2015

Mine Subsidence Predictions Using a Mechanistic Modelling Approach

Detlef Bringemeier
Golder Associates, University of Queensland

Publication Details

Detlef Bringemeier, Mine Subsidence Predictions Using a Mechanistic Modelling Approach, 15th Coal Operators' Conference, University of Wollongong, The Australasian Institute of Mining and Metallurgy and Mine Managers Association of Australia, 2015, 212-219.

MINE SUBSIDENCE PREDICTIONS USING A MECHANISTIC MODELLING APPROACH

Detlef Bringemeier[1&2]

ABSTRACT: A practical, predictive method, based on closed form solutions for displacement and strain around longwall panels, is proposed to facilitate the assessment of subsidence and changes to ground conditions above longwall mining. The displacement discontinuity method is employed to simulate the displacement and strain field around a single longwall panel in a three-dimensional transversely isotropic medium. The analytical solutions are effectively combined and implemented in MATLAB language, which allows for deriving key information to subsidence predictions. Predictive accuracy, applicability and efficiency of the code are demonstrated using data from collieries in New South Wales. Close agreement was achieved between the key parameters maximum tensile strain, maximum tilt, maximum convex and minimum concave curvature derived by empirical methods, the proposed displacement discontinuity method and survey records. However, there is still scope for improvements in this approach and additional testing is recommended in order to further validate the proposed method and evaluate its potential for practical longwall mining impact and risk assessments.

INTRODUCTION

Longwall mining is the preferred mining method for coal resources in Australia if geological conditions are favourable and overburden to seam thickness ratios render coal extraction by open-cut methods uneconomical. One of the few drawbacks of the longwall mining method is the potential of mining induced ground movement interfering with aquifers, surface water sources and engineered structures, including transport ways, drainage and bridges.

In the Southern and Newcastle Coalfields, the operating mines are predominantly longwall operations extracting mostly gently dipping coal seams. Panels range between 163 m and 400 m in width and are up to 4065 m long. Vertical and horizontal displacement and tensile and compressive strain develop above, below and adjacent to the extracted panel due to the caving of the extracted coal cavity. This in turn alters the characteristics of the rock above and below the extracted seams and causes vertical and horizontal displacement at the ground surface. A variety of empirical and numerical techniques have been developed in an attempt to describe and quantify the observed changes at the ground surface above a completed panel or a panel undergoing extraction. In their review of mine subsidence prediction methods, Bahuguna, *et al.,* (1991) differentiate between (i) empirical techniques, (ii) influence functions, and (iii) theoretical modelling. Empirical techniques use observations to derive physically based correlation functions between observables (e.g. ground movement parameters) and mining parameters. The ground movement parameters are either single value parameters (e.g. maximum of an observable measured across or along the longitudinal centre line of a panel) or spatial parameters (e.g. profile functions). Methods using theoretical models are based on the application of stochastic or deterministic mechanics to relate rock mass behaviour to loading due to coal extraction and environmental influences. Theoretical models are based on a set of assumptions and the predictive performance of the model is strongly dependent on how well these assumptions are met by the system under investigation.

The aim of this paper is to present a mechanistic method to estimate ground displacement due to longwall extraction in a three-dimensional transversely isotropic medium. Computed results are compared with published data from the Southern and Newcastle Coalfields and results obtained with well-established empirical methods, demonstrating the validity and limitations of the proposed method.

DETERMINATION OF SUBSIDENCE AND THE STRESS FIELD AROUND A LONGWALL PANEL

Displacement Discontinuity approach

The caving of a longwall panel strip can as a first approximation be modelled as a discrete displacement along and normal to a rectangular roof and floor section with a uniform displacement small compared to

---

[1] *Golder Associates, 147 Coronation Drive, Milton, Queensland 4064, Australia, E-mail: dbringemeier@golder.com.edu*, Tel: *(+61) 7 3721 5400*

[2] *School of Civil Engineering, The University of Queensland, St Lucia, QLD 4072, Australia*

the panel width and depth. In this treatment it is assumed that the panel is horizontal and the ground surface can be represented by a traction-free plane surface of a three-dimensional transversely isotropic, elastic half-space with displacement and stress at equilibrium and zero at infinity. This problem requires the solution of a set of 15 coupled partial differential equations comprising three equations of equilibrium, six stress – strain relations and six strain – displacement relations. Using the displacement formulation, the set of equations can be reduced to three coupled second-order partial differential equations in terms of displacement. For a homogeneous transversely isotropic half-space, the equilibrium equations in terms of displacements can be further simplified by introducing potentials or harmonic displacement functions (Love, 1906).

