Analysis of excavated clay slopes: a multi-stage finite element approach

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ANALYSIS OF EXCAVATED CLAY SLOPES: A MULTI-STAGE FINITE ELEMENT APPROACH

A thesis submitted in (partial) fulfilment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

from

THE UNIVERSITY OF WOLLONGONG

by

P. A. Gray, B.Sc., M.Sc.
Civil and Mining Engineering Department
1981
DECLARATION

This is to certify that the work presented in this thesis has not been submitted to any other University for the award of a degree or other qualification

P. A. Gray
ACKNOWLEDGEMENTS

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P.A. GRAY

Wollongong,

December, 1981
1. ABSTRACT

The work presented in this thesis is the result of investigations into the stability of excavated slopes in overconsolidated clay using the finite element method. The finite element method of stress analysis was chosen because it has many advantages over conventional stability analysis techniques. Conventional stability analysis techniques are described and their suitability for particular stability problems is outlined. Suggested mechanisms of slope failure are then reviewed, and in particular, attention is given to ideas concerning progressive failure in both soil and rock slopes. Some examples of progressive failure are also given.

Following initial investigations it was concluded that a complete solution could only be given by using a finite element method of stress analysis. Published work using the finite element method to analyse excavated slopes is reviewed and attention is given to the limitations of various techniques. In order to obtain realistic stress distributions and assessments of stability, a multi-stage finite element method using a total/effective stress approach was finally adopted. This enabled the simulation of excess pore water pressures including high negative pore water pressures during the excavation process.

Examples are given of the effect of high or low values of $K_o$ on the extent of failure within a slope and attention is given to the variation of $K_o$ with depth which represents every real situation. Four case histories are then described. These are failures of excavated slopes for which well-documented data on
strength parameters, in-situ stresses and pore water pressures is available. These are the Sudbury Hill, Northolt, New Cross and Bradwell sites in London clay. The Bradwell case was a short-term failure (5 days), New Cross was a short/medium term failure (3 years), and Northolt and Sudbury Hill were both long-term failures (19 and 46 years respectively). Therefore the case histories cover a wide range of times to failure as well as different slope geometries.

The factor of safety at any stage of excavation was obtained in terms of actual effective stresses computed at that stage of the excavation on the basis of a new total/effective stress approach incorporated in a finite element method. Many trials were made to locate the most critical slip surface associated with the minimum factor of safety at a given stage of excavation. The results for the selected case histories are consistent with known facts and the developed model can explain both short-term failures and long-term failures. Considering the assumptions and limitations of the model, specific results (e.g. end-of-excitation factor of safety or long-term factor of safety as the case may be) are in excellent agreement with known facts concerning individual slides.

The results are, in general, consistent with the conclusion of Skempton (1977) that the mobilised shear strength at failure of excavated slopes in London clay approaches the softened shear strength, although it is assumed in this thesis that the soil is at its peak strength before excavation starts. The results support recent findings that delays in long-term failures are due to
delays in pore water pressure equilibration (Vaughan & Walbancke, 1973). Thus the postulate that the magnitude of shear strength parameters decreases with time (however slowly), is not necessary to explain long-term failures. Most importantly, the paramount importance of high in-situ stresses in the mechanism of failure is properly explained in relation to the selected case histories.
2. INTRODUCTION

2.1. PROBLEMS WITH CONVENTIONAL ANALYSIS TECHNIQUES

Conventional techniques for slope stability analysis make generalized assumptions in regard to the stresses acting within a slope which, in many cases, may not be correct. Conventional analysis techniques are those which usually derive normal and shear forces on the basis of the concept of limit equilibrium or, in the simplest of these methods, on the basis of gravity (overburden) loading. Pore pressures are derived from either a static water table or from a specified \( r_u \) value which may not be accurate (where \( r_u \) is defined as \( \frac{\gamma WH}{\gamma Z} \), in which \( \gamma_w \) is the water density, \( \gamma \) is the soil density, \( H \) is the height of the water table above the point under consideration, and \( Z \) is the height of the soil surface above the same point). In Chapter 3, commonly used conventional methods of slope stability analysis are reviewed and attention is drawn to their limitations.

The stresses acting on any point in a slope may change in a complex manner during an excavation process. In the case of clays, pore water pressures play a significant role due to dramatic changes which often occur during and after excavation. Conventional methods of slope stability analysis cannot model these complex stress changes. As outlined in Chapter 5, in-situ stresses can be very different from simple overburden loads and principal stress directions generally do not correspond to the vertical and horizontal directions. Consequently the stresses used in a conventional stability analysis will be incorrect. Furthermore the reduction in stress caused by the process of excava-
tion will cause a reduction in pore pressures. In some cases pore pressures will become negative, and this cannot be simulated by overburden or static water level considerations alone.

