowable stress in tension
$F_{tf}$ = maximum permissible tensile stress
$F_{tf}$ = maximum permissible shear stress
$F_a$ = maximum tensile strength
$F_{yf}$ = stress at 2% offset
$G$ = shear modulus
$GPa$ = giga Pascal
$h$ = rise of truss
$I$ = second moment of area
$j$ = number of joints
$J$ = torsion constant
$K$ = the stiffness matrix
$kN$ = kilo Newton
$L$ = length of member
$l$ = span of truss
$l/r$ = slenderness ratio
θ = middle chord subtended angle
m = number of independent inextensional mechanisms
M_p = Plastic moment
MPa = mega Pascal
σ = stress
σ_y = yield stress
P_{cr} = critical load
P_e = Euler load
P_y = squash load
r = radius of gyration
r = middle top chord radius of curvature
r = the rank of the equilibrium matrix
R = edge top chord radius of curvature
R = the degree of redundancy or statical indeterminacy
ρ = radius of curvature
s = number of the states of self-stress
s = curved length
U = vector of nodal displacements
y = distance from the centroidal axis
Z_p = plastic modulus

The following acronyms have also been used in this thesis:

ANSYS = a commercial finite element program
AS = Australian Standard
CHS = Circular Hollow Section
FLD = Force limiting device
FPS = Frame prestrain
GCBVST = Gently Curved Barrel-Vault Space Truss
GCBVTG = Gently curved Barrel-Vault Trussed Grid
GCDSST = Gently Curved Dome-Shaped Space Truss
GCDSTG = Gently Curved Dome-Shaped Trussed Grid
GCHST = Gently Curved Hypar Space Truss
GCHTG = Gently Curved Hypar Trussed Grid
MPS = Member prestrain
NASTRAN = a commercial finite element program
RHS = Rectangular Hollow Section

xx
RMG = Rectangular Mesh Grid
SCBVST = Sharply curved Barrel-Vault Space Truss
SCDSST = Sharply Curved Dome-Shaped Space Truss
SCHST = Sharply Curved Hypar Space Truss
SCST = Single-Chorded Space Truss
SHS = Square Hollow Section
SMG = Square Mesh Grid
STRAND5 = a commercial finite element program
UB = Universal Beam
1 Introduction

This thesis is primarily concerned with the formation of curved space trusses of various forms derived from a flat configuration by means of post-tensioning. In order to obtain a fairly general picture of the shape-formation process by means of post-tensioning, the geometrical forms investigated cover a wide scope, rather than one type in detail. Three types of curved space trusses are considered: barrel vaults, domes, and hypars. The structural performance and ultimate load behaviour of the models are also investigated as a secondary objective. The results of experimental and theoretical work on the shape formation of curved space trusses by means of post-tensioning will be presented herein.

The shape formation process, referred to as self-erection, can lead to significant economies in the construction of large-span lightweight structures by eliminating or minimizing the need for scaffolding and heavy cranes.

The base models consist of single-chorded space trusses (SCSTs) assembled on the floor from single-layer mesh grids of top chords and pyramidal units of web members. For ease of fabrication, the top chords of the steel models are continuous and the joints between the top chords and the web members are bolted connections. There are no chords in the bottom layer except for the edges, and in some cases the diagonal of the models as well. These edge bottom chords are cut shorter than the original distance between the panel points of the SCSTs, and, therefore, create gaps between the lower edge panel points of the models in the flat position. Prestressing wire is passed through these shorter tubular chords and also through the hubs (joints) of the models along the lower edges.

The structures are devised to be near mechanisms as long as the gaps are not closed. They are near mechanisms as only the flexural stiffness of the top chords provides any resistance to deformation. Therefore, no appreciable force is induced in the members of the structures during the shape formation process by means of post-tensioning. The models are then post-tensioned in order to close the gaps, thereby curving the space trusses as required.

The shape formation process bends the continuous top chords in order to produce the design geometrical shapes. But this bending, as will be shown, is not detrimental to
the structural performance of the models due to the fact that the resulting structures, in their final forms after post-tensioning, act as space trusses. In the extreme case where the top chords are bent beyond the yield strain of their material (steel) in order to form the structures into sharply curved geometrical forms, they can be considered as pin-ended columns. The prevalent axial action of the members, due to the triangulated geometry of the models, still governs the overall behaviour of the structures as space trusses.

The behaviour of the space truss models is compared with that of curved single-layer grids with peripheral trusses. It will be seen that, as far as the formation of the design geometrical shapes is concerned, curved single-layer grids can also be formed from flat models by means of post-tensioning. However, their overall structural behaviour under external load is characterised by far less stiffness and ultimate load capacity in comparison with curved space truss models of similar dimensions formed by means of post-tensioning. This decrease in the structural performance of the curved single-layer trussed grids formed by means of post-tensioning is attributed to the prevalent flexural (and not axial) action of their members, particularly in the central portion of the grids.

The thesis consists of eleven chapters. Chapter 2 presents a review of the literature on space structures, in general, and the methods used to form and erect them, in particular. Chapter 3 deals with the linear and nonlinear analyses of space trusses and the assumptions made in modelling the physical experiments for analysis.

Chapter 4 gives a general introduction of the experimental investigation towards the shape-formation of the model space trusses, and gives general information on the total number of the test models. A detailed account of the basic design of the test models, joints and members as well as the test rig and the measurement apparatus is given in this chapter.

The remaining chapters of the thesis are classified according to the geometrical categories of the models formed during the tests. Chapter 5 gives the results of theoretical and experimental work on the shape-formation and ultimate load behaviour of the gently curved barrel-vault space truss (GCBVST) and the gently curved barrel-vault trussed grid (GCBVTG). Chapter 6 deals with the sharply curved barrel-vault space truss (SCBVST) formed by means of post-tensioning.
Chapter 7 presents the details of the tests and analyses on the shape-formation and ultimate load behaviour of the gently curved dome-shaped space truss (GCDSST) and the gently curved dome-shaped trussed grid (GCDSTG). Chapter 8 deals with the shape-formation of the sharply curved dome-shaped space truss (SCDSST).

Chapter 9 gives the results of experimental and analytical work on the gently curved hypar space truss (GCHST) and the gently curved hypar trussed grid (GCHTG) formed by means of post-tensioning. Finally, Chapter 10 presents the details of laboratory and computer work on the shape-formation of the sharply curved hypar space truss (SCHST).

The thesis concludes with a brief chapter on the principal points and with suggestions for future research. There are two appendices to the thesis. Appendix A gives the details of material and member tests and, Appendix B contains the references.
2 An overview of space structures

2-1 History

The history of space structures dates back to thousands of years ago. Some of the famous ancient space structures are: the Treasury of Atreus, Mycenae (corbelled dome, 1325 BC), the Pantheon, Rome (concrete dome, span = 44 m, 123 AD), the Hagia Sophia Mosque, Istanbul (brick dome, span = 32.6 m, rise = 14 m, 537 AD), and the Kasra Arch, Persia (adobe barrel vault, about 400 AD).

In order to resist the hoop tension at the base of the ancient domes, heavy buttresses were built next to the domes, as iron had not yet been used as tie rods. A new era started in the history of space structures with the introduction of iron into construction.

2-2 Classification

Space structures can be classified into the following categories:
1- Space frames (including space trusses)
2- Folded plate structures
3- Rigid shell structures
4- Soft shell structures (i.e. cable nets, membranes, and pneumatic structures).

The reason why space structures, in general, and space trusses, in particular, are so popular today has been attributed to several factors, including: the widespread use of computers, development of efficient standardised connections, and in-depth research into the elastic and non-elastic behaviour of space trusses.

It has been mentioned that with the advance of technology, other structural systems such as steel space frames are becoming more cost effective than concrete shells for large span construction [Popov, 1991].

Among the most remarkable space frames are: the Toronto SkyDome, Canada (retractable dome with a diameter of 203 m), the Fukoka Dome, Japan (retractable dome with a diameter of 222 m, the Florida Suncoast Dome, U.S.A. (tensegrity
dome with a span of 210 m), the dome for the Barcelona Olympics Sports Hall, Spain (Pantadome 106 m $\times$ 128 m), the hypar steel structure of the Islamic University in Riyadh, Saudi Arabia (MERO hypar steel space frame with a clear span of 150m and an overhang of 55 m), the timber dome of Takoma, U.S.A. (with a clear span of 162 m and a rise of 49 m) [Makowski, 1993], the Split Stadium, Czechoslovakia (twin cantilevered MERO barrel vaults each with a span of 213 m at the edge) [Klimke et al., 1985], the Astrodome in Texas (with a span of 200 m), the Superdome in New Orleans (with a span of 213 m), the Olympic Games Sports Palace dome in Mexico (with a clear span of 134 m), the Indraprastha Sports Stadium dome in India (with a diameter of 150 m), the double-layer geodesic dome for the US pavilion for Expo '67, Montreal, Canada (with a diameter of 76 m and a height of 61 m), the stressed-skin aluminium dome at Schipol Airport, Amsterdam (with a clear span of 60 m), the Baton Rouge geodesic stressed-skin steel dome, U.S.A. (with a diameter of 117 m and a rise of 35.7 m) [Makowski, 1984].

One of the outstanding hypar space structures is the fan-shaped twin-hypar space frame built for the Expo '85, Japan (80 m $\times$ 54 m) [Shimzu et al., 1984].

Among the most outstanding folded plate structures are: the folded plate roof for a store in Tampa, U.S.A. [Popov, 1991] and the folded plate roof of the Law School, St. Louis, U.S.A. [Gould, 1988].

Among the most elegant rigid shell structures are: the Kingdome (the largest RC dome with a clear span of 201.6 m and a rise of 33.5 m), the Ehime Prefecture Hall, Japan, the Algeciras Market Hall, Algeria, St. Mary's Cathedral, Tokyo, and Nervi's Plazzetto, Rome [Popov, 1991], and free form shells by Heinz Isler in Switzerland [Ramm et al., 1991], the intersecting barrel shells at the St. Louis Airport, U.S.A., and the vertical hyperbolic paraboloids at a San Fransisco church, U.S.A. [Gould, 1988].

Among the most outstanding cable nets are: the Raleigh Arena, U.S.A., the Calgary Saddledome, Canada, Tokyo Olympics gymnasia [Popov, 1991], the flat roofed circular building in the Madison Square, New York, the Scandinavium in Gothenburg, Sweden, and the Palazzo dello Sports in Milan, Italy [Makowski, 1984].
One of the remarkable pneumatic structures is the air-supported membrane roof at the Dalhousie University, Canada, with the outstanding feature of having a stainless steel membrane [Springfield, 1985].

2-3 Space trusses

Space trusses are used to cover large clear spans where there is a need to avoid columns. Due to the triangulated arrangement of their discrete members, the forces induced in space trusses under load are principally axial. This axial action leads to a more efficient use of the material. Because of their regular pattern, space trusses lend themselves to modular assemblage.

There have been numerous papers and reports published on the analytical and experimental studies of space trusses. Many of them concern small-scale tests of space trusses. Full-scale tests have been limited, though, due to being expensive, especially if the trusses are to be tested to failure.

The members of a triangulated truss are light because they are only subjected to tension or compression. On the other hand, this type of structure is not always economical as a roof because assembling the joints in the air is complicated and besides, the roof still needs cladding. Although space trusses could use less material than equivalent structural solid systems to cover the same span, nevertheless, they need more careful detail designing and field erection because of their numerous joints, which are sensitive to stresses and deformations [Zetlin et al., 1975].

Space trusses have other uses, too. Tall towers and poles are usually designed as space trusses. The main advantage of space trusses over the traditional "column-beam" systems lies in their ability to distribute the loads applied to them more widely. As a result, the stresses induced in the directly loaded members are decreased and the stresses in the distant members increased, leading to a balanced stress distribution in the whole structure.

Another advantage of space trusses is their aesthetics, which are due to the interesting three-dimensional pattern of their members, and also due to a greater freedom of the architect in their design because of their rigidity.
The most important parts of any space truss system are the joints. Simple connections are a significant factor in reducing the overall cost of a space truss, but other factors influencing the overall cost of a space truss are the weight, ease of assembly and erection, type of covering, etc.

Most space trusses have concentric joints, i.e. the centroids of all the members meeting at a joint, pass through a common point which is the centre of the joint. Some space trusses, including those discussed herein, have eccentric joints. This eccentricity causes local bending of the joints and members which should be considered in design. However, if the top chords are continuous at the joints, the continuity can compensate for the disadvantages of eccentricity as is the case for the joints used in this project.

Space trusses with eccentric joints and continuous chords have been successfully built in Australia [Codd, 1984].

The large demand for space trusses to cover sports stadia, exhibition halls, swimming pools, etc., has led to the development of numerous commercial systems for their manufacturing, including: MERO, Triodetic, Space Deck, Unibat, and NODUS.

There are various types of space trusses, including: double-layer grids which are constructed in different forms such as planar (flat), barrel vault, and dome. A single-layer grid works mainly in flexure and torsion under load, whereas, a double-layer grid is principally under the action of axial forces. Single-layer grids are suitable for spans of up to 10 metres. Double-layer grids can cover spans of up to 100 metres and more. To increase the stiffness of double-layer grids, triple-layer grids have been used to cover large spans.

It has been shown that despite the apparent high degree of statical indeterminacy of double-layer grids, they are not 'fail-safe' and the loss of a few critical members may result in their collapse due to the formation of a mechanism [Affān et al., 1986]. When the design is based on the elastic limit, the loss of one member could result in 20% to 30% reduction in the load carrying capacity of a double-layer grid. This reduction in load carrying capacity can be augmented to as much as 50% due to imperfections such as lack of fit, initial member out-of-straightness, and joint slip [Hanaor, 1988].
A single-layer braced dome made up of triangulated bars connected to each other with pins is stable due to its especial geometrical form. But a single-layer grid with pinned connections will collapse. In order to be stable, flat triangulated grids with pinned connections should have at least two layers connected to each other with triangulated bars.

Barrel-vault space trusses are especially suitable for aircraft hangars, railway stations, and industrial buildings. Barrel vaults and domes have been of architectural interest since ancient times. Most of the domes built in practice are spherical, covering circular areas. However, as mentioned by others [Makowski, 1984], it is also possible to produce domic surfaces generated by the translation or sliding of a plane curve on another plane curve.

Braced domes can be divided into the following main groups: (1) frame or skeleton-type single-layer domes; (2) truss-type domes and double-layer domes; (3) stressed skin type; and (4) formed surface type domes.

The post-tensioned domes discussed herein fall into the second category, i.e. truss-type domes. Most domes are in the first group, i.e. single-layer domes. Single-layer domes are used for smaller spans. Double-layer domes are used for very large spans.

Elastic linear analysis of braced barrel vaults can now be carried out easily by means of computers. However, it is still difficult to obtain a reliable estimate of the buckling loads of these structural systems. That is why large spans are usually covered with double-layer braced barrel-vault space trusses which are almost immune to overall buckling instability due to their rigidity.

It has been shown that space trusses, despite a high degree of statical indeterminacy, usually have a brittle-type failure due to the buckling of their critical members in compression. However, using force limiting devices (FLD) as a means of artificial ductility can enhance the load carrying capacity and the post-buckling behaviour of a space truss [Schmidt et al., 1979]. A novel soft member has recently been introduced as a FLD for space trusses [Parke, 1993].

As another way to avoid the brittle-type failure of space trusses and also to reduce the cost of material, composite space trusses have been proposed with the top layer
of chords replaced with a continuum, such as a reinforced concrete slab [Al-Bazzaz, 1981; El-Sheikh et al., 1993].

According to previous research work [Makowski, 1985], the determination of stresses in the members of braced barrel vaults due to wind action, still requires further research.

2-4 Shape formation and erection of metal structures

In order to form a curved truss from an initially flat configuration (Fig. 2-1a), either of the two methods shown in Figures 2-1 and 2-2 can be employed. The first method is to replace the top-chord members with loose cables (see Fig. 2-1b), and then draw the roller support towards the fixed support to such an extent that the desired curvature is obtained and the cables are stretched tightly enough, and then to tie the two supported ends with a tie rod (see Fig. 2-1c). This method has been used by others to construct a braced steel arch [Saar, 1984].

![Figure 2-1 Forming a curved truss by tensioning the top chord.](image)

The second method, which will be elaborated on later in this thesis, is to leave the top-chord members at their true length, and introduce instead a certain lack of fit or gap into the bottom chords proportionate to the desired curvature. Then, by post-tensioning this mechanism (see Fig. 2-2a), the final curved truss can be obtained (Fig. 2-2b). This method has been used by others to erect a large number of large-span steel structures [Ellen, 1986; Clarke et al. 1989]. The method was primarily used to erect planar trusses which were longitudinally connected by purlins. The resulting structures were, therefore, a series of trussed portal frames connected by flexural members (see Fig. 2-3).
The Strarch frames [Clarke, 1992] have been studied through extensive experimental and theoretical work. A geometric and material nonlinear finite element analysis has been developed which models the behaviour of the top chord rigorously and applies an initial strain to the bottom chord to simulate the erection procedure. Very good agreement has been reported between the experiments and the nonlinear analysis [Clarke, 1992]. The bottom chord has been assumed to be continuous and homogeneous. The prestressing wire has not been modelled independently in the Strarch frames. A more precise modelling of the bottom chord and the prestressing wire to include the gaps and the independent sliding movement of the prestressing wire inside the bottom-chord tubes, and the possible opening up of the gaps under external load, has been postponed to future research [Clarke, 1992].

This thesis expounds on the application of the second method to a novel three-dimensional structure in order to form curved space trusses, with the dual advantage of economy in construction and shape formation through post-tensioning. The novel three-dimensional structure is a single-chorded space truss (SCST) which depends on a combined torsional and flexural rigidity for its transverse stiffness to applied load [Schmidt, 1985]. Although the SCST is a stable load carrying structure [Schmidt, 1985], once it lacks sufficient boundary restraints, which is the case during the post-tensioning operation discussed herein, it works as a near mechanism without offering appreciable resistance. Flexure of the continuous top chords, friction and self-weight form the resistance to post-tensioning.
Schmidt has outlined the geometry of the SCST needed to form three curved surfaces, ie. barrel vault, dome, and hypar. The position of the bottom chords and the prestressing cables, and the length of the bottom chords determine the final curvature to be gained after post-tensioning [Schmidt, 1989].

A developable surface such as a barrel vault is produced by curving a flat configuration in such a way as to restrict deformation to bending only, without any extension in the surface. All line lengths and angles on the original planar surface are kept unchanged during curving process to form a developable surface [Duncan et al, 1982].

Camber can be introduced in many space truss systems. Positive camber can be obtained either by increasing the fabricated lengths of the top chord elements, decreasing the fabricated lengths of the bottom chord elements, or putting shims between the mating surfaces of the top chord joints. The Space Deck system allows the inducement of camber by means of tightening the tie bars which have right- and left-hand threads at their ends [Whitworth, 1981].

Using the existing standardised systems of space frame construction, double-layer grids and braced barrel vaults which have a developable surface, can be erected by putting identical elements together. However, domes, which are not among developable surfaces, cannot be constructed by putting together elements of the same size.

The method of post-tensioning space trusses discussed herein, makes it possible to form a wide range of interesting architectural shapes, including barrel vaults, domes, hypars, and conoids, by using members of the same size.

Bini has referred to the erection of reinforced concrete shells and metal structures from a flat configuration by pneumatic means [Bini, 1993]. His automated erection procedure is claimed to offer significant economies over conventional construction methods despite the expensive equipment and jointing details involved. The telescopic pipes used for the erection of the 'Binistar' metal structures still need manual fixture after the erection procedure when the final shape of the structure has been obtained. This manual fixation has been chosen as a cheaper and more reliable alternative to the expensive self-locking system that was used earlier in his structures [Bini, 1993].
Isler has used the fabrication of laboratory-scale models for the shape-finding and optimisation of concrete shells [Isler, 1993]. His physical models are made with a very high degree of accuracy so that direct proportions can be drawn between the physical model and the real structure in terms of the load-carrying capacity per unit area [Isler, 1993].

Montero has formed different structural shapes, including domes and folded plates, from flat configurations [Montero, 1994]. Some of his models are formed from flat single-layer grids and some others work as deployable foldable double-layer grids [Montero, 1993].

Deployable structures have been studied extensively over the past decade due to their initial compactness and fast erection procedure. One category of deployable structures includes the assemblage of discrete members connected in pairs in scissor-like hinged configurations [Clarke, 1984]. Two-dimensional scissors are called 'duplets' (see Fig. 2-4) and the 3-D scissors are called 'trissors' (Fig. 2-5). The resulting structures which resemble a pantograph are referred to as 'p-structures'. Different geometrical forms, including barrel vaults and domes can be obtained from the assemblage of duplets [Escrig, 1985].

![Figure 2-4](image1.png)

(a) A duplet  
(b) A pair of neighbouring duplets  
Figure 2-4 The basic components of deployable structures.

![Figure 2-5](image2.png)

Figure 2-5 A trissor.

P-structures which can be folded into a bundle, are classified into compatible and incompatible groups. In a compatible p-structure, no internal deformation or stress is developed in the uniplets during deployment, while internal deformations and
stresses are induced in an incompatible p-structure as a result of the deployment process [Shan, 1992]. The geometry of compatible p-structures can be generated by means of Formex algebra [Shan, 1992]. The main forces induced in the members of an incompatible p-structure are flexural [Shan, 1992]. Figure 2-6 shows an incompatible deployable structure, a geodesic dome, in which bending deformations are induced during the deployment process [Clarke, 1984].

The deployment process of the dome shown in Figure 2-6 has been modelled using a large displacement - small strain finite element procedure available in ADINA [Gantes et al., 1989]. The nonlinear analysis has shown a very unbalanced distribution of member forces and a snap-through type behaviour of the structure during deployment [Gantes et al., 1989].

![Figure 2-6 A deployable dome.](image)

Study of a deployable ring structure has shown that high geometric accuracy is needed in its fabrication, as stresses might build up during deployment due to small errors during fabrication and assembly [Pellegrino et al., 1993].

Two designs for 'compatible' deployable structures are based on the concepts of tessellations of regular polygons as well as regular and semi-regular polyhedra. The resulting flat and spherical structures are self-standing and stress-free in both fully folded and fully deployed configurations [Gantes, 1993].

Several roofing possibilities have been proposed for deployable structures. The roofing may be incorporated into the structure either in the fully folded configuration, or in a slightly unfolded state, or at a nearly complete state of deployment [Valcarcel et al., 1993].

One of the numerical methods developed to simulate the folding of space structures gives not only the final folded shape, but also the fastest sequence of the folding
which could imply the quickest process for the deployment of these structures [Kawaguchi et al., 1993].

Stiffness formulations have been developed for planar kinematics which seem to be suitable to model the deployment of 'compatible' deployable structures in a mode of stress-free rigid body motion [Fujii et al., 1991].

Another type of deployable structure uses a different basic module for its fabrication. The module is called an octet element which is made up of a regular octahedron and two regular tetrahedrons (see Fig. 2-7). There are also various alternative forms of the basic octet element which are called modified octet elements. A structure made from the assemblage of octet elements is called an octet truss [Natori et al., 1986]. In order to fold these deployable structures, the diagonal members of the octet trusses are made in a telescopic fashion. Curved surfaces can also be obtained from the octet system by varying the lengths of the diagonal members [Natori et al., 1986].

Adaptive structures are another type of deployable structures. The shape and the physical properties of an adaptive structure can be changed in a controlled manner. One type of adaptive structure is made up of cube elements with telescopic diagonal members assembled together to form a partially stiffened or statically determinate deployable structure which can change its shape from a flat double-layer grid (see Fig. 2-8c) into a barrel vault, a paraboloid, or a hypar [Natori et al., 1986]. The change in the geometry and the inherent structural characteristics of adaptive structures is due to the inclusion of 'active' members in them with actuators and sensors which respond to external stimuli such as electrical, magnetic, or thermal fields. This characteristic makes adaptive structures suitable for space missions such as the large deployable reflector (LDR) [Wada, 1990].

![Figure 2-7 An octet element.](image-url)
It has been shown by others that a double-layer grid with folded corners can give an enhanced architectural performance as the folded corners act as columns (see Fig. 2-9) [Wendel, 1984].

Various methods are used for the erection of space trusses, including: the scaffolding method in which the whole truss is assembled in the air by means of scaffolding, the block method in which the truss is assembled on the ground in small blocks that are then hoisted to their final position by means of a crane, the sliding method in which the truss members are assembled in small blocks at an end portion of the roof and then slid into their final position, and the lift up method in which the whole truss is assembled at ground level and then lifted into its final position [Iwata et al., 1984].
One of the methods for the erection of braced domes is the Pantadome method in which the structure is first assembled in a folded shape close to the ground level and then lifted in its central portion by means of hydraulic jacks to obtain the final configuration (see Fig. 2-10). No stays or bracing are needed for the lateral stability of the central portion of a Pantadome structure during erection. Among the outstanding domes constructed by means of the Pantadome method are the Barcelona Olympic Sports Hall and the Singapore National Indoor Stadium [Kawaguchi, 1991].

Figure 2-10  The Pantadome method for the erection of braced domes.

2-5 Post-tensioning

Although post-tensioning has been used mainly in concrete structures, its application to steel structures has gained acceptance over the past two or three decades. Belenya
mentions a number of cases wherein the use of prestressing or post-tensioning has led to savings on cost ranging from 12 to 30 percent. In particular, he has referred to the application of prestressing to the roof trusses of industrial buildings by the displacement of the middle bearing (support), which resulted in 12% saving on steel and 29% saving on erection costs [Belenya, 1977].

Cuoco refers to the use of post-tensioning as one of the areas of future trends for space frame roof structures, and makes mention of a successful application of the principles of space frame post-tensioning in the construction of a 260 feet by 263 feet two-way truss system in Philadelphia [Cuoco, 1981].

Substantial increase in load carrying capacity or reduction of weight has been reported via the use of lack of fit as a means of post-tensioning double-layer grids [Hanaor et al., 1986]. While random lack of fit usually reduces the load carrying capacity of a space truss, imposing lack of fit on selected members in a controlled manner can increase the load carrying capacity of a space truss and/or reduce its weight [Hanaor et al., 1985].

The main significance of the present work, however, is to emphasise the powerful shape-forming aspect of post-tensioning rather than its effectiveness in increasing load carrying capacity or reducing the costs of material (e.g. steel) through a more suitable distribution of stress. Therefore, more attention is paid to the geometry of the shapes of space trusses that lend themselves to post-tensioning. Comparisons will be made, in terms of the tolerances achieved, between the theoretical and measured geometries of the shapes that were formed during tests on model space trusses.

Prestressing has also been used to shape a flat grid made from a series of interconnected two-dimensional trusses, without bottom chords, into a singly curved atrium wall. The prestress was applied to the vertical posts of the trusses through expanding joints [McConnel et al., 1993].

Fuller introduced the idea of fabricating free-standing self-stressed structures from a combination of discontinuous compression members (bars) and continuous tension members (cables). The original idea belonged to the artist Snelson with whom Fuller had worked for a while. Fuller called these structural forms tensegrity.
structures. The smallest tensegrity structure is called a simplex (see Fig. 2-11) [Motro, 1984].

Various shapes have been made from the tensegrity structural system, including barrel vaults and domes. Tensegrity structures have a unique architectural appearance although they are relatively heavy due to their long discontinuous bars as compared with double-layer grids. The long bars also cause the problem of buckling for tensegrity shells.

Tensegrity double-layer grids have the disadvantage of large deflections under vertical load [Hanaor, 1991]. However, they are highly statically indeterminate such that the failure of a member does not significantly affect the load carrying capacity of the whole structure. Their design should be based on a nonlinear analysis, as a linear analysis of double-layer tensegrity grids generally results in an unsafe design [Hanaor, 1991]. The flexibility of tensegrity structures has been attributed to the existence of infinitesimal modes with low stiffness in the network of their members [Calladine, 1978].

The discontinuous nature of tensegrity structures makes them suitable for deployment [Hanaor, 1993]. For the tensegrity structures to be viable as lightweight structures, it seems to be inevitable to abandon the idea of discontinuous members which might affect the deployability of tensegrity systems at the same time [Hanaor, 1993].

Figure 2-12 shows the cross sections of two different tensegrity domes designed by Fuller and Vilnay. Fuller's design uses shorter bars which are less susceptible to buckling but may interfere with each other and reduce curvature as the span increases [Hanaor, 1993]. Figure 2-13 shows the perspective view of a shallow tensegrity dome [Vilnay, 1985].
As most tensegrity structures are flexible, the incorporation of a flexible membrane into their design as roof covering can enhance their structural performance [Hanaor, 1993] and reduce the cost by eliminating heavy anchoring [Voigt, 1986].

The formation of hypar tensegrity shells has been proposed as an area of future research [Motro, 1989].

The concept of prestressing has also been implemented by introducing prestrain into selected members of a roof truss in order to reduce stresses and deflections under dead weight. Two methods called member prestrain (MPS) and frame prestrain (FPS) have recently been introduced [Nakashima et al., 1993].

The MPS method includes the removal of the corner bottom-chord members of a flat two-dimensional roof truss, applying horizontal prestrain to the corner lower nodes of the truss by means of hydraulic jacks, and then replacing the removed members back in position after the desired camber is obtained. This method results in a curved shape for the roof truss.
The FPS method includes the removal of the middle top-chord member, applying vertical prestrain to the middle lower node, and replacing the middle top-chord member back in position. The final shape obtained by this method is a pitched roof [Nakashima et al., 1993].

Miura has introduced the concept of tension trusses for the construction of lightweight structures such as large deployable antenna reflectors [Miura, 1989]. The 3-D tension truss consists of a triangulated cable network which forms a rigid geodesic shape (see Fig. 2-14).

![Figure 2-14 A tension truss structure.](image-url)
3 Analysis

There are two general groups of mathematical modelling processes for the analysis of curved braced space structures: the equivalent shell method and the discrete structure method. With the widespread usage of computers today, the discrete structure method has become almost the only method used in the design and analysis of space structures, although the equivalent shell method is still used by some designers for the early stages of design.

In the equivalent shell method, the behaviour of the structure is approximated by that of an equivalent shell. There are two theories within this method: the finite difference theory and the orthotropic shell theory [Mullord, 1984].

The discrete structure method can be classified into two methods. In one, the space truss is assumed to have pinned joints, and in the other, the joints of the space truss are assumed to be continuous.

One of the main aspects of the analysis of space structures, particularly those with curved and complex geometries, is the generation of the computer model. The Formex algebra has been developed as a very effective data generating tool for the analysis of space structures [Nooshin, 1984].

An efficient mesh generation procedure has also been developed for structures whose load carrying ability is mainly determined by the shape of the surface, including cable networks, space trusses, concrete shells, and tensioned fabric structures. The method can generate regular curved surfaces such as hypars and cones as well as more complex shapes through a powerful smoothing procedure [Kneen, 1993].

Like all other structural analyses, the analysis of space trusses using the discrete structure method can be classified into the two groups of linear and nonlinear methods.

The model space trusses studied herein were analysed using both linear and nonlinear methods.

In the linear analysis, the models were assumed to be in their final forms for both the post-tensioning analysis and the vertical loading analysis.
In the nonlinear analysis, the models were assumed to be in their original (i.e. flat) configurations before starting the shape-formation (post-tensioning) analysis, and were then subjected to vertical load in their final forms obtained from the shape-formation analysis.

The assumptions made for each of these two types of analyses in modelling the experimental trusses are described in the following sections.

3-1 Linear analysis

A simple linear elastic analysis together with suitable allowable stresses can check for all types of local member or joint failure which might include yield, member buckling, or fracture.

The most commonly used linear method for the analysis of space trusses is the stiffness method in which a large set of simultaneous linear equations are solved by means of matrix methods to find the unknown displacements, and thereby, the member forces.

In a linear static analysis, the equations of equilibrium are formed for the original configuration of the structure (before loading), and are then solved to find the nodal displacements.

The equilibrium equations formed in the stiffness method are in the general form of $Kx = b$ where $K$ is the stiffness matrix, $x$ is the vector of unknown nodal displacements, and $b$ is the vector of applied load. These simultaneous equations are solved in two general methods: the direct method and the iterative method.

The most common direct method is the Gaussian elimination method which includes a forward elimination and backward substitution to give the displacements and subsequently, the member forces. Other common variations of the Gaussian elimination are the Gauss-Jordan method, in which the stiffness matrix $K$ is transformed into a diagonal matrix, and the Choleski method in which $K$ is decomposed into the product of upper and lower triangular matrices.

The iterative methods are in the general form of $Ax = Bx$. In these methods the stiffness matrix is decomposed into the sum of lower and upper triangular matrices and a diagonal matrix, i.e. $K = L + D + U$. Thus, we will have $(L + D + U)x = b$ or
Dx = b - (L + U)x which leads to Dx_{n+1} = b - (L + U)x_n and is called the Jacobi method. Or we can have (L + D)x = b - Ux which leads to (L + D)x_{n+1} = b - Ux_n and is called the Gauss-Seidel method. The Gauss-Seidel method converges for most structural problems. The iterative methods are generally slow for large structural problems and accelerating techniques are usually employed with them to speed up the convergence of the solution.

The barrel-vault model space trusses herein were analysed using the linear static stiffness method. For each model, two linear analyses were carried out. In the first analysis, the members and joints of the model were assumed to be concentric. In the second analysis, the member and joint eccentricities were taken into account.