The general solution of this problem for an infinite medium was first suggested by Elliot and Mott (1948) and Shield (1951) in the form of harmonic displacement functions Φ_1 and Φ_2 that represent the solutions of the system of partial differential equations

$$\left(\frac{\partial^2}{\partial {x_1}^2}+\frac{\partial^2}{\partial {x_2}^2}+\frac{\partial^2}{\partial {(x_3^k)}^2}\right)\Phi_k(x_1,x_2,x_3^k)=0,\quad x_3^k=x_3/\sqrt{\nu_k}\quad \text{for}\quad k=1,2 \tag{1}$$

where $\nu_1$and $\nu_2$are the roots of

$$c_{1111}c_{4444}\nu^2+(c_{1313}(2c_{4444}+c_{1313})-c_{1111}c_{3333})\nu+c_{3333}c_{4444}=0\ . \tag{2}$$

The displacement parallel to the Cartesian coordinates are expressed by the harmonic displacement functions

$$u_1=\frac{\partial}{\partial x_1}(\phi_1+\phi_2),\ u_2=\frac{\partial}{\partial x_2}(\phi_1+\phi_2),\ \text{and}\ u_3=\frac{\partial}{\partial x_3}(q_1\phi_1+q_2\phi_2) \tag{3}$$

with

$$\phi_i=\frac{1}{1+q_i}\Phi\quad \text{for}\ i=1,2 \tag{4a}$$

$$q_i=(c_{1111}\nu_i-c_{4444})/(c_{1313}+c_{4444})\quad i=1,2 \tag{4b}$$

and c_{ijkl} are the components of the 4th order symmetric stiffness tensor with five independent parameters for a transversely isotropic material which are related to well-known engineering elastic moduli.

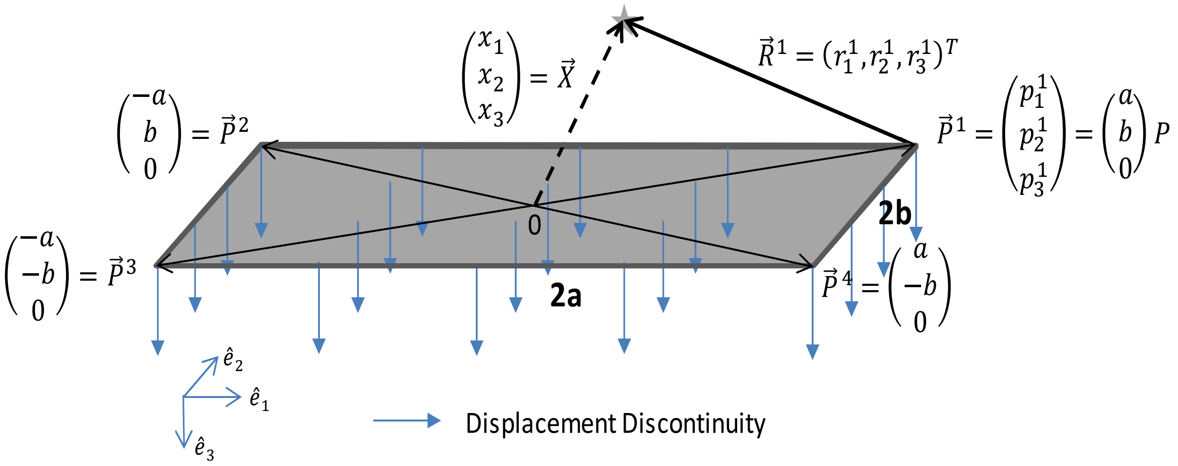


Figure 1: Representation of an extracted longwall panel using a Displacement Discontinuity