Chapter 5 gives detailed information regarding in-situ stresses in soils and rocks and the importance of the actual state of stress for realistic stability computations is emphasised. Although one cannot determine the influence of all significant parameters at different stages of the life of a slope, many slope failures can only be explained satisfactorily by considering the effect of in-situ stresses. Back analyses of slope failures using conventional analysis techniques have in some cases used different strength parameters at different positions along the failure surface, but still have not been able to satisfactorily explain failure. To do so they have used techniques such as decreasing strength with time, which in reality may reflect equilibration of pore pressures with time rather than a decrease in the values of strength parameters. Therefore if in-situ stresses are to be considered in detail in a slope stability analysis, a stress analysis method must be adopted.

Stress analysis methods are not without difficulties as is explained in Chapter 7 and Chapter 10. Most stress analysis procedures for slopes simulate excavation as a single-stage process, and consequently many of the problems associated with a multi-stage analysis are not encountered. However, a single-stage analysis is not realistic and does not lead to satisfactory results. In this thesis a multi-stage method for simulation of excavation is presented. The problems associated with such a method
are defined and solutions to them found. The application of the method to some case histories is then demonstrated.

2.2. SCOPE OF RESEARCH

As a result of the problems associated with conventional slope stability analysis techniques, alternative methods of analysis were investigated. The finite element method has been used for stress analysis for about two decades but its use for slope stability analysis has been infrequent, and the solutions developed have not been comprehensive enough to deal with the complexities of real slope problems, and especially those concerning excavations. It was therefore decided to develop a consistent finite element method which could be used to simulate excavation as a real multi-stage process and which could be used to assess both short-term and long-term stability under a wide range of conditions.

Initially, conventional methods of slope stability were reviewed so that their advantages and limitations could be assessed. This review is presented in Chapter 3. It became important to understand alternative mechanisms of failure. Most slope failures are preceded by progressive changes and often movements, and it is, therefore, very important to give attention to progressive failure. Ideas concerned with progressive failure are therefore presented in Chapter 4. Progressive failure is not a single "clear-cut" phenomenon and different slopes may undergo different types or processes of progressive failure. In the context of this thesis the progressive transfer of stress from failed zones to non-failed zones within a slope is of paramount
importance to stability. Consequently it is necessary to know the effect of in-situ stresses on the development of failed zones. Discussion of in-situ stresses and their role is included in Chapter 5. Initial input data for a finite element excavation analysis can only be generated on the basis of known or assumed in-situ stresses. Previous finite element work on excavations was noted and is described in Chapter 6. However it became apparent that for meaningful results to be obtained, case histories should be studied where detailed information on strength parameters, water pressures, and in-situ stresses was available. When such case histories were found, initial finite element analyses were performed. It was then apparent that the problem was fairly complex, and that the model should try to simulate the actual excavation procedure stage by stage. To do this, a multi-stage excavation analysis procedure was devised but there were considerable additional problems associated with this and these are described in Chapter 7. The model was then initially applied to case histories, further refinements were made, and this work is presented in Chapters 8 and 9. The model in its final form was then applied to four case histories and detailed investigations were made of slope stability both for the overall slope and for the slope after each excavation stage and this work is presented in Chapter 9.

Although it became apparent that there are many problems associated with the adjustment of long-term pore water pressures, the simulation of the time rate of pore pressure equilibration after completion of excavation is outside the scope of this thesis.
The performance of the model was evaluated and comparisons made with results from conventional analysis techniques.

2.3. METHOD

The finite element method used in this thesis is a standard two-dimensional displacement formulation for plane strain conditions that has been modified to incorporate a stage excavation procedure and the adjustment of pore water pressures. The stage excavation procedure has seven stages, six of which are excavation stages; the final stage is used to adjust water pressures to the final long-term values. The gradual adjustment of pore water pressures to their final long-term values in excavated slopes in London clay has been studied by Vaughan and Walbancke (1973) and Walbancke (1976), and this aspect is discussed in Chapter 8.

The failure criteria used in this program model the peak strength of a clay and a lower strength (e.g. residual or softened strength) as shown in fig. 4.1. A stress release and transfer system is then used to transfer excess shear stress from failed elements onto adjacent elements. The clay soil is idealised as a perfectly brittle material, an assumption which simplifies the simulation procedure considerably.