3-1-1 Linear concentric analysis
The following assumptions were made in the linear concentric analysis of the model barrel-vault space trusses.

• The members were straight between panel points.
  The members were assumed to be pin-ended in one case and fixed-ended in another case.
• The joints were assumed to be concentric.
• The material (i.e. steel) was assumed to have a linear elastic behaviour.

3-1-2 Linear eccentric analysis
The following assumptions were made for the linear eccentric analysis of the barrel-vault model space trusses.

• All members were modelled as beam elements.
• Each top-chord member between the panel points was modelled as two straight beam elements connected to each other midway between the panel points. The measured rise of the curved top-chord members between panel points was included in the coordinates of the middle point. In other words, each curved top-chord member was modelled as two straight beam elements joining at an angle.
• The eccentricities between the top-chord members and the webs at the top-layer joints were modelled as short beam elements linking together the different members meeting at the joints (see Fig 4-6).
• The eccentricities between the prestressing wire, the bottom chords, and the webs were also modelled as short beam elements.
• The material was assumed to have a linear stress-strain curve.
3-2 Nonlinear analysis

In order to check for instability effects involving more than one member or geometry change and also to trace the post-buckling behaviour of the structure so as to utilise any reserve of strength, a nonlinear analysis needs to be carried out.

Nonlinear effects can be divided into material and geometric nonlinearities. There are three general methods of nonlinear analysis: the plastic mechanism method, the bifurcation method, and the incremental method. In the incremental method, the load is applied in increments. At each increment, the structure stiffness matrix is updated to include changes in member stiffness and structure geometry.

Most nonlinear analysis methods comprise a series of linear analyses carried out in iterations. Each iteration gives a closer approximation of the final solution of the system. The first iteration is basically a linear analysis after which the next iterations involve the formation of the structural stiffness matrix based on the displaced shape of the structure and the nonlinear stress-strain curve of the material resulting from the previous iterations.

The most commonly used iterative method for the solution of nonlinear structural problems is the Newton-Raphson method (see Fig. 3-1). Depending upon the method used for the solution of the simultaneous equilibrium equations along with the Newton-Raphson iterative method, the composite method may be called the Newton-Gauss, the Newton-Jacobi, the Newton-Choleski method, etc. [Supple, 1984].

The Newton-Raphson method is usually used in an incremental-iterative scheme to solve highly nonlinear problems. The load is applied in increments and the stiffness matrix is updated after each iteration. In each load step, after convergence is reached within a load step through successive iterations, the load is incremented to the next load step and the procedure is repeated.

The equation to be solved in the Newton-Raphson method is:

\[ K_{i-1} \Delta U_i = F^A - [F_{NR}^T]_{i-1} \]

where \( K_{i-1} \) is the tangent stiffness matrix based on the deformed structural geometry from iteration \( (i-1) \); \( \Delta U_i \) is the incremental displacement vector which is equal to \( U_i - U_{i-1} \); \( U_i \) is the displacement vector at the current iteration; \( F^A \) is the vector of the applied load; and \( [F_{NR}^T]_{i-1} \) is the Newton-Raphson restoring load for iteration \( (i-1) \).
The difference between the applied external load and the Newton-Raphson restoring load in each load step is called the out-of-balance force or the compensating load which has to be reduced, through successive iterations, to a certain negligible value, referred to as the convergence value, before the next load step is started.

The tangent stiffness matrix is checked throughout the procedure and whenever its determinant is negative, it indicates that there is a loss of stability.

An elastic structure with nonlinear unstiffening (softening) behaviour may lose its stability either due to snap through or due to bifurcation buckling.

The post-buckling behaviour of a space truss depends very much on the post-buckling curve of its critical members. It has been shown that different member behaviours result in different structural behaviours after the limit point (see Fig. 3-2) [Schmidt et al. 1980].

The snap-through or limit point instability involves a sudden decrease of the load together with large displacements in the structure.
A bifurcation buckling involves a sudden overall rotational change in the geometry of the structure due to unsymmetrical imperfections in the geometry or an unsymmetrically applied load.

A bifurcation buckling may finally lead to snap through. The bifurcation buckling and snap-through modes of instability may interact in a structure and may affect either all of the structure or only a local zone of it.

The Newton-Raphson method slows down in convergence near the limit or bifurcation point because the tangent stiffness matrix tends to become singular (see Fig 3-1). It generally fails to go past the limit point to trace the post-buckling behaviour of the structure because the structural matrix becomes singular at the limit point.

There are generally four theoretical techniques to detect the post-buckling behaviour of the structure: the method of suppressing equilibrium iterations, the method of fictitious springs, the displacement control method, and the constant-arc-length method of Riks/Wempner and modified by Crisfield and Ramm [Papadrakakis, 1987].

The displacement control method cannot trace the post-buckling path of a space truss if the structure shows a snap-back behaviour (see curves c and d, Fig. 3-2).

Space trusses with high degrees of statical indeterminacy such as double-layer grids are not generally "fail-safe" as was once considered the case. It has been shown that the failure of a few members in a double-layer grid can lead to the collapse of the structure [Affan et al., 1986].

3-2-1 Assumptions
The following assumptions were made in the nonlinear analysis of the model space trusses shaped by means of post-tensioning:

- The members between panel points were assumed to be straight.
- The members were assumed to be pin-ended. In other words, the continuity of the members was not taken into account.
- The members and joints were assumed to be concentric.
Based on the evidence obtained from the experimental stress-strain curve of the steel, the material was assumed to have a multilinear kinematic hardening behaviour, i.e. a stress-strain curve with more than two slopes.

3-2-2 Modelling

All of the curved model space trusses studied herein were formed from original flat configurations by means of post-tensioning. The top chords were continuous and the bottom chords had pre-defined gaps, in the flat position, which were closed during the post-tensioning process. The post-tensioning operation was carried out by means of tensioning the prestressing wires which passed through the tubular bottom chords placed along the edges and/or the diagonal of the model trusses, depending on the final shape desired.

For the nonlinear analysis, the model space trusses were introduced to the computer program in their original flat configuration. The different elements of the model space trusses were modelled as follows:

The top-chord and the web members were modelled as truss elements capable to accept material and geometric nonlinearities.

The bottom-chord members were modelled as gap bars which work only in compression. The original gap for each member was input as gap per unit length.

The prestressing wire was modelled as a cable element which can work only in tension. As the prestressing wire passed through the bottom chords in the physical models, the cable elements were introduced without any eccentricity with respect to the gap bars. This concentricity was modelled by introducing, for each cable element, end nodes with the same coordinates as those of the end nodes of the corresponding gap bar, but with different node numbers.

In order to model the relative movement (i.e. sliding) of the prestressing wire inside the tubular bottom chords, the transverse degrees of freedom of the end nodes of each cable element and its corresponding gap bar were linked (coupled) so that the end nodes of the two elements would have the same transverse displacements but different longitudinal movements. In other words, the end nodes of the cable elements and the gap bars could have relative longitudinal movement with respect to each other. At the corners of the models, of course, where the prestressing wire was
anchored, all of the translational degrees of freedom for the end nodes of these two types of elements, including the longitudinal movement, were linked to each other.

The nonlinear analysis of the model space trusses was carried out in two stages, i.e. in two load cases. Load case 1 was the shape-formation analysis, and load case 2 was the vertical loading analysis.

The models were introduced in their flat configurations. In load case 1, the corner bottom nodes were given suitable displacements obtained from the measurement of the final spans after post-tensioning the physical models.

The shape-formation process in load case 1 of the nonlinear analysis necessitated the shortening of all the bottom-layer members, thereby inducing compression in both the gap bars and the cable members. However, the cable members were only subject to tension in the physical models. Therefore, using a feature of the finite element program used (ANSYS), the cable elements had to be deactivated for load case 1 and later reactivated for load case 2. In other words, the prestressing wire could only be modelled for the vertical loading analysis. Nevertheless, this point was not considered to be a major drawback in the analysis as the physical models were near mechanisms during the shape-formation process and therefore, did not need large prestressing forces to form them.

Due to the large displacements involved, particularly in the shape formation of the models, a full Newton-Raphson method of solution which updated the stiffness matrix after each iteration was adopted (Fig. 3-1). The ANSYS program uses the Gaussian elimination method to solve the equations of equilibrium. Consequently, the overall solution method could be called the Newton-Gauss method.

The shape analysis of structures under a given displacement mode, which is the case of the shape-formation analyses of the space trusses studied herein, has been described as the reverse procedure of the stress analysis, to the effect that the deformation and the state of stress of a structure are sought under specified geometric constraints and boundary conditions for a given number of structural nodes [Hangai, 1993].

4 Experimental Investigation

4-1 The basic design of test models

In order to carry out the experimental work on post-tensioned curved space trusses, a single-chorded planar space truss (SCST) with a square mesh top chord grid connected to pyramidal units of webs was designed as the basic model (Fig. 4-1). A single-chorded space truss acts as a mechanism, or near mechanism, as long as it is not sufficiently restrained at its supports [Schmidt, 1983]. If subjected to negative bending moments in the plane of section A-A (see Fig 4-1b), the SCST will not show appreciable resistance except for that shown by its weight, the friction of its joints, and the flexing of its top chords. Therefore, it could be an ideal base model for the formation of curved space trusses through post-tensioning.

![Figure 4-1 A single-chorded planar space truss (SCST).](image)

Depending upon the final curved shape desired, bottom chords with high-tensile prestressing wire passing through them were placed at the edges or the diagonal of the SCST to form a barrel vault, a dome, or a hypar after post-tensioning. These bottom chords were tubes cut shorter than the original distance between the bottom nodes of the SCST in its flat position (Fig. 4-2).

The gaps thus created would close up in the course of the post-tensioning of the wires that ran across the edges, or the diagonal, of the model, thereby resulting in the achievement of the required final shape at the end of the post-tensioning operation. The structure would then change from a mechanism to a kinematically determinate structure, and thereby would be able to carry external load.

Previous research has shown that the out-of-plane web system in the resulting structure prevents overall instability under unsymmetrical loading conditions [Hoe et al., 1986].
The test program was to shape and test to failure model space trusses of each geometrical category, i.e. barrel vault, dome, and hypar. Table 4-1 shows some general information on the type and total number of model space trusses tested.

Table 4-1  The test program (1=yes; 0=no; c= continuous; p=pinned)

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Shape</th>
<th>Member material</th>
<th>Joint detail</th>
<th>Joint eccentricity</th>
<th>Basic no. of panels</th>
<th>Top chord span/rise ratio</th>
<th>Overall top chord layer dimensions (m)</th>
<th>General category</th>
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<td>steel</td>
<td>c</td>
<td>1</td>
<td>6 by 6</td>
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</tr>
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<td>c</td>
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</tr>
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</table>

Notes: GCBVST= Gently Curved Barrel Vault Space Truss; SCBVST = Sharply curved Barrel Vault Space Truss; GCDSTST = Gently Curved Dome-Shaped Space Truss; GCBVTG = Gently Curved Barrel-Vault Trussed Grid (i.e. the same as the GCBVST but without intermediate webs); GCDSTG = Gently Curved Dome-Shaped Trussed Grid; SCDSST = Sharply Curved Dome-Shaped Space Truss; GCHST = Gently Curved Hypar Space Truss; SCHST = Sharply Curved Hypar Space Truss; GCHTG = Gently Curved Hypar Trussed Grid.
With regard to the dimensions of the testing frame and the space available at the laboratory, the final span of the (steel) models to be load tested was designed to be about 2400 mm. For purposes of easy and fast fabrication, all the top chords and webs of the model (steel) space trusses were designed to have the same length, i.e. about 500 mm, with the top chords to be made of square hollow section (SHS) and the webs and bottom chords to be made of circular hollow section (CHS) tubes.

4-2 Joint design

4-2-1 Top chord joints
Several designs were considered for the top chord joints. The first design was made up of continuous SHS tubes perpendicular to each other and bolted to a square plate placed in between them with four webs connected at the corners of the plate and bolted to it from underneath (see Fig. 4-3). The webs and the top chords then had eccentricities with respect to each other.

The second design comprised an orthogonal twofold top chord grid bolted to each other and to the four webs connected at the corners and lying with flattened ends between the perpendicular top chords (see Fig. 4-4). The elimination of the plate gave more flexibility to the joint, but eccentricities still existed between the top chords and the webs.
The third design consisted of a twofold top chord grid bolted together to a plate placed underneath them. The four webs connected at the corners were bolted to the plate from underneath with separate bolts (see Fig. 4-5).

The fourth design consisted of a top chord grid of orthogonal continuous SHS tubes connected to each other with vertical bolts, while the four webs were bolted to the sides of the lower top chord (see Fig. 4-6).
Figure 4-5  The third design for top chord joints.

Figure 4-6  The final design adopted for top chord joint.
Of the above four designs, the last one was chosen for the model space trusses because of its lesser number of bolts which would make it simpler and faster to fabricate. The eccentricities existing in this joint are shown in Figures 4-6(a) and (b).

The bolted joints were designed according to AS1250, AS1252, and AS1538, with the following considerations:

The maximum allowable stress in tension on the effective area of the connected member, given in AS 1250, Rule 7.1: \( F_{at} = 0.60 F_y \) \( (F_y = \sigma_y) \)

For mild steel where \( F_y = 250 \text{ MPa} \), the maximum allowable stress \( F_{at} \) is 150 MPa.

The bolts chosen for the top node connections were 6 mm high-tensile bolts of grade 8.8 with a shank area of 28.3 mm², and possessed the following design data according to AS 1252:

Minimum tensile strength, \( F_{uf} = 800 \) MPa

Stress at 2% offset, \( F_{yf} = 800 \times 0.8 = 640 \) MPa

Maximum permissible tensile stress, \( F_{uf} = \) The lesser of \( 640 \times 0.6 \) and \( 800 \times 0.45 = 360 \) MPa.

Maximum permissible shear stress, \( F_{vf} = \) The lesser of \( 640 \times 0.33 \) and \( 800 \times 0.25 = 200 \) MPa.

Considering the maximum permissible stresses mentioned above, the safe loads for 6 mm high-tensile bolts are: axial tension @ 360 MPa = 10.18 kN; and shear @ 200 MPa = 5.65 kN (threads excluded).

4-2-2 Lower joints

The lower ends of all web members were designed to be welded to cylindrical steel hubs. At the lower edges of the model, these hubs were drilled to accommodate the tapped ends of other steel hubs with a square cross section which had indented round grooves on two of their opposite faces in order to hold the edge bottom chords in position after post-tensioning (Fig. 4-7).

4-2-3 Experimental calibration of bolt tightening

In order to have a consistent behaviour of top chord joints, an experiment was carried out to calibrate the manual tightening of the 6 mm high-tensile cap screws used for the top chord layer joints. The experiment was done by means of a torque wrench which was calibrated beforehand. Six specimens of top chord joints were tested by applying torque to the cap screws gradually until failure. Except for the first and sixth specimens, the others had washers placed between the SHS tubes and the cap screw head.
The first specimen failed due to bolt shear under a torque of 30 Nm. The second bolt tightening experiment was stopped due to the squashing of the corners of the bolt nut at 30 Nm. In the 3rd and 5th specimens, failure occurred due to bolt shear at 30 Nm. The 4th and 6th specimens failed due to the stripping of the nut threads at 30 Nm and 25.5 Nm, respectively.

This experiment showed that the average ultimate torque capacity of the 6 mm high-tensile cap screws was 30 Nm. Therefore, tightening them with a torque of 15 Nm would give a safety factor of 2. By means of an Allen key which was used to tighten the cap screws during the fabrication of the model space trusses, 10 more cap screws were tightened, each time measuring the torque after tightening by means of the torque wrench in order to obtain a calibrated "feel" for this routine manual action.

Finally, the author's hand was felt to have been calibrated for tightening the cap screws up to 15 Nm with the Allen key. This calibrated hand was used to fabricate the model space trusses.

4-3 Member sizes and properties

The following assumptions were made in the preliminary design of the model steel space trusses to be formed by means of post-tensioning:

Modulus of elasticity for steel ($E$) = 200 GPa.
Yield point for steel ($\sigma_y$) = 250 MPa. (The true $\sigma_y$ was later found, from member tests, to be 450 MPa for top chords and 440 MPa for 13.5 x 2.3 CHS; see Appendix A1).

The member sizes and properties are given in Table 4-2, where $l$ is the length, $A$ the cross-sectional area, $I$ the second moment of area, $J$ the torsion constant, $r$ the radius of gyration, and $l/r$ the slenderness ratio of the members. The lengths are based on measurements made on the fabricated models. The length of bottom chords given in the table applies only to the gently curved barrel-vault model space trusses and the first sharply curved barrel-vault model (tests 1-5).

<table>
<thead>
<tr>
<th>Member type</th>
<th>Type</th>
<th>$l$ (mm)</th>
<th>$A$ (mm$^2$)</th>
<th>$I$ (mm$^4$)</th>
<th>$J$ (mm$^4$)</th>
<th>$r$ (mm)</th>
<th>$l/r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top chord (curved)</td>
<td>Tubing</td>
<td>520</td>
<td>70.9</td>
<td>1300</td>
<td>2530</td>
<td>4.29</td>
<td>121.2</td>
</tr>
<tr>
<td>Top chord (straight)</td>
<td>Tubing</td>
<td>510</td>
<td>70.9</td>
<td>1300</td>
<td>2530</td>
<td>4.29</td>
<td>118.9</td>
</tr>
<tr>
<td>Web</td>
<td>13.5 x 2.3 CHS</td>
<td>500</td>
<td>80.9</td>
<td>1320</td>
<td>2640</td>
<td>4.04</td>
<td>123.8</td>
</tr>
<tr>
<td>Bottom chord</td>
<td>13.5 x 2.3 CHS</td>
<td>460</td>
<td>80.9</td>
<td>1320</td>
<td>2640</td>
<td>4.04</td>
<td>113.9</td>
</tr>
</tbody>
</table>

Table 4-3 shows the critical compressive and tensile forces for the members of the model space trusses as obtained from theory (using the measured $\sigma_y$) and member tests (see Appendix A), assuming for each member the simplified end conditions closest to reality.

The theoretical critical compressive force, $P_{cr}$, for the curved top chords, was 5.2 kN according to the Perry-Robertson formula: $\sigma_y = P_{cr} \left[ \frac{1 + \frac{\delta c}{r^2}}{1 - \frac{P_{cr}}{P_e}} \right]$, where $A$ is the cross-section area, $c$ is the distance from neutral axis to extreme fibre, $r$ is the radius of gyration of the cross section, and $P_e$ is the Euler load, i.e. $\frac{\pi^2 EI}{L^2}$. A 6.5 mm out-of-straightness, $\delta$, was assumed (i.e. the rise of the curved top chord at midspan between two panel points in the gently curved model barrel vaults; see Fig. 4-8). However, this value was found to be too conservative when compared with test results. Therefore, the curved top chord between each two panel points was assumed to be straight and the theoretical critical compressive force for that condition, i.e. 9.5 kN, is shown in Table 4-3. It can also be observed in Figure 4-8 that there is a negative eccentricity of 13 mm between the centre lines of the orthogonal top chords which is to the advantage of the curved top chords.
Also, for the case of the straight top chords, the theoretical compressive force assuming a 13 mm eccentricity, \( e \) (see Fig. 4-8) was calculated according to the secant formula:

\[
\sigma_y = \frac{P_{cr}}{A} \left[ 1 + \sec \left( \frac{\pi P_{cr}}{P_{e}} \right) \right],
\]

where the parameters are as defined before. Again the result (i.e. 5.7 kN) was found to be too conservative and therefore, the theoretical value for the straight top chord (i.e. 9.9 kN) is shown in Table 4-3.

The compressive test result for the webs (i.e. 14.3 kN) was found to be closer to the theoretical value for a column with both ends pinned (i.e. 10.4 kN), rather than that for a fixed-pinned column (i.e. 21.3 kN). In the tensile tests, the web specimens failed due to the tearing off of their bolted end at 19.2 kN before the full yield capacity of the cross section (i.e. 35.6 kN) was reached. Finally, the bottom edge chords showed a behaviour in member tests close to that for an elastically restrained column [Chen et al, 1987], i.e. a column with both ends neither fully fixed nor fully pinned, with a theoretical critical compressive force of

\[
P_{cr} = \frac{21.03EI}{L^2} = 26.2 \text{ kN}.
\]

<table>
<thead>
<tr>
<th>Member type</th>
<th>End condition</th>
<th>In compression (kN)</th>
<th>In tension (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top chord (curved)</td>
<td>pinned-pinned</td>
<td>9.5</td>
<td>31.9</td>
</tr>
<tr>
<td>Top chord (straight)</td>
<td>pinned-pinned</td>
<td>9.9</td>
<td>31.9</td>
</tr>
<tr>
<td>Web</td>
<td>fixed-pinned</td>
<td>21.3</td>
<td>35.6</td>
</tr>
<tr>
<td>Bottom chord</td>
<td>elastically restrained</td>
<td>26.2</td>
<td>35.6</td>
</tr>
</tbody>
</table>

4-4 The test rig

The model space trusses load-tested to failure were mounted on a testing frame with a span of about 2400 mm. The lower nodes of the model to be loaded were connected, through a whiffletree system, to a hydraulic jack and a load cell in series mounted on a supporting beam at the floor level (Fig. 4-9). The deflection of the testing frame under the maximum load conditions was 0.5 mm which was found

\[...\]
negligible in comparison with the deflections of the test models. This value was obtained experimentally by applying one quarter of the maximum ultimate load capacity of the test models (i.e. 15.5 kN) to the middle of one of the beams of the testing frame and measuring its deflection. The model with the largest load carrying capacity was a gently curved dome-shaped space truss supported on its four corner nodes which had an ultimate load capacity of 61.7 kN in Test 13.

A load versus deflection plot was obtained for each test by means of a plotter connected to the load cell at the base of the loading jack and to an LVDT mounted on the supporting frame with its moving end resting on the base beam of the whiffletree (Fig. 4-9).

Therefore, the deflections shown on the plots were those of the middle portion of the base beam of the whiffletree and were thus an average of the deflections of the lower nodes loaded through the whiffletree. The load cell and the LVDT were calibrated before the tests.
A survey level was also used to read the deflections of a number of nodes during each test from scales suspended from the nodes of the model space trusses (Fig. 4-10). This would eliminate errors that could result from any possible slack or deformation in the whiffletree.

Figure 4-10 Measurement of nodal deflections from suspended scales.

From Test no. 4 onwards, electrical resistance strain gauges were also used on critical members to find the forces induced in the members of the models and check them against theoretical values obtained from computer analyses. A pair of strain gauges was used for each member (see Fig. 4-11).

Figure 4-11 Strain gauges on the top and bottom surface of critical members.

The strains measured by means of the two strain gauges on each member, referred to as $\varepsilon_1$ and $\varepsilon_2$, would give the axial strain, $\varepsilon_a$, and the flexural strain, $\varepsilon_b$, as follows:

$$
\varepsilon_a = \frac{\varepsilon_1 + \varepsilon_2}{2} \quad \varepsilon_b = \frac{\varepsilon_1 - \varepsilon_2}{2}
$$

Also, in order to plot the post-buckling path of the model space trusses by means of a displacement-controlled loading, a needle valve was fitted into the hydraulic pump connected to the loading jack from Test 4 onwards.
4-5 The whiffletree design

For the purpose of loading the model space trusses during tests to failure, it was decided to load the bottom nodes (i.e. the apices of the inverted pyramidal units) so that they would distribute the load to the top chords through the webs. This would give a more widely distributed loading. A whiffletree system was designed to distribute the load, to be applied to its base by a hydraulic jack, to the desired bottom nodes. Four were used in the first test. Figures 4-12 shows the whiffletree layout.

Figure 4-12 The sketch of the whiffletree for Test 1.

Figure 4-13 shows the whiffletree used in Test 8.

Figure 4-13 Whiffletree used for loading the model space truss in Test 8.
5 Barrel-vault space trusses

5-1 Gently curved barrel-vault space trusses

The first model to be formed by means of post-tensioning and tested under load was chosen to be a gently curved barrel-vault space truss. The radius of curvature was designed to be just enough to induce the originally assumed yield stress (i.e. $\sigma_y = 250$ MPa) in the topmost fibres of the curved top chords.

5-1-1 The geometry

Based on the above assumption and the measurements made on the pyramidal units which were fabricated for the assembly of the first model space truss, the length needed for each edge bottom chord was found to be 460 mm. This length induced a radius of curvature equal to 5200 mm in the top chords for the formation of the gently curved barrel-vault space truss by means of post-tensioning. The design geometry is shown in Figure 5-1.

The truss was designed to be originally flat with the distance between every two adjacent lower nodes equal to 520 mm in the X direction and 510 mm in the Y direction (see Fig. 5-2), while a gap of 36 mm was left in each of the lower edge panels to be post-tensioned (see Fig. 4-2).

5-1-2 Preliminary analysis

After post-tensioning, the gaps would close up, reducing the final length of the free lower edge chords on each side to 460 mm. The 6 by 6 panel space truss dimensioned as above was considered for preliminary analysis.

The model was assumed to be loaded on four of its lower nodes. Because of having two axes of symmetry in plan view, only one fourth of the model was analysed (see Fig. 5-2) with the nodal coordinates shown in Table 5-1.

All members were assumed to be pin-ended, concentric, and straight. A linear static analysis was carried out on the idealised model using both the STRAND5 and NASTRAN computer packages (analysis no. 1). The results obtained from both packages were the same. Comparison between the results of the analyses of one quarter of the model and the whole model showed that they were identical.
Therefore, the correctness of the boundary conditions assumed for 1/4 of the model was confirmed.

Based on the assumptions made for the design, the results of computer analysis were compared with the critical forces of different members given in Table 4-3, section 4-3.

\[ d = 360 \text{ mm} \]

\[ h' = \text{Bottom chord} \]

\[ \frac{I}{I} = 30 \times 30 \times 30 \text{ mm} \text{ hubs with } 3 \text{ mm-deep grooves on two faces} = \frac{2}{3} \times 30 + 2 \times 3 = 460 \text{ mm} \]

\[ E = 200000 \text{ MPa} \]

\[ (\text{Original assumption}): \sigma = 250 \text{ MPa} \]

\[ \rho = \frac{E \gamma}{{\sigma}^2} = \frac{200000 \times 6.5}{250} = 5200 \text{ mm} \]

\[ s = \text{curved top chord length} = 6 \times 520 = 3120 \text{ mm} \]

\[ \theta = \frac{3120}{5200} = 0.6 \text{ rad} = 34.38^\circ \]

\[ \theta' = \frac{5 \theta}{6} = 0.5 \text{ rad} = 28.65^\circ \]

\[ \rho' = \rho - d = 5200 - 360 = 4840 \text{ mm} \]

\[ s' = \rho' \theta' = 4840 \times 0.5 = 2420 \text{ mm} \]

\[ \text{Final length of each edge bottom chord:} \]

\[ X_{20} = \rho \sin \frac{\theta'}{2} = 1197 \text{ mm} \]

\[ Y_{20} = 0 \]

\[ Z_{20} = \rho' \left(1 - \cos \frac{\theta'}{2}\right) = 144 \text{ mm} \]

The coordinates of other nodes were calculated in a similar way and are shown in Table 4-1.

Figure 5-1 The geometry of the gently curved barrel vault.
The results showed that under a total load of 55.2 kN, top chord no. 19 (Fig. 5-2b) would reach its experimental critical compressive force, i.e. 10.8 kN and buckle first. At the same time, one of the webs connected to the loaded node (i.e. member 43) showed the largest tensile force in it, i.e. 10.4 kN.

Table 5-1  The calculated and measured nodal coordinates of 1/4 of the gently curved barrel-vault model space truss (All dimensions are in mm).

<table>
<thead>
<tr>
<th>Node no.</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Node no.</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
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<td>-345</td>
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</tr>
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<td>1275</td>
<td>1275</td>
<td>272</td>
<td>272</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5-2  The plan view of 1/4 of the gently curved barrel-vaults in flat position
As a result, the bolt connecting the two webs 43 and 37 to the top chord 5 would be subjected to large shearing and axial forces, i.e. 7.3 kN shear and 5.2 kN axial tension. Comparing this with the maximum safe shear in the bolts of grade 8.8, i.e. 5.65 kN (see section 4-2-1), it was observed that the bolt connecting webs 43 and 37 would also be in a critical condition under a total load of 55.2 kN on the model space truss.

On the whole, calculations showed that the bolt connecting web members 37 and 43 to top chord 5 would reach a critical condition simultaneously with the buckling of the first top chord(s) under vertical loading.

Nevertheless, because the first model space truss had been fabricated by that stage using high-tensile bolts of grade 8.8, it was decided to carry out the test in order to obtain a general idea of the actual behaviour of the model under load and the accuracy of linear analysis to predict it.

5-1-3 First model fabrication
During the fabrication of the first model space truss, measurements made on the model while still in the flat position (before post-tensioning) showed that the top chords formed not square panels, but rather rectangular panels. The dimensions of each panel were 510 x 520 mm.

Because the difference was small and as it was intended to reuse the same pyramidal units for the next tests if their deformations remained within the elastic range, and also because the same jig was intended to be used to fabricate any other pyramidal unit if needed, therefore, the same dimensions were decided to be kept for the analyses as well as the tests to be carried out afterwards.

5-1-4 Measurements on the model before post-tensioning
Figure 5-3 shows the first model before post-tensioning.
The following measurements were made on the first model space truss before post-tensioning:

The length of top chords in X direction = 520 mm.
The length of top chords in Y direction = 510 mm.
Depth of the space truss = 360 mm.

5-1-5 Measurements on the shaped model after post-tensioning

The post-tensioning force used to shape the first model space truss was 8.5 kN. After post-tensioning, the dimensions of the final shape of the model space truss (Fig. 5-4) were measured. Both calculated and measured nodal coordinates are shown in Table 5-1. The calculated and measured top chord radii of curvature are also discussed in section 5-1-14.
Comparison between the calculated and measured dimensions of the gently curved barrel-vault model space truss showed good agreement, i.e. less than 2% difference between the calculated and measured dimensions. The slight differences could be attributed to errors in measurement and also due to the fact that the model had been post-tensioned beyond the point of just closing up the gaps, causing some axial forces and nodal displacements in the model due to post-tensioning, as an analysis of the model acted upon by post-tensioning forces showed (analysis no. 2).

5-1-6 Analysis for the measured dimensions
The measured dimensions were introduced into the computer and the results of the analysis (no. 3) showed almost identical deflections and axial forces as compared with the results of the preliminary analysis (no. 1) which was based on the calculated dimensions. Therefore, the measured dimensions were adopted as the basis for computer analysis.

Also, a simple analysis was carried out to study the post-failure behaviour of the model space truss. In this analysis, the top chord members 51, 53, 162, and 164 (see Fig. 5-5) which, according to the previous analyses, reached their experimental critical compressive force (i.e. 10.8 kN) first during vertical loading, were removed and replaced with their residual compressive forces obtained from member tests (i.e. 1/3 of the ultimate load capacity of the members in compression tests).

These residual forces were applied to the end nodes of the removed members. Then, vertical load was applied to the four bottom nodes of the reduced analytical model to see which member(s) would fail next. The results showed extremely large displacements (on the order of $10^{13}$) which indicated the overall instability and total collapse of the model after the loss (i.e. local buckling) of four of its top chord members.

This indicated that only 4 members had to be removed to transform the structure into a mechanism, despite an overall degree of static redundancy of 19, as given from Maxwell's rule:

$$b + c \geq 3j,$$

where $b$ is the total number of members (i.e. 238), $c$ is the number of kinematical constraints (i.e. 36), and $j$ is the number of joints (i.e. 85).

This means that at collapse the structure is statically indeterminate (i.e. redundant) as well as kinematically indeterminate (i.e. a mechanism).

The question of statical indeterminacy is dealt with in more detail in section 5-2-6.
5-1-7 Test 1

For the first test, the model space truss was supported at 12 bottom nodes along its two horizontal edges. The supports were provided by hinge-like rod ends allowing rotations about three axes and preventing translation (see Fig. 5-6).

Vertical load was gradually applied to four bottom nodes (Fig. 5-7a) while the vertical displacements of 9 nodes were read with a survey level, and a load vs. deflection plot was recorded by means of a plotter (Fig. 5-7b) as indicated earlier. The nodes, whose deflections were read, are shown in Fig. 5-7a. During vertical loading, the middle edge bottom chords became free under a total load of 6 kN.
Under a total load of 62.3 kN, shear failure occurred in the same bolt that had been predicted as being critical, ie, the bolt connecting webs 37 and 43 to top chord 5 (Fig. 5-2b).

Figure 5-6 The hinge-like supports of the model space trusses provided by rod ends.

Figure 5-7 The first test on gently curved barrel-vault model space truss.
Figure 5-8 shows the model space truss after failure in Test 1. Also, closer observation showed that a small portion of the threaded length of the bolts was still contained in the top chord joints and therefore, what should have been originally considered in the design calculations, was the safe load for the 6 mm high-tensile bolt with threads included.

Nevertheless, the result of the first test confirmed, to some extent, the reasonable estimate given by linear static analysis in predicting the ultimate load capacity of the post-tensioned gently curved barrel-vault model space truss.