The explicit solution obtained by Berry and Sales (1962) for the derivatives of the displacement functions at any point $\vec{X}=(x_1,x_2,x_3)^T$ in an infinite space is proportional to the height T of the panel excavation and a function

$$\Phi=\frac{T}{4\pi}\frac{(1+q_1)(1+q_2)}{q_1-q_2}\sum_{i=1}^{4}(-1)^i\vec{R}^i\cdot\vec{F}^i \tag{5}$$

of the harmonic coordinate vector

$$\vec{R}^i = \left(r_1^i, r_2^i, r_3^i\right) \tag{6}$$

with

$$r_j^i = \left(x_j - p_j^i\right), \quad j = 1,2,3 \quad \text{and} \quad \left\|\vec{R}^i\right\| = R^i = \sqrt{\sum_{j=1}^{3}\left(r_j^i\right)^2}$$

and the function

$$\vec{F}^i = \left(\log\left(R^i - r_2^i\right), \quad \log\left(R^i - r_1^i\right), \quad \tan^{-1}\left(\frac{r_1^i r_2^i}{r_3^i R^i}\right)\right)^T. \tag{7}$$

Figure 1 presents an oblong shaped displacement discontinuity with corner points $\vec{P}^1 = (p_1^1, p_2^1, p_3^1)^T$, $\vec{P}^2, \vec{P}^3$ and $\vec{P}^4$. The stress field is derived by partial derivation of equations (3) and (5), and the strain – displacement and stress – strain relations.

It is noted that the presented solutions for the stress and displacement field are limited to real valued roots of equation (2) and do not extend to the case when the roots obtain complex conjugated values or become zero (isotropic material).

Berry and Sales (1962) extended the solution for an oblong shaped discontinuity in an infinite medium to a half-space with the discontinuity parallel to the traction free half-space boundary (e.g. panel extracted in a horizontal seam). They obtained a solution by mirroring the discontinuity and its properties at an infinite plane parallel to the discontinuity and thus creating a plane along which all shear stresses τ across that plane are equal but of opposite sign. Compressive and tensile stresses σ are continuous across this plane. The solution for the pair of displacement discontinuities is attained by exchanging $\Phi(x_1, x_2, x_3^k), k = 1,2$ in equation (4a) by the sum

$$\Phi(x_1, x_2, x_3^k + D^k) + \Phi(x_1, x_2, x_3^k - D^k) + \Phi^*(x_1, x_2, x_3^k), \; k = 1,2 \tag{8a}$$

with D the distance of the panel to the half-space boundary. The symmetry of the solution ensures that on the half-space boundary $x_3 = 0$:

$$\sigma_{33}^l = \sigma_{33}^u, \quad \tau_{13}^l + \tau_{13}^u = 0, \quad \tau_{23}^l + \tau_{23}^u = 0$$

and the harmonic function

$$\Phi^*(x_1, x_2, x_3^k) = -\frac{2}{\alpha_1 - \alpha_2}\left(\alpha_1 \Phi(x_1, x_2, x_3^k - D^1) - \alpha_2 \Phi(x_1, x_2, x_3^k - D^2)\right), \; k = 1,2 \tag{8b}$$

cancels out the remaining normal traction on z=0. The indices u and l are denoting the upper and lower half-space.

The displacement vector field at the half-space boundary, the z-component of such representing the ground surface subsidence above a mined panel, is found by means of equations (3), (4) and (8) with setting $x_3^k = 0$.

Transversely isotropic material description

Stratified inter- and overburden rocks typical for the Southern and Newcastle Coalfields and many other coal basins in Australia can in many cases be represented by an equivalent transversely isotropic medium. The linear stress-strain state is completely determined by the symmetric effective stiffness tensor $\hat{c}_{ijkl}$ with the components of the tensor derived by the method of asymptotic averaging (Vlasov *et al.*, 2003) and taking the form

$$\hat{c}_{ijkl} = \langle c_{ijkl} \rangle + \langle c_{ijm1} c_{m1n1}^{-1} \rangle \langle c_{n1p1}^{-1} \rangle^{-1} \langle c_{p1q1}^{-1} c_{q1kl} \rangle - \langle c_{ijm1} c_{m1n1}^{-1} c_{n1kl} \rangle \tag{9}$$