Initial analyses were performed using an effective stress approach, and failure zones and stability analyses were calculated using effective stresses. However it became apparent that there were major problems with the determination of pore water pressures at different excavation stages if analysis was made exclusively in terms of effective stresses. It is well known that pore water pressures decrease during an excavation procedure (re-
fer to fig. 7.12), and then finally increase to their final long-term values with time. If an effective stress approach is used, this decrease in pore water pressure cannot be simulated using static water table levels. Since water pressures have a significant effect on failure and overall stability, it is essential that pore pressures be estimated accurately throughout the excavation procedure. In order to determine the pore pressures, reliance was placed on the concept of pore pressure parameters and especially on Skempton's pore pressure parameters A and B. The changes in pore water pressure at different points can be determined on this basis provided the changes in total stress at these points are known, and provided the values of the pore pressure parameters A and B are reliable. Consequently it became necessary to calculate stresses in terms of total stress to determine pore pressures. However failure was still defined in terms of effective stress and these were calculated by simply using the principle of effective stress. In general, principal total stresses and principal effective stresses are thus known at each stage. This method has been called a total/effective stress approach and the final analyses in this thesis are determined using this method.

Trial slip surfaces or sliding surfaces (approximated as circles in cross section) were then analysed at each excavation stage and effective stress stability calculations made using the actual normal and shear stresses computed from the foregoing stress analysis procedure (total/effective finite element analysis). Stresses were actually calculated at a discrete number
of points (generally 30 to 50 in number) along any trial circle.

It became apparent using the stage excavation procedure that one has to be very careful in choosing a failure circle for a stability analysis. Initial work was done using an automatic search type procedure to locate the critical circle. However there are dangers in using an automatic search type method to locate critical failure circles because very shallow failure circles give very low factors of safety using this stress analysis procedure. If the method is correct then these circles would only be very small slumps of material and would not be significant in terms of overall slope stability. Hence circles were chosen which cut through most of the slope exposed up to that stage. However, for the early excavation stages only a small amount of slope is exposed, and if local tensile or shear failure has occurred, then the factor of safety for that particular stage may appear to be very low. As excavation proceeds, the failure zone may not be so significant in relation to a larger failure circle and hence the factor of safety may actually increase for the slope exposed at that stage (refer to the New Cross example figs. 9.28 to 9.31).

However the stability of the final slope is of greater significance and the critical circle for the final slope geometry is one which cuts through the whole slope. Hence at the first excavation stage, this circle is nearly a complete semi-circle and thus has a very high factor of safety. As excavation proceeds, the length of the failure circle is reduced and the factor of safety drops dramatically. The relevant figures which show this are figs. 9.12, 9.23, 9.34 and 9.45 for the four case histories.
considered. For long-term failures, the factor of safety approaches 1.0 at stage 7, whereas for short-term failures the factor of safety was close to 1.0 at stage 6. Moreover, the results support the contention of Skempton (1977) that a decrease in shear strength parameters to residual values is not necessary to explain first-time slides of excavated slopes in London clay. It is particularly significant that a realistic variation of $K_o$ with depth, based on published work, was used to obtain these results. The use of constant $K_o$ with depth leads to unrealistic results. It should also be noted that realistic undrained modulus values, increasing with depth, were used for the case studies on the basis of published results.
3. REVIEW OF CONVENTIONAL METHODS OF STABILITY ANALYSIS
(LIMIT EQUILIBRIUM)

3.1. INTRODUCTION

Conventional methods of stability analysis are generally based on the assumption that the resisting and disturbing forces in a slope are in a state of limiting equilibrium. Therefore if these conventional analyses give a factor of safety of less than unity then the assumption has been made that the forces are theoretically in a state of equilibrium, although this may not be the case in practice. Moments as well as forces can be used to determine a factor of safety, and generally a large number of different failure surfaces are investigated before the critical failure surface is obtained. Failure surfaces are usually circular for soil slopes and non-circular for rock slopes, although methods have been developed to cater for any material type and shape of failure surface. Log spiral failure surfaces have been investigated both by Taylor (1948) and by Chen (1975) and found to give results very similar to circular failure surfaces, and therefore for most practical purposes, are not widely used. The exact shape of the failure surface is controlled by the path of least resistance relative to imposed shear stress. Therefore in most soft or relatively homogeneous materials (e.g., in clay slopes, highly weathered rock or spoil dumps, etc.), the failure surface will most likely be circular. However materials which are highly anisotropic or have a strong joint pattern (such as most rocks) will most likely have a planar or multi-planar failure surface. Examples of this have been described by many au-
thors (Coates, 1965; Skempton & Hutchinson, 1969; Hoek & Bray, 1974; Sage, 1976). All methods require, however, that the shape of the failure surface be specified before the analysis is undertaken.