5-1-8 Comparison between experimental and theoretical results
The collapse load in Test 1 was 12.9% larger than that given by linear static analysis. This showed that the assumption of pin-ended conditions for the members of the model space truss gave a lower bound for the collapse load of the truss. Although the linear analysis assuming pin-ended concentric members could give a reasonable estimate of the ultimate load capacity of the model space truss, nevertheless, the deflections of the nodes measured during the test were four to five times as large as those given by linear analysis. This difference was attributed to the built-in eccentricity of the joints of the space truss and possible joint slip at connections.

Therefore, another analysis was carried out using beam elements (instead of truss elements) to introduce the eccentricities by means of short links at the location of the joints (analysis no. 4). Also, the curved top chords were approximated by two line elements connected at the middle point of the curved chords. The resulting
deflections from the analysis considering eccentricities were 44% of those measured in the test.

5-1-9 Test 2

In order to prevent bolt shear failure from happening again in the next tests, two measures were taken: first, to distribute the load to a larger number of nodes, and second, to use high-tensile bolts of a higher grade and longer shanks. The diameters of the bolts could not be increased because of the small sizes of the cross sections of the tubes. The bolts chosen were 6 mm cap screws of grade 12.9 with a shank length of 21 mm. Other data for cap screws of grade 12.9 are:

Shank area \(= 28.3 \text{ mm}^2\)

Minimum tensile strength \(= 1200 \text{ MPa}\)

Stress at 0.2% offset \(= 1080 \text{ MPa} = 1200 \times 0.9\)

Maximum permissible tensile stress = 540 MPa.

Maximum permissible shear stress = 300 MPa.

Safe axial tension @ 540 MPa = 15.3 kN, and safe shear @ 300 MPa = 8.5 kN.

These measures proved effective as the second test was successful in terms of obtaining the maximum load capacity with top chord failure. The support conditions in Test 2 were the same as those in Test 1, i.e. the model was supported on 12 edge bottom nodes along its two parallel straight sides, with freedom of rotation and suppressed translations (see Fig. 5-7).

Linear analysis (no. 5) showed that under a total vertical load of 36 kN, applied to eight lower nodes, top chord member 24 (see Fig. 5-2b) would reach its experimental axial compressive critical force (i.e. 10.8 kN; see Table 4-2) and would buckle first, therefore causing a complete collapse. At the same time, the tensile force in web 43 would reach 8.6 kN inducing a shear force of 7.5 kN and an axial force of 4.3 kN in the same bolt which was critical in test 1. These forces were less than the above-mentioned safe loads for the Grade 12.9 cap screws. The resulting shear and tensile stresses in other connections were also checked according to AS1250 and AS1252 and proved satisfactory.

The experimentally determined ultimate load for the second gently curved barrel-vault space truss was 52.3 kN, including the weight of the whiffletree which was 1.8 kN. The second model failed due to the buckling of three top chords in its middle portion. The order of failure of the top chords could not be noticed as the failure was
sudden. Figure 5-9 shows the plan view and the load vs. deflection plot for the central top chord node of the model space truss in Test 2.

![Plan view](image)

(a) Plan view  
(b) The deflection of central top chord node

**Figure 5-9** The pattern of failure and load-deflection curve for Test 2.

Figure 5-10 shows the model GCBVST after failure in Test 2.

![Model GCBVST](image)

**Figure 5-10** The model GCBVST after failure in Test 2.

### 5-1-10 Test 3

The third test was aimed at confirming the results of the second test and measuring the forces induced in some of the members of the model space truss by means of electrical resistance strain gauges. The support conditions were the same as those in Tests 1 and 2. The third gently curved barrel-vault model space truss carried an
ultimate load of 45.8 kN upon which failure occurred due to successive buckling of top chords (see Fig. 5-11).

![Plan view and load-deflection curve](image)

(a) Plan view (b) The deflection of the central top chord node

Figure 5-11 The pattern of failure and the load-deflection curve in Test 3.

The residual load after failure was 14 kN. The patterns of failure in tests 2 and 3 were similar to each other (see Figures 5-9 and 5-11), though Test 2 showed 14% greater ultimate load capacity. Figure 5-12 shows the model GCBVST after Test 3.

![Model GCBVST after Test 3](image)

Figure 5-12 The model GCBVST after Test 3.
5-1-11 Test 4

The fourth test was carried out with an aim to investigate the effect of a larger post-tensioning force used to shape the model and also to measure the forces induced in different members during both the shape formation and the vertical loading stages by means of electrical resistance strain gauges. The support conditions were the same as those in the previous tests (see Fig. 5-7).

The ultimate load for the fourth gently curved barrel-vault model space truss was 51.8 kN where failure occurred due to a web member buckling which was followed later by successive top chord buckling (see Fig. 5-13) at lower load levels. The residual load after failure was 20 kN.

(a) Plan view
(b) The deflection of the central top chord node

Figure 5-13 The pattern of failure and the load-deflection curve obtained in Test 4.

Figure 5-14 shows the model GCBVST after Test 4.
5-1-12 Calculated and measured top chord radii of curvature
Measurements made on the gently curved barrel-vault model space trusses shaped by means of post-tensioning showed that good tolerances had been obtained in comparison with the design. The measured rise and span of the curved top chords were 230 mm and 3065 mm, respectively. This gave a span/rise ratio of 13.3. As the length of the top chords in the curved direction was 3120 mm, the measured radius of curvature was, therefore, 5221 mm which was close to the design radius of curvature of 5200 mm.

In addition to finding the radius of curvature of the top chords by measuring the overall dimensions (i.e., the span/rise ratio) of the shaped model, the data from the pairs of strain gauges placed on the top and bottom surface of the top chords, were also used to give another measure of the radius of curvature. The flexural strain was obtained by taking the average of the difference between the readings for each pair of strain gauges. The radius of curvature, $\rho$, was then found from the relationship $\rho = \frac{y}{\varepsilon}$, where $y$ is the distance from the centroidal axis and $\varepsilon$ is the strain.

Table 5-2 shows the radius of curvature of the top chords in the gently curved barrel-vault model space trusses, as obtained from calculation and measurements.

<table>
<thead>
<tr>
<th>The radius of curvature of top chords (mm)</th>
<th>Calculated based on design</th>
<th>Measured from span/rise ratio</th>
<th>Measured by strain gauges</th>
</tr>
</thead>
<tbody>
<tr>
<td>5200</td>
<td>5221</td>
<td>5128</td>
<td></td>
</tr>
</tbody>
</table>

Differences were attributed to errors in both measurements and fabrication.

During the shape formation of the gently curved barrel-vault model space trusses, no attempt was made to measure the changes in the span/rise ratio of the models with the increase in the post-tensioning force.

5-1-13 Loosening of the middle edge bottom chords
During the tests, it was observed that some of the edge bottom chords, especially those in the middle of the curved sides, became loose or free at some stage of the vertical loading. This loosening was due to the tensile forces induced in those
members, as the analysis for vertical loading showed. Part of the tensile force induced in those edge bottom chords offset the original compression left in them from the post-tensioning stage, and the rest was totally transferred to the high-tensile prestressing wire which passed through the chords. Consequently, the chords became loose.

The stage during vertical loading wherein these bottom chords became free, depended upon the magnitude of the final post-tensioning force used for shape formation. This stage was visible in the load-deflection plots obtained during the tests, as a slight change occurred in the slope of the curves, which represents the change in vertical stiffness of the model space trusses.

5-1-14 Post-tensioning forces

The post-tensioning forces used to shape the models were measured by reading the pressure dial gauge mounted on the hydraulic pump connected to the jack used for post-tensioning. The jack and the dial gauge were calibrated twice in a standard testing machine. The post-tensioning force was applied to the prestressing wire which, in turn, transferred it to the corners of the model through the wedge-shaped end anchorages used for prestressing (see Fig. 5-16).

The final post-tensioning force used to shape the first, second, and third models was 8.5 kN. During vertical loading, the middle edge bottom chords of the second and the third model space trusses became free under a total vertical load of 18 and 10 kN, respectively.

The post-tensioning force used to shape the fourth model was 12.6 kN. The edge bottom chords became noticeably free under a total vertical load of 20 kN. However, the change in the overall stiffness of the model, as observed in the load-deflection plot occurred at a lower load level, i.e. 7 kN (see point A in Fig. 5-13b).

This discrepancy could be attributed to the inaccuracy of the visual and manual examination of the bottom chords during the tests to see when they became loose, and also to other imperfections such as joint slip.

The larger post-tensioning force used for the shape formation of the fourth model was probably the cause of its premature failure due to web buckling under vertical loading in Test 4.
Comparing the results of Tests 3 and 4, it was concluded that there must be an optimum post-tensioning force beyond which the shaped model would show an undesirable mode of failure (premature shear) different from the desirable flexural mode of failure in which the maximum load capacity of the space truss can be reached by initiating top chord buckling.

If the post-tensioning forces applied during shape formation were beyond the optimal limit, extremely large forces could be induced in some of the webs in the vicinity of the post-tensioned corners of the model space truss and this could lead to a premature failure of the model under vertical loading as occurred in Test 4.

5-1 Further observations
The shape formation and vertical loading stages of the 3rd and 4th gently curved barrel-vault model space trusses were video-taped in order to observe the behaviour of the models more carefully.

The sequence of the members that failed during the tests was checked by reviewing the tapes and it was observed that the first top chord member to fail was the same as that shown by analysis.

It was also observed in the video-tape review that the buckling of the top chords between panel points occurred before any instability in their adjacent joints. This showed the satisfactory behaviour of the top chord joints on the one hand, and indicated, on the other hand, that the principal forces induced in the members of the models were axial, as is expected from a space truss due to the triangulated configuration of its members.

5-2 Analysis of gently curved barrel-vault space trusses

5-2-1 Linear analysis assuming pinned concentric joints
The post-tensioning stage was modelled by applying the final forces - applied with a hydraulic jack to the corners of the test models - to the computer model. Two equal compressive forces were applied to each curved side of the model to simulate the post-tensioning forces (Fig. 5-15).

![Figure 5-15](image-url)
Because post-tensioning was applied before placing the space trusses in their final position on the testing frame for vertical loading, the boundary conditions for the model in the post-tensioning analysis (analysis no. 2) were assumed to allow sliding movement for one side of the model while preventing rigid body motion at the same time.

The results of the post-tensioning analysis assuming pin-ended concentric members showed that the forces in the space truss at the final stage of post-tensioning were mainly induced in the bottom edge chords through which the post-tensioning cables passed. Also, some axial forces of lesser magnitude were induced in the top chords and the webs of the two opposite tensioned edges of the space truss.

The assumption of pin-ended members for the post-tensioning analysis gave virtually zero axial forces in the rest of the top chords and webs of the space truss. However, because the top chords were continuous in reality, those in the curved direction of the model had flexural stresses induced in them. These stresses are equal to \( \sigma = \frac{Ey}{\rho} \), where \( y \) is the distance of the fibres from the neutral axis of the SHS top chord tube, and \( \rho \) is the radius of curvature.

For the gently curved barrel-vault model space trusses, the measured maximum flexural stress was: \( \sigma = \frac{200000 \times 6.5}{5128} = 254 \text{ MPa} \) (see Table 5-2). Because the flexural stresses are almost equal but opposite in sign for the top and bottom fibres of the curved SHS top chords in the post-tensioned models, the resultant axial force induced in them was negligible, which was what the analysis (no. 2) with the assumption of pin-ended members showed. Nevertheless, what should be considered in the final analysis, is the superposition of the stresses induced in the members of the space truss from both cases of post-tensioning and vertical loading.

Therefore, to find the final stresses of the curved top chords, the above-mentioned stress (i.e. 254 MPa) should be added to (or deducted from) the stresses shown by the vertical loading analysis. Doing this for the curved top chord(s) which showed the largest axial force (among the curved members only) in vertical loading analysis, i.e. member 12 (see Fig. 5-2b), it was seen that the superposition of stresses was still below the experimental yield stress for the SHS tubes:

\[
\sigma = 254 + \frac{9637}{70.9} = 386 < 450 \text{ MPa}
\]
Another critical member to check was the corner edge bottom chord (member 25) which showed axial compressive forces induced in it in both post-tensioning, $F_p$, and vertical loading, $F_d$, analyses. The superposition of these forces in this member was below the critical load obtained from the axial compression tests of individual CHS members (see Table 4-2).

$$\sum F = F_p + F_d = 10.6 + 7.1 = 17.7 \text{kN} < 26.8 \text{kN}$$

According to analysis, the forces induced in the other edge bottom chords (ie, members 26 and 27) under vertical loading were tensile. This indicated that the tensile forces induced in these bottom chords during gravity loading might offset the compressive forces induced in them by post-tensioning and they would become free, at which stage the tensile forces would be carried by the post-tensioning cable.

However, the superposition of forces for the middle edge bottom chords (ie, member no. 3) showed: $F_p + F_d = 13.3 - 3.7 = 9.6 \text{kN}$ (compression). Therefore, according to static linear analysis assuming straight pin-ended concentric members, all edge bottom chords would still be in compression under a total vertical load of 36 kN.

To study the case of premature failure in Test 4 which was shaped using larger post-tensioning forces (i.e. 12.6 kN), these forces were used in the post-tensioning analysis. The superposition of forces for the web member (no. 37) which buckled first in Test 4, showed:

$$F_p + F_d = -0.5 + 8 = 7.5 \text{kN}$$

which was less than the experimental critical compressive force for the webs (ie, 14.3 kN). Therefore, the reason why this member failed first in Test 4 could not be shown by the linear analysis assuming pin-ended concentric members. It could be attributed though, to initial imperfections in the webs of the model space trusses, as the pyramidal units were reused to fabricate other models after the first test.

Overall, the linear analysis assuming pin-ended members gave a reasonable estimate of the ultimate load capacity of the GCBVST. Previous research had also shown that simple pin-ended analysis could give a reasonable estimate of the peak load capacity of a barrel-vault space truss of almost the same dimensions [Hoe et al., 1986].
5-2-2 Linear analysis assuming fixed concentric joints

An analysis (no. 6) was carried out considering the model as a space frame composed of beam elements welded together concentrically. The resulting deflections and member forces were very close to those obtained from the analysis with pin-ended (truss) assumption under the same load.

This agreement highlighted the fact that the main forces induced in the members of the model are axial due to its triangulated configuration which makes it act as a space truss. For example, analysis no. 6 showed the maximum deflection was 5.15 mm occurring at central top chord node under a total load of 36 kN. It also showed the maximum compressive force under the same total load would be 10.8 kN induced in top chord member 24. These values were exactly what the analysis assuming pin-ended members (analysis no. 5) had also shown.

5-2-3 Analysis considering joint eccentricities

In order to obtain more realistic results from the analyses, particularly in terms of the deflections of the model space trusses under vertical loading, the eccentricities existing in the models were introduced into the analytical model. Figures 4-6(a) and (b) show these eccentricities for the centre lines of the members meeting in a top chord joint. The lower ends of the webs had no eccentricity with respect to each other as they were concentrically welded to cylindrical bottom hubs.

Previous work [Schmidt et al. 1981] had shown that a fair estimate of the deflections had been made for the nodes of a space truss sub-assemblage by introducing the eccentricities into the analytical model.

Using STRAND5, the members of the space truss were introduced as beam elements with end releases applied where appropriate. The 13 mm eccentricity between the two perpendicular top chords (see Fig. 4-6) was introduced as an element having the properties of the 6 mm high-tensile bolt connecting the two in the actual model. The 20 mm eccentricity between the pairs of webs meeting at the top chord was introduced as a member with the same properties as the top chords.

For the vertical loading analysis, because the middle edge bottom chords became free during loading and their portion of tensile axial force was transferred to the post-tensioning wire passing through them, therefore, the two members representing them in the analytical model were given the same properties as the 5 mm prestressing wire
used for post-tensioning. For the post-tensioning analysis though, the middle edge bottom chords were given their own properties as they acted in compression. Also two end conditions were tried for all edge bottom chords, pinned and fixed, in order to evaluate the difference.

The rotations for one end of the webs were released in the computer model to simulate the bolted end of the webs at the top chord level. This was tried in two cases: In one case, all rotations about all the three local axes of the webs were released at one end, and in the other case, only the rotations about the two principal local axes of the elements were released at one end without any release for twist (i.e. rotation about the longitudinal axis of the member). Also, to take account of the curved top chords, an extra node was introduced in the middle of each curved member with an eccentricity of 6.5 mm (in the plane of curvature), as both calculations and measurements had shown.

At the location of the supports, the webs met at a level 60 mm higher than the level of the supports. Also, the edge bottom chords were positioned slightly lower than the point of convergence of the webs, thus introducing another eccentricity (see Fig. 5-16). These eccentricities were taken into account in the computer model by introducing short members with the properties of the 25 mm cylindrical hubs that were used in the model space trusses.

![Figure 5-16 The eccentricities in the bottom edge nodes.](image)

Attempts were also made to simulate the behaviour of each top chord high-tensile bolt by releasing the rotations of its adjacent top chord members about the vertical (Z) axis. Another time the rotations were released of the bolt about its two cross-sectional principal axes, and another time by giving it a zero torsional rigidity (GJ). The first assumption, i.e. releasing the rotations of the bolt about its principal cross-sectional axes, gave odd results (i.e. extraordinary deflections and member forces). This could also be expected as bolts with big hole tolerances would increase the
degrees of freedom of the space truss and cause it to undergo extremely large deformations under load.

One quarter of the model space truss was analysed. The vertical displacement of the central top chord node as measured in tests 2, 3, and 4, and also as given by linear static analysis, with both assumptions of concentric and eccentric joints, is given in Table 5-3. Various parameters were changed in the analyses considering joint eccentricities, each giving results slightly different from the others.

A static nonlinear analysis was also carried out on the GCBVST assuming concentric joints. Both material and geometric nonlinearities were taken into account. Details of the nonlinear analysis are given in section 5-2-5.

Table 5-3 The experimental and theoretical deflection of the central top chord node (node no. 16, Fig. 5-2) under a total vertical load of 36 kN.

<table>
<thead>
<tr>
<th>Test or analysis</th>
<th>Deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1- Test 2*</td>
<td>30.0</td>
</tr>
<tr>
<td>2- Test 3</td>
<td>35.5</td>
</tr>
<tr>
<td>3- Test 4</td>
<td>29.5</td>
</tr>
<tr>
<td>4- Concentric space truss analysis</td>
<td>5.05</td>
</tr>
<tr>
<td>5- Concentric space frame analysis</td>
<td>5.00</td>
</tr>
<tr>
<td>6- Eccentric space frame analysis (rotations released for webs about principal axes only, edge bottom chords pin-ended, no rotation released for top chords)</td>
<td>12.1</td>
</tr>
<tr>
<td>7- Eccentric space frame analysis (same as 6 and GJ = 0 for top chord bolts)</td>
<td>12.2</td>
</tr>
<tr>
<td>8- Eccentric space frame analysis (rotation released for the bolted end of webs about principal axes only, rotation released for edge top chords, edge bottom chords assumed as fixed-ended)</td>
<td>10.7</td>
</tr>
<tr>
<td>9- Eccentric space frame analysis (same as 8 and middle edge bottom chords removed due to becoming free)</td>
<td>12.4</td>
</tr>
<tr>
<td>10- Eccentric space frame analysis (same as 6 and rotation released for top chords about their local vertical principal axis)</td>
<td>12.2</td>
</tr>
<tr>
<td>11- Eccentric space frame analysis (rotation and twist released for webs, no rotation for top chords, bottom chords</td>
<td>12.9</td>
</tr>
</tbody>
</table>
(Table 5-3, continued)

<table>
<thead>
<tr>
<th>Test or analysis</th>
<th>Deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pin-ended)</td>
<td></td>
</tr>
<tr>
<td>12- Eccentric space frame analysis (rotation and twist released for webs, no rotation for top chords, bottom chords fixed-ended)</td>
<td>10.7</td>
</tr>
<tr>
<td>13- Eccentric space frame analysis (rotation and twist released for webs, corner edge bottom chords fixed and others pinned, the bottom edge chords next to the middle one also assumed to become free)</td>
<td>11.8</td>
</tr>
<tr>
<td>14- Eccentric space frame analysis (same as 13, all bottom chords pin-ended)</td>
<td>13.0</td>
</tr>
<tr>
<td>15- Eccentric space frame analysis (all rotations released for webs, including twist, all bottom chords pin-ended, rotation released for top chords)</td>
<td>12.4</td>
</tr>
<tr>
<td>16- Eccentric space frame analysis (no twist for webs, rotation released for top chords, bottom chords pin-ended)</td>
<td>11.9</td>
</tr>
<tr>
<td>17- Eccentric space frame analysis (no rotation and twist for webs, bottom chords pin-ended, rotations released for top chords)</td>
<td>10.1</td>
</tr>
<tr>
<td>18- Eccentric space frame analysis (no rotation released for members, bottom chords pin-ended)</td>
<td>9.9</td>
</tr>
<tr>
<td>19- Eccentric space frame analysis (no rotation released for members, bottom chords fixed-ended)</td>
<td>9.1</td>
</tr>
<tr>
<td>20- Eccentric space frame analysis (same as 14, but the flexural stiffness of top chord bolts reduced to 1/10 of the nominal value)</td>
<td>15.5</td>
</tr>
<tr>
<td>21- Eccentric space frame analysis (same as 14, but the flexural stiffness of top chord bolts reduced to 1/100 of the nominal value)</td>
<td>27.8</td>
</tr>
<tr>
<td>22- Eccentric space frame analysis (same as 21, but the torsional stiffness of top chord bolts reduced to zero)</td>
<td>28.0</td>
</tr>
<tr>
<td>23- Nonlinear (geometric and material) space truss analysis; concentric joints</td>
<td>5.3</td>
</tr>
</tbody>
</table>

* Test 1 is not included as bolt shear occurred.
According to Table 5-3, out of the analyses 4 to 19 carried out for the gently curved barrel-vault model space trusses, analysis no. 14 gave the largest deflection for the central top chord node. This analysis assumed all rotations released for the webs about their three local axes at their bolted ends, all edge bottom chords pin-ended, with the middle edge bottom chords plus their adjacent chords given the properties of the prestressing wire, as they lost their grip and became free under vertical loading. Nevertheless, the deflection given by this analysis was only 41% of the average deflection of the central top chord node measured in the tests under 36 kN external load.

It is interesting to note that when rotations are released for the top chords about their local 2-2 axis (ie, the axis originally parallel to the global Z axis; see Fig. 5-2), the deflection of the central top chord node (analysis no. 15) becomes less than the case when the top chords are given no released rotation at all (analysis no. 14).

Although the top chord bolts were tightened hard, it was observed during the fabrication of the models that moving the top chord rectangular grid, before connecting the pyramidal units of webs to it, would cause its cells to take diamond shapes because of the rotation of the joints. Therefore, considering released rotation for the top chords about their vertical (Z) cross-sectional axis, was found to be closer to reality.

The main points observed were as follows.

- The eccentric analyses, without modified bolt stiffness, showed that the straight top chord member 24 (see Fig. 5-2b) would reach its experimental critical axial force under the same total load as that shown by the concentric analysis, i.e. 36 kN. However, the eccentric analysis, with modified bolt stiffness, showed that this would happen under a total load of 33 kN. The ultimate load predicted by the eccentric analysis with modified bolt stiffness was 33 kN.

- The eccentric analyses with and without modified bolt stiffness showed that the tensile forces induced in the middle edge bottom chords were 78% and 40% larger than those given by the concentric analysis, respectively. Nevertheless, the computed tensile forces in these chords were only 36% of the computed compressive forces induced in them by post-tensioning. Therefore, none of the linear concentric or eccentric analyses indicated that the middle edge bottom chords would become free during vertical loading.
• Another point noted from the eccentric analysis was that besides the compressive force of 7.2 kN, a bending moment of 48 Nm was also induced in web member 37 under a total load of 36 kN, indicating why this member failed first in Test 4.

• The eccentric analysis without modified bolt stiffness (i.e. no. 15; Table 5.3) also showed that under a total load of 36 kN, the axial tensile force in top chord member 87 (Fig. 5-5) was 10.3 kN which was 213% larger than that shown by the concentric analysis (i.e. 4.7 kN). Meanwhile, the eccentric analysis with modified bolt stiffness (no. 21) showed this force would be 3.4 kN tensile, under the ultimate load predicted by this analysis, i.e. 33 kN. This was 21% less than that shown by concentric analysis (no. 4). Although this point was not verified in the tests because no strain gauges were placed on that member, nevertheless, inspection of the top chord joints at the ends of member 87, and the members lying in symmetrical positions with respect to it, showed no indication of the deformation of the bolts or joints. Therefore, it could be concluded that the estimates of the force induced in member 87, as computed by the eccentric analysis no. 21 and the concentric analysis no. 4, were more realistic.

• Also, another analysis was carried out for the post-tensioning stage, taking into account the eccentricities. The results showed that the forces induced in the members of the gently curved barrel-vault model space truss due to the final post-tensioning force were close to the forces obtained from the concentric analysis. The edge bottom chords were under axial compression, ranging from 15 kN in the corner edge chord(s) to 19.5 kN in the middle one. The forces in the curved top chords ranged from 3 kN (tensile) in the chords just above the prestressed edges to 0.4 kN tensile forces in the inner top chords. The forces in the rest of the members (i.e. the straight top chords and the webs) were alternately compressive and tensile, ranging from 2.4 kN near the edges to 0.02 kN towards the middle.

• In the eccentric analysis for post-tensioning, all web and top chord rotations were released while bottom chords were assumed fixed-ended due to the fact that at the final stage of post-tensioning and before vertical loading, all edge bottom chords, including the middle ones, were tightly locked in position.

• It was only for the vertical loading analysis that the middle edge bottom chords and the edge bottom chords just next to them were assumed to have lost their grip and become free, and therefore, they were given the properties of the prestressing wire only in that analysis.
• Another analysis showed that when the flexural stiffness of the bolts connecting the orthogonal continuous top chords was reduced to 1/10 of their nominal flexural stiffness in order to take account of the 1 mm tolerance of the bolt holes, the central top chord node deflection increased to 15.5 mm (i.e., 49% of test average) with only a 2% change in the distribution and magnitude of forces in the members of the space truss.

• By using a series of trial sensitivity analyses [Schmidt et al. 1981], it was found that if the flexural stiffness of the top chord bolts was reduced to 1/100 of the nominal value, the eccentric analysis would give a central top chord node deflection of 27.8 mm which was 87% of the average experimental value.

• Meanwhile, it was observed that if, in addition to the assumptions made in the above-mentioned analysis, the torsional stiffness of the top chord bolts was assumed to be zero (analysis no. 22), the resulting top chord node deflection would be 28.0 mm which showed less than 1% increase in comparison with the previous case (i.e. analysis no. 21).

5-2-4 Nonlinear analysis
Another analysis was carried out taking into account the material and geometrical nonlinearities using the general purpose finite element program ANSYS. The model was assumed to be made up of concentric truss elements. The edge bottom chords were modelled using special elements called gap bars. The prestressing wire was modelled using cable elements. In order to carry out a nonlinear buckling analysis, only the top chord member 89 - i.e. the member shown by previous analyses to reach its critical force first under the vertical loading (see Fig. 5-5) - was modelled as a beam element.

The degrees of freedom for the nodes defining the ends of the cable members and the gap bars were linked together in all directions except for the longitudinal (X) direction in order to model the longitudinal movement of the prestressing wire inside the bottom chord tubes.

The material nonlinearity was modelled by introducing the experimental stress-strain curve obtained from axial tensile tests (see Fig. 5-17). The type of material behaviour chosen was the rate independent plasticity using the "multilinear kinematic hardening" option which is characterised by irreversible straining beyond a certain
level of stress and the Bauschinger (kinematic hardening) effect. The yield criterion used by the program for the above option is the von Mises criterion, the flow rule is associative, the hardening rule is kinematic hardening, assuming the yield surface to remain constant in size while the surface translates in stress space with progressive yielding, and the material response is multilinear.

The model was introduced into the program in its flat position. The analysis was carried out in two steps or load cases. The first load case (load case 1) was to generate the final shape of the model after post-tensioning. The second load case (load case 2) was to simulate the vertical loading to obtain the ultimate load capacity.

The type of analysis was static with nonlinear geometric and material effects using the incremental iterative full Newton-Raphson method. For the solution to converge, particularly in load case 1, the load was applied gradually in 10 steps.

In load case 1, it was first attempted to apply concentrated forces equal to the experimental value to the corners of the model to simulate the post-tensioning force. However, this attempt was unsuccessful as the point loads could not rotate with the model and remained in their original direction in the analysis. Therefore, the post-tensioning or shape-formation loading was applied in the form of controlled displacement which was also more numerically stable than the force loading.
Because the existence of the cable elements in the model would cause them to be compressed in load case 1, which was contrary to what happened in reality as they were subject to tensile stresses, they were deactivated in load case 1. This was feasible as the type of element chosen for the cables allowed a "birth and death" option. The program would consider a very small value for these deactivated elements in the stiffness matrix, and when they were activated at a later stage during the solution it would assign to them their nominal stiffness value. However, the solution time for load case 2, i.e. vertical loading, would increase remarkably due to the sudden change induced in the structural stiffness matrix caused by activating these elements.

At the end of the solution, the final results for these elements would show only the forces induced in them by vertical loading. However, deactivating the cable elements in load case 1 would not be theoretically wrong because the structure acted ideally as a mechanism and no appreciable forces were induced in its members during the post-tensioning stage. Nevertheless, the experimentally measured prestressing forces were added to the theoretical cable axial forces obtained from load case 2 in order to obtain a true estimate of the total stress induced in the cable elements. Because, as mentioned earlier, there were some axial forces induced in the cable due to the post-tensioning force which was needed to overcome the flexure of the continuous top chords, the weight of the structure, and the friction of the joints, during the shape formation process.

According to the measured final span of the model GCBVSTs, the displacements applied to the corners of the flat model could be determined, i.e. 102.5 mm about the vertical centreline. Also, the gap per unit length of the gap bars was introduced, i.e. 0.069 (see section 5-1-2). In a normal design procedure where no experimental values are available for the particular span/rise ratio, the values for the forced displacement of the corners and the gap per unit length for the edge bottom chords can be determined from the overall geometry of the model as explained earlier.

The boundary conditions assumed for load case 1 were to restrain vertical (Z-direction) movement for the edge bottom nodes along the two opposite straight sides of the model, and to apply horizontal forced displacements equal to 102.5 mm from each side, to the four lower corner nodes of the model. The shaped model is shown in Figure 5-18. Table 5-4 shows the computed and measured nodal coordinates for 1/4 of the GCBVST.
Figure 5-18 The deformed shape of the GCBVST versus its original flat shape.
Table 5-4  Nodal coordinates for 1/4 of the GCBVST (see Fig 5-2a).

<table>
<thead>
<tr>
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<th>Y (mm)</th>
<th>Z (mm)</th>
<th>Node no.</th>
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<th>Y (mm)</th>
<th>Z (mm)</th>
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The computed coordinates given in Table 5-4 are from the shape formation analysis, with the origin based at node 17 (see Fig. 5-2a).

According to the figures given in Table 5-4 and Table 5-1, good agreement was observed between the (ANSYS) computed, the experimentally measured, and the geometrically calculated values found for the coordinates of the nodes of the GCBVST after shape formation.

Figure 5-19 shows the displacement of the central top chord node of the model according to the nonlinear analysis. The horizontal axis shows the loading history in Fig. 5-19(a) and the total load in Fig. 5-19(b), respectively, while the vertical axis shows the displacement in the vertical (Z) direction in both (a) and (b). As the curves show, the main displacement of the node occurs between 0 (i.e. the start of load case 1) and 1 (i.e. the end of load case 1), it is upward, i.e. the node moves up from an original flat position during shape formation (Fig. 5-19a), and this movement occurs under zero total load (Fig. 5-19b). Between 1 and 2, i.e. load case 2, the node moves downward due to vertical loading. This downward movement, however, is only 6% larger than that given by the linear concentric analysis (see Table 5-3) under an equivalent vertical load (i.e. 36 kN).
Based on the results given in Table 5-3, it could be concluded that the main factors which caused the displacements of the nodes under vertical loading to be larger than those given by the concentric linear and nonlinear analyses, were the eccentricity of the joints and the bolted connections of the model barrel-vault space trusses. In other words, the large deflections of the model space trusses under vertical loading occurred because the connections were neither concentric nor truly pinned and frictionless. It is known that bolted joints in steel structures show a nonlinear M-θ behaviour, because even for the best designed pinned joints some moment occurs except where the joint is truly frictionless, and some rotation occurs even for the best designed rigid joint. That is why structures containing bolted connections show a nonlinear load-displacement behaviour [Vanderbilt, 1974].

Previous research work [Schmidt, 1983] on a single-chorded flat space truss with concentric joints (using the Triodetic system) had also shown good agreement between test and theory for the peak load capacity of the model test, but poor agreement between the elastic stiffnesses obtained from test and theory. This poor agreement for stiffnesses was attributed to slip effects in the joints of the model. The above-mentioned causes, that is, eccentricity and bolted connections were nonexistent in that model. Making mention of this case is not, of course, meant to draw a comparison between the behaviour of the two models, as the SCST tested in the previous research work lacked edge bottom chords and its geometry was also different from that of the GCBVSTs discussed herein. It was just to give an example of the discrepancy between the results of test and theory which could occur even for concentric jointing systems as far as nodal displacements are concerned.
Figure 5-19  The displacement of the central top chord node (see Fig. 5-5).
Figure 5-20 The axial force induced in member 89 (the straight middle top chord).