where $c_{ijkl} = \frac{E}{2(1+\nu)}\left[\frac{2\nu}{(1-2\nu)}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right]$ are the components of the stiffness tensor of an isotropic, linear elastic material and δ_{ij} is the Kronecker delta with $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$. Summation with respect to repeating subscripts is implied here. The indices i,j,k,l,m,n,p,q X {1,2,3} are in conformity with the axes of the Cartesian coordinate system x_1, x_2, x_3 with x_1, x_2 parallel and x_3 oriented normal to the bedding planes. *E* and n are the Young's modulus and Poisson ratio, respectively. The c_{ijkl}^{-1} are the components of the inverse stiffness or compliance tensor and

$$\langle * \rangle = \frac{1}{B}\sum_{i=0}^{N} b_i(*)\,, B = \sum_{i=0}^{N} b_i \qquad (10)$$

denotes the weighted average of the variable * with the layer thickness b_i representing the weights and B being the total thickness of the N layers composing the layered material.

There is a vast amount of data on the geomechanical properties of the Southern and Newcastle Coalfields sedimentary strata in public documents and geoengineering consulting reports. Some of the data relevant to this study has been collated and a summary is presented in Table 1.

Table 1: Typical material properties for selected stratigraphic units of the Southern and Newcastle Coalfields, NSW, Australia

	E/E_m [GPa]	υ [-]	C [MPa]	φ [°]	σ_T [MPa]	Thickness [m]
Hawkesbury Sst	14/7.3	0.29	9.7	37.25	3.58	0 – 187
Newport Formation	11.6/6.1	0.25	8.85	35.00	3.4	18 – 50
Bald Hill Cst	10.4/5.4	0.46	10.60	27.80	2.9	6 – 67
Bulgo Sst	18/9.4	0.23	17.72	35.4	6.55	90 – 180
Stanwell Park Cst	19.2/10	0.26	14.57	27.8	4.83	5 – 23
Scarborough Sst	20.6/10.7	0.23	13.25	40.35	7.18	16 – 40
Wombarra Shale	17/8.8	0.37	14.51	27.8	4.81	7 – 30
Coal Cliff Sst	23.8/12.4	0.22	19.4	33.3	7.87	10 – 20
Munmorah Conglomerate	19/9.9	*0.25*	5	42	8	15 – 120
Interbedding of Sst, Ust, Cst	6 – 50/13	*0.26*	n/a	n/a	n/a	17 – 36
Wallarah Seams	3/1.6	*0.3*	n/a	n/a	n/a	1.4 – 4.6
Awaba Tuff	16 – 18/8.8	*0.29*	n/a	15 – 20	n/a	0.5 – 18
Laminite	7/3.6	*0.26*	n/a	n/a	n/a	0.3 – 5
Teralba Conglomerate	21/10.9	0.25	5	42	8	10 – 18

Note: E – Young's Modulus, υ – Poisson's Ratio, c – Cohesion, φ – Friction Angle, σ_T – Tensile Strength, Sst – Sandstone, Ust – Mudstone, Cst – Claystone, n/a – not available
E_m – rock mass modulus assuming $\mathrm{E_m} = \mathrm{E}\left(0.02 + 1/\left(1 + \mathrm{e}^{(60-\mathrm{GSI})/11}\right)\right)$ and GSI of 60 for all strata
Italic – Inferred from tests results of similar material
Sources: Keilich (2009), Colin and Bryce (2013), Pells (2004), Ditton (2013)

The effective stiffness tensor for a transversely isotropic rock mass typical for the overburden of collieries in the Southern and Newcastle Coalfields is computed using equations (9), (10) and the thicknesses and material properties listed in Table 1. Components of the effective stiffness tensor are transformed back into moduli and Poisson ratios for comparison purpose and are summarised in Table 2. This data set forms the basis for ground displacement computation outlined in the following section.

Table 2: Set of rock engineering parameters derived for transversely isotropic overburden above longwall panels in the Southern and Newcastle Coalfields

Parameter Set	$E_\perp$ [GPa]	$E_\parallel$ [GPa]	$\nu_{\parallel,\perp}$ [-]	$\nu_{\parallel,\parallel}$ [-]	$G_\perp$ [GPa]
Southern – Newcastle Coalfields	1.38	0.17	0.14	0.30	0.46

Note: $\mathrm{E}_\perp$ – Young's modulus across bedding planes, $\mathrm{E}_\parallel$ - Young's modulus along bedding planes, $\nu_{\parallel,\perp}$– Poisson's ratio ps, $\nu_{\parallel,\parallel}$ - Poisson's ratio pp, $\mathrm{G}_\perp$ - Shear modulus across bedding planes.