The various limiting equilibrium methods can give widely differing answers to the same problem and this is because the calculated factors of safety are often defined in different ways. Some methods define the factor of safety as the ratio of total resisting forces to the total disturbing forces, whereas other methods define it as the ratio of the total available shear strength to the total mobilised shear strength, and the two definitions may give different answers. The limiting equilibrium methods are also not precise mathematically and all methods have to make certain assumptions particularly in regard to the forces acting on the side of slices. Another problem with the limit equilibrium methods is that the factor of safety is assumed to be constant around the failure surface. In practice this is not the case, since localised failure may occur in a slope (either shear or tensile failure) even though the overall slope is stable. Therefore the factors of safety would vary over a potential failure surface.

3.2. STABILITY CHARTS

Short term failures in uniform clay slopes are probably the simplest slopes to analyse and there is no need to resort to more complex methods of analysis such as the method of slices. Taylor (1937, 1948) studied this problem and presented stability charts so that a rapid assessment of stability could be made. Fig. (3.1)
shows a simple clay slope with a potential failure surface. The stability of this slope can then be assessed in terms of moments about the centre of the circle 0. The factor of safety is given by:

\[ F = \left(\frac{CR^2\theta}{Wx}\right) \]  \hspace{1cm} (3.1)

where \( c \) is the undrained cohesion value (assuming \( \phi = 0 \), \( c \) would equal half the unconfined compressive strength), \( R \) is the radius of the potential failure surface, \( \theta \) is the angle described by the end points of the failure surface and the centre of the circle, \( W \) is the mass of the potential sliding material and \( x \) is the horizontal distance from the centre of the circle to the centre of gravity of the sliding block. This method assumes that the undrained shear strength remains constant around the failure surface and there is therefore no need to consider normal stresses acting on the failure surface. In practice, the void ratio of a clay would decrease with increasing normal pressure and hence the undrained shear strength and the undrained modulus would increase with increasing confining pressure. Nevertheless the increase in undrained strength with depth may not be significant and this method would therefore still give accurate results.

In order to determine the minimum factor of safety it is necessary to try a large number of potential failure surfaces which is a laborious process. To overcome this problem Taylor (1937, 1948) developed some stability charts in which stability is based only on the slope angle and a depth factor. The depth factor is included so that if a stronger material underlies the
Stability of uniform clay slope considering moments. Undrained analysis only

\[ W = \text{weight of sliding mass} \]
\[ \theta = \text{the angle enclosed by the failure surface} \]
\[ x = \text{the horizontal distance between } W \text{ and centre of failure circle} \]
\[ C = \text{undrained cohesion} \]
\[ R = \text{radius} \]

Fig. 3.1
clay layer and is at relatively shallow depth, it is quite probable that failure may occur at the interface between the two materials. For slope angles above 54 degrees the depth factor has no effect and failure would always be through the toe of the slope and hence Taylor introduced two charts one for slope angles up to 53 degrees and one for slope angles of 54 degrees or more. Once the correct point is found on either chart, a value for $c_m/\gamma H$ is read off the chart from one axis and $c_m$ (the mobilised cohesion) is found by substituting $\gamma H$. The factor of safety is then determined by:

$$F = \frac{c_a}{c_m} \frac{\text{(available cohesion)}}{\text{(mobilised cohesion)}} \quad (3.2)$$

Taylor's charts are probably still the best way to make a rapid assessment of short-term stability of clay slopes and have been reproduced in many text books (e.g., Lambe and Whitman, 1969; Chowdhury, 1978).