Figures 5-20 to 5-27 show the axial forces induced in different members of the GCBVST during loading history. The negative values indicate compression and the positive values tension. The horizontal axes in Figures 5-20 (a) and (b) represent the loading history - i.e.load case 1 (shape formation) and load case 2 (vertical loading) -
and the total load, respectively, while the vertical axes in both figures represent the same value, i.e. the axial force in member 89 (see Fig. 5-5).

As these figures show, there is virtually no force induced in the members of the model space truss during shape formation (load case 1) as the model is still a mechanism, as it is meant to be. In fact, the prime objective of the shape-formation process by means of post-tensioning is to utilise mechanisms within the structure, in the flat configuration, to obtain the desired shapes without inducing initial stresses in the resulting structures.

The distance between 1 and 2 on the horizontal axis indicates the theoretical total vertical load, i.e. 40 kN, applied statically to the model before it indicated collapse. Figure 5-20 shows that the total force induced in the critical member before buckling (member 89, Fig. 5-5) is 12 kN (compression). As compared with the Euler load for a pin-ended column of the size of this member, i.e. 9.9 kN (see Table 3, Chapter 4), this value is 20% larger. This increased load capacity indicates the effect of the partial restraint caused by the rest of the structure attached to member 89 (see Fig. 5-5).

Figure 5-21 shows the axial force induced in the corner edge bottom chord. According to the figure, the displacement-controlled loading applied in load case 1 goes a little bit further than the closure of the gaps causing a sudden compressive force (10 kN) to develop in the member just before applying the vertical load, as is evident in Figure 5-21. The same phenomenon can be observed in Figure 5-27 which shows the development of the axial force in the middle edge bottom chord. But this force is not realistic as a physical model is formed by means of applying concentrated forces to its corners with a hydraulic jack. Nevertheless, the second portion of the curves corresponding to the vertical loading is still valid.
It was considered a weakness of the nonlinear analysis not to be able to predict the closure of the gaps in load case 1 so as not to go beyond that point. Nevertheless, this matter did not negate the validity of the axial forces output by the nonlinear analysis for the other members, i.e., the top chords and webs further away from the end curved panels, as the axial forces in these members still remained zero after the closure of the gaps. Besides, the main feature of the analysis, that is, its shape formation accuracy was still unchanged.
Figure 5-23, of which only the second portion is valid because the cable members were deactivated in load case 1, shows that the axial force in the prestressing wire at midspan still remains zero even after vertical loading. This is again unrealistic as the total axial force in the corresponding gap bar (i.e., the middle edge bottom chord) at the end of load case 2 is compressive only because of the large fictitious compression left in it from load case 1 (see Fig. 5-27).

In real testing situations where the post-tensioning forces are not applied much further beyond the closure of the gaps, the tension induced in the middle edge bottom chord due to vertical loading (see Fig. 5-27) can compensate the compression from load case 1, and therefore, any additional tension will be transferred to the prestressing wire passing through it after this bottom chord becomes free, as it occurred in the tests.

Figure 5-23  The axial force in member 241 (the prestressing wire in the middle).

The sudden increase in the axial forces of web members 34 and 41 just before the start of load case 2, as seen in Figures 5-24 and 5-25, is again due to the closure of all the gaps which causes the model to transform from a mechanism into a **kinematically** determinate structure.
Figure 5-24  The axial force in member 34 (web member near the corner).

Figure 5-25  The axial force in member 41 (the corner web member).
Figure 5-26 The axial force in member 88.

Figure 5-27 The axial force induced in member 3 (the middle edge bottom chord).
5-2-5 Experimental and theoretical member axial forces

It should also be noted that the member forces given by both the concentric linear and nonlinear analyses as well as the linear eccentric analyses were, overall, close to those measured during Test 4 by means of strain gauges (see Fig. 5-28).

Figure 5-28 The calculated and measured member axial forces.
However, as far as the edge bottom chords were concerned, there were some discrepancies between the theoretical and experimental results as depicted in Figure 5-29. The axial forces obtained for the edge bottom chords from the nonlinear analysis were presented separately in section 5-2-5. The reason was that they could not be compared directly with the experimental results as well as the results obtained from the other analyses due to the fact that, contrary to the actual condition in the tests where post-tensioning was applied in the form of concentrated loads, they were based on a displacement-controlled loading for their first portion from load case 1 (i.e., the shape formation stage). Therefore, for these results to be comparable with the other results, only the second portion of the axial forces obtained from the nonlinear analysis for the edge bottom chords (i.e., from load case 2 or vertical loading) should be added to the compressive forces induced in the edge bottom chords by the prestressing wire.
5-2-6 An attempt to detect the post-buckling behaviour

In order to follow the post-buckling behaviour of the gently curved barrel-vault model space trusses, a simple analysis was carried out by eliminating, in the computer model, the top chord members which were shown by the first analysis to fail first, and replacing them with their residual load capacities which had been obtained from member tests (i.e. \( \frac{1}{3} \) of the ultimate load capacity of the members in compression tests; see Appendix A).

The residual forces were applied to the end nodes of the removed members in opposite directions to simulate the residual axial compression that the failed members applied to their end nodes. This analytical model was solved again. The results showed extremely large displacements on the order of \( 10^{13} \) mm which obviously indicated the overall instability of the structure.
Therefore, it was concluded that the loss of the first two top chord members 89 and 126 (see Fig. 5-5) was sufficient to cause the model space truss to collapse. The tests also showed that the model space trusses collapsed without carrying any further increased load after the failure of the first top chords.

Using Maxwell's rule, the necessary condition for stability was checked for the GCBVST with all members present: 

\[ R = b - (3j - c) \]

where \( R \) is the degree of statical indeterminacy, \( b \) is the total number of bars, i.e. 238; and \( j \) is the number of joints, i.e. 85, and \( c \) is the number of restraints at the supports, i.e. \( 12 \times 3 = 36 \).

Therefore, the model space truss appears to be 19 times statically indeterminate. However, the effect of the loosening of the edge bottom chords under vertical loading has not been taken into account in this calculation. As three edge bottom chords became free on each side of the model during vertical loading, the statical indeterminacy, according to Maxwell's rule would reduce to 13 (i.e. 19-6). Nevertheless, the model space truss collapses (i.e. it has a mechanism) even though it is statically indeterminate at collapse.

Therefore, it was concluded that a closer treatment of the subject should be carried out by looking into the rank of the equilibrium matrix, \( r \), the number of independent states of self-stress, \( s \) (\( \geq 0 \)), the number of independent inextensional mechanisms, \( m \) (\( \geq 0 \)), and the degree of redundancy or statical indeterminacy \( R \), as depicted in these expressions [Pellegrino and Calladine 1986]:

\[ R - s + m = 0 \quad s = b - r \]

Without trying to find the rank of the equilibrium matrix in order to find \( s \) and \( m \) from the above relations, it was attempted to find states of self-stress in terms of repeated modules like those mentioned by Affan and Calladine [1986] (see Fig. 5-30). It could be easily observed that -at least in the central portion of the model GCBVST where there is no bottom chord - there is no state of self-stress because any state of self-stress in such a configuration needs the bottom chords to be involved, too. In fact, the smallest module of self-stress would need at least two adjacent rows of bottom chords [Affan and Calladine, 1986] which were non-existent in the model space trusses discussed herein. Of course, Affan and Calladine considered a truss with only four supports, but here the number of supports is much larger. Hence,
states of self-stress of a different kind to those found by Affan and Calladine will be possible. The lack of a state of self-stress in the central portion of the model space truss was also confirmed by the experimental and analytical evidence that the axial forces, due to post-tensioning, in all the top chords and web members of the model, positioned away from the tensioned edges, were virtually zero. In other words, post-tensioning the edges of the model induced no axial forces in the members located away from the edges, whereas it should have caused axial forces in those members had the structure had a state of self-stress. This point explains why the buckling of the first top chord member led to the formation of a failure mechanism in the models.

![Figure 5-30 The smallest unit of self-stress in a double-layer grid.](image)

Looking at the question of statical indeterminacy from another point of view, it was observed that although the number of external constraints (i.e. supports) was increased after shape formation by suppressing the translation of the lower edge bottom nodes along the two straight sides of the space truss for vertical loading, nevertheless, the internal degree of statical indeterminacy was virtually zero as the structure could go no further than a statically determinate state after the closing up of all the gaps during the post-tensioning stage which caused it to change its status from a mechanism into a kinematically determinate structure.

In the nonlinear buckling analysis, too, an attempt was made to chase the post-buckling behaviour of the structure. First, the ultimate load of the structure was found to be 40 kN by applying vertical load to the analytical model in load case 2. Then in another solution, vertical load was applied up to 36 kN in load case 2, and then by using the restart option in the program, further load was applied to the model in load case 3 in the form of a controlled displacement (applied vertically to the central top chord node). Figure 5-31 shows the GCBVST after failure according to the nonlinear analysis.
The post-buckling path can be found by a displacement control method [Papadrakakis, 1986] as the load control method can show the behaviour only up to the point of collapse and does not allow the structure to shed load. However, load case 3 could go no longer than the collapse point either, which was marked by a halt in the solution and an error given by the program indicating a negative main diagonal value appearing in the stiffness matrix of the structure. The collapse load in load case 3, found by adding up the vertical reactions of the supports, was equal to the value found before, i.e. 40 kN.

The reason why the displacement control method could not find the post-buckling path of the model space truss was attributed to the possible snap-back failure of the model upon the buckling of the first member. Previous research [Schmidt, 1980; Supple et al., 1981; Collins, 1984] has also shown that the failure of space trusses made up of chords of the same slenderness ratio range as that used in the present work, is characterised by a snap-back. The displacement control method is generally unable to follow a snap-back failure [Duxbury, 1989].

It has also been shown that space trusses are sensitive to the effect of dynamic member snap-through and, therefore, the results of static post-buckling analyses should be considered with caution, as they might be erroneous [Morris, 1993].

Also a study on the initial post-buckling behaviour and imperfection sensitivity of space trusses with perfectly pinned joints which do not transmit moment has shown that the worst shape of imperfection in which the load drops most rapidly at the bifurcation point, initially involves the buckling of only one member [Peek, 1993].
5-2-7 Comparison between ultimate load capacities in test and theory

Using the average ultimate load capacity (i.e. 22.4 kN) of the individual top chords which had been obtained from the axial compression testing of 3 fixed-ended SHS tubes (see Appendix A2), the theoretical ultimate load capacity of the gently curved barrel-vault model space trusses in Tests 2, 3, and 4 (with 8 lower nodes loaded vertically) was found to be 74.7 kN.

This value was considered to be an upper bound for the ultimate load capacity of the models as the experimental fixed-ended member compressive load capacities were used in the computer analysis. The lower bound, assuming the space truss made up of pin-ended members (using an average of 10.8 kN from pin-ended compressive member test results; Appendix A2) was found to be 36 kN, as mentioned earlier. Figure 5-32 shows the load-deflection curves for the central top chord node in theory and experiment.

Comparing the average ultimate load capacity of the models in tests 2, 3, and 4, i.e. 49.9 kN, with the above theoretical upper and lower bounds, it was seen that the average of the tests was 33% below the upper bound and 28% above the lower bound, i.e. lying almost half way in between. Therefore, it was observed that the continuity of the top chords had, by far, compensated for the deleterious effects of joint eccentricities as far as the ultimate load capacity of the model space trusses was concerned.
It could also be concluded that the assumption of pin-ended concentrically connected members was a safe way to design the gently curved barrel-vault space truss with eccentric joints.

As far as the deflections are concerned, neither linear nor nonlinear static analyses assuming pin-ended concentric members gave a good estimate. Therefore, an analysis considering joint eccentricities should be carried out for this purpose with modifications applied to the flexural stiffness of the top chord bolts, as mentioned earlier in section 5-2-3.

5-3 Gently curved model barrel-vault trussed grid

5-3-1 Model barrel vault without intermediate webs
For purposes of comparison, a gently curved barrel-vault space truss without intermediate webs was fabricated by means of post-tensioning. Because the middle portion was made up of an orthogonal grid of top chords surrounded by pyramidal units along the four edges, this model was referred to as a gently curved barrel-vault trussed grid (GCBVTG), rather than a space truss. The surrounding pyramidal units could not be eliminated because they were needed to hold the prestressing wire, and also to support the structure after shape formation.

It might appear at first sight that the intermediate webs, particularly those which are not directly connected to a loaded point, do not play any significant role in giving stiffness and strength to a gently curved barrel-vault space truss. However, it was known from previous research work [Schmidt, 1983] that, even in its flat form, such a structure would act as a single-chorded space truss (SCST) provided it was sufficiently restrained at its edges. Nevertheless, an experiment was needed to determine the different responses of shaping, stiffness, ultimate load capacity, and failure mechanism.

A model dimensioned exactly like the previous gently curved barrel-vault model space trusses, but without the intermediate webs (see Fig. 5-33a), was fabricated. The only other difference was that the corner webs of the corner pyramids (i.e. members 32, 41, 205 and 214; see Fig. 5-5) were made of 20 mm rigid round bars and the web members 34, 39, 207 and 212 (see Fig. 5-5) as well as all the edge
bottom chords were made of $17 \times 3.2$ CHS, in order that no premature shear mode of failure would occur under the vertical load test.

A new whiffletree was designed for the vertical loading of this model after shape formation because of the different number and location of the nodes to be loaded directly. The details of the whiffletree will follow.

5-3-5 Details of the whiffletree designed for Test 14

In order to load the model "space trussed grid" in a manner similar to the gently curved barrel-vault model space trusses, 21 top chord nodes had to be loaded proportionally (see Fig. 5.35a). This led to the design of the five-layer whiffletree shown in Figure 5-33.

The sizes of the beams and the number required of each are given below.

- 1st layer from the top: 16 beams made of $40 \times 40 \times 2$ SHS, each 600 mm long.
- 2nd layer: 8 beams made of $50 \times 50 \times 2.5$ SHS, each 600 mm long.
- 3rd layer: 4 beams made of $750 \times 50 \times 3$ RHS, two of them 700 mm and the other two 1600 mm long.
- 4th layer: 2 taper flange I beams $100 \times 45$ mm
- 5th layer (i.e. the base beam of the whiffletree): one 150UB, 1500 mm long.

![Figure 5-33 Layout of the whiffletree used in Test 14.](image)
The whiffletree used in Test 14 is shown in Figure 5-34. The first layer of the whiffletree was connected, from the top, to the model by means of 6 mm stranded steel cables hooked to the loaded points of the model, and from the bottom, to 8 mm threaded bars which were connected to the second layer of the whiffletree. The threaded bars were used to facilitate fine adjustment for the levelling of the beams of the whiffletree.

The connections for the second and third layers of the whiffletree were also provided by means of 8 mm and 12 mm threaded bars, respectively. The total weight of the whiffletree was 1.3 kN which was included in calculating the ultimate load of the model GCBVTG.

The total weight of the whiffletree was 1.3 kN. The deflections of the nodes of the model were measured before and after hanging each layer of the whiffletree because of the anticipated low stiffness of the model.

Figure 5-34 The whiffletree used in Test 14.

5-3-2 Test 14
The model was fabricated and formed by means of post-tensioning. The final post-tensioning force was 7 kN. Measurements made on the model showed that good tolerances had been obtained as far as the shape formation was concerned.
The model for Test 14 was vertically loaded after hanging the whiffletree. Although the test was carried out later in the sequential order of the tests, it was deemed appropriate to mention the results here in this chapter for purposes of comparison.

The ultimate load in Test 14 was 11.4 kN (Fig. 5-35b). Very large deflections started to occur, especially in the central "grid" portion of the model as early as 2 to 3 kN in the vertical loading history. Finally, the model continued to deflect under the above-mentioned constant ultimate load. Closer observation showed that the curved top chords at the location of nodes 41 and 45 (see Fig. 5-35a) were starting to tear apart because of extreme flexural distortion.

The measured deflection of the central top chord node was 230 mm (see Fig. 5-35b) as compared with 4.5 mm, 5.5 mm, and 5.5 mm in Tests 2, 3, and 4, respectively under an equal total vertical load. Therefore, the experiment proved the efficiency of the intermediate webs in enhancing the stiffness of the structure and highlighted the identity of the GCBVST as a space truss as distinguished from grids and trussed portal frames.

The strain gauges placed on one of the middle edge bottom chords in Test 14 showed that the middle edge bottom chord did not loosen under vertical loading contrary to the case of the middle edge bottom chords in the GCBVST. Figure 5-36 shows the model GCBVTG after Test 14.

The test showed that shaping a single-layer barrel vault is possible by the post-tensioning process.

![Figure 5-35 Test 14](image)
5-3-3 Failure mechanism
The failure mechanism of the GCBVTG, as observed in Test 14, was in the form of the bending of the top chords and the sway of nodes characteristic of flexural systems such as single-layer grids.

This mode of failure was different from the failure mechanism of the GCBVST, observed in Test 3, which was initiated by the buckling of an individual top chord member between panel points, and was then followed by the sway of nodes adjacent to the failed top chord member. The latter is referred to as a patch mode of failure or an interaction between member buckling and node buckling in the general classification of the failure mechanisms of space trusses [Gioncu et al., 1985].

The failure mechanism of the models in Tests 2 and 3 (see Figs. 5-9 and 5-11) indicated that the GCBVST acts as a space truss with primarily axial member forces.

5-3-4 Analysis
A linear concentric analysis was carried out on the model GCBVTG in its final form. The post-tensioning analysis showed a pattern of force distribution almost like the one for the gently curved barrel-vault space trusses (with all members present). The main forces were axial compressive forces induced in the edge bottom chords, ranging from 8.2 kN in the corner bottom chord to 10.2 kN in the middle one.
There were also some axial forces induced in the top chords and the webs just above the edge bottom chords, but their maximum magnitude was only 16% of the forces induced in the edge bottom chords. The axial forces induced in the rest of the members were trivial according to the analysis (their maximum being only 0.4% of the axial forces in the edge bottom chords).

However, it should be noted that the pin-ended truss idealisation assumed in the analytical model, did not take account of the flexural stresses induced in the curved top chords during post-tensioning.

The model was also analysed for vertical loading. For the analysis of the space trussed grid under vertical loading, the model had to be considered as made up of beam elements, as otherwise the grid would be unstable.

Because each of the loaded (intermediate) pyramids in the previous tests on the GCBVSTs (i.e. Tests 2, 3 and 4) distributed the vertical load applied to its apex to 4 top chord nodes; in the analytical model for the GCBVTG, the load was applied to the corresponding top chord nodes to simulate the 8-point loading of the previous models. A total of 21 top chord nodes were loaded. Some of the nodes were given a larger share of the load because they corresponded to those nodes which were connected to 2 or 4 loaded webs in the previous model space trusses.

The total load applied in the analysis was 11.4 kN. The results showed significant bending moments at the ends of the top chords in the "grid" portion of the model, particularly in the middle portion. The resulting deflections were very large as compared with the theoretical deflections of the GCBVST. For example, the central top chord node showed a vertical displacement of 154 mm, whereas its deflection in the GCBVST assuming concentric joints, was only 1.7 mm under the same load.

The post-tensioning analysis of the space trussed grid using truss elements, however, showed reasonable results, indicating the stability of the model under the final post-tensioning forces. This was attributed to the fact that the post-tensioning forces were applied to the "truss" part of the model, not its grid part, in contrast to the vertical load case wherein the forces were applied to the "grid" part of the model.
5-3-5 Comparison
Comparing the ultimate load capacity of the GCVBTG, i.e. 11.4 kN, with the average ultimate load capacity of the GCBVSTs in Tests 2, 3, and 4, i.e. 49.9 kN, it was observed that the former is 23% of the latter (see Fig. 5-37).

Also, the central top chord node deflection of the model in Test 14 under a total load of 11.4 kN was 192 mm as compared with 7.5 mm deflection for the same node in Tests 2, 3 and 4. This result shows that the stiffness of the gently curved barrel-vault model trussed grid was 26 times less than that of the gently curved barrel-vault model space trusses.

![Graph](https://via.placeholder.com/150)

Figure 5-37 Comparison between GCBVST and GCBVTG.

5-4 Time schedule

For purposes of comparison of fabrication and erection times with other similar projects, a record was kept of the time spent to fabricate and assemble the individual parts of a 6 by 6-panel GCBVST as well as the post-tensioning and erection time. The time schedule was determined as follows:

- Cutting to size, marking out and drilling the 13 × 13 × 1.8 SHS top chords: 7 hours.
- Making a jig for fabricating the pyramidal units: 10 hours.
- Making 36 pyramids: cutting, drilling, and welding: 45 hours.
- Making 20 hubs: cutting, machining to size, putting screw threads, milling slots, and counter boring: 49 hours.
- Making 10 spacing tubes (ie, bottom chords): cutting to length, cutting to angle (ie, chamfer): 4 hours.
- Assembly of the space truss: 14 hours.
- Strain-gauging time: 20 hours.
- Post-tensioning: 2 hours.
- Placing the model in position (ie, on the testing frame): 1 hour.

------------------------ Total time = 152 hours (i.e.22 days or one month).

The total time spent on the fabrication of a model GCBVST, was compared with the time spent in previous research work to fabricate a similarly dimensioned model by assembling the discrete members in space [Hoe et al. 1986/87]. The time spent in fabricating and erecting the GCBVST was less than 1/5 of the time spent on the previous project [Hoe et al. 1986/87]. Therefore, the comparison shows that, in terms of the model size scales, the post-tensioning technique had reduced the total time needed to fabricate a curved space truss by conventional methods by a factor of 5. This could be considered as a significant achievement from an economical point of view.
6 The sharply curved barrel-vault space trusses

6-1 The aim of the tests

Another series of tests was carried out on, what are referred to as, sharply curved barrel-vault model space trusses shaped by means of post-tensioning. The purpose was, firstly, to see the feasibility of forming a sharply curved barrel-vault shape by the post-tensioning technique described herein, and secondly, to observe its behaviour under load.

6-2 Design of test models

With regard to the limited laboratory space available, it was decided to use almost the same span for the sharply curved models as that used for the gently curved barrel vaults. The final span of the model after post-tensioning was designed to be 2700 mm on the lower chord level.

Contrary to the case of the design of the gently curved barrel-vault space trusses in section 5-2 wherein the radius of curvature of the top chords and the bottom chord span (i.e. the distance between the corner supports) were assumed as known values, in the case of the sharply curved barrel-vault model space truss, the only known value was the bottom chord span, while the top chord radius was unknown.

Therefore, the question as to how many panels, bays, or modules to choose in the curved direction (certainly more than 6) to give a sharply curved barrel-vault space truss for the given span, was a matter of choice and depended on the desired truss rise or height. The span/rise ratio for the lower edge chords was assumed to be 2.1.

Therefore, a 10 by 6 panel sharply curved barrel-vault space truss was designed using basically the same member sizes as those of the gently curved models (see Fig. 5-1). The size and chamfer of the edge bottom chords were calculated accordingly. Figure 6-2 shows the geometry of the sharply curved barrel-vault model space truss designed for testing.
6-3 Yielding of top chords

Because of the sharper curvature of this barrel-vault model space truss, it was expected that the curved top chords would yield during post-tensioning. The strain at the top and bottom faces of the curved top chords at the end of post-tensioning would be 0.37\% (i.e. \( \varepsilon = \frac{\gamma}{\rho} = \frac{6.5}{1733} = 0.0037 \)), which is beyond the 0.2\% offset yield stress of the cold-formed SHS top chords.

![Support points](image)

**Figure 6-1** The SCST which was used to make the sharply curved barrel-vaults.

Although the first sharply curved barrel-vault model space truss showed little signs of yield in the top chords when looked upon as a whole (see Fig. 6-3), nevertheless, closer observation and measurements showed that the curved top chords had yielded at the location of the bolts connecting the orthogonal SHS tubes. The signs of yielding in the top chord joints was more visible at the two end panels where the curved top chords lacked continuity (see Fig. 6-4).
Bottom chord rise/span (assumed) = 0.4
Top chord rise/span = 0.46
s = curved top chord length = 5200 mm
ρ = Top chord radius of curvature = 1733 mm

\[ \rho' = \text{radius of curvature of bottom chords} = 1733 - 360 = 1373 \text{ mm} \]

\[ l' = 2 \rho' \sin \frac{\theta}{20} = 411 \text{ mm} \]

Allowing for 30 x 30 mm hubs with 4 mm grooves on both faces:
Bottom chords cut to: 411 - 30 + 8 = 389 mm with 81.4° chamfer.
The original gap in each lower edge chord panel = 520 - 411 = 109 mm

Figure 6-2 The geometry of the sharply curved barrel-vault model space truss.

Figure 6-3 The sharply curved barrel-vault model space truss after post-tensioning.
Figure 6-4 The curved top chords near the edges showed signs of yielding.

6-4 Post-tensioning

As in the case of the shape formation of the gently curved barrel-vault model space trusses, the formation of the sharply curved barrel-vault models was also carried out by post-tensioning each edge to a certain extent at a time, applying the same amount of post-tensioning to the other edge, and repeating this cycle three to four times until the final shape was gradually formed and the edge bottom chords were completely locked in position. The original gap for each edge bottom chord was 109 mm.

Strain gauges were used on critical members to measure the forces induced in them during both of the post-tensioning and vertical loading stages. Measurements on the shaped model showed that good tolerances had been obtained as compared to the design as far as the overall geometry was concerned. However, because partial plastic hinges had been formed at the location of the top chord joints, the curved top chord between each two panel points had less curvature than what it would have had if the continuous top chords had remained elastic at the joints. Therefore, the radius of curvature for each panel of the top chords was greater than the radius of curvature of the top chord as a whole, due to the rotations that had taken place at the plastic hinge locations (top chord joints) which tended to reduce the curvature of the top chord in each panel and make the whole circular curve as a series of straight lines connected at the joints.
The sharply curved barrel-vault models did not perfectly recover their originally flat shape after the prestressing wires were cut in order to dismantle them. The model was supported on 12 edge bottom nodes along its two parallel straight edges (see Fig. 6-1). The supports allowed rotations and suppressed translations. The measured rise and span of the curved top chords were 1347 mm and 3420 mm, respectively (measured on the level of the supports). Because the top chord rise and span were measured at a level higher than that of their extreme ends, these figures are smaller than the corresponding design dimensions shown in Figure 6-2.

The measured rise and span of the model, at the edge bottom chord level, were 1070 mm and 2700 mm, respectively. The distance between the supports in the curved direction was, however, a little shorter than 2700 mm, i.e. 2630 mm. This difference was due to the rotation of the bottom edge hubs (connected to the apices of the edge pyramids) which introduced an eccentricity (i.e. a lever arm) between the supporting points (i.e. rod end hinges) and the apices of the pyramids (see Fig. 6-5). This eccentricity was taken into account in the eccentric truss analysis.

![Figure 6-5 The eccentricity at the supported edges.](image)

### 6-5 Test 5

The final post-tensioning force used to shape the model for Test 5 was 10.7 kN. Test 5 was carried out on the first sharply curved barrel-vault model space truss. Eight lower nodes near the central part of the truss were loaded vertically (see Fig. 6-6a). Under a total load of 36.8 kN, premature failure occurred due to the buckling of one of the corner edge bottom chords (see Fig. 6-6).

Subsequently, the load dropped to 23.8 kN, then rose to a 28.8 kN, and dropped again due to the buckling of another corner edge bottom chord located opposite to the first one (Fig. 6-6a). After that, the load continued to decrease gradually until it reached a constant residual value of 12 kN under which the model space truss...
continued to deform (Fig. 6-6b). During vertical loading, the middle edge bottom chords lost their compressive prestress and became free under a total load of 10 kN.

Figure 6-6 The sharply curved barrel-vault model space truss in Test 5.

Figure 6-7 shows the model SCBVST after Test 5.

Figure 6-7 The model SCBVST after Test 5.
6-6 Test 6

Test 6 was carried out on a sharply curved barrel-vault model space truss similar to the model used in Test 5, but with edge bottom chords replaced with a larger size tubing, i.e., 17.1 x 3.2 CHS seamless tube in order to prevent premature bottom chord failure.

The final post-tensioning force used to shape the model for Test 6 was 18 kN. The support conditions in Test 6 were the same as those for Test 5. During the vertical loading, a middle edge bottom chord and its adjacent edge bottom chord became free under a total vertical load of 20 kN. But the middle edge bottom chord on the other side of the model became free later at a total load of 30 kN. However, when tested to failure, the model failed at the same location and under the same load as that in Test 5 (see Fig. 6-8).

![Figure 6-8 Test 6](image)

Upon this unwanted failure, the model was unloaded to keep other members in the elastic range so that only the failed members needed to be replaced for the next test. Figure 6-9 shows the model after Test 6.

The unwanted failure was attributed to the fact that although the tubing used for the edge bottom chords in Test 6 had a substantially larger cross-sectional area, nevertheless, because it was hot-rolled (seamless), its yield stress (i.e., 267.5 MPa, see Appendix A) was much lower than that of the replaced cold-formed tubing used in Test 5 (i.e., 440 MPa). The slenderness ratio of the edge bottom chords in Test 6 was 92 as compared with l/r = 113.9 for the edge bottom chords in Test 5. For this
slenderness ratio, the Euler load \( P_e = 32.5 \text{ kN} \) and the squash load \( P_y = 34.8 \text{ kN} \). Therefore, the end result was that no increase in the axial load-carrying capacity of the edge bottom chords had been obtained.

Figure 6-9  The model SCBVST after Test 6.

6-7 Test 7

The final post-tensioning force used to shape the model for Test 7 was 18 kN. During vertical loading, the middle edge bottom chords became free at 35 kN. It was expected that using larger post-tensioning forces would prevent the middle edge bottom chords from becoming free in the early stages of vertical loading.

The third sharply curved barrel-vault model space truss was made by replacing the edge bottom chords of the model tested in Test 6 with a larger size tubing, i.e. 26.7x5.6 CHS seamless tube. The support conditions were the same as those for Tests 5 and 6. Finally, Test 7 was successful in terms of obtaining a flexural mode of failure with the buckling of top chords.

The model carried a total load of 49.3 kN before it failed due to the buckling of one of its central curved top chords just next to the midspan top chord node (see Fig. 6-10).
Consequently, the load dropped to 30 kN, then rose again to 39.3 kN upon which two other curved top chords near the central portion of the truss buckled. The load dropped afterwards and finally retained a constant residual value of 26.8 kN on further increase of the displacement-controlled loading. Figure 6-11 shows the model after failure in Test 7.

6-8 Failure mechanism

The failure mechanism of the model SCBVST under the ultimate symmetrical vertical load, as observed in Test 7, was in the form of the buckling of a middle top-chord member between panel points followed by the buckling of two other top chords near the middle. In comparison with the case of the GCBVST models, the
sway and deflections of the nodes in the failure zone of the model SCBVST seemed to be more localised and less significant. Contrary to the case of the GCBVSTs, the failure of the SCBVST was not associated with a severe dynamic jump. The reason is discussed below.

A braced barrel vault supported along its straight sides, as is the case for the model barrel vaults discussed herein, works principally as an arch structure and is relatively a soft structure [Mullord, 1985].

There are generally two types of instability modes for this type of structure: bifurcation buckling and snap through. The bifurcation buckling mode of instability occurs in the form of a rotationally unsymmetrical mode of deformation and is very sensitive to imperfections in the structure and the lack of symmetry in the applied load. Snap through occurs in the form of extremely large deformations under symmetrically applied load and generally starts with a dynamic jump the severity of which increases with the span-to-rise ratio of the arch structures [Supple, 1985]. Therefore, shallow arches and domes are more susceptible to snap through than sharply-curved arches and domes. Bifurcation buckling is an unstable mode and generally leads to snap through.

Meanwhile, it has been stated by others that these two modes of instability may interact with each other. They may also affect only a partial area of the structure, resulting in the formation of a localised 'dimple' in some cases [Supple, 1985].

The above points explain why the failure mode of the sharply-curved barrel vault space truss in Test 7 was more localised and associated with no severe dynamic jump, as compared with the gently-curved barrel vault space trusses in Tests 2, 3, and 4. In both cases the failure was initiated by the buckling of top-chord members which could be considered here as a local bifurcation buckling under imperfect conditions (which resulted in a buckling load less than the ideal bifurcation buckling load). Then, as the bifurcation buckling is an unstable mode, the member buckling in both cases led to local snap through. This snap through was very significant and associated with a severe dynamic jump in Tests 2, 3, and 4 due to the larger span-to-rise ratio of the GCBVST models which classified them as shallow barrel vaults. On the other hand, the member buckling in the model SCBVST which had a smaller span-to-rise ratio was followed by an insignificant and very localised snap in the form of the sway of adjacent nodes. Therefore, the sway of nodes and the localised
area of failure covered by the snap through in the GCBVST models were larger than those observed in the SCBVST model in Test 7.

6-9 Calculated and measured top chord radii of curvature

Because of being curved beyond the yield point, the curved top chords of the sharply curved barrel-vault model space trusses lost their smooth continuous curvature to some extent (though it could not be detected by eye while looking at the completed models) and yielded at the location of the top chord joints.