Comparison of ground movement parameters derived FROMempirical and the displacement discontinuity model

The development of a subsidence trough above an extracted longwall panel results in differential vertical (tilt and curvature) and horizontal displacements (strain). Tilt refers to the change in subsidence over a given horizontal distance and is defined in units of mm/m. Tilt indicates the magnitude of surface gradient change across and along a subsidence trough. Curvature refers to the change in tilt over a given horizontal distance and is defined in units of mm/m^2 or 1/km. The curvature indicates the magnitude of surface bending across and along a subsidence trough. The inverse of curvature is the radius of curvature or bending (Figure 2).

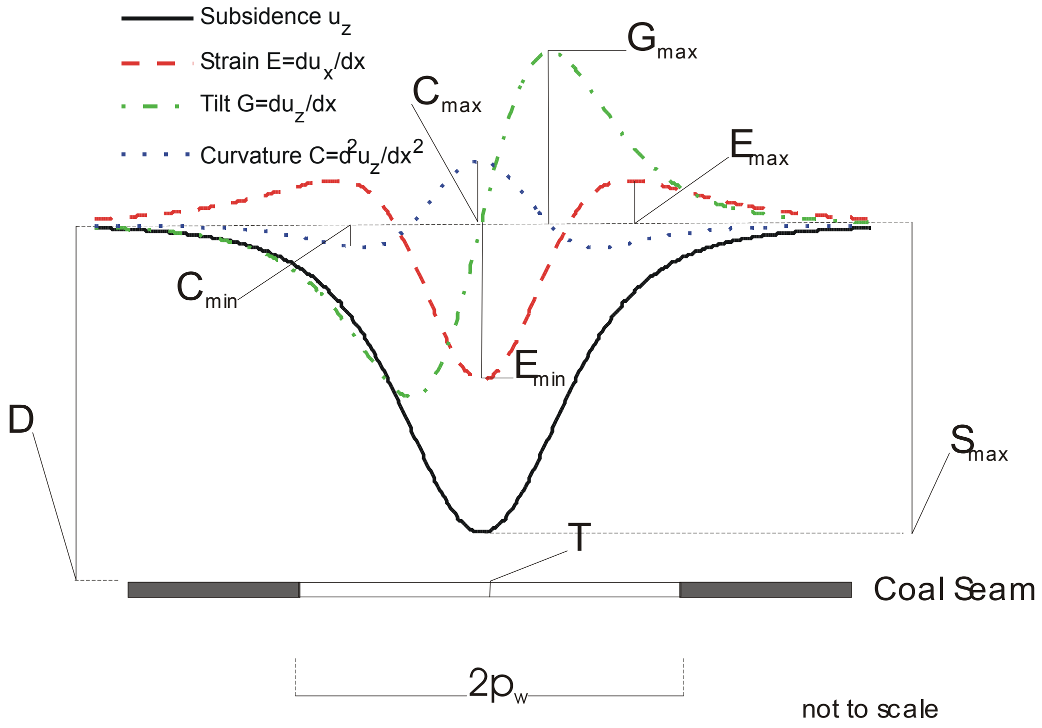


Figure 2: Subsidence and overburden distortion above an extracted longwall panel. Vertical scale strongly exaggerated. T: Extraction height of a panel, D: depth of panel, S_{max}: maximum subsidence and $2p_w$: panel width

The validity of the displacement discontinuity model is exemplified by the ability to accurately resemble empirical relationships and ground movement parameter values obtained for the Southern and Newcastle Coalfields. In Figure 3 the computed maximum subsidence normalised by the panel extraction height is plotted against the panel width in terms of panel depth for a range of panel width to depth ratios typical for longwall mining in the Southern and Newcastle Coalfields. Data from both coal fields are well approximated for critical and super critical panels with width to depth ratios exceeding 0.8. For the lower width to depth range the model overestimates subsidence as a consequence of arching effects and partial closure typical for sub-critical panels, but not represented by the displacement discontinuity model. The large spread of data points can be partially attributed to variations in geomechanical parameters of the overburden, to changes in the depth to panel along the survey lines, to survey lines only partially crossing a panel and to the effects of multiple panel extraction.

In NCB (1975) linear correlations are established between the ratio of maximum subsidence, $\mathrm{S_{max}}$, to depth of panel below ground surface, D, and observed (1) maximum convex and minimum concave strain, $\mathrm{E_{max}}$ and $\mathrm{E_{min}}$; (2) maximum tilt $\mathrm{G_{max}}$ and (3) maximum convex and minimum concave curvature, $\mathrm{C_{max}}$ and $\mathrm{C_{min}}$. The empirical relationships

$$K1 = \frac{E_{max}D}{1000S_{max}}\ , \qquad K2 = \frac{E_{min}D}{1000S_{max}}, K3 = \frac{G_{max}D}{1000S_{max}}, \quad K4 = \frac{C_{max}D}{1000E_{max}} \qquad \text{(10 a – d)}$$

are based on data collected from British Coalfields.

Holla (1985) and Holla (1987) confirmed the linear relationships of equations (10 a) to (10 c) for subsidence parameters observed across longwall panels in the Southern and Newcastle Coalfields, respectively. More recent data are used in Holla and Barclay (2000) to derive a range of values for the proportionality factors of equations (10 a) to (10 c) with K1 ranging between 0.2 and 0.9, K2 between -0.9 and -1.5, K3 between 1 and 3 for panel width to depth ratios between 0.2 and 4. A value for K4 of 22 is reported in Keilich (2009) for the Southern Coalfield.

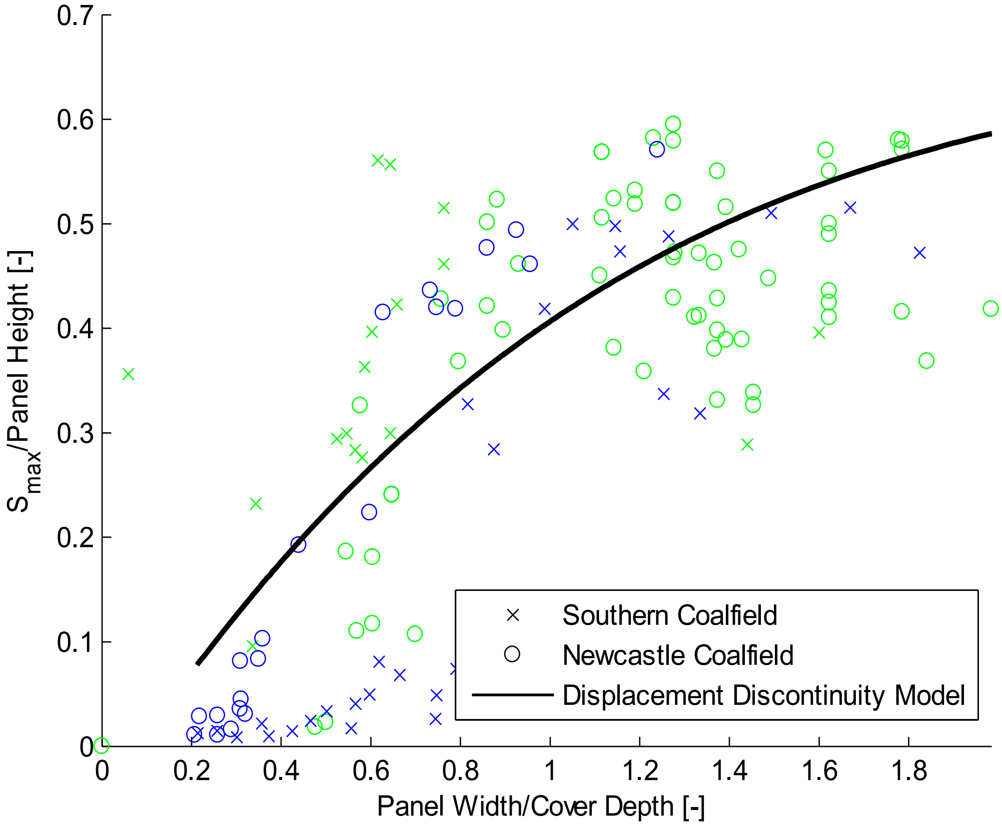
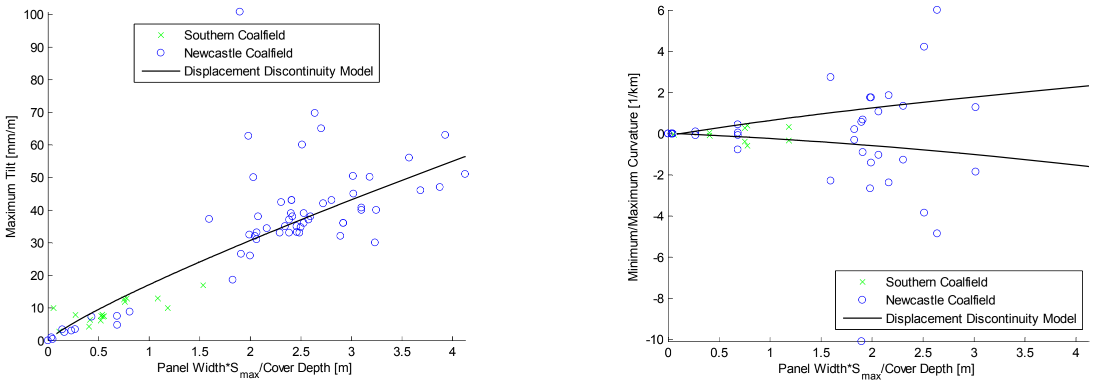