This yielding caused the formation of partial plastic hinges in the curved members at the location of the top chord joints. The plastic moment being applied to the ends of the top chords between panel points was proportional to the plastic section modulus of the drilled section at the joints as shown in Figure 6-12.

Based on the yield stress of the top chords found from tensile tests (see Appendix A1), the plastic moment of the weaker (drilled) sections at the location of top chord joints was calculated. Then, it was assumed that as the bending moment, applied to the continuous top chords during post-tensioning, reached this value of $M_p$, there would be no further increase in the bending moment. Therefore, it could be assumed that the portions of the top chords between panel points, acted like a simple beam acted upon by $M_p$ at both ends (see Fig. 6-12b). (Zero axial force was assumed because the structure is a near-mechanism during shape formation. Otherwise, $M'_p$ should be used instead of $M_p$).

\[
M_p = \frac{\sigma_y A}{2}(y_1 + y_2) = \sigma_y Z_p
\]

\[
M_p = 450 \times [276 - 6 \times 1.8 \times (13 - 1.8)]
\]

\[
M_p = 450 \times 155 = 69768 \text{N}\cdot\text{mm}
\]

\[
\delta = \frac{M l^2}{8EI} = \frac{69768 \times (520)^2}{8 \times 200000 \times 1300} = 9.0 \text{mm}
\]

\[
\frac{1}{\rho} = \frac{M}{EI} = \frac{69768}{200000 \times 1300}
\]

\[
\therefore \rho = 3727 \text{mm}
\]

Figure 6-12 Calculation of the top chord radius of curvature from the $M_p$ of the drilled section.
The deflection of the middle point of the top chord in each panel, calculated based on this assumption (i.e. 9.0 mm), was in good agreement with that measured in the experiments (i.e. 8 to 10 mm).

Strain gauges were also attached to the members of the model SCBVST for measuring member axial forces and top chord radius of curvature (see section 5-1-14).

Table 6-1 shows that there was a good agreement between the top chord radius of curvature, obtained from a calculation subjecting a top chord to these plastic end moments, the top chord radius of curvature obtained from measuring the rise/span ratio of each portion of the top chords between two adjacent panel points, and the top chord radius of curvature obtained from strain gauges.

Table 6-1  Top chord radius of curvature in the sharply curved barrel-vault space trusses.

<table>
<thead>
<tr>
<th>Top chord radius of curvature (mm)</th>
<th>Overall</th>
<th>Members between panel points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designed for overall span/rise ratio</td>
<td>Measured from the overall span/rise ratio</td>
<td>Measured from strain gauges</td>
</tr>
<tr>
<td>1733</td>
<td>1733</td>
<td>2870</td>
</tr>
</tbody>
</table>

The top chord radius of curvature obtained from strain gauge measurements was 76% of that obtained from the direct span/rise measurement of the top chord panels. The difference could be attributed to the errors originating from the data acquisition system used for strain gauge readings. The strain gauge measurements for the gently curved model barrel vaults were carried out by means of manually operated digital strain meters which were found to be more accurate.

6-10 Further observations

During the shape formation of the sharply curved model barrel-vault space trusses, it was observed that, as the post-tensioning force was being applied gradually by the hydraulic jack on to the prestressing wire, there did not occur a uniform shortening of the gaps in the edge bottom chord panels. The gaps at the end panels closed up first
early in the post-tensioning operation. Then the gaps closer to the end panels closed up and finally, the middle panel gaps were the last ones to close up.

In other words, at each stage of the post-tensioning process, upon movement from the ends towards the middle of the post-tensioned edges, the gaps exhibited a differential rate of closing. This indicated a loss of the post-tensioning force in the prestressing wire due to friction. It also showed that the curvature of the top chords was not uniform along their length during the post-tensioning operation, though at the final stage when all gaps would close up, the curvature would become uniform. The end panels of the top chords gained their final curvature in the early stages, while the middle panels were still relatively flat and the panels in between, had varying curvatures, increasing on movement from the middle towards the ends.

As mentioned before, the gap introduced into each edge bottom chord panel of the sharply curved model barrel-vault space trusses was 109 mm, which was 49 mm longer than the gap needed to cause the formation of plastic hinges in the top chords at the location of the joints connecting the orthogonal top chords. Therefore, it can be concluded that early in the post-tensioning operation, the ends of the continuous curved top chords yielded at the location of the bolted joints along the border line between the first and the second panels. This yielding was observed in the tests (see Fig. 6-13).

The same phenomenon was expected to occur in other panel points along the curved top chords as the bottom chords were gradually locked into position with the closing up of the gaps. Although this was not markedly observed in the experiments, nevertheless, measurement of the rise/span ratio of each panel showed that the curved top chords had yielded at the location of all joints (see section 5-3-8).

Figure 6-13 Top chord panel points yielding during post-tensioning.
6-11 Analysis

As with the case of the GCBVST, three types of analysis were carried out on the SCBVST: linear concentric, linear eccentric, and nonlinear concentric. Table 6-2 shows the ultimate load capacity and the deflection of the central top chord node under the respective ultimate load according to these analyses, and their comparison with the experimental values.

In the analysis considering the eccentricities of the joints, the curvature of the curved elements was also taken into account by introducing a middle node for each element with an eccentricity of 8 mm (according to measurements on the models) with respect to its ends.

The eccentric analysis was repeated with a modified stiffness for the top chord bolts by reducing their flexural stiffness to 1/100 of the nominal value in order to obtain a better estimate of the nodal displacements.

Table 6-2  The SCBVST ultimate load and central node deflection according to analysis and experiment.

<table>
<thead>
<tr>
<th>Type of analysis</th>
<th>The ultimate load capacity (kN)</th>
<th>The deflection of the central node (mm)</th>
</tr>
</thead>
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<tr>
<td>Linear concentric analysis</td>
<td>41.5</td>
<td>3.6</td>
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<tr>
<td>assuming pinned joints</td>
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<td></td>
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<tr>
<td>Linear eccentric analysis</td>
<td>43.4</td>
<td>5.1</td>
</tr>
<tr>
<td>without modified bolt stiffness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear eccentric analysis</td>
<td>48.0</td>
<td>22.5</td>
</tr>
<tr>
<td>with modified bolt stiffness</td>
<td></td>
<td></td>
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<tr>
<td>Nonlinear concentric analysis</td>
<td>47.6</td>
<td>4.2</td>
</tr>
<tr>
<td>Test 7</td>
<td>49.2</td>
<td>26</td>
</tr>
</tbody>
</table>

Figure 6-14 shows the theoretical and experimental load-deflection curves for node 69 (see Fig. 6-15).
The deflection of node 69 in theory and experiment.

As observed in Table 6-2, the best estimates for both the ultimate load capacity and the central node deflection of the SCBVST were given by the linear eccentric analysis with modified bolt stiffness; 98% and 87% of the experimental values, respectively.

The nonlinear concentric analysis gave a 97% estimate of the experimental ultimate load, but its resulting deflection for the central top chord node was only 16% of the experimental value. Nevertheless, as with the case of the GCBVST, the main feature of the nonlinear analysis performed was its shape formation capability which could predict the sharply curved barrel-vault space truss from its original flat position with good accuracy.

6-12 Shape formation

Table 6-3 shows a comparison between the values of the nodal coordinates of the SCBVST as obtained from overall geometrical calculation (labelled overall), ANSYS shape formation analysis (comp.), and direct measurement (meas.). The origin is assumed to be based at node 12 (see Fig. 6-15).
Figure 6-15  The node numbers of the SCBVST.

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<td>-</td>
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</tbody>
</table>

Note: The blank cells in the Table indicate that there was no measured data available.
Figures 6-16 to 6-19 show some of the graphical results of the shape-formation analysis (load case 1).

Figure 6-16 The SCBVST after shape formation; side view.

Figure 6-17 The SCBVST in plan view; the final shape versus the original shape.

Figure 6-18 The SCBVST in perspective view.

Figure 6-19 The SCBVST shape formation process.
Figure 6-20 shows the vertical displacement of the central top chord node (ie, node 69; see Fig. 6-15) during load cases 1 and 2 in the nonlinear analysis.

(a) The vertical displacement of node 69 during the loading history.

(b) The vertical displacement of node 69 versus the total load.

6-13 Axial forces

Figures 6-22 to 6-28 show the axial forces in different members of the SCBVST according to the nonlinear analysis (see Fig. 6-21).
Figure 6-21  Half of the SCBVST considered in the nonlinear analysis.

Figure 6-22 (a) The axial force in the member 147 versus the total load.
Figure 6-22 (b)  The axial force in member 147 during the loading history.

Figure 6-23  The axial force in member 198 according to nonlinear analysis.
Figure 6-24  The axial force in member 1 according to nonlinear analysis.

Figure 6-25  The axial force in member 5 according to nonlinear analysis.
Figure 6-26  The axial force in member 137 according to nonlinear analysis.

Figure 6-27  The axial force in member 146 according to nonlinear analysis.
Figures 6-28 to 6-31 show the vertical reaction forces of support nodes 12, 33, and 54 (see Fig. 6-15).

Figure 6-28  The axial force in member 5 versus the total load.

Figure 6-29  The vertical reaction force of node 12 according to nonlinear analysis.
Figure 6-30 The vertical reaction force of node 33 according to nonlinear analysis.

Figure 6-31 The vertical reaction force of node 54 according to nonlinear analysis.

Figure 6-32 shows the axial force curves versus the total vertical load for different members of the SCBVST as obtained from analyses and experiment. Each curve starts from the equivalent experimental or theoretical value of the axial force induced in the member from the post-tensioning stage.
Figure 6-32  Axial forces in the members of the SCBVST according to theory and experiment.
Test 7
- Linear concentric analysis
- Linear eccentric analysis
- Nonlinear concentric analysis

(e) Axial force in member 146 (kN)

(f) Axial force in member 208 (kN)

(g) Axial force in member 52 (kN)

(h) Axial force in member 53 (kN)

(Figure 6-32 continued)
(Figure 6-32 continued)
7 Gently curved dome-shaped space trusses

7-1 The basic design

A series of tests was carried out in order to form the gently curved dome-shaped space trusses (GCDSST) by means of post-tensioning, and then to test them to failure.

The flat single-chorded space truss (SCST) which was used as the mechanism to be post-tensioned to form the GCDSST was basically the same as the one used for the formation of the gently curved barrel-vault model space trusses (GCBVST). It was a 6 by 6-bay SCST. The only difference was that the originally shorter edge bottom chords were placed all around the perimeter of the model (see Fig. 7-1).

![Figure 7-1 The SCST with loose bottom chords all around to form a dome.](image)

The purpose was to form a surface with positive Gaussian curvature (i.e., a dome) from the originally flat top chord layer. It was expected that the largest deformations during post-tensioning would occur in the corner pyramidal units where the originally orthogonal top chords would form into diamond shapes.

The edge bottom chords used in the models had the same length as those used in the GCBVST models, i.e., 460 mm. This left a gap of 36 mm in the X (longer) direction and 26 mm in the Y (shorter) direction in each bottom chord panel in the flat position before post-tensioning. All member sizes and dimensions were the same as those used before for the fabrication of the GCBVSTs. Strain gauges were attached to different members for measuring axial forces and top chord radii of curvature.
Figure 7-2 shows the model built to be formed into a GCDSST just before post-tensioning.

![Figure 7-2 The model to be formed into a GCDSST.](image1)

Figure 7-3 shows the model GCDSST after post-tensioning.

![Figure 7-3 The model GCDSST after post-tensioning.](image2)
7-2 The geometry

Because the continuous top chords were of the same length in each direction, they had to gain varying radii of curvature in order to be formed into a dome (i.e. a surface with positive Gaussian curvature). The final shape could not be part of a sphere, of course, as the lengths of the top chords in each direction were equal, contrary to a portion of a sphere. As a result, the top chord surface would look like that shown in Figure 7-4 from a side view.

![Figure 7-4 The geometry of the top chord layer in the dome-shaped space truss.](image)

7-2-1 Observations from measurements

The following points could be observed from measurements made on the gently curved dome-shaped model space trusses formed by means of post-tensioning:

1- Each of the top chords lay in a plane. The middle top chords lay in vertical planes. The planes of the other top chords were at an angle to the vertical plane. This angle increased on movement from the middle to the edges of the GCDSST.

2- The centre of curvature of each top chord lay at some point between the middle top chord centre of curvature O (Fig. 7-4) and the crest of the top chord arc.

3- The radii of curvature of the top chords varied from a minimum at the edges to the maximum in the middle of the dome-shaped space truss.

4- The whole shape was symmetrical.
7-2-2 Derivation of basic geometrical relations for the top chord surface
(Case 1) Top chords with equal lengths in both directions (i.e. the base model has a square mesh grid of top chords before post-tensioning)

With regard to the above observations, the following relations can be obtained between the half-span subtended angles of the middle and edge top chords and their radii of curvature (see Fig. 7-4) for a dome-shaped space truss formed by means of post-tensioning an originally square SCST.

Because the top chords are equal in length in both directions, we have:

\[ r \theta = \frac{1}{2} = R \beta \quad \Rightarrow \quad R = \frac{\theta}{\beta} r \]  \hspace{1cm} (7-1)

where \( r \) and \( \theta \) are the radius of curvature and the subtended angle of the middle top chord, \( R \) and \( \beta \) are the radius of curvature and the subtended angle of the edge chord, and \( l \) is the length of the (continuous) top chord as shown in Figure 7-4.

Projecting different lengths on the plane of the middle chord (i.e. the vertical plane) gives:

\[ [r - r(1 - \cos \theta) - R(1 - \cos \beta) \cos \theta] \tan \theta = R \sin \beta \cos \theta \]  \hspace{1cm} (7-2)

Substituting \( -\frac{r}{\beta} \) from equation (7-1) for \( R \) in equation (7-2), we get:

\[ r - r(1 - \cos \theta) - \frac{\theta}{\beta} r(1 - \cos \beta) \cos \theta \] \[ \sin \theta / \cos \theta = \frac{\theta}{\beta} r \sin \beta \cos \theta \]

Dividing both sides by \( r \) gives:

\[ 1 - (1 - \cos \theta) - \frac{\theta}{\beta} (1 - \cos \beta) \cos \theta \] \[ \sin \theta / \cos \theta = \frac{\theta}{\beta} \sin \beta \cos \theta \]

\[ \sin \theta \left[ 1 - \frac{\theta}{\beta} (1 - \cos \beta) \right] = \frac{\theta}{\beta} \sin \beta \cos \theta \]

Dividing both sides by \( \sin \theta \), gives:

\[ 1 - \frac{\theta}{\beta} (1 - \cos \beta) = \frac{\theta \sin \beta}{\beta \tan \theta} \]

or:

\[ 1 = \frac{\theta}{\beta} \left( 1 - \cos \beta + \frac{\sin \beta}{\tan \theta} \right) \]  \hspace{1cm} (7-3)

Equation (7-3) can be verified by assuming \( \theta = \beta \):

\[ 1 = \frac{\theta}{\theta} \left( 1 - \cos \theta + \frac{\sin \theta}{\sin \theta} \cos \theta \right) \Rightarrow 1 = 1 \]

which is correct.
For each design case, once the rise and span of the dome-shaped space truss - either at the edges or in the middle- are given, the radius of curvature, the subtended angle, and the length of the top chord can be calculated for the edge or the middle top chord from simple geometrical relations. Then, from equations (7-1) and (7-2), the subtended angle and the radius of curvature for the other (ie, middle or edge) top chord can be found.

Relations similar to (7-1) and (7-2) hold between the subtended angle and the radius of curvature of the middle top chord and the subtended angle and the radius of curvature of the other top chords lying between the middle and the edges of the dome-shaped space truss formed by means of post-tensioning. In fact, the centres of curvature for all top chords lie on different points along the vertical line passing through the middle of the dome-shaped space truss, with the centres of curvature of the edge and middle chords marking the top and bottom limits on that line, respectively (see Fig. 7-4).

Despite what might appear at first, measurements show that it is not correct to consider a linear change between the two extremes (ie, the edge and middle top chord radii of curvature) to find the radii of curvature and the subtended angles of the intermediate top chords.

Equations (7-1) and (7-3) apply to a post-tensioned dome-shaped space truss with equal radii of curvature and subtended angles for the edge and middle top chords in both (orthogonal) directions. By trial and error, it is seen that equation (7-3) reduced to \( \theta = \frac{\beta}{2} \). Substituting this into equation (7-1), gives \( r = 2R \).

Therefore, for a dome-shaped space truss formed by means of post-tensioning an originally square SCST, the relation between the radii of curvature and the subtended angles of the edge and middle top chords in the two orthogonal directions reduces to:

\[
\beta = 2\theta \quad \text{and} \quad r = 2R \tag{7-4}
\]

It should be mentioned that for the case of a dome-shaped space truss with equal curvatures and subtended angles in both directions, it can be seen that the formed shape as defined by equation (7-4) has an upper limit in terms of the subtended angles \( \theta \) and \( \beta \). The extreme case is a dome with circular edge top chords (\( \beta = 180^\circ \); Fig. 7-4) on the four sides and two orthogonal semi-circular middle top chords (\( \theta = \frac{\beta}{2} = 90^\circ \)). This seems, of course, to be a purely geometrical limit, regardless of
whether the physical properties of the top chords, webs and joints would allow such a form to be shaped practically or not.

(Case 2) Top chords with different lengths in the two directions
In a general case, the top chord radii of curvature and subtended angles might be different in the two orthogonal directions, either due to the fact that the original gaps in the edge bottom chords may be chosen to be different in the two directions, or due to the lack of squareness of the top chord grid in the flat position (i.e. with a rectangular top chord grid), or even due to a combination of both these factors, as was the case for the test models which were used to shape the gently curved dome-shaped space trusses (GCDSSST).

In such a case, equation (7-1) still holds between the top chord radii of curvature and subtended angle for each direction because the lengths of the top chords are identical in each direction. Therefore, we can still write (see Fig. 7-5): \[ R = \frac{\theta}{\beta} \] (7-5), and

\[ R' = \frac{\theta'}{\beta'} \] (7-6)

Figure 7-5 Dome top chord radii of curvature and subtended angles in the two directions.

If the lengths of the top chords are the same in both directions, i.e. for a case of an originally square SCST, but with different gap lengths in the edge bottom chords in the two directions, we will also have: \[ \frac{1}{2} = \frac{l'}{2} \Rightarrow r\theta = r'\theta' = R'\beta' = R\beta \]

But in a general case, we might have: \( l \neq l' \). Referring to Figure 7-5(a), we have:
\[ r - r(1 - \cos \theta) - R'(1 - \cos \beta') \cos \theta \tan \theta = R \sin \beta \cos \theta' \]
and, from Figure 7-5(b) we can find:
\[ r' - r'(1 - \cos \theta') - R(1 - \cos \beta) \cos \theta' \tan \theta' = R' \sin \beta' \cos \theta \]

The above expressions reduce to:
\[ r - R'(1 - \cos \beta') \sin \theta = R \sin \beta \cos \theta' \quad (7-7) \]
\[ r' - R(1 - \cos \beta) \sin \theta' = R' \sin \beta' \cos \theta \quad (7-8) \]

Equations (7-5), (7-6), (7-7), and (7-8) show the relations between the radii of curvature and the subtended angles of the edge and middle top chords in the two orthogonal directions for dome-shaped space truss formed by means of post-tensioning an originally flat single-chorded space truss (SCST).

7-2-3 Comparison between the geometrical formula and measurements

From measurements made on the gently curved dome-shaped model space trusses, the following data were obtained regarding the edge and middle top chords radii of curvature and subtended angles:

1. \[ \frac{1}{2} = \frac{3060}{2} \Rightarrow 1530 \text{ mm} \]
   \[ \tan \alpha_1 = \frac{215}{3015/2} \Rightarrow \alpha_1 = 8.12^\circ \]
   \[ \gamma_1 = 90 - \alpha_1 = 81.88^\circ \]
   \[ \beta = 180 - 2\gamma_1 = 16.23^\circ = 0.283 \text{ rad} \]
   \[ R = \frac{1}{2} = 5400 \text{ mm} \]

2. \[ \frac{1}{2} = \frac{3120}{2} \Rightarrow 1560 \text{ mm} \]
   \[ \tan \alpha'_1 = \frac{255}{3060/2} \Rightarrow \alpha'_1 = 9.46^\circ \]
   \[ \gamma'_1 = 90 - \alpha'_1 = 80.54^\circ \]
   \[ \beta' = 180 - 2\gamma'_1 = 18.92^\circ = 0.33 \text{ rad} \]
   \[ R' = \frac{1}{2} = 4723.0 \text{ mm} \]

Then, the radii of curvature and subtended angles obtained from measurements were substituted in equations (7-5) to (7-8) in order to check the accuracy of these equations in predicting the shape formed. The results are given below.

Equation (7-5) \[ 5400 \approx \frac{1.90}{16.23} \times 8043.6 = 5401.0 \text{ mm} \] OK.
Equation (7-6) \[ 4723 \approx \frac{13.33}{18.92} \times 6704 = 4723.3 \text{mm} \quad \text{OK.} \]

Equation (7-7):
\[ [8043.6 - 4723(1 - \cos18.92^\circ)] \sin10.90^\circ \approx 5400 \times \sin16.23^\circ \times \cos13.33^\circ \]
\[ \therefore 1472.5 \approx 1468.6 \text{ mm} \quad \text{OK.} \]

Equation (7-8):
\[ [6704 - 5400(1 - \cos16.23^\circ)] \sin13.33 \approx 4723 \times \sin18.92^\circ \times \cos10.90^\circ \]
\[ \therefore 1496.0 \approx 1503.8 \quad \text{OK.} \]

The above results show that the derived equations are in good agreement with the measurements as far as the radii of curvature and subtended angles of the middle and edge top chords of the GCBVST are concerned.

Equations (7-5) to (7-8) show that there are 8 unknowns altogether as far as the shape of the middle and edge top chords of the post-tensioned dome-shaped space truss are concerned. Given the rise and span in the two orthogonal directions will eliminate four of the unknowns; the rest can be found from these four equations.

Therefore, for the original design of the geometry of a dome-shaped space truss formed by means of the post-tensioning method described herein, depending upon the desired rise and span of the edge top chords, the gaps in the edge bottom chords of the original SCST can be proportioned by assuming the edge top and bottom chords to be approximately in the same plane, and consequently, the final rise and span of the middle top chords can be found from equations (7-5) to (7-8).

7-2-4 A comparative study of the shape
Despite what might appear at first, the dome-shaped space truss formed by means of post-tensioning is not part of a sphere. If we assume that the continuous top chords lie on a surface cut from a sphere by four vertical planes (see Fig. 7-6), and that each top chord is a circular arc lying in a vertical plane, then in such a case the radii of curvature of the top chords will vary (i.e. decrease from the middle towards the edges), the lengths of the top chords will not be equal and the centres of curvature will lie on two horizontal lines perpendicular to each other. This is in contrast to the (gently-curved) post-tensioned self-erected dome wherein the top chords have equal lengths and their centres of curvature lie on a vertical line passing through the middle of the dome, although all of the top chords do have circular arcs and varying radii of curvature (which also decrease from the middle towards the edges).
Therefore, as compared with a spherical surface bounded by four vertical planes (Fig. 7-6), the difference of the GCDSST lies in the fact that its top chords lie in planes at an angle to the vertical (see Fig. 7-4) and have equal curved lengths in each direction.

Equations (7-5) to (7-8) define the middle and edges of the top chord surface of the post-tensioned dome-shaped space truss with good accuracy. However, for practical purposes, especially for the generation of nodes defining the geometry of the model for finite element computer analysis, an attempt was made to approximate the surface by other means or formula in order to evade the tedious calculation of the coordinates of each top chord node by finding its relevant radius of curvature and subtended angle.

The equation of an elpar (i.e. elliptic paraboloid; Fig. 7-7) was found to give a fair estimate of the nodal coordinates in a case when the dome has large radii of curvature in the two orthogonal directions, i.e. for a gently curved dome-shaped space truss [Schmidt et al, 1993]. Nevertheless, the elpar formula is inaccurate for the corner nodes of the GCDSST though it gives accurate results for the middle top chord nodes and the middle nodes along the edges.

Using the geometry generation facilities of NASTRAN, an attempt was made to produce the dome-shaped space truss by simply introducing a single pyramid in the horizontal (flat) position first, then giving it two rotations about the two orthogonal X and Y axes, then repeating the rotated pyramid along a predefined circular curve in either of the X or Y directions in order to obtain a row of pyramids, and finally
repeating this curved row of pyramids along the other predefined circular curve perpendicular to the first on, in order to obtain the whole dome.

\[ Z = -\left(\frac{C_X}{a^2}\right)X^2 - \left(\frac{C_Y}{b^2}\right)Y^2 \]

Figure 7-7 An elliptic paraboloidal surface.

The attempt to generate the dome-shaped space truss by mean of NASTRAN was not successful because when the single row of pyramids was expanded (i.e. repeated) along the predefined curve perpendicular to it, a series of redundant nodes was also generated on the top chord layer with each repetition which had to be deleted manually in order to obtain a smooth curved surface for the top chord layer at the end. This was due to the fact that circular arches (curves) were being used to generate a surface of translation, when in fact, the generated curves should have been parabolic, similar to the case of an elpar which is a surface of translation generated by sliding a parabola along another parabola perpendicular to it (Fig. 7-7).

As a comparison, the difference between the Z-coordinates of nodes 1, 4, and 13, and 16 (see Fig. 7-5), as obtained from measurement and calculation, are given below.

Node 1 (ie, the corner top chord node):
Measurement: \( Z_{16} - Z_1 = 400 \) mm
Derived equations: \( Z_{16} - Z_1 = r(1 - \cos \theta) + R'(1 - \cos \beta') \cos \theta = 395.6 \) mm
Elpar formula: \( Z_{16} - Z_1 = -\left(\frac{145}{3015}\right)^3\left(\frac{3060}{2}\right)^2 - \left(\frac{180}{3060}\right)^2\left(\frac{3015}{2}\right)^2 = 324 \) mm
NASTRAN-generated geometry: \( Z_{16} - Z_1 = 214 \) mm
Node 4 (i.e., the middle edge top chord node in X direction):
Measurements: $Z_{16} - Z_4 = 180$ mm
Derived equations: $Z_{16} - Z_4 = r'(1 - \cos \theta') = 180.6$ mm
Elpar formula: $Z_{16} - Z_4 = 180$ mm
NASTRAN: $Z_{16} - Z_4 = 214$ mm

Node 13 (i.e., the middle edge top chord node in Y direction):
Measurement: $Z_{16} - Z_{13} = 145$ mm
Derived equations: $Z_{16} - Z_{13} = r(1 - \cos \theta) = 145.1$ mm
Elpar formula: $Z_{16} - Z_{13} = 145$ mm
NASTRAN: $Z_{16} - Z_{13} = 220$ mm

Measurements carried out on the test truss showed that for each continuous top chord, the diagonal distances from the middle to the ends were effectively equal (see Fig. 7-8). This indicated that the curved shape of the top chords was fairly symmetrical after post-tensioning.

![Figure 7-8 Measurements on the continuous edge top chord in Y direction.](image)

It was also observed that the diagonal distances between the middle and the ends of the top chords were almost equal for all of the top chords in each direction, with a maximum variation of 5 mm from the edges towards the middle. This diagonal distance (see Fig. 7-8) was about 1520 mm in the shorter (Y) direction and 1540 mm in the longer (X) direction. This was due to the gentle curvature of the top chords and that was why the surface could be approximated by an elliptic paraboloid as a result of the sliding (i.e., translation) of a gently curved parabola (which approximates a circular arc; [Gould, 1988]) over another gently curved parabola.

7-2-5 Calculating the nodal coordinates
In order to calculate the nodal coordinates of the GCDSST, the centre of the largest circular arc, i.e., the centre of curvature of the middle top chord in the shorter (Y) direction (point O in Fig. 7-5; also see Fig. 7-1) was chosen as the origin. Some sample calculations are given below.
Node 1 (see Fig. 7-4):
\[ Y_1 = -R \sin \beta = -1509 \text{mm} \]
\[ X_1 = -R \sin \beta = 1530 \text{mm} \]
\[ Z_1 = r \cos \theta - R' \left( 1 - \cos \beta' \right) \cos \theta = 9200 \text{mm} \]

Node 15:
\[ X_{15} = 0 \]
\[ Y_{15} = -r \sin \frac{\theta}{3} = -510 \text{mm} \]
\[ Z_{15} = r \cos \frac{\theta}{3} = 8028 \text{mm} \]

Node 2:
\[ X_4 = -r' \sin \theta' = -1542 \text{mm} \]
\[ Y_4 = 0 \]
\[ Z_4 = r' \cos \theta' + r - r' = 7864 \text{mm} \]
\[ X_2 = -\left[ r' - 0.385R \left( 1 - \cos \beta \right) \right] \sin \theta' = -1523 \text{mm} \]
\[ Y_2 = -R \sin \frac{\beta}{3} = -1014 \text{mm} \]
\[ Z_2 = Z_4 - R \left( 1 - \cos \frac{2\beta}{3} \right) \cos \theta' = 7771 \text{mm} \]

Node 13:
\[ X_{13} = 0 \]
\[ Y_{13} = -r \sin \theta = -1521 \text{mm} \]
\[ Z_{13} = r \cos \theta = 7899 \text{mm} \]
\[ X_3 = -\left[ r' - 0.1 \times R \left( 1 - \cos \beta \right) \right] \sin \theta' = -1537 \text{mm} \]
\[ Y_3 = -R \sin \frac{\beta}{3} = -509 \text{mm} \]
\[ Z_3 = Z_4 - R \left( 1 - \cos \frac{\beta}{3} \right) \cos \theta' \]

Node 16:
\[ X_{16} = 0 \]
\[ Y_{16} = 0 \]
\[ Z_{16} = r = 8044 \text{mm} \]

Node 14:
\[ X_{14} = 0 \]
\[ Y_{14} = -r \sin \frac{\theta}{3} = -1017 \text{mm} \]
\[ Z_{14} = r \cos \frac{\theta}{3} = 7979 \text{mm} \]
Node 5:

\[ X_5 = R' \sin \frac{2\beta'}{3} = 1030\text{mm} \]
\[ Y_5 = Z_5 \tan \theta = -1500\text{mm} \]
\[ Z_5 = R' \cos \frac{2\beta'}{3} \cos \theta + r - R' \cos \theta - r(1 - \cos \theta) = 7787\text{mm} \]

Node 9:

\[ X_9 = R' \sin \frac{\beta'}{3} = 518\text{mm} \]
\[ Y_9 = Z_9 \tan \theta = 1516\text{mm} \]
\[ Z_9 = R' \cos \frac{\beta'}{3} \cos \theta + r - R' \cos \theta - r(1 - \cos \theta) = 7871\text{mm} \]

*Note:

\[ Z_4 - Z_3 = 226.4\text{mm} \quad Z_9 - Z_8 = 139\text{mm} = 0.61 \times (Z_4 - Z_3) \]
\[ Z_4 - Z_3 = 22.4\text{mm} = 0.1 \times (Z_4 - Z_3) \quad Z_4 - Z_3 = 0.385 \times (Z_4 - Z_3) \]
\[ Z_3 - Z_2 = 65\text{mm} \]

The coordinates of the intermediate top chord nodes, i.e., nodes 6, 7, 10, and 11 were found by calculating the radii of curvature of the intermediate top chords first. For example, the radius of curvature and subtended angle of the continuous top chord (2-14) was calculated as follows (Fig. 7-9).

\[ \hat{\alpha}'_2 = \arctan \left( \frac{208}{1523} \right) = 7.78^\circ \]
\[ \hat{\gamma}'_2 = 90 - \hat{\alpha}'_2 = 82.22^\circ \]
\[ \hat{\beta}'_2 = 180 - 2\hat{\gamma}'_2 = 15.55^\circ = 0.271\text{rad} \]
\[ \sin \beta'_2 = \frac{1523}{R'_2 \cos \frac{2\theta}{3}} \Rightarrow R'_2 = 5681\text{mm} \]
\[ \hat{R}'_2 = 5681 \times \cos \frac{2\theta}{3} = 5635\text{mm} \]
\[ \beta'_2 = \frac{\hat{\beta}'_2}{\cos \frac{2\theta}{3}} = 15.68^\circ = 0.27\text{rad} \]

Figure 7-9  Calculating an intermediate top chord radius of curvature.

From similar calculations, the radii of curvature and subtended angles of the other intermediate top chords were obtained. The results are shown in Figure 7-10.
The calculated top chord radii of curvature and subtended angles did not show a linear variation from the edges towards the middle in each direction. An example is given below.

\[
\begin{align*}
R_2' - R_2 &= 6704 - 4723 = 1981 \text{mm} \\
\beta' - \theta' &= 18.9 - 13.3 = 5.6^\circ \\
R_2' &= 5681 \text{mm} \neq R' + \frac{1981}{3} = 5383 \text{mm} \\
\beta_2' &= 15.68^\circ \neq \beta' - \frac{5.6}{3} = 17.0^\circ \\
R_2' &= 6411 \neq R' + \frac{2 \times 1981}{3} = 6404 \text{mm} \\
\beta_3' &= 13.9^\circ \neq \beta' - \frac{2 \times 5.6}{3} = 15.2^\circ
\end{align*}
\]

The angles between the planes of the top chords and the vertical plane were also measured. These angles are generally different from the values \(\beta, \theta, \beta', \theta'\) which indicate the subtended angles between the ends of the top chords and the centreline (ie, Z axis; Fig. 7-5). Although it seems from Figure 7-5(a) and (b) that the angle of the edge top chord plane in each direction, with respect to the vertical, is equal to the subtended angle of the middle top chord in the other direction (ie, angle \(\theta\) or \(\theta'\), nevertheless, measurements showed they were different. The measured angles between the planes of the top chords and the vertical are given below. Each plane is represented by the corresponding top chord nodes.