Figure 3: Relationship between maximum subsidence and panel width in terms of depth. Color of markers represents different data sources: green - Ditton and Frith (2003), blue - Holla and Barclay (2000), red - this study)

The empirical relationships are tested with ground movement parameter values computed using the displacement discontinuity model described by equations (1) to (7) and a set of effective stiffness tensors derived above (Table). Both, the empirical linear and the mechanistic model relationships, are plotted in Figure 4 and Figure 5 together with subsidence parameter values obtained from subsidence monitoring of recent longwall mining and data reported in Ditton and Frith (2003).

(a) (b)

Figure 4: Relationships between maximum tensile strain (a) and compressive strain (b) to maximum subsidence in terms of cover depth of longwall panel. Data sources: Ditton and Frith (2003) and collected field data



(a) (b)

Figure 5: Relationships between maximum tilt (a), minimum and maximum curvature (b) to panel width and maximum subsidence in terms of cover depth for longwall panels with a panel width to cover depth ratio between 0.2 and 4. Data sources: Ditton and Frith (2003), Holla and Barclay (2000), and author's database

Solutions obtained from the displacement discontinuity model for a set of panel width to cover depth ratios between 0.2 and 4 are approximated using polynomial and potential functions to more easily compare the results with the empirical linear relationships of equations (10 a) to (10 d). The following non-linear approximation functions are a good fit, with a coefficient of determination R^2 close to 1 (R^2>0.98):

$$\boldsymbol{E_{max} = 376.9\frac{S_{max}}{D}}$$

(11a)

$$G_{max} = 29.8\left(\frac{p_w S_{max}}{D}\right)^{0.8418} \tag{11b}$$

$$C_{max} = 0.0724\left(\frac{p_w S_{max}}{D}\right)^3 - 0.3804\left(\frac{p_w S_{max}}{D}\right)^2 + 1.6163\left(\frac{p_w S_{max}}{D}\right) - 0.076 \tag{11c}$$

$$C_{min} = -0.209\left(\frac{p_w S_{max}}{D}\right)^2 - 0.4257\left(\frac{p_w S_{max}}{D}\right) + 0.024 \tag{11d}$$

The modelled relationship between maximum tensile strain and maximum subsidence in terms of cover depth is in close agreement with the empirical linear relationship and a K1 value of 0.38 was obtained. No such close agreement between the models is found for the maximum compressive strain with the model results plotting along a line for S_{max}/D below 0.0025 correlating well to data from the Southern Coalfield, but data points spreading more and more apart for S_{max}/D above 0.0025 with little correlation at values exceeding 0.01. Computed maximum tilt, maximum (convex) and minimum (concave) curvature do not correlate well with S_{max}/D as suggested by the empirical equations (10c) and (10d). However, when plotted against $p_w S_{max}$/D computed as well as observed data points are in close agreement for maximum convex, minimum concave curvatures and maximum tilt.