**Top chords in shorter (Y) direction:**
- Plane (1-2-3-4) lay at 5° to the vertical plane.
- Plane (5-6-7-8): 2.1°
- Plane (9-10-11-12): 0°
- Plane (13-14-15-16): 0°

**Top chords in longer (X) direction:**
- Plane (1-5-9-13) lay at 5° to the vertical.
- Plane (2-6-10-14): 1.9°
- Plane (3-7-11-15): 0.75°
- Plane (4-8-12-16): 0°
As for the coordinates of the bottom nodes of the GCDSST, it was confirmed by measurements that the edge bottom nodes on each side lay on a circular curve the radius of curvature of which was equal to the radius of curvature of the edge top chord on the same side minus the depth of the truss, i.e. 360 mm. The subtended angle of this curve was 5/6 of the subtended angle of the top chord above it. Measurements also showed that the bottom nodes in the (third) row just next to the middle in each direction lay on a circular curve with radius of curvature almost equal to the that of the middle top chord minus the depth of the truss and a subtended angle equal to 5/6 of that of the middle top chord in the same direction. As for the bottom nodes lying between the middle and edges of the GCDSST, it was found to be reasonably accurate to assume that they lay on a circular curve with a radius of curvature equal to the average of the radii of curvature of the top chords just above them in the same direction, and a subtended angle equal to 5/6 of the average of those of the same top chords.

The middle and edge lower nodes radii of curvature and subtended angles were substituted in the equations found for the top chords (i.e. equations 7-6 to 7-9) and good agreement was observed. Therefore, the coordinates of the lower nodes were also calculated in a manner similar to the top chord nodal coordinate calculation.

7-3 Shape formation by means of nonlinear analysis

Similar to the case of the GCBVSTs, a nonlinear analysis was performed to generate the shape of the GCDSST and subject it to vertical loading subsequently. First, the geometry of the model in the flat position was input to with 456 mm-long gap bars all around the model. Then, each corner bottom node of the model was subjected to forced displacements of the absolute value of 120 mm in the X direction and 96 mm in the Y direction. The length of the gap bars and the final span of the corner nodes in X and Y directions were equal to the values measured on the physical model. The solution method adopted was a full Newton-Raphson method updating the stiffness matrix in each iteration in order to account for both material and geometric nonlinearities. Figure 7-11 shows the plan view of the GCDSST with node numbers. The nodal coordinates of the GCDSST as resulted from the nonlinear shape-formation analysis, are given in Table 7-1 which also shows the nodal coordinates based on measurements and the geometrical calculations described in section 7-2-5.
Figures 7-12 to 7-14 show different views of the model after the shaped formation analysis.

Figure 7-11  The model for nonlinear shape-formation analysis; node numbers.

(a) Plan view

Figure 7-12  The GCDSST after shape formation analysis.
Figure 7-12 continued

(b) side view

(c) Perspective view

Figure 7-13  Different stages during the shape formation analysis of the GCDSST.
Figure 7-14  Perspective views of the GCDSST after the shape formation analysis.
Table 7-1 The nodal coordinates of one quarter of the GCDSST (see Fig. 7-11)

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Nonlinear analysis</th>
<th>Geometrical calculation</th>
<th>Measurement</th>
</tr>
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<td>Z</td>
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<td>1179</td>
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</table>

Note: The blank cells indicate that there was no measured data available for those locations.

Table 7-1 shows good agreement between the measured nodal coordinates and both of the computer generated (by means of the nonlinear analysis) and geometrically calculated nodal coordinates. The Z-coordinates of the nonlinear results seem to be closer to the measured values.
7-4 Top chord radii of curvature

Table 7-2 shows a comparison of the radii of curvature of the top chords of the GCDSST, as obtained from geometrical calculations, overall measurements on the models, and strain gauges (see section 5-1-14).

<table>
<thead>
<tr>
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<tr>
<td>15</td>
<td>5937</td>
<td>5937</td>
<td>-</td>
<td>9630</td>
</tr>
</tbody>
</table>

(Note: The blank cells indicate that there was no data available either from overall measurement or strain gauging.)

Table 7-2 shows that, overall, the radii of curvature derived from strain gauge data were larger than those obtained from overall geometrical measurement of the span/rise ratios of the top chords and the calculations.

7-5 Further observations on the top chord layer

In the shape formation of the GCDSST models by means of post-tensioning, it was observed that in each cell of the top chord rectangular grid mesh (RGM), on movement from the centre towards the edges, and on movement from the middle of each edge towards the corners, the originally equal diagonals of each rectangular cell started to become different in length, reaching their maximum difference at the
corners. In other words, the rectangular cells of the top chord layer started to take diamond shapes.

For the GCDSST models, the original diagonal length of each top chord cell was equal to $\sqrt{510^2 + 520^2} = 728$ mm. Measurements showed that for the top chord cells just next to the central top chord node, both diagonal lengths were 727 mm. These lengths changed to 726 mm and 731 mm in the middle of the shorter (Y-dir.) edge and to 726 mm and 733 mm in the middle of the longer (X-dir.) edge. The diagonals of the corner diamond-shaped cells were 723 mm and 735 mm (see Fig. 7-15).

![Figure 7-15 GCDSSTs top chord layer deformation due to post-tensioning.](image)

Also, as shown in Figure 7-15, there was a $4^\circ$ reduction in the originally square angle between the orthogonal top chords at the corners of the GCDSST model after post-tensioning. There was a corresponding, but reverse change in the angle between the orthogonal edge bottom chords at the corners; the angle increased to $94^\circ$.

### 7-6 Test 8

Test 8 was carried out on a GCDSST. The SCST used as the base model was basically the same as those used for the GCBVST models, i.e. a RGM of SHS 13 × 13 × 1.8 top chords with 13.5 × 2.3 CHS tubes bolted to it in the form of pyramidal units. The edge bottom chords placed around the SCST were 456 mm - long 17 × 3.2 CHS seamless tubes.
During the shape formation of the model in Test 8, first a post-tensioning force of 4.8 kN was applied to all four sides of the model. Then it was increased to 8.6 kN. Then an attempt to increase the post-tensioning force to 18 kN failed due to the buckling of an edge bottom chord, situated second from the corner, and the slight bending of another tube in a symmetrical position with respect to it. At this stage, the shape formation operation was stopped, the bent tubes were replaced, and then all four sides of the model were post-tensioned to 8.5 kN. But it was observed that the middle edge bottom chords on two sides of the model were still loose at 8.5 kN. Therefore, only those two sides were post-tensioned up to 11.8 kN to have all their edge bottom chords locked in position.

After taking measurements on the geometry of the model, it was placed on the testing frame for vertical loading. The model was supported only on its four corner bottom nodes. Hinge-like "rod ends" were used as the supporting points in order to allow rotation in three directions and suppress translation in all directions. Figure 7-16 shows the model before load testing.

![The GCDSST model before vertical loading.](Image)

Vertical load was applied to 8 lower nodes of the model (Fig. 7-17 a). The load was statically increased in increments of 5 kN. During the test, two of the middle edge bottom chords became free under a total load of 5 kN. The other two middle edge bottom chords became free under a total load of 15 kN.
The model failed under a total load of 29.3 kN, due to the buckling of a corner web member connected to the support node (Fig. 7-17 a). Subsequently, the load dropped to 13.8 kN. It increased again up to 22.2 kN when another corner web member in a symmetrical position with respect to the first one buckled and the load dropped to 11.9 kN. Then the model was unloaded. Figure 7-18 shows the model after failure in Test 8.

After cutting the prestressing wires, it was observed that, except for the two buckled web members, the rest of the model was undistorted, as it had remained within the elastic range (see Fig. 7-19). Therefore, the corner web members had to be reinforced in order to obtain the full load carrying capacity of the GCDSSST and prevent a premature failure.
Test 9

The model for Test 9 was made by replacing all the corner web members of the previous model with a larger size tubing, i.e. $17 \times 3.2$ seamless tubes. The model was then formed into a GCDSST by post-tensioning all its four sides up to 10.3 kN. Then the model was subjected to vertical loading which was applied to it in increments of 5 kN.

During the test, one of the middle edge bottom chords on the shorter side (i.e. Y direction) became free at 5 kN, while the other three middle edge bottom chords were still tightly locked in their positions. The next middle edge bottom chord became free at 10 kN on the longer side (i.e. X direction). At 20 kN, the third middle edge bottom chord (in the shorter direction) became free, and at 30 kN, all the middle edge bottom chords were free.

The maximum load in Test 9 was 41.8 kN at which two corner web members buckled (Fig. 7-20). Consequently, the load dropped to 23.4 kN, and then the model was unloaded. Figure 7-21 shows the model after Test 9.

It was again observed, after cutting the prestressing wires, that except for the buckled corner web members, the rest of the model was undistorted.
Figure 7-20  Test 9.

Figure 7-21  The GCDSST model after Test 9.

7-8 Test 10

The failed members as well as the other corner web members of the model of Test 9 were replaced with 20 mm round solid bars. The model was then post-tensioned to 10.3 kN on the two longer sides and 11.8 kN on the shorter sides until all the edge bottom chords were locked in position. Then, vertical load was applied to the model.

During the test, the two middle edge bottom chords on the shorter sides (i.e. Y direction) became free at 5 kN, while the other two middle edge bottom chords were
still tightly locked in position. One of the middle edge bottom chords in the longer (X) direction became free at 15 kN and the other one became free at 25 kN.

Under a total load of 51.7 kN, a web member, located on an edge panel next to the corner panel, buckled (Fig. 7-22). Consequently, the load dropped to 26.7 kN and then the model was unloaded.

Closer examination after Test 10 showed that a base plate bolted to the testing frame to support the GCDSST model at one of the corners, had moved (slipped), while the rest of the corner supports of the model were on firmly welded supports (Fig. 7-23). Therefore, it was decided to weld that base plate, replace the buckled web member, and repeat the test.

![Diagram](image1.png)

**Figure 7-22** Test 10.

![Graph](image2.png)

**Figure 7-22** Test 10.

![Image](image3.png)

**Figure 7-23** The support movement in Test 10.
7-9 Test 11

Test 11 was carried out after replacing the failed members in the previous model and shaping it with a post-tensioning force of 8.5 kN. During vertical loading, three middle edge bottom chords, -two in the shorter direction and one in the longer direction-, became free under a total load of 5 kN. The other middle edge bottom chord in the longer direction became free at 20 kN.

The remedial measure taken turned out to be of no avail. The model failed again at the same location and under the same total vertical load, i.e. 51.8 kN. Consequently, the load dropped to 24.9 kN and then the model was unloaded (Fig. 7-24).

Figure 7-24 Test 11.

Figure 7-25 shows the model after Test 11.

Figure 7-25 The model GCDSST after Test 11.
7-10 Test 12

In order to carry out Test 12, the web member which had buckled in the model for Test 11, as well as the other 7 web members in similar positions with respect to the model's axes of symmetry, were replaced with a larger size tube, i.e. 17 × 3.2 CHS. The model was formed by means of post-tensioning its four edges up to 8.5 kN.

Under vertical loading, three middle edge bottom chords, -two in the shorter direction and one in the longer direction-, became free at 7 kN. The last (fourth) middle edge bottom chord became free at 30 kN.

Finally, Test 12 was successful in terms of reaching the ultimate load capacity of the GCDSST by having a top chord failure. The collapse load for Test 12 was 56.7 kN. The first member to fail was one of the middle top chords in the shorter (Y) direction (Fig. 7-26). Consequently, the load dropped to 17 kN. It could be increased again to 23.7 kN, when the failure of other top chords and nodes next to the first failed member (Fig. 7-26) occurred and caused increasing deflections in the model under an almost constant (residual) load of 21 kN. Figure 7-27 shows the model after failure in Test 12.

![Figure 7-26 Test 12.](image)

The irregular pattern in the loosening of the middle edge bottom chords in Tests 8 to 12 could be attributed to an inaccuracy in the fabrication of the model. It was found out by inspection that one of the rows of top chord panels in the longer (X) direction, just next to the axis of symmetry, was 10 mm shorter than the others which were 520 mm spacing.
7-11 Test 13

Because Tests 8 to 12 had been carried out on the same model by replacing the failed members for each new test, it was decided to do another test on a GCDSST model, with a new top chord grid, in order to compare the results with those of the previous tests and to also see to what extent the successive stressing and de-stressing of the top chords had affected the ultimate load capacity of the GCDSST obtained in Test 12.

Whereas the check on the loosening of the middle edge bottom chords in Tests 8 to 12 had been carried out by frequent visual inspection, it was decided to add an extra checking tool by placing strain gauges on the middle edge bottom chords in each direction in order to find the time of loosening in the loading history more accurately.

The new model was formed by post-tensioning all of its four sides to 8.5 kN. During vertical loading, three of the middle edge bottom chords, -two in the longer direction and one in the shorter direction-, became free at 5 kN. The last middle edge bottom chord became free at 15 kN. Although the inaccuracy mentioned in Test 12 had been eliminated in the fabrication of the model for Test 13, nevertheless, there was still an apparent inconsistency observed in the loosening of one of the middle edge bottom chords under vertical loading.
The ultimate load obtained in Test 13 was 61.7 kN, which showed a 5 kN (i.e. 8.8%) increase as compared with Test 12.

The pattern of the failed members was like that of Test 12, i.e. a middle top chord buckled first under a total load of 61.7 kN. As a result, the load dropped to 25 kN and rose again to 30 kN at which bending of the continuous top chords at the location of nodes adjacent to the first failed member, led to large increasing deflections under an almost constant (residual) load of 24 kN (Fig. 7-28).

Figure 7-28 Test 13.

Figure 7-29 shows the model after failure in Test 13.

Figure 7-29 The GCDSST model after Test 13.
7-12 Failure mechanism
The failure mechanism of the GCDSST models under ultimate symmetrical vertical load, as observed in Tests 12 and 13, was a patch mode of failure consisting of the initial buckling of individual top-chord members followed by a local snap through which was associated with a sudden dynamic jump. This localised snap-through failure caused the sway of nodes adjacent to the buckled members (see Figs. 7-26 and 7-29).

The localised snap buckling in a shallow dome is generally more severe than that in a shallow barrel vault due to the high rigidity of the shape of a dome which makes it impossible to be developed onto a plane without distortion and severe dynamic effects [Gioncu, 1993].

The experimental evidence that the buckling of the top-chord members preceded the flexural sway of the nodes highlighted the point that the model domes acted as space trusses with member behaviour determined by axial action primarily.

7-13 Comparison between the experimental ultimate loads of the GCDSST and GCBVST models

The models used in the tests on the GCBVSTs and the GCDSSTs were of the same scale in terms of the sizes of the top chords and most of the web members as well as the span in the longer (X) direction. Although there were slight differences in the sizes of the edge bottom chords and some of the corner webs, and in the post-tensioning forces used to shape the models, nevertheless a rough comparison was drawn between the experimental ultimate load capacities of these two categories of the post-tensioned space trusses.

Test results showed that the average ultimate load capacity of the GCDSST models, obtained from Tests 12 and 13, was 59.2 kN. This was 18.6% more than the average ultimate load capacity of the GCBVST models, obtained from Tests 2, 3, and 4. The post-tensioning force used in Test 4 was 12.6 kN as compared with 8.5 kN used in Test 13.

The deflection of the central top chord node in Tests 12 and 13 was 45.7 and 46 mm, respectively, as compared with the 41.5 and 42.3 mm deflection of the same node measured in Tests 2 and 4, respectively.
Overall, it could be concluded that the GCDSST models, which were supported only at four corner points, showed a remarkable increase in the ultimate load capacity without a significant loss of overall stiffness, as compared with the GCBVST models, which were supported at 12 points along their parallel straight edges.

7-14 Nonlinear analysis

As mentioned earlier in section 7-3, a nonlinear analysis was carried out for both shape formation and vertical loading of the GCDSST. The results of the shape formation analysis were already given in Table 7-1. The physical models which were subjected to vertical load had curved members and initial elastic stresses in their continuous top chords from the shape-formation process. The analytical model considered for the vertical load analysis was assumed to be made up of straight pin-ended members with no initial stresses.

In order to analyse the structure under vertical load, the prestressing wire elements were also needed to be reactivated to take up any tension that might be induced in them as a result of the loosening of the middle edge bottom chords under vertical load. This caused the number of elements to exceed the maximum allowed by the program, i.e. 250. Therefore, half of the model was analysed using symmetry (Fig. 7-30).

After the model was formed in load case 1, vertical load was applied to 4 (i.e. 8 nodes in the complete model) of its lower nodes in load case 2, while being supported only at its lower corner nodes (see Fig. 7-31) and the nonlinear buckling solution was restarted. The theoretical ultimate load obtained for the model GCDSST from the nonlinear analysis was 47.5 kN based on a chord member assumed to act as a pin-ended member, and whose ultimate load was derived from test results. Figure 7-32 shows the theoretical and experimental load-deflection curves for the central top chord node (node 43 in Fig. 7-30a).
Figure 7-30  Half of the GCDSST analysed for post-tensioning and vertical loading.

Figure 7-31  Applying vertical load to the model GCDSST in load case 2.
In order to follow the post-buckling path of the structure, another analysis was carried out by first, applying to the model, in load case 2, a total vertical load close to the ultimate load obtained (46 kN), and then restarting the solution with a displacement-controlled loading applied to the central top-chord node of the model, referred to as load case 3. Figure 7-33 (a) shows the vertical displacement of node 43 in load cases 1 to 3. Figure 7-33 (b) shows the deflection of node 43 versus the total vertical load.

According to the analysis, the model collapsed after the buckling of the middle top chord (in the shorter direction) without any further increase in the load carrying capacity beyond the ultimate load of 47.5 kN.

The nonlinear analysis procedure used was considered efficient, specially in terms of generating the final geometry of the models. Therefore, no further attempt was made to carry out a linear analysis on the model GCDSST, because this would necessitate the using of the output of the nonlinear analysis as the geometrical input for the linear analysis, which was contrary to the general assumption of the simplicity and less time-consumption features of linear analyses.

Figures 7-34 to 7-46 show some of the member axial forces obtained from the nonlinear analysis.
Figure 7-33 (a) The vertical displacement of node 43 during shape formation and vertical loading according to nonlinear analysis.

Figure 7-33 (b) The vertical displacement of node 43 versus the total load.
It should be noted that the change in the slope of the axial force curves which is seen after load case 2 in Figures 7-34 to 7-45, is not due to the material nonlinearity. It is only because of the different values of the total vertical load applied to the computer model in each load case - i.e. 40 kN in load case 1, 6 kN in load case 2 and 1.5 kN (resulting from the displacement-controlled loading) in load case 3 - while the loading history axis in the figures is equally scaled for all the load cases irrespective of the total load corresponding to each load case (compare Figs. 7-34 a and b). This point is highlighted by the fact that, although material nonlinearity could be considered by the program for the truss elements, nonetheless, the above-mentioned change of slope is seen even in the curves for the gap bars and the cables, which were treated as elastic elements by the analysis. In fact, it was observed from the computer results that, due to their high slenderness ratios, none of the members entered the phase of material nonlinearity during the loading history. Figure 7-34 (a), i.e. the axial force versus total load for the critical top chord member 89, confirms this point.
Figure 7-34 (b) Axial force induced in member 89 during the loading history.

Figure 7-35 Axial force in member 69 according to nonlinear analysis.
Figure 7-36 Axial force in member 83 according to nonlinear analysis.

Figure 7-37 Axial force in member 130 (the middle edge bottom chord in Y direction).
Figure 7-38  Axial force in member 128 (the corner edge bottom chord in Y direction).

Figure 7-39  Axial force in member 129 (the edge bottom chord between the middle and the corner in Y direction).
Figure 7-40  Axial force in member 4 (the edge bottom chord between the middle and the corner in X direction).

Figure 7-41  Axial force in member 3 (the middle edge bottom chord in X direction).
Figure 7-42 Axial force in member 5 (the corner edge bottom chord in X direction).

Figure 7-43 Axial force in member 119 (the middle top chord in X direction).
Figures 7-46 to 7-48 show the theoretical axial force induced in the prestressing wire running along the edges of the model inside the edge bottom chords. In these figures, only that portion of the curves which is after load case 1, is valid, because in the analysis the members representing the prestressing wire had to be deactivated for the shape-formation stage of the model in load case 1. Otherwise, the wires would
have been subjected to compression in load case 1 due to the displacement controlled loading which caused the corner supports of the model in its flat position to be driven towards each other up to the (measured) final span(s) in order to attain the final shape of the model. Such a compressive force would have, of course, been contrary to what happened in the post-tensioning of the physical model which subjected the prestressing wire to tension. Therefore, the nonlinear analysis procedure adopted herein could give the axial forces induced in the prestressing wire only under vertical loading. Nevertheless, this was considered not to be in contradiction with the assumption that the models, in their flat position, acted as near mechanisms, and did not generally require large post-tensioning forces for being shaped into their final forms.

Figure 7-46 shows that theoretically, there is no significant axial force induced in the prestressing wire in the X direction during the vertical loading of the model GCDSST.

Looking at the horizontal axis in Figure 7-46, the interval between 0 and 1 shows load case 1, i.e. the shape-formation or post-tensioning stage; the interval between 1 and 2 shows load case 2, i.e. the first vertical load stage in the analysis wherein 40 kN was applied to the model; the interval between 2 and 3 shows load case 3 in which an additional 6 kN was applied to the model, making the total load equal to 46 kN at the end of load case 2; and finally, from 3 onwards is load case 3 in which the model was loaded in a displacement-controlled manner in addition to the 46 kN already being
applied to it. The results showed that the model failed during load case 3 under a total load of 47.5 kN.

From Figures 7-37 and 7-47, it can be seen that even after the loosening of the middle edge bottom chord (member 130, Fig. 5-37), no appreciable axial force is induced in the prestressing wire passing through it (i.e. member 136, Fig. 7-47).

![Figure 7-47 Axial force in member 136 (the middle prestressing wire in the Y dir.).](image)

![Figure 7-48 Axial force in member 135 (the prestressing wire between the middle and the corner in Y direction).](image)
7-15 Comparison of theoretical and experimental axial forces

Figure 7-49 shows the axial forces induced in different members of the GCDSST model according to theory and experiment. It should be noted that the numbers of some of the members is not seen in the element numbering diagram (i.e. Fig. 7-30b). The reason is that they were positioned symmetrically with respect to the half of the model shown in Figure 7-30a. The axial forces of those members were measured with strain gauges in order to check the symmetrical behaviour of the models. The number of the corresponding symmetrical member has been mentioned under the graph in each case.

Figure 7-49 Member axial forces according to theory and experiment.
Test 13
Nonlinear analysis

Test 8
Test 10
Test 12
Test 13
Nonlinear analysis

Axial force in member 120 (kN)

Axial force in member 119 (kN)

Axial force in member 82 (kN)

Axial force in member 126 (kN)

(Figure 7-49 continued)
Figure 7-49 continued

(g) Axial force in member 83 (kN)

(h) Axial force in member 156 (kN) (in symmetry with member 82)

(i) Axial force in member 157 (kN) (in symmetry with member 83)

(j) Axial force in member 125 (kN) (in symmetry with member 88)
Tension

Axial force in member 33 (kN)

(k)

Axial force in member 34 (kN)

(l)

Axial force in member 1 (kN)

(m)

Axial force in member 239 (kN)

(n)

(Figure 7-49 continued)

(167)
(o) (the corner edge bottom chord in the X direction)  
(p) (the middle edge bottom chord in the Y direction)  
(in symmetry with member 1)

(Figure 7-49 continued)
7-16 The gently curved dome-shaped trussed grid (GCDSTG)

As mentioned earlier in Chapter 5, Test 14 was carried out on a gently curved barrel-vault trussed grid (GCBVTG) and the results were given in that chapter for purposes of comparison because of the similarity in the general geometrical shapes of the GCBVST and the GCBVTG models.

Test 15 was carried out on a gently curved dome-shaped space truss without intermediate webs, or to call it more accurately, a GCDSTG. Like the case of the GCBVTG, the purpose of this test was to compare the experimental ultimate load capacities of the two structural systems, i.e. the dome-shaped space truss and the dome-shaped "trussed grid", without any reinforcement of the "grid" part of the model. Therefore, this test would show, in fact, how effective the intermediate webs were, in terms of enhancing the stiffness and the ultimate load capacity of the dome-shaped models.

7-16-1 Test model
The model for Test 15 was made with the same size members as those used in Tests 12 and 13, except for the fact that the intermediate webs were eliminated (Fig. 7-50).

![Figure 7-50 The model GCDSTG during post-tensioning.](image)

All the four edges of the model were post-tensioned to 8.1 kN during the shape formation stage. The measured and calculated top chord radii of curvature for the model used in Test 15 were already mentioned in Table 7-2. Figure 7-51 shows the GCDSTG model after post-tensioning.
7-16-2 Test results
After shape formation, the model was tested under vertical models tested. The loading started with increments of 3 kN. The layout of the whiffletree, while being supported at its four lower corner nodes, like the GCDSTG rea was the same as that used in Test 14, except for modifications made in the lengths of some of the beams because of the change in the geometry (from a barrel vault to a dome). Details of the whiffletree were already given in section 5-3-5.

The model showed large deflections in its grid part from the very early stages of loading. As with all of tests, the deflections of the model were measured both before and after hanging the whiffletree. Moreover, because of the anticipated low stiffness in the model, the deflections were also measured after assembling each layer of the whiffletree. Because the deflections were read directly from scales hanging from the nodes of the model by means of a survey level, the measured values could not be affected by any possible slack in or deformation of the whiffletree itself. The total weight of the whiffletree was 1.3 kN which was included in calculations.

During the test, the middle edge bottom chords in the shorter (Y) direction became loose from the very beginning just after hanging all the layers of the whiffletree. The other two middle edge bottom chords (in the longer direction) remained tightly locked in their positions till the end of the test.

The ultimate load capacity of the model GCDSTG was 14.8 kN. The measured deflection of the central top chord node at the ultimate load was 349 mm (Fig. 7-
52b). The model failed due to the very large deflections in its central "grid" part which caused extremely large distortions in the continuous top chord joints on the border lines between the "trussed" edge panels and the "grid" part of the model (Fig. 7-52a).

![Graph and Diagram]

**Figure 7-52 Test 15.**

Figure 7-53 shows the model GCDSTG after Test 15.

![Image of the model GCDSTG after Test 15]

**Figure 7-53 The model GCDSTG after Test 15.**
7-16-3 Failure mechanism
The failure mechanism of the GCDSTG, as observed in Test 15, was in the form of the bending of members and sway of nodes characteristic of single-layer grids and other flexural systems. This mode of failure is distinct from the failure mechanism of the GCDSST which acts as a space truss with primary axial member forces.

7-16-4 Analysis
The GCDSST was analysed for post-tensioning and vertical loading. The analysis was linear and the model was assumed to be in its final form. The top-chord members were modelled as straight beam elements between panel points and the joints were assumed to be concentric.

The post-tensioning analysis showed axial forces were mainly induced in the peripheral trussed portion while mainly bending and torsional moments were induced in the central grid part of the model.

The vertical load analysis showed large deflections in the model GCDSTG. The central top-chord node had a vertical displacement of 177 mm under 14.8 kN. Therefore, the stiffness of the model GCDSTG according to the linear analysis was 50% of that measured in Test 15.

7-16-5 Comparison
The ultimate load capacity of the GCDSTG model was 25% of the average ultimate load capacity of the GCDSST obtained from Tests 12 and 13, while its stiffness was 30 times less than that of the GCDSST models. Figure 7-54 shows the deflections of the central top chord node versus the total load applied in the tests carried out on the GCDSST and the GCDSTG models.

Also, as compared with the GCBVTG model loaded in Test 14, the ultimate load capacity of the GCDSTG was 31% larger. The measured deflection of the central top-chord node in the GCDSTG model under 11.4 kN was 215, as compared with 237 mm for the GCBVTG under the same load.
Figure 7-54 Comparison of the deflection of the central top chord node in Tests on the GCDSST and GCDSTG models.
8 Sharply curved dome-shaped space trusses

8-1 Sharply curved aluminium domes - Experiment

The experiments on the dome-shaped space trusses were continued with an attempt to form a sharply curved dome-shaped space truss (SCDSST) by means of post-tensioning. The aim of this part of the experiment was only to study the feasibility of forming the shape. Therefore, the models were not load tested after shape formation.

Also, in order to apply the idea of forming metal structures by means of post-tensioning, to joints and members of a different type, a commercial jointing system was chosen, namely, the Triodetic system for space frame fabrication. This type of joint is concentric; cylindrical aluminium hubs with serrated key slots accommodate the crimped ends of the members (see Fig. 8-1). The joints can be considered as pinned for rotations about the Z-Z axis but are semi-rigid for rotations about any axis perpendicular to the member's longitudinal axis and are coplanar with the circular cross-section of the hub. This rigidity is due to the swaged ends of the tubes which act like haunches (Fig. 8-1b). During the shape-formation of the aluminium domes, the rotations were principally in X and Y directions thereby utilizing the flexural rigidity of the joints and not their pinned mode of behaviour.

A 13 × 13 panel single-chorded space truss was assembled from aluminium members and Triodetic aluminium joints (Fig. 8-2). Then edge bottom chords, shorter in length than the other members, were placed all around the lower edges of the model, with prestressing wire passing through them and through steel hubs that were bolted to the aluminium hubs along the edges (Fig. 8-3).
All of the members were made of 12.7 × 1.5 CHS aluminium tubes. They all had the same length, i.e. 304 mm, except for the edge bottom chords that were 250 mm long for the first test and 225 mm long for the second test on the model aluminium truss (i.e. Tests 16 and 17). The properties of the aluminium tube used for the members are given below.

Young's modulus: $E = 67000 \text{ MPa}$

$$r = \frac{I}{A} = 4.00 \text{ mm}$$

$A = 52.8 \text{ mm}^2$

Nominal slenderness ratio: $\frac{1}{r} = \frac{304}{4} = 76$

$I_x = I_y = 842.4 \text{ mm}^4$

Transitional slenderness ratio: $\left(\frac{1}{r}\right)_T = \pi \sqrt{\frac{E}{\sigma_y}} = 47$

$J = 2I = 1684.8 \text{ mm}^4$

Yield stress: $\sigma_y = 294 \text{ MPa}$
The overall truss dimensions, in the flat position, were 3952 x 3952 mm. The depth of the truss was 215 mm. There was no eccentricity in the joints of the top chords and the web members. The only eccentricity in the model was that between the edge bottom chords and the web members, which had been inevitably caused by the addition of the steel hubs to the model in order to hold the prestressing wire and the bottom chords (Fig. 8-4). This eccentricity was equal to 41 mm. The cylindrical aluminium hubs were 58 mm deep and had a diameter of 38 mm. The steel hubs had a prismatic shape with a 25 x 25 mm square base and a height of 40 mm. Two faces of each steel hub had round grooves 15 mm in diameter and 4 mm deep to hold the originally free ends of the edge bottom chords after the closure of the gaps at the end of the shape formation process. Joint slip was generally possible for all of the joints.

![Figure 8-4](image)

**Figure 8-4** The eccentricity between the edge bottom chords and the web members.

### 8-2 Test 16

#### 8-2-1 Design geometry

As mentioned earlier, the length of each edge bottom chord in the model fabricated for Test 16 was 250 mm. This produced an original gap of 37 mm in each edge bottom chord panel in the flat position.

Contrary to the case of the steel GCDSST models where the lengths of the top chords were slightly different from each other, the aluminium model had identical dimensions and gaps in both orthogonal directions. Therefore, it was expected to obtain equal top chord radii of curvature in both directions after the closure of the gaps at the end of post-tensioning.

According to equation 7-4 derived in section 7-2 for a dome-shaped space truss formed from a square SCST, the relation between the subtended angles and radii of curvature of the edge top chords and the top chords next to the middle of the aluminium model after shape formation, could be simply expressed as: $\beta = 2\theta$ and
\[ r = 2R, \text{ where } \beta \text{ and } \theta \text{ are the subtended angles and } R \text{ and } r \text{ are the radii of curvature of the edge top chords and the top chords next to the middle of the dome-shaped space truss, respectively, as shown in Fig. 8-5.} \]

From the assumption of the original gaps in the edge bottom chord panels of the aluminium model, \( \beta \) and \( R \) could be calculated as follows:

Curved top chord length \( s = \) original top chord length in the flat position = 13 \times 304 = 3952 mm.