CONCLUSIONS

As demonstrated by this study, the displacement discontinuity model of a single longwall panel in a transversely isotropic half-space can achieve results that are in close agreement with empirical model results and ground movement parameters obtain from monitoring data of the Southern and Newcastle Coalfields despite the simplicity of the mechanistic model. Differences between modelled and observed maximum subsidence in terms of panel height and maximum modelled and observed tilt are due to arching effects and partial closure typical for sub-critical panels, but not represented by the displacement discontinuity model.

The mechanistic method presented herein has the advantage over the empirical and influence functional methods in that it can be used for estimating ground movements due to panel extraction equally as for estimating subsurface stress and strain conditions. The method saves time, during repeated iterations of the analytical solutions, making Monte-Carlo simulations for uncertainty assessment a feasible tool which would otherwise be impractical when used with numerical modelling (e.g. Flac3D, 3DEC). Last but not least the monitoring of ground displacement during extraction of a coal seam panel can be understood as a rock mechanical test and the displacement discontinuity model provides a means of inverse modelling for geomechanical parameter estimation of a large scale rock mass. However, there is still scope for improvements in this approach and additional testing is recommended in order to further validate the proposed method and evaluate its potential for practical longwall mining impact and risk assessments.

REFERENCES

Bahuguna, P P, Srivastava, A M C and Saxena N C. 1991, A critical review of mine subsidence prediction methods, *Mining Science and Technology,* 15: 369-382.

Berry, D S and Sales, T W. 1962, An elastic treatment of ground movement due to mining – III three dimensional problem, transversely isotropic ground, *J. Mech. Phys. Solids*, 10: 73-83.

Colin, R W and Bryce, F J K. 2013, Background paper on New South Wales geology with a focus on basins containing coal seam gas resources, *Report issued to Office of the NSW Chief Scientist and Engineer by Unisearch Expert Opinion Services, UNSW Gobal Australia*, Sydney, 87 pp.

Ditton, S. 2013, Subsidence predictions and general impact assessment for the Mandalong Southern Extension Project, *Report prepared by DGS for Centennial Mandalong Pty Ltd, DGS Report No*, MAN-001/1, 12 August 2013

Ditton, S and Frith, R C. 2003, Review of industry subsidence data in relation to the impact of significant variations in overburden lithology and initial assessment of sub-surface fracturing on groundwater, *ACARP Project No. C10023*.

Elliot, H A and Mott N F. 1948, Three-dimensional Stress Distributions in Hexagonal Aeolotropic Crystals, *Mathematical Proceedings of the Cambridge Philosophical Society*, 44 (4): 522-533.

Holla, L. 1985, Mining Subsidence in New South Wales. 1. Surface Subsidence Prediction in the Southern Coalfield, *Department of Mineral Resources*, 32 pp.

Holla, L. 1987, Mining Subsidence in New South Wales. 2. Surface Subsidence Prediction in the Newcastle Coalfield, *Department of Mineral Resources*, 30 pp.

Holla, L and Barclay, E. 2000, Mine Subsidence in the Southern Coalfield, NSW, Australia, New South Wales, *Department of Mineral Resources*, 16 pp.

Keilich, W. 2009, Numerical modelling of mining subsidence, upsidence and valley closure using UDEC, PhD thesis, University of Wollongong, 355 pp.

Love, H. 1906, *A Treatise on the Mathematical Theory of Elasticity*, 551 pp (The University Press: Cambridge).

National Coal Board. 1975, *Subsidence Engineers' Handbook*, 2nd ed., 111 pp (National Coal Board: London).

Pells, P J N. 2004, Substance and Mass Properties for the design of Engineering Structures in the Hawkesbury Sandstone, *Australian Geomechanics*, 39:1 – 21.

Shield, R T. 1951, Notes on problems in hexagonal aeolotropic materials, *Mathematical Proceedings of the Cambridge Philosophical Society*, 47(2):401-409.

Vlasov, A N, Merzlyakov, V P and Ukhov, S B. 2003, Determination of deformation and strength properties of layered rock by asymptotic averaging, *Soil Mechanics and Foundation Engineering*, 40(6):197-205.