Curved edge bottom chord length \( s' = (250 + 25 - 8) \times 12 = 3204 \) mm

Distance between top and bottom chords = 260 mm

R = \( \rho + 260 \), where \( \rho \) is the edge bottom chord radius of curvature

\[ \beta = \frac{13\omega}{12}, \text{ where } \omega \text{ is the edge bottom chord subtended angle and is equal to} \]

\[ \frac{s'}{\rho} = \frac{3204}{1876} = 1.71 \text{ rad} = 97.8^\circ \]

Therefore, the design edge top chord radius of curvature and subtended angle were:

\[ R = 1876 + 260 = 2136 \text{ mm} \quad \text{ and } \quad \beta = \frac{s}{R} = 1.85 \text{ rad} = 106^\circ, \text{ and the top chord next to the middle was expected to have } \theta = \frac{\beta}{2} = 0.925 \text{ rad} = 53^\circ \text{ and } r = 2R = 4272 \text{ mm.} \]
8-2-2 Post-tensioning
The model aluminium SCDSST was formed in Test 16 by means of post-tensioning. First, all the four sides of the model were post-tensioned up to 1.9 kN. Then the maximum post-tensioning force needed, i.e. 3.8 kN, was applied to model. At the end of post-tensioning, all the edge bottom chords of the model SCDSST were locked in position.

8-2-3 Measurements on the shape
The final bottom chord spans (i.e. the distances between adjacent corner supports) of the aluminium SCDSST after post-tensioning in Test 16 were 2870 mm in the X direction and 2885 mm in the Y direction (Fig. 8-8). The total weight of the model SCDSST was 1.2 kN. The ratio of the post-tensioning force to the weight of the model was 3.2. Figure 8-6 shows the model SCDSST after post-tensioning.

Figure 8-6 The aluminium model SCDSST after post-tensioning in Test 16.

The span and rise of the edge top chords of the model aluminium space truss were measured at different stages during the post-tensioning operation. Figure 8-7 shows the experimental curve for the edge post-tensioning force obtained from these measurements. The variation of the post-tensioning force with the span was the same for all of the edges of the model.
Measurements on the aluminium SCDSST after shape formation showed that the edge top chords lay in curved surfaces rather than planes, and the top chords next to the centrelines lay in vertical planes.

The measured edge top chord radius of curvature and subtended angle (in a plane approximating the curved surface of the edge top chord) were 2021 mm and 112°, as compared with the design values of 2136 mm and 106°. The measured radius of curvature and subtended angle of the top chord next to the middle were 5951 mm and 38°, as compared with the design values of 4272 mm and 53°.

Therefore, it was observed that although the measured and design edge top chord radius of curvature and subtended angle were close to each other (i.e. within 5% of difference), nevertheless, the corresponding values for top chords next to the middle were within 28% of difference from each other. This matter together with the fact that the edge top chords and the top chords next to them did not lie in planes, indicated that equation 7-4 which was derived for the dome-shaped space trusses formed by means of post-tensioning did not apply to the aluminium SCDSST model.

The measured angles between the top chord planes (for the edge top chords and the top chords next to them, the approximating planes) and the vertical planes of symmetry were: 18.0, 13.5, 5.9, 5.1, 1.4, 0.8, and 0.0 degrees, respectively, on movement from the edges towards the middle.
Figure 8-8 shows the measured top chord spans in the two directions.

Figure 8-8 also shows the measured diagonals and angles of the corner diamond-shape top chord panels of the SCDSST after post-tensioning.

8-2-4 Analysis
The model SCDSST had 1088 members. In order to carry out the shape formation analysis, 1/8 of the model had to be considered using symmetry (Fig. 8-9), so that the number of the elements would not exceed the maximum allowed by the program, i.e. 250.

All members were assumed as truss elements except for the edge bottom chords (members 1 to 6; Fig. 8-9a) which were modelled as gap bars. As there was no intention to analyse the model for vertical load, the prestressing cable was not modelled. Because it had to be deactivated for the shape formation analysis, anyway, and therefore, it would make no difference whether to include it in the stiffness matrix with a deactivated (i.e. close to null) value or not to include it at all.
Figure 8-9 1/8 of the aluminium model space truss considered for the shape formation analysis.
Because the top chords were cut by the line of symmetry (see Fig. 8-9b), fictitious members had to be added to the model in the form of web members, with one end connected to a lower node on the centreline and the other to a top chord node on the line of symmetry (see Fig. 8-10), in order to restrain the otherwise free ends of the top chords. The stiffness of these fictitious additional members was assumed to be 1/8 of that of the web members so that they would not affect the overall stiffness of the structure.

Also, the properties of the web members on the other (diagonal) line of symmetry were halved because they were cut by that line. A local coordinate system was introduced for the nodes on the diagonal line of symmetry. The freedom conditions assumed for the nodes on both lines of symmetry were suppressed translations in the directions perpendicular to these lines, while translations along those lines as well as in the Z direction were free to occur (see Fig. 8-11). There was no need to suppress the rotations about the axes of symmetry, as the program itself considered no rotational degree of freedom for the type of elements assumed (i.e. 3-D truss elements).

Figure 8-10  The fictitious auxiliary web members on the line of symmetry.

In order to shape the final form of the aluminium SCDSST, the corner support (node 9; see Fig. 8-9a) was given a translation of 544 mm along the diagonal line of symmetry, with components on the X and Y axes equal to the measured values of the total movement of that node from the flat position to the final shaped position after post-tensioning in the two directions, i.e. 390 mm in the X direction and 380 mm in the Y direction.
The model was analysed with a nonlinear solution using the full Newton-Raphson method updating the stiffness matrix after each iteration. Only the geometric nonlinearity was considered. Figures 8-12 to 8-14 show different views of the deformed shape versus the original shape of the model according to the analysis.

The nodal coordinates obtained from the nonlinear analysis are given in Table 8-1 together with the measured nodal coordinates of the model aluminium SCDSST in Test 16. The origin is assumed to be based at node 9 (i.e. the corner support node; see Fig. 8-9a) after shape formation. The measured values were obtained by projecting the position of each node of the model onto the floor (i.e. the X-Y plane) with a plumb bob and also by measuring the height (i.e. the Z coordinate) of each node.

Figure 8-15 shows a perspective view of the aluminium SCDSST in which it can be seen that the edge top chord lies in a curved surface according to the shape formation analysis, as was observed in the experiment, too.
Figure 8-12  The plan view of the final shape of 1/8 of SCDSST versus the original shape according to nonlinear analysis.

Figure 8-13  The side view of 1/8 of the aluminium model SCDSST before and after the shape formation analysis.
As it can be seen in Table 8-1, the nonlinear shape-formation analysis could predict the final shape of the aluminium SCDSST model in Test 16 with good accuracy. The differences between the measured and computed values of nodal coordinates were attributed to joint rigidity of the real truss and errors in measurement.
Table 8-1  The computed and measured nodal coordinates of the SCDSST in Test 16 (mm).

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8-3 Test 17

Test 17 was carried out with the objective of forming an aluminium dome with a sharper curvature than that used in Test 16 by means of post-tensioning.

As there were some distortions observed in some of the corner web members of the model in Test 16 (see Fig. 8-16a), the cross sections near the ends of the corresponding web members in the model for Test 17 were reduced by cutting off some part of the tube (Fig. 8-16b) in order to give more flexibility to those web members and make them behave more like pin-ended members.

The length of each edge bottom chord for the model in Test 17 was 225 mm, i.e. 25 mm shorter than that of the bottom chords used in Test 16. Figure 8-17 shows the model aluminium space truss before Test 17.

![Figure 8-16](image1.png)  
(a) The corner web members with grooves near the joints to make them more flexible.

![Figure 8-17](image2.png)  
(b) The model space truss before post-tensioning in Test 17.
8-3-1 Post-tensioning
After assembling the model for Test 17 using the same members as for Test 16, it was post-tensioned in order to form the desired shape. The final post-tensioning force applied to all the four sides of the model was 6.7 kN, after which all the edge bottom chords became locked in position. Figure 8-18 shows the model SCDSST after post-tensioning in Test 17.

Figure 8-18 The model SCDSST after post-tensioning in Test 17.

8-3-2 Measurements
The lower chord spans (i.e. the distance between the corner edge bottom nodes) of the SCDSST after shape formation were 2250 mm and 2200 mm in the two directions. The curvatures of the top chords located in similar positions in plan view (see Fig. 8-8) were equal in both directions.

Measurements on the model SCDSST showed that it formed a complex shape made up of a central top chord surface with positive but varying Gaussian curvature, and four saddle-shaped or hypar corners [Schmidt et al., 1993]. The curvature of the central part had its maximum at the edges and tended to decrease on movement towards the middle or the central point of the dome. Each of the top chords lay in a curved surface which had an angle to the vertical plane. The curvature of this surface and its angle with respect to the vertical plane tended to decrease on movement from the edges of the sharply curved dome towards the middle.
Figure 8-19 shows the diagonal lengths of the top chord panels of the model as a measure of the deformation of the originally square panels into diamond shapes. The maximum deformation occurred in the corner panels.

Figure 8-19  The deformation of the top chord panels of the model SCDSST in Test 17.

8-3-3 Analysis

Similar to the case of the model for Test 16, a nonlinear analysis was carried out for the shape formation of the model in Test 17. The only difference between the two models was the length of the edge bottom chords which were modelled as gap bars. The analysis was performed by applying forced displacements corresponding to the final measured span of the model to the corner support (in 1/8 of the model). Figures 8-20 to 8-23 show some of the graphic results of the shape formation analysis.

The nonlinear analysis could predict the negative Gaussian curvature which occurred at the corners of the physical model during shape formation. This negative curvature is marked by the change in the curvature of the top chords near the corner of the model in Figure 8-20.

The computed and measured nodal coordinates of the SCDSST model for Test 17 are given in Table 8-2 for the purpose of comparison. The origin is assumed to be based at node 9 (i.e. the corner support node, Fig. 8-8a) after shape formation.
As Table 8-2 shows, there is good agreement between the values given by the nonlinear shape formation analysis and those measured after post-tensioning the model.

Figure 8-20  The plan view of the SCDSST versus the original shape.

Figure 8-21  The side view of the model SCDSST after the shape formation analysis.
Figure 8-22  The side view of the SCDSST from another angle.

Figure 8-23  The perspective view of the SCDSST model of Test 17 according to the shape formation analysis.
Table 8-2  The computed and measured nodal coordinates of the SCDSST in Test 17 (mm).

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### 8-4 The shape formation analysis of a steel SCDSST

A hypothetical 6 by 6 panel steel single-chorded space truss, with the same dimensions as the those of the models used in the formation of the GCDSST in Tests 8 to 13, was analysed for the formation of a SCDSST.

A model was assumed in the flat position with edge bottom chord gap per unit lengths equal to 0.32 and 0.26 in the X and Y directions. Each corner support was given a displacement of 550 mm and 450 mm in the X and Y directions, respectively. Figure 8-24 shows the plan view of the resulting sharply curved dome versus the original configuration. The negative Gaussian curvature of the top chord surface can be seen near the corners of the model.

![Diagram of the SCDSST formed by means of the nonlinear analysis.](image)

**Figure 8-24** The SCDSST formed by means of the nonlinear analysis.

Figure 8-25 shows the side view of the steel SCDSST model formed according to the nonlinear analysis. The sequence of perspective views in Figure 8-26 shows the gradual change of the top chord shape from a positive to a negative Gaussian curvature on movement from the middle towards the corners of the SCDSST.
Figure 8-25  The steel SCDSST after the shape formation analysis.

Figure 8-26  The gradual change of the sign of top chord curvature near the corners.
It was also attempted to find out how far the program could go in forming a sharply curved dome from the original flat position, or in other words, how close it could approach the theoretical limit of forming the edge top chords of the sharply curved dome-shaped space truss into circles. The computer model assumed for this purpose was characterised in the flat configuration by edge bottom chords with gaps per unit length in the longer (X) and shorter (Y) directions equal to 0.72 and 0.74, respectively. Displacement-controlled loading was applied to the corner supports of...
the model, equal to 1231.7 mm and 1211.5 mm in the X and Y directions, respectively. The solution could go as far as implementing 0.837 of the forced corner displacements, i.e. 1031 mm in the X direction and 1014 mm in the Y direction for each corner support. When the solution reached this stage the program gave an error message and halted the solution indicating that the displacements had exceeded a maximum limit allowed by the program. Figure 8-27 shows the plan view of this dome versus the original flat shape. The negative Gaussian curvature (saddle shape) can be seen in the figure near the corners of the model.

Figure 8-27  The plan view of the SCDSST formed by means of nonlinear analysis.

Figures 8-28 to 8-29 show other views of the SCDSST resulting from the nonlinear shape-formation analysis.
From the experimental and theoretical results given in Chapters 7 and 8, it can be generally concluded that by means of the post-tensioning process, referred to as a self-erection procedure herein, structural forms generally of positive Gaussian
curvature (i.e. domes) can be shaped from an originally flat position. Although these domes are not spherical or of any other regular geometrical shape, nevertheless, they are structurally sound and can also be of architectural interest.
9 Gently curved steel hypar space trusses

A series of tests was carried out to form hypar (saddle-shaped) space trusses from originally flat steel models by mean of post-tensioning. After shape formation, the model hypar space trusses were tested to failure under symmetrically applied vertical load.

9-1 Test models

The single-chorded space truss (SCST) used as the base model to form the hypars was basically the same as those used to form the GCDSST models in Tests 12 and 13. Bottom chords made of 17 x 3.2 CHS seamless steel tube were placed all around the SCST at the lower edges between the apices of the inverted pyramidal modules (Fig. 9-1). These edge bottom chords were cut to a size to fit between the hubs at the lower edges of the SCST and were held in position by means of prestressing wires which passed through them and through the hubs. These bottom chords were included in the model in order to give it the torsional rigidity which was needed for the shape formation. Also, bottom chords made of 17 x 3.2 CHS seamless tube were placed along one diagonal of the SCST model. These diagonal bottom chords were cut shorter than the distance between the apices of the inverted pyramidal modules in the diagonal direction. The original gap for each bottom chord was 27 mm. They were also held in position by means of a prestressing wire which ran across the diagonal of the SCST and passed through them and through the hubs at the apices of the pyramids.

Figure 9-1 The base model used to form the GCHST.
Figure 9-2 shows the physical model in the flat position before being post-tensioned to form the GCHST.

![Image: The GCHST before post-tensioning.](image)

**Figure 9-2** The GCHST before post-tensioning.

## 9-2 Post-tensioning

After assembling the SCST and placing the bottom chords in position, the model was post-tensioned. First, the edges were post-tensioned up to 2 kN just to remove any lack of fit or slack from the edge bottom chords and the wires which passed through them. Then, the model was post-tensioned along the diagonal with shorter bottom-chord members so as to remove the gaps and thereby curve the model along that diagonal. Simultaneously, and consequently, the model curved inversely along the other (unstressed) diagonal. Because of the existence of the edge bottom chords, the model was not able to remain straight along its unstressed diagonal and had to bend in the opposite direction. Therefore, the end result was a shape with positive and negative Gaussian curvatures along its two diagonals and rectangular in plan, i.e. a saddle-shaped hypar. Figure 9-3 shows the model hypar space truss after post-tensioning.
9-3 Observations

During the post-tensioning operation, it was observed that after the closure of the middle gap, the other gaps along the diagonal of the model hypar truss did not close despite increasing the post-tensioning force. This matter was attributed to the fact that, due to the increased number of the elements in the model hypar, as compared with the barrel-vault and dome models, there was only one member needed to be added (i.e. to be locked in position) to the active members in order to transform the flat model from a mechanism into a statically determinate structure. This point is shown by Maxwell's rule: \( R = b - (3j - c) \) where \( R \) is the degree of statical indeterminacy, \( b \) is the total number of members excluding the gap bars, i.e. 248; \( j \) is the number of joints, i.e. 85, and \( c \) is the number of restraints, i.e. a minimum of 6 to prevent rigid body motion. Substituting these values into Maxwell's equation, we obtain \( R = -1 \), i.e. the model is only one degree below the state of kinematic determinacy before the closure of the gaps. Therefore, any further attempt to post-tension the model after the closure of the first gap, would induce significant stresses in the structure and have little success in closing the rest of the gaps.

The impact of this matter on the shape-formation process was that the ideal hypar with straight line generators (see Fig. 9-4) could not be achieved. Instead, the resulting shape was what could be termed a "partial" hypar, i.e. a pseudo hypar with curved line generators and lacking uniform curvatures along its two diagonals (see
Nevertheless, it was aesthetically pleasing and, as will be seen later, was also structurally sound. So, with a little compromise, a saddle-shaped structure had been formed with considerable ease and speed in fabrication.

Figure 9-5 The gently curved hypar space truss after post-tensioning.

9-4 Shape formation analysis

A nonlinear analysis was carried out to model the post-tensioning process and generate the shape of a gently curved hypar space truss from an originally planar configuration. Due to the symmetry of the model with respect to its diagonals, only half of the model was analysed. Figure 9-6 shows the node and element numbering of half of the hypar model for analysis. The measured dimensions of the experimental model and the displacements of the corner supports were input and a nonlinear static solution was started including both material and geometric nonlinearities.
Figures 9-7 to 9-10 show some of the views of the generated hypar shape versus the original geometry.

Figures 9-11 and 9-12 show similar views of the model hypar space truss in an inverted position as obtained from the shape formation analysis and the post-tensioned physical model, respectively.
Figure 9-7  The hypar space truss in plan view.

Figure 9-8  The perspective view of the hypar space truss output by the nonlinear analysis.
Figure 9-9 Side views of the hypar space truss.

Figure 9-10 Different stages during the hypar shape formation analysis.

Figure 9-11 The hypar space truss in an inverted position.
Figure 9-12 The model hypar space truss in an inverted position.

Table 9-1 shows the nodal coordinates of the GCHST, as obtained from the exact hypar formula (i.e. $z = -\frac{f}{ab}$ xy, see Fig. 9-4), measurement on the experimental model, and the shape-formation analysis. The origin is assumed to be based at the central top chord node (i.e. node 37, see Fig. 9-6a).

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9-5 Test 18

Test 18 was carried out on the first model GCHST formed by means of post-tensioning. The final post-tensioning force applied to the diagonal of the model was 21.7 kN. This large post-tensioning force could close four of the five gap bars, while the last bottom chord at one corner in the diagonal direction was still free.

After post-tensioning, the model was placed on the testing frame for vertical loading. The model hypar space truss was supported at its four corner points. The two corner bottom nodes of the model along the tensioned diagonal acted as hinge supports allowing rotations and suppressing translations in all directions. The other two corners along the unstressed diagonal acted as roller supports, allowing all rotations as well as the translations in the horizontal plane but suppressing vertical (i.e. Z) translation (see Fig. 9-13).

![Figure 9-13 The supports of the GCHST.](image-url)
To provide the roller support condition, round solid bars were pin-connected to the testing frame from one end and to the two corner bottom nodes along the unstressed diagonal from the other, keeping the model in a balanced position (Fig. 9-13).

Vertical load was applied proportionally to eight bottom nodes of the model GCHST (Fig. 9-14). The maximum load in Test 18 was 26.7 kN when a premature failure occurred due to the buckling of two corner web members on the tensioned diagonal (see Fig. 9-14). Subsequently, the model was unloaded.

![Graph showing load vs. deflection for Test 18](image)

**Figure 9-14** Test 18.

**Figure 9-15** shows the model hypar after Test 18.
Test 19

After unloading the model hypar in Test 18 and cutting the prestressing wires in order to release tension, it was observed that apart from the failed corner web members, the rest of the model had remained undistorted and within the elastic range due to the relatively small vertical load applied in Test 18. Therefore, the failed web members were replaced with a larger size bar (i.e., 20 mm round solid bar) and the experiment was repeated. The final post-tensioning force in Test 19 was 10 kN. The maximum vertical load was 26.7 kN when one of the corner bottom chords (made from 17 x 3.2 CHS) on the diagonal buckled (Fig. 9-16). Subsequently, the model was unloaded.

Figure 9-16 Test 19.

Figure 9-17 shows the model hypar space truss after Test 19.

Figure 9-17 The model hypar space truss after Test 19.
9-7 Test 20

After de-stressing the model hypar used in Test 19, it was observed again that except for the failed member, the rest of the model had remained undistorted. Therefore, all of the bottom chords on the diagonal of the model were reinforced, by welding $30 \times 30 \times 3$ angles to the $17 \times 3.2$ CHS tubes, and the experiment was repeated.

The final post-tensioning force applied in Test 20 was 10 kN. Under the vertical load, the middle bottom chord became free at 10 kN and the corner gaps closed up at 35 kN. The maximum vertical load that could be reached in Test 20 was 49.2 kN. At this load, one of the corner web members on the tensioned diagonal buckled (see Fig. 9-18). Subsequently, the model was unloaded.

Figure 9-18 Test 20.

Figure 9-19 shows the model hypar space truss after Test 20.

Figure 9-19 The model gently curved hypar space truss after Test 20.
9-8 Test 21

The failed corner web member as well as the other member located in a symmetrical position in the model used for Test 20 were replaced with larger size bars, i.e. 25 mm solid bars with 30 x 30 x 3 mm angles welded to one side of them. The model was post-tensioned again up to 10 kN and then subjected to vertical load. The maximum load in Test 21 was 56.7 kN when a web member situated on the tensioned diagonal buckled (see Fig. 9-20). Subsequently, the load dropped to 23.7 kN and then the model was unloaded.

Figure 9-20 Test 21.

Figure 9-21 shows the model hypar space truss after Test 21.

Figure 9-21 The model hypar space truss after Test 21.
9-9 Test 22

In order to reach the ultimate load of the GCHST by initiating a top chord failure, the failed web member as well as all of the other web members along the tensioned diagonal of the model (except for the corner webs which were made of 25 mm solid bars with an angle welded to them) were replaced with a larger size tubing, i.e. 17 × 3.2 CHS. Also, because Tests 18 to 21 had been carried out basically on the same model by replacing the failed members each time, a new top chord grid with the same dimensions was fabricated and the pyramidal units of web members were bolted to it. The bottom chords of the model along the tensioned diagonal were made of 26.7 × 5.6 CHS seamless tubing in order to prevent any bottom chord failure.

The new model hypar was post-tensioned along its diagonal up to 12.6 kN. Then the vertical load test was started.

The ultimate load of the model GCHST in Test 22 was 45.7 kN when three top chord members in the middle part of the space truss buckled simultaneously (see Fig. 9-22). Subsequently, the load dropped to 26.7 kN. The load was increased again up to 34.7 kN when the buckling of another top chord member near the middle and also a top chord member near the corner of the model occurred, and then the model was unloaded.

![Figure 9-22 Test 22](image)

**Figure 9-22** Test 22.
The reason why the model in Test 22 showed less ultimate load capacity than the model hypar of Test 21 was attributed to the larger post-tensioning force used to shape the model in Test 22. The steel hypar space trusses were found to be sensitive to the level of the post-tensioning force applied to them during the shape-formation process. The closure of the first gap transformed the model hypars from near mechanisms into statically determinate structures. Therefore, any further increase in the post-tensioning force after the closure of the first gap induced initial stresses in the members of the hypar models, thereby reducing the ultimate load capacity of the models under vertical load.

9-10 Further observations

It was observed during the tests on the hypar model space trusses that as the vertical load increased, the middle bottom chord in the diagonal direction became free and the gaps in the corner bottom chords along the diagonal of the model tended to close up gradually due to the overall deflection of the model. Also, some of the edge bottom chords which had no original gaps, became loose during the post-tensioning and vertical loading stages (see Fig. 9-23).

![Figure 9-23](image)

Figure 9-23 The loosening of some of the edge bottom chords during the hypar tests.
9-11 Failure mechanism

The mechanism of the failure of the model GCHST under ultimate load conditions, as observed in Test 22, was in the form of individual top-chord member buckling between panel points (see Fig. 9-22). In contrast to the failure mechanisms of the model barrel-vault and dome-shaped space trusses tested earlier, no significant bending and sway of the top chord nodes adjacent to the failed members was observed in the failure pattern of the model hypar space truss in Test 22 following the buckling of the top chords (see Fig. 9-24). This point was attributed to the fact that in a hypar space truss of this configuration, one of the two top-chord members intersecting at a joint is generally in tension and, therefore, has a stiffening effect on the joint when the other top-chord member buckles due to axial compression [Gioncu, 1993].

![Figure 9-24](image)

Figure 9-24 The failure mechanism of the model steel hypar space truss in Test 22.

9-12 Vertical load analysis

The model GCHST was analysed for vertical load in a nonlinear analysis including both material and geometric nonlinearities. The model was assumed to have concentric members and joints. As mentioned in section 9-4, only half of the model was analysed due to symmetry. After the shape-formation stage of the analysis, i.e. load case 1, vertical load was applied to the lower nodes of the model in load case 2 (see Fig. 9-25).
The support conditions considered for the model in load case 2 were to have the corner bottom nodes on the tensioned diagonal fixed (i.e., pinned to the supports) and the two corner bottom nodes along the other diagonal on rollers, that is allowing translations in the X and Y directions but suppressing the Z translation of these corner nodes after the shape formation stage, i.e., load case 1 (see Fig. 9-26).

Figure 9-25 Vertical load analysis (load case 2).

Figure 9-26 The constraints applied to the model hypar space truss in load case 2.

Figure 9-27 shows that the central top chord node moved upwards about 250 mm during the shape formation stage (i.e., load case 1; Fig. 9-27a) and then moved down 5.3 mm under a total vertical load of 40.2 kN (Fig. 9-27b).
It should be pointed out that the nonlinear buckling analysis was not carried out on the model hypar in load case 2 due to a program error after replacing the critical top chord member with a beam element and carrying out a nonlinear buckling analysis (as had been carried out before for the cases of the barrel vaults and the domes). The error could not be debugged. Therefore, the ultimate theoretical load of the hypar space truss was found as the total vertical load under which the first member reached
its experimental critical load. According to the nonlinear concentric analysis, this
ultimate load was 40.2 kN at which member 95 (see Fig. 9-6b) reached its
experimental buckling load of 10.8 kN. It should also be noted that in load case 2 a
total vertical load of 48 kN (i.e. 24 kN for half of the model) was applied to the
model to see when the first member would reach its experimental critical load.
Therefore, in the figures obtained from the nonlinear analysis, only the portions of
the curves up to the total load of 40.2 kN or a loading history value of 1.8 (where 2
corresponds to 48 kN) have been plotted.

It should also be mentioned that in order to apply the vertical load to the model in
load case 2, the corner bottom node (i.e. node 8, Fig. 9-6a) had to be restrained
against translation in the Z direction to act as a roller support. As node 8 had already
moved 450 mm in the Z direction in load case 1 according to the shape formation
analysis, the Z-displacement value for this node was set to 450 mm in load case 2
before restarting the solution for vertical loading.

But after the solution, the results showed that setting the Z-displacement to the above
value for node 8 was interpreted by the program as starting its displacement from
zero again in load case 2.

Therefore, in another run, the final Z-displacement of node 8 was input to the
program from the beginning and the shape formation and vertical loading analyses
(i.e. load cases 1 and 2) were repeated.

Introducing this controlled displacement for node 8, however, made the model
statically determinate because, as mentioned earlier, the model needed only one
additional member or restraint to become statically determinate.

While this matter (i.e. the statical determinacy of the model) was true after the
closure of the first gap, i.e. the gap in the middle bottom chord (see Fig. 9-28a), the
member axial force results shown by the program before the closure of the first gap
were invalid, as there was basically no axial forces induced in the members of the
model at that stage and the model worked as a mechanism. That is why in the
figures to follow, the computer produced graphs which have their horizontal axes
labelled as the loading history, show some false values which should be ignored for
member axial forces in load case 1 before the closure of the middle bottom chord
gap, i.e. before the value of 0.92 on the load case axis (see Fig. 9-28).
An example of the correction which should be considered for these graphs is shown with a dashed line in Figure 9-29a. The valid portions of the axial force graphs are marked with letters A, B, and C. Also those axial member graphs which have their horizontal axes labelled as the total load, are generally valid. The only point to be noted about them is that in each case the axial force starts from zero (the point is marked with a dark circle on the graph in Figure 9-29b) and increases (in compression or tension) only in one direction on the line of zero total load (i.e. on the vertical axis). Therefore, the other portion of the graph on the vertical axis which falls above or below zero on that axis and which is not immediately connected to the rest of the curve after zero, is invalid and should be ignored. In these graphs, the valid portions are marked with letters A, B, and C. The point where the axial force curve departs from the vertical axis marks the axial force induced in the member at the end of the shape formation stage.

Figures 9-29 (a) and (b) show the axial forces induced in the top chord member 95 of the GCHST during the shape-formation stage of the nonlinear analysis, referred to as load case 1. As it is seen in these figures, the analysis showed that there was a sudden increase in the axial force of the middle top chord member 95 (Fig. 9-29a) due to the closure of the gap in the middle bottom chord member 133 (Fig. 9-28) while the other bottom chords were still free (Figs. 9-30 and 9-31). This point confirms the discussion on the statical determinacy of the model hypar after the closure of the first gap.
Figure 9-28 The axial force in the middle bottom chord.
Figure 9-29 The axial force in the middle top chord member.
9-13 Axial forces

Figures 9-32 to 9-40 show the theoretical axial forces induced in selected members of the model GCHST during the loading history, i.e. the shape formation and vertical loading stages, referred to as load cases 1 and 2. According to the analysis, there
was no force induced in the prestressing wire under vertical load (see Fig. 9-32) because the middle bottom chord did not become free (see Fig. 9-28).

Figure 9-32 The axial force in the prestressing wire.

Figure 9-33 The axial force in member 77.
Figure 9-34  The axial force in the corner web member.
Figures 9-35 to 9-40 show that the axial forces induced in the edge bottom chord members of the model GCHST during the shape formation and vertical loading stages are tensile. As these members were not connected to the joints and were only tightly held in position by means of prestressing before the shape formation process, the tension shown in them by the analysis necessitates that these edge chords should become free and the tension be transferred to the prestressing wire passing through them. This point confirms the experimental observation mentioned in section 9-10.
Figure 9-35  The axial force in member 2.
Figure 9-36 The axial force in the corner edge bottom chord.

Figure 9-37 The axial force in member 3 (kN).
Figure 9-38 The axial force in member 126.

Figure 9-39 The axial force in member 127.
Figures 9-41 and 9-42 show the vertical reaction forces at the supports of the model hypar. The values in Figure 9-42 represent the vertical reaction force of node 13 in the complete model. In these figures, the true zero for the vertical reaction force starts from the point where the curve departs from the vertical axis, which is marked with a dark circle. Therefore, only the portions marked AB are valid.

As mentioned earlier, the assumption of the controlled vertical displacement for node 8 from the beginning of load case 1 created a condition of statical determinacy in the model. This artificial determinacy is why the other values shown at the intersection of each curve and the vertical axis are false and should be ignored, because there is basically no vertical reaction induced in the support nodes under zero vertical load.

As it is seen in the figures, the sum of the reaction forces of nodes 8 and 13 times 2 gives the total load on the model, i.e., 40.2 kN.

Figure 9-43 shows the vertical displacement of node 8 during the loading history.
Figure 9-41  The vertical (Z) reaction force of the corner support along the unstressed diagonal.

Figure 9-42  The vertical (Z) reaction of the corner support along the stressed diagonal.
9-14 Comparison between test and theory

The theoretical ultimate load, i.e. 40.2 kN, was 88% of the experimental ultimate load of the model hypar space truss (where top chord failure was initiated) obtained in Test 22, i.e. 45.7 kN. Figure 9-44 shows the load-deflection curves for node 28 (see Fig. 9-6a), as obtained from Tests 21 and 22 and the nonlinear concentric analysis. The theoretical deflection of the top chord node 28, according to the nonlinear analysis assuming concentric joints and members, was 11% of that measured in Test 22 under a total load of 40 kN.

The deflection of node 37, i.e. the central top chord node could not be completely measured in Tests 21 and 22 as the scale hanging from the node went out of the sight of the survey level which was used to read the deflections during the tests. In comparison with the deflections measured in an earlier test on the hypar model, i.e. Test 20, the measured deflection of node 28 was 89% of that of node 37 under a total load of 46.7 kN.
Figure 9-44 The deflection of node 28 in theory and experiment.

Figure 9-45 shows the theoretical and experimental axial force curves versus the total vertical load for some of the members of the model hypar space trusses. Each curve starts from a value indicating the axial force left in the member from the post-tensioning stage, whether theoretical or experimental.

For example, Figure 9-45(a) shows that the measured axial force in the top chord member 77 (see Fig. 9-6b) just before applying the vertical load in Test 22 was -2.7 kN (compression) as compared with -1.2 kN measured for the same member in Test 21. This point indicates why the maximum load obtained in Test 22 was less than that reached in Test 21.

The reason why the model hypar in Test 22 carried less load than the model in Test 21 was attributed to the 26% larger post-tensioning force used to form the former model. The post-tensioning force in Test 22 was 12.6 kN as compared with 10 kN used in Test 21. As mentioned earlier, the model hypars became statically determinate after the closure of the first bottom chord gap during the shape-formation stage and therefore, any further increase in the post-tensioning force...
beyond the closure of the first gap resulted in axial forces in the members of the model due to the post-tensioning process.

Figure 9-45  Member axial forces in theory and experiment.
(Figure 9-45 continued)
9-15 Gently curved hypar trussed grid

Test 23 was carried out in order to study the feasibility of forming a flat single-layer grid with peripheral and diagonal trusses (see Fig. 9-46) into a hypar, and then to load it to failure in order to compare the results with those of the hypar space trusses shaped by means of post-tensioning.

9-15-1 Post-tensioning
The model used for the shape formation of the gently-curved hypar trussed grid (GCHTG) was basically the same as those used to shape the GCHSTs, except for the fact that the intermediate webs were eliminated wherever possible (Fig. 9-46).

After assembling the trussed grid model on the floor, prestressing wires were passed through the bottom chords along the edges and one diagonal. Each of the bottom-chord members along the diagonal of the model had an original gap of 27 mm. Figure 9-47 shows the model before post-tensioning in Test 23.
Before starting the shape-formation process, the lower edges of the model were first prestressed up to 2 kN in order to hold the edge bottom chords, which had been cut to fit in between the panel points, tightly in position. The model was then post-tensioned along its diagonal.

The final post-tensioning force used in Test 23 was 12.6 kN. At the end of post-tensioning, all of the bottom chords along the diagonal became locked in position except for one of them near the corner of the model.

Measurements on the model GCHTG showed the same dimensions and geometrical features as those observed in the model GCHST. The overall shape of the model
looked like a hypar, but the top chords were slightly curved and also the two curvatures across the diagonals of the model were not equal, i.e. the model achieved greater curvature along the tensioned diagonal than along its other diagonal. Figure 9-48 shows the GCHTG after post-tensioning.

![Figure 9-48 The model GCHTG after post-tensioning.](image)

9-15-2 Vertical loading

Vertical load was applied to the model GCHTG after the post-tensioning stage. A whiffletree was used to distribute the load proportionally to the nodes of the model so that the general pattern of loading would be similar to that of the GCHST models. Figure 9-49 shows the loaded points of the model GCHTG.

![Figure 9-49 The loaded points of the model GCHTG in Test 23.](image)
From the early stages of the vertical loading, the model GCHTG showed large deflections especially in its grid region. The ultimate load obtained for the model GCHTG in Test 23 was 19.3 kN at which load a top chord member near the corner of the model buckled (see Fig. 9-50).

The failure mechanism of the model GCHTG as observed in Test 23 was in the form of very large deflections in the grid portions of the model followed by a single top-chord member buckling.

Figure 9-51 shows the model GCHTG after Test 23.
Figure 9-52 shows the vertical displacements of nodes 28 and 37 (see Fig. 9-6a) versus the total load.

9-15-3 Analysis
The model GCHTG was analysed for post-tensioning and vertical load. A linear analysis was carried out on the model in its final form assuming concentric joints and beam elements.

The experimentally measured post-tensioning force (i.e. 12.6 kN) was applied to the corner nodes of the model along the tensioned diagonal. All of the bottom chords were assumed to behave as beam elements without any gaps. The maximum axial force from the post-tensioning linear analysis was 9 kN induced in the middle bottom chord.

The vertical load analysis showed a maximum axial force of 33 kN in the corner bottom chords and a maximum deflection of 18 mm in nodes 27 and 57 (see Fig. 9-50) under a total load of 19.3 kN. The measured deflections of nodes 27 and 57 under the same total load (i.e. 19.3 kN) in Test 23 were 154 mm and 173 mm, respectively. Therefore, the maximum deflection given by the linear concentric analysis was only 11 % of that measured in testing the GCHTG.
9-15-4 Comparison

The ultimate load capacity of the model GCHTG in Test 23 was 42% and its stiffness was 10% of those of the model GCHST in Test 22. Figure 9-53 shows a comparison of the load-deflection curves of the model hypars in Tests 21, 22, and 23.

![Figure 9-53](image)

Figure 9-53 The deflection of node 28 in Tests 21, 22, and 23.
10 Sharply curved hypar space truss

Test 24 was carried out on a single-chorded aluminium space truss in order to form it into a sharply curved hypar space truss (SCHST). The post-tensioning shape-formation method was applied to a space truss fabrication system with concentric joints, namely, the Triodetic system, in order to see whether this refinement could improve the precision of the final shape in terms of obtaining a true hypar with straight line generators and equal curvatures (with opposite signs) along the two diagonals of the model.

10-1 Test model

The aluminium SCHST was fabricated to the Triodetic system and had the same dimensions and member sizes as those used in Tests 16 and 17. Steel tubes (13 x 2.3 CHS) were placed all around the edges of the model with prestressing wires passing through them and through the hubs at the apices of the pyramidal units. These edge bottom chords were cut to a size to fit between the hubs along the sides of the model and had no original gap. Other bottom chords, also made of 13 x 2.3 CHS steel tube, were placed along one of the diagonals of the model for post-tensioning. These diagonal bottom chords were cut shorter than the distance between the hubs in the diagonal direction in order to create gaps that would close up in the process of shape formation. The gap for each bottom chord was 71 mm long. The model had a total number of 365 nodes and 1100 members. Figure 10-1 shows the plan view of the base model used to form the aluminium hypar.

Figure 10-1 The base model used to form the aluminium hypar.
Figure 10-2 shows the aluminium model space truss before post-tensioning in Test 24.

### 10-2 Test 24

After assembling the aluminium model with steel bottom chords on the floor, the prestressing wires which had been passed through the bottom chords and the hubs along the edges of the model, were prestressed up to 1.5 kN in order to remove any lack of fit and hold the edge bottom chords tightly in position. Then the model was post-tensioned along the diagonal having shorter tubes. The post-tensioning force started from zero and was increased gradually. At a post-tensioning force of 8.1 kN, the model started to change form (see Fig. 10-3).
The post-tensioning force was then increased to 10 kN when the gaps in the two middle bottom chords closed up. Further increase in the post-tensioning force up to 13.6 kN seemed to be of little effect in closing the other gaps and changing the shape of the structure. Like the case of the model GCHST, the model SCHST seemed to become statically determinate after the closure of the first bottom chord gap. Maxwell's rule confirmed this point: \[ R = b - (3j - c) \] where \( R \) is the degree of statical indeterminacy, \( b \) is the total number of members, \( j \) is the total number of joints, and \( c \) is the total number of restraints. Substituting 1088 for \( b \) (i.e. the total number of active elements excluding the gap bars), 365 for \( j \), and 6 for \( c \) (i.e. the minimum restraints required to prevent rigid body motion), and assuming that \( R = 0 \), the number of mechanisms is \( M = 1 \). The model became statically indeterminate after the simultaneous closure of the two middle gaps.

Figure 10-4 shows the model SCHST in its final shape after the shape formation process.

Figure 10-4  The model SCHST in its final form after Test 24.

10-3 Measurements

Measurements on the model SCHST showed that the final shape obtained in Test 24 was not a true hypar. The top chords were curved and the curvatures along the two diagonals of the model were not equal (regardless of the sign of curvatures). In fact the model had remained almost flat along its unstressed diagonal except for the
corners which were bent upwards (see Fig. 10-5). Also, the curvature along the tensioned diagonal was not uniform due to the differential closure of the gaps. The maximum curvature was in the middle where two of the bottom chords closed up and the minimum curvature (along the tensioned diagonal) was towards the corners where the size of the gaps remained almost unchanged. On movement from the middle towards the corners of the model along the tensioned diagonal, there was an increase in the sizes of the gaps left at the end of post-tensioning. Therefore, the shape of the model remained almost straight near the corners along the tensioned diagonal (see Fig. 10-4).

Figure 10-5 The unstressed diagonal of the SCHST.

Figure 10-6 shows some measured dimensions of the top-chord layer of the SCHST after the shape-formation test. The final values of the angles of the corner panels are also shown in the figure.

Figure 10-6 The top chord measured dimensions after Test 24.
Figure 10-7 shows the measured dimensions of the bottom layer of the model after shape formation.

![Image](image.png)

Figure 10-7 The measured dimensions of the bottom-chord layer of the SCHST.

The measured span/rise ratios of the top-chord layer of the model along the stressed and unstressed diagonals were 4.6 and 9.6, respectively.

10-4 Further observations

After first series of measurements on the model aluminium hypar space truss, a further increase in the post-tensioning force, in order to close the remaining gaps, resulted in the failure of a top chord joint on one of the edges of the model at a post-tensioning force of 14 kN. One of the top-chord members connected to that joint pulled out of its slot. No significant change in the geometry of the model was observed following this joint failure (Fig. 10-8). This matter was attributed to the statical determinacy of the model after the elimination of a top-chord member from the structure as a result of this failure. The model was already one degree statically indeterminate before this failure (see section 10-2).

An inspection of the model after the failure showed that there was an extra original slot in the failed aluminium hub which might have weakened the joint and triggered its failure. Therefore, the model was de-stressed in order to replace the failed joint. The model SCHST completely recovered its original flat form after cutting the...
prestressing wire along its tensioned diagonal. Figure 10-9 shows the model after destressing.

Figure 10-8 The SCHST after the failure of a top chord joint during post-tensioning.

Figure 10-9 The model SCHST after cutting the prestressing wire.

After replacing the failed hub and member, the model aluminium space truss was post-tensioned again in an attempt to approach the shape of a true hypar by closing more of the bottom-chord gaps. At a post-tensioning force of 17 kN, the two middle bottom chords became locked in position. But at 19 kN a top-chord member fractured in tension at one of its ends. Then another top-chord member and one web member near the first failed member fractured at its crimped end near the joint in a mode of tensile failure (see Fig. 10-10). The failure of these members was followed by the distortion of the model and the buckling of three corner web members at the
other side of the model (see Fig. 10-11). This general failure was due to the fact that the model turned into a mechanism again after the loss of the first three members.

Figure 10-10 The failure of top chords and web members in the model SCHST.

Figure 10-11 The distorted model SCHST after failure.

10-5 Shape-formation analysis

A nonlinear analysis was carried out to form the SCHST from an originally flat configuration. Only the geometric nonlinearity was taken into account, as the model was not expected to develop large forces in its members before the closure of the gap bars.
Only 1/8 of the model was analysed due to symmetry. Figure 10-12 shows the node and element numbering of the model in the analysis.

The model was assumed to be symmetric about the tensioned diagonal (i.e., line 1-63 in Fig. 10-12a) and anti-symmetric about the X axis (Fig. 10-12). The anti-
symmetric condition allowed rotation about the X axis but restrained translations along it. Figure 10-13 shows the boundary conditions of the model assumed in the analysis. Like the case of the aluminium dome-shaped space truss analyses, additional fictitious elements with a reduced stiffness were assumed to hold the free ends of the top-chord members cut on the line of anti-symmetry (see Fig. 10-14). The stiffness of these fictitious web elements was assumed to be equal to 1/8 of the stiffness of the top chords and web members of the model so as not to affect significantly the overall stiffness of the structure.

Figure 10-13 The boundary conditions assumed for 1/8 of the model SCHST.

Figure 10-14 The fictitious web members assumed on the line of anti-symmetry for the shape-formation analysis of the SCHST.

Node 9 was given a displacement along the tensioned diagonal equal to the value measured after the shape-formation experiment in Test 24. All of the members of
the model were assumed to be pin-ended truss elements except for the bottom chords along the diagonal which were assumed to be gap bars, with a gap per unit length of 0.165 for each bottom chord based on the experimental measurement. A full Newton-Raphson method was adopted for the shape-formation analysis because of the significant geometrical nonlinearity of the model which necessitated the updating of the stiffness matrix after each iteration. Figures 10-15 to 10-20 show the graphical results of the shape formation analysis.

Figure 10-15 The plan view of the SCHST versus the original form.

Figure 10-16 The side view of the SCHST versus the original flat configuration (Looking in the A direction; see Fig. 10-15).
Figure 10-17  The side view of the SCHST after the shape-formation analysis (looking in the B direction; see Fig. 10-15).

Figure 10-18  The SCHST in perspective view.

Figure 10-19  Different stages during the shape-formation analysis of the SCHST.
10-6 Nodal coordinates

Table 10-1 shows the nodal coordinates for 1/8 of the model SCHST as obtained from the hypar formula (see Fig. 9-4, section 9-3), the shape-formation analysis, and measurement. The origin is assumed to be on the top-chord surface at a point just over node 63 (see Fig 10-12a).

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A review of the results of the shape-formation tests on the hypar space trusses showed that there was closer agreement between the measured geometry of the steel GCHST models and the nodal coordinates obtained from the hypar formula and the shape-formation analysis results (see Table 9-1) than a similar comparison between the geometry of the aluminium SCHST model and the respective nodal coordinates obtained from the hypar formula and the shape-formation analysis (see Table 10-1). The differences were attributed to the joint rigidity of the aluminium model in one direction (see Fig. 8-1).
11 Conclusions

It has been shown that it is possible to shape single-chorded flat space trusses into multiple curvature forms. By this curving process, the structures can be considered to be self-erecting. This process holds potential future realisation of significant economies for the fabrication and erection of large-span space structures.

In this thesis curved space trusses of the forms of a barrel vault, a dome, and a hypar were shaped from an originally flat position by means of post-tensioning. This process can eliminate the need for scaffolding and large cranes in the construction of lightweight space trusses.

11-1 Experimental observations

A total of 24 curved space trusses were formed, including 7 barrel vaults, 8 domes, and 6 hypars as well as one space grid of each of these three categories of shapes. In forming the gently curved barrel vaults and domes, it was observed that the design shapes could be formed from planar base models of both single-chorded space trusses and single-layer space grids within a general tolerance of 6% as compared with the results of geometrical calculations and computer-based shape-formation predictions. The measured dimensions of the gently-curved and the sharply-curved hypars, as compared with the geometrical design calculations and the shape-formation analysis, were within tolerances of 20% and 40%, respectively. This large discrepancy between the measured and calculated dimensions was due to the different curvatures along the diagonals of the models and also due to the inconsistent closure of the gap bars.

The typical ratio of the post-tensioning force to the self-weight of the models was 3, as measured for the aluminium SCDSST models.

Of the 24 shaped models, all of the gently-curved steel models, which totalled 18 in number, were roughly 3 by 3 meters in plan view (see Table 3-1) and 360 mm deep. The 3 sharply curved steel barrel vaults, were 3.5 by 3 meters in plan and 360 mm deep. The 2 sharply-curved aluminium domes were 3.3 by 3.3 meters and 2.4 by 2.4 meters, respectively, and each had a depth of 260 mm. The aluminium hypar was 3.6 by 3.6 meters in plan and 260 mm deep.
The shaped steel models were tested under symmetrical vertical load to study their ultimate load behaviour. The results of the tests showed that the continuity of the top-chord members in the steel models tended to offset the negative effects of the curvature of the members and the eccentricity of the joints.

All of the gently-curved steel models as well as all the aluminium models regained their original flat configuration after cutting the prestressing wires. This indicated that the shape-formation process kept these models within the elastic range of their material. The sharply-curved barrel-vault models did not regain their flat configuration completely as the continuous steel top chords had been curved beyond the elastic limit of the material near the corners.

The failure mechanism of the gently-curved barrel-vault space truss models under symmetrical vertical load was generally in the form of top-chord member buckling failure between panel points, followed by local snap through. The local snap-through failure was accompanied by a sudden dynamic effect which caused the sway of the nodes adjacent to the buckled member(s). The overall pattern of failure could be referred to as a patch mode of failure, ie, an interaction between member buckling and node sway, in the general classification of the failure mechanisms of space trusses. The models showed no further increase in the load carrying capacity after the failure of the first member(s), as the trusses were statically determinate.

The failure mechanism of the sharply-curved barrel-vault space truss model was also in the form of individual top-chord member buckling. But this member buckling was not followed by a significant sway of the adjacent nodes because the span-to-rise ratio of the model was small and thus the model was not susceptible to local or overall snap through.

The failure mechanism of the gently-curved dome-shaped space truss models was generally similar to that of the GCBVSTs except for the fact that the dynamic effect which caused the sway of nodes adjacent to the buckled top-chord members was more severe in the case of the GCDSSTs. This point was attributed to the larger rigidity of the shape of the dome which, contrary to the barrel vault, is not a developable surface and, therefore, requires a more severe dynamic effect to reverse its curvature into a snap-through mode of failure.
The failure mechanism of the gently-curved hypar space truss model was in the form of top-chord member buckling without any significant sway of adjacent nodes. This point was attributed to the fact that from the straight top-chord members of a hypar, intersecting at a joint, one is generally in compression and the other is in tension under the effect of symmetrically applied vertical load. Therefore, when the compression member buckles, the tension member has a stiffening effect on the joint and prevents it from swaying in a dynamic snap-through mode of failure.

The failure mechanism of the gently-curved barrel-vault, dome-shaped, and hypar trussed single-layer grid models was generally in the form of the overall buckling of the continuous top chords in the grid regions of the models associated with sway of the nodes which caused very large deflections in the central "grid" portion of the models and the distortion of the joints along the borderlines between the "truss" and "grid" portions of the models.

The space truss models showed significantly larger load carrying capacity and stiffness in comparison with the trussed grid models with the same overall dimensions. In the case of the domes, the average load carrying capacity and stiffness of the model GCDSSTs were 4 and 30 times as large as those of the model GCDSTG, respectively.

The sharply curved domes formed from flat SCSTs by means of post-tensioning were not spherical nor of any other simple regular geometrical shapes, but they were complex shapes made up of positive and negative Gaussian curvatures. Nevertheless, it is assumed the unique geometry of this type of complex dome can be of architectural significance from functional and aesthetic points of view.

The hypars formed by means of post-tensioning were "partial" hypars to the effect that the curvatures obtained were not constant across the model surfaces.

The steel hypar models seemed to be sensitive to any further increase in the post-tensioning force beyond the closure of the first gap in the diagonal bottom chords. Attempting to close the remaining gaps led to the inducement of axial forces in the members of the model hypars, particularly, in the top chord, thereby reducing the ultimate load capacity of the hypar space truss.
11-2 Analysis

A procedure was developed for the shape-formation, or self-erection, of curved space trusses, formed by means of the post-tensioning method described herein, on a personal computer using commercially available finite element software packages (eg. ANSYS). The analytical method used included both material and geometric nonlinearities, but did not include the eccentricities. The members were assumed to be pin-ended. The nonlinear analysis gave good results in terms of nodal displacements in the first stage, i.e. shape formation. The shapes formed by means of the nonlinear analyses were in good agreement with the experimental measurements.

Linear and nonlinear load analyses were carried out on the barrel-vault space truss models and the results were compared with those obtained from the experiments.

The linear analysis assuming pin-ended concentric members gave reasonable results in terms of the ultimate load and member axial forces, but poor results in terms of nodal displacements under vertical load. The member axial forces, nodal displacements, and ultimate load capacities obtained from the linear analysis assuming pin-ended straight members were 80%, 16%, and 72% of the experimentally measured values, respectively.

The linear analysis including the eccentricities of the members and joints and with modified flexural stiffness of the top chord bolts was found to give good results in terms of nodal displacements under vertical loading, member axial forces, and the ultimate load. The axial forces, nodal displacements, and ultimate load capacity obtained from the linear analysis with eccentric joints and approximating the curved members with two straight members meeting at an angle, were 90%, 88%, and 66% of the experimentally measured values, respectively.

The type of the analysis carried out on the dome-shaped and hypar space trusses was nonlinear as the prediction of the final shapes of these models necessitated. The nonlinear analysis could model the behaviour of the bottom chords with original gaps, the prestressing wire with freedom of movement inside the bottom chord tubes. The top chords were modelled as pin-ended truss elements except for the critical top-chord member in each case, which was modelled as a beam element in order to enable a nonlinear buckling analysis and to detect the post-buckling path.
Under the vertical load, the axial forces and ultimate loads obtained from the nonlinear analyses for the dome-shaped steel space trusses were 85% and 80% of the experimentally measured values, respectively, but poor results were obtained in terms of nodal displacements in the second stage, i.e., under vertical load. The nonlinear analysis assumed concentric joints and straight members.

The ultimate load capacity of the hypar steel space trusses as obtained from the nonlinear analysis was 88% of the experimentally measured value. The nonlinear analysis confirmed the experimental observation that the axial forces and the ultimate load capacity of the steel hypar space trusses were sensitive to the post-tensioning operation as the model hypars became statically determinate structures after the closure of the first gap and therefore, any further increase in the post-tensioning force would induce axial forces in all the members and reduce the ultimate load capacity by causing a premature failure.

It could be concluded that the main parameters which caused the discrepancy between the experimentally measured nodal displacements under external vertical load and those obtained from linear and nonlinear analyses assuming concentric joints and members, were the eccentricities of the joints and, particularly, the flexural stiffness of the bolts connecting the continuous top chords.

The gently-curved barrel-vault, dome-shaped, and hypar trussed grid models were analysed for post-tensioning and vertical loading. The analyses were linear and the trussed grid models were assumed to be in their final forms. The top-chord members were modelled as straight beam elements between panel points. The results of the analyses were in general agreement with the experimental observations, i.e., large deflections occurred in the 'grid' portions of the models under vertical load.

Looking into the mechanics of the model space trusses, it was observed that the model barrel vaults and domes had no states of self-stress in their central portions and, therefore, the buckling of the first top-chord member caused the formation of a mechanism in the structures and led to collapse. The hypar models, however, had a state of self-stress after the closure of the first gap and, therefore, needed at least two members to buckle in order to collapse due to the formation of a failure mechanism.

The post-buckling path of the model space trusses could not be traced by means of a displacement control method. This point was attributed to a possible snap-back of
the models after the peak load due to the slenderness ratio of the top-chord members which was about 120.

11-3 Future research

More detailed information can be obtained by including the eccentricities of the members and joints in the nonlinear analysis using beam elements, instead of truss elements.

In addition to the formation of barrel vaults, domes, and hypars, the method of post-tensioning described herein can be applied to the formation of other shapes, such as conoids. The shaped conoids would have an oblique generatrix. Analytical and experimental work will be needed to examine the accuracy of the formed conoids and their ultimate load behaviour.

Further analysis of the post-tensioned curved space trusses can be carried out by considering a variety of load cases such as unsymmetrical loading, wind and earthquake loading, etc.

Further detailing can be carried out by modelling the covering of the shaped surfaces and including it in the analysis. The covering could be made from Kevlar, nylon, etc. It is assumed that this inclusion will enhance the structural performance of the space trusses.

Further investigation of the shape formation and structural performance of model space trusses can be carried out by including differential (instead of uniform) gaps in the edge bottom chords.

Further investigation can be carried out by adding more members (ie. bottom chords) to the models after post-tensioning with an aim to increase the statical indeterminacy and possibly enhance the structural performance of the shaped space trusses.

The method of post-tensioning described herein can be applied to base models (SCSTs) having shapes other than the square on square mesh grid, such as hexagonal grids, which may give rise to other final shapes. The reverse process is in need of investigation, viz., to specify the final shape first and then flatten it to determine a suitable member layout.
Optimisation techniques can be employed to find the optimal span/rise ratios for the range of the shaped space trusses discussed herein.

Further detailing can be carried out by grouting the post-tensioning tubes (edge/diagonal bottom chords) and modelling them in the analysis.
Appendix A - Member tests

In order to find the yield stress of the members of the model steel space trusses, tensile tests were carried out on 3 specimens of each member type by means of a standard testing machine. The yield stress of the cold-formed sections without a clearly defined yield point was found as a proof stress for an offset strain of 0.002.

Also, individual members, cut to their true lengths as used in the steel model space trusses, were tested in axial compression in order to find their critical compressive loads under different boundary conditions. The results of the member tests are given below.

A-1 Tensile tests

A-1-1 Top chords (13 × 13 × 1.8 SHS cold-formed tube)
Three specimens of the 13 × 13 × 1.8 SHS tube, each with a gauge length of 70 mm, were tested under axial tension. Their average yield and ultimate stresses are given below.

\[
\sigma_y = \frac{468 + 445 + 445}{3} = 452.7 \text{ MPa, } \sigma_{ult} = \frac{519 + 513 + 508}{3} = 513.3 \text{ MPa, } \frac{\sigma_y}{\sigma_{ult}} = \frac{452.7}{513.3} = 88% 
\]

Figure 1 shows the stress-strain diagram obtained from a tensile test on a 13 × 13 × 1.8 SHS specimen.

![Figure 1 The stress-strain diagram from a tensile test on a top chord specimen.](image-url)
Another tensile test was also carried out on a strip cut from the wall of the 13 × 13 × 1.8 SHS. The dimensions of the strip were: 4.6 × 1.8 × 40.0 mm. The yield and the ultimate stresses of the strip specimen were found to be 396 MPa and 439 MPa, respectively, with $\frac{\sigma_y}{\sigma_{ult}} = 90\%$. However, because of the small size of the strip specimen and the inaccuracy involved in both cutting and measuring it, only the results of the tensile tests carried out on the complete cross section were used in the analyses.

A-1-2 The 13.5 × 2.3 CHS cold-formed tube (used for web members and bottom chords in Tests 1 to 5)
Three specimens of the 13.5 × 2.3 CHS tube which was used for all of the web members and bottom chords of the models in Tests 1 to 5, were tested in axial tension. The gauge length of each specimen was 70 mm. The results are as follows.

\[
\sigma_y = \frac{437 + 440 + 442}{3} = 439.7 \approx 440 \text{ MPa}
\]
\[
\sigma_{ult} = \frac{451 + 451.3 + 453}{3} = 451.8 \text{ MPa}
\]
\[
\frac{\sigma_y}{\sigma_{ult}} = \frac{439.7}{451.8} = 97\%
\]

Figure 2 shows the stress-strain curve obtained from one of the tensile tests on the 13.5 × 2.3 CHS specimens.

A-1-3 The 17.1 × 3.2 CHS seamless tube (used for bottom chords and some web members from Test 6 onwards)
Three specimens of the 17.1 × 3.2 CHS tube, which was used for bottom chords in
Test 6 as well as in the tests carried out on the gently curved domes and hypars, and also for some corner web members from Test 9 onwards, were tested in axial tension. The gauge length of each specimen was 255 mm. The results are given below.

\[ \sigma_y = \frac{270.6 + 265 + 267}{3} = 267.5 \text{ MPa} \]
\[ \sigma_{\text{ult}} = \frac{417 + 414 + 416}{3} = 415.7 \text{ MPa} \]

\[ \frac{\sigma_y}{\sigma_{\text{ult}}} = \frac{267.5}{415.7} = 64\% \]

Figure 3 shows the stress-strain diagram for one of the specimens.

Figure 3 The stress-strain diagram for a 17.1 x 3.2 CHS specimen.

A-1-4 The 26.7 x 5.6 CHS seamless tube (used for bottom chords in Tests 7, 20, and 24)

Three specimens of the 26.7 x 5.6 CHS seamless tube, each with a gauge length of 250 mm, were tested in axial tension. The results of the tests are as follows:

\[ \sigma_y = \frac{327.3 + 324.6 + 325}{3} = 325.6 \text{ MPa} \]
\[ \sigma_{\text{ult}} = \frac{493.5 + 487 + 490}{3} = 490.2 \text{ MPa} \]

\[ \frac{\sigma_y}{\sigma_{\text{ult}}} = \frac{325.6}{490.2} = 66\% \]

Figure 4 shows the stress-strain diagram obtained from one of the tensile tests on the 26.7 x 5.6 CHS specimens.
A-1-5 The 5-mm prestressing wire

Three tests were carried out on the prestressing wire which was made of the 5 mm hard-drawn mechanical wire manufactured to standard AS1472, Range 2. The gauge length of each specimen was 300 mm. The results are given below. Figure 5 shows the stress-strain diagram for the prestressing wire.

\[
\sigma_y = \frac{1149 + 1249 + 1167}{3} = 1188 \text{ MPa}
\]

\[
\sigma_{\text{ult.}} = \frac{1583.5 + 1506.5 + 1523}{3} = 1538 \text{ MPa}
\]

\[
\frac{\sigma_y}{\sigma_{\text{ult.}}} = \frac{1188}{1538} = 77\%
\]
A-1-6  Web member tensile tests
Three specimens made of 13.5 × 2.3 CHS tube with the same length and boundary conditions as those of the web members of the model steel space trusses (i.e. l = 500 mm; one end bolted and the other end welded) were tested in axial tension. The average ultimate load of these specimens is given below. All of the specimens failed due to the tearing off of the flattened part of their bolted ends.

$$P_{cr} = \frac{18.0 + 20.4 + 21.9}{3} = 20.1 \text{kN}$$

A-2  Compression tests

A-2-1  Top chords
Axial compression tests were carried out on specimens made of 13 × 13 × 1.8 SHS steel tube cut to the true length as used in the model space trusses. The tests were carried out with different end conditions for the members.

(a) Both ends fixed (i.e. gripped in the jaws of the testing machine):
Gauge length = 520 mm
Average of 3 tests: $$P_{cr} = \frac{21.8 + 23.6 + 21.7}{3} = 22.4 \text{kN}$$

Figure 6 shows the load versus axial shortening curve obtained from one of the tests.

Figure 6 The load versus axial shortening curve obtained from the compression test of a top-chord member with both ends fixed.
(b) Both ends pinned (i.e. supported on knife edges; see Fig. 7):

Gauge length = 520 mm

Average of 3 tests: \[ P_{cr} = \frac{8.3 + 8.6 + 9.8}{3} = 8.9 \text{ kN} \]

Figure 7  Top chord specimens with pinned (on knife edges) and bolted ends.

Figure 8 shows the load versus axial shortening curve for one of the specimens.

Figure 8  The load versus axial shortening curve for a pin-ended top-chord member.
(c) Both ends bolted (to solid end pieces with enough tolerance to allow rotation; see Fig. 9):

Gauge length = 520 mm. Average of 3 tests: \[ P_{cr} = \frac{12.0 + 9.4 + 10.9}{3} = 10.8 \text{ kN} \]

![Figure 9](image)

**Figure 9** A top chord specimen with bolted ends during a compression test.

Figure 10 shows a representative experimental curve for these specimens.

![Figure 10](image)

**Figure 10** The load-axial shortening curve for a top-chord member with bolted ends.
(d) Both ends bolted (to solid end pieces with very little tolerance to rotate; see Fig. 7):
Gauge length = 520 mm
Average of 3 tests: \[ P_{cr} = \frac{15.8 + 16.7 + 17.2}{3} = 16.6 \text{ kN} \]

(e) Both ends bolted (eccentrically to end pieces made of the same size SHS; i.e. similar to the case of the model space trusses):
Gauge length = 520 mm
Average of 3 tests: \[ P_{cr} = \frac{11.4 + 10.3 + 10.7}{3} = 10.8 \text{ kN} \]

(f) Both ends bolted and loaded eccentrically (with an eccentricity of 13 mm):
Gauge length = 510 mm
Average of 3 tests: \[ P_{cr} = \frac{9.0 + 10.1 + 10.5}{3} = 9.9 \text{ kN} \]

A-2-2 Compression tests on the edge bottom chords
Axial compression tests were also carried out on the bottom chords. The results are given below.

(i) 13.5 x 2.3 CHS; gauge length = 456 mm (used for Tests 1-4):
1- Both ends fixed:
Average of 3 tests: \[ P_{cr} = \frac{29.3 + 28.1 + 28.7}{3} = 28.7 \text{ kN} \]

Figure 11 shows the load-axial shortening curve for one of the fixed-ended specimens.

![Figure 11 The load-axial shortening curve for a fix-ended bottom-chord member.](image-url)
2- Both ends in grooved counterbored hubs:
Average of 3 tests: \( P_{Cr} = \frac{27.8 + 25.8 + 18.5}{3} = 24.0 \text{ kN} \)

Figure 12 shows the experimental curve obtained for one of the specimens.

![Figure 12](image)

Figure 12 The load-axial shortening curve for a bottom-chord member with both ends in grooved hubs.

(ii) 13.5 x 2.3 CHS; gauge length = 382 mm (used for Test 5):
1- Both ends fixed:
Average of 3 tests: \( P_{Cr} = \frac{30.3 + 29.6 + 28.9}{3} = 29.6 \text{ kN} \)

2- Both ends in grooved hubs:
Average of 3 tests: \( P_{Cr} = \frac{30.7 + 29.3 + 27.8}{3} = 29.3 \text{ kN} \)

(iii) 17.1 x 3.2 CHS seamless tube; gauge length = 382 mm (used for Test 6):
1- Both ends fixed:
Average of 3 tests: \( P_{Cr} = \frac{35.5 + 34.5 + 33.7}{3} = 34.6 \text{ kN} \)

2- Both ends in grooved hubs:
Average of 3 tests: \( P_{Cr} = \frac{24.7 + 27.2 + 26.4}{3} = 26.1 \text{ kN} \)

(iv) 26.7 x 5.6 CHS seamless tube; gauge length = 385 mm (used for Test 7):
1- Both ends fixed:
Average of 3 tests: \( P_\text{cr} = \frac{122.8 + 123.9 + 121.5}{3} = 122.7 \text{ kN} \)

2- Both ends in grooved hubs:
Average of 3 tests: \( P_\text{cr} = \frac{124.6 + 113.6 + 114.8}{3} = 117.7 \text{ kN} \)

A-2-3 Web members
Three specimens made of 13.5 \( \times \) 2.3 CHS, bolted at one end and welded at the other to end pieces, were tested in axial compression to simulate the condition of the web members in the model space trusses. The average critical load of the 3 specimens is given below.

\( P_\text{cr} = \frac{15.7 + 12.9 + 13.6}{3} = 14.1 \text{ kN} \)

Figure 13 shows the load-axial shortening curve obtained from the compression test on one of the specimens.

![Figure 13](image)

Figure 13 The load-axial shortening curve obtained for a web member.
Appendix B - References


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