3.3. FRICTION CIRCLE METHOD

The Friction Circle Method is a method of analysing the stability of uniform slopes incorporating both $c$ and $\phi$ parameters and was introduced by Taylor (1948). The method assumes that the stresses along a potential failure surface can be replaced by three resultant forces and these are shown in fig. 3.2. These are: the resultant of cohesion $R_c$, the resultant of normal effective stress $N'$, and the resultant of friction $R_\phi$. The unknowns in the method are the factor of safety $F$, the magnitude of $N'$, the angle describing the line of direction of $N'$, $B$, and the dis-
Fig. 3.2 (a) stresses in a uniform clay slope, (b) forces, (c) forces considered in friction circle method
tance from the centre of the circle at which $R$ acts, $r$. Generally the assumption is made that $r$ equals the failure circle radius. This is equivalent to assuming that all the normal stress is concentrated at a single point along the failure arc. Lambe and Whitman (1969) have looked at this method in detail and have shown that assuming $r = r$ gives a lower value for the factor of safety, whereas an upper value of the factor of safety can be obtained if the effective stresses are assumed to be concentrated at the end points of the failure surface. Frohlich (1954) worked through an actual example slope problem using the friction circle method and found that the minimum possible factor of safety was found if the normal force was assumed to be concentrated at one point, and the maximum possible factor of safety was found if the normal force was concentrated at the two end points of the failure surface. These two extreme limits for the factor of safety which he determined, were 1.387 and 1.537. For an example quoted by Lambe and Whitman (1969) the upper and lower values for the factor of safety were 1.62 and 1.27. Therefore the assumption of how the normal force is distributed around the failure surface has a major effect on the calculated factor of safety. Taylor (1948) made the assumption that the normal stresses would be distributed along a potential failure surface in a particular way and he derived a relationship between $r / r$ and the central angle of the failure arc (CA) in which $r / r = 1.0$ when CA = 20 degrees, increasing to $r / r = 1.11$ when CA = 120 degrees. Lambe and Whitman (1969) applied this relationship to their example and found a factor of safety of 1.34. They state that any
value between 1.30 and 1.36 is equally correct from a consideration of static forces alone and therefore Taylor's relationship for \( r^\phi / r \) is reasonable.

The friction circle method involves the calculation of both a factor of safety in terms of cohesion \( F_c \), and a factor of safety in terms of the friction angle \( F_\phi \). Different values are substituted for \( c \) and \( \phi \) until \( F_c = F_\phi \) which then equals the final factor of safety. Taylor's charts enable a fairly rapid assessment of this final factor of safety. Taylor (1948) considered various pore-water conditions but Chowdhury (1978) has pointed out that their use is strictly only applicable to total stress conditions.

Stability charts to cater for both \( c \) and \( \phi \) values and \( \phi \) values in a slope have also been introduced by subsequent authors and all have some limitations. Bishop and Morgenstern (1960) introduced stability charts that covered a range of slope angle from 11 - 27 degrees but required considerable "interpolation and extrapolation to determine the factor of safety" (Chowdhury, 1978). Spencer (1967) introduced stability charts for embankments for slopes angles from 14 - 34 degrees (for toe circles only), which had a maximum error of up to 4% depending on \( r_u \) and slope angle values. Janbu (1967) introduced stability charts which were also for toe circles only, but he used a dimensionless parameter:

\[
\lambda' c \phi = (1 - r_u)(\gamma H \tan \phi')/c'
\] (3.3)

which was used subsequently by Cousins (1977). However Cousins'
charts had restrictions on slope angles above 17.5 degrees, and \( \gamma_u \) values of greater than 0.25 could not be used on slope angles greater than 25 degrees. Hoek and Bray (1974) have prepared stability charts for five different water table conditions and also include a tension crack in their analysis. However for slopes which do not correspond to these conditions (e.g., no tension crack, different pore pressure conditions), the results have to be interpreted. Also for low values of \( \phi \) it is difficult to accurately interpret values from their stability charts. A detailed description of most of these stability charts has been given by Chowdhury (1978).

Chowdhury (1978) also pointed out that care must be taken in performing a total stress analysis on overconsolidated clays since these clays expand during shearing, create suction pressures and increase in water content, therefore the undrained shear strength decreases. Also care must be taken in the choice of undrained strength parameters so that they are representative of in-situ conditions.

3.4. METHOD OF SLICES

3.4.1. Advantages over Stability Charts

There are several major limitations with stability charts which have caused engineers to use other methods of analysis. Firstly, slopes generally consist of more than one material with different strength properties, secondly, the distribution of normal stress along the failure surface may not be modelled accurately by stability charts, thirdly, failure surfaces may be
non-circular, and lastly, programmable calculators and computers make it easy to use more sophisticated methods of analysis.

The method of slices involves dividing the potential failure mass into a series of slices with vertical parallel sides. The number of slices is not critical, although a minimum number is desirable to maintain accuracy of the method and usually between 10 and 50 slices are used. Above about 30 slices there is very little increase in accuracy of the method (Spencer, 1967).

Fig. 3.3 shows the set of forces acting on a particular slice (after Lambe & Whitman, 1969), and several assumptions have to be made in any of the different slices methods. If the distance $a_i$ in fig. 3.3 is so small that it can be assumed to be zero, then Lambe and Whitman point out that there are $4n-2$ unknowns and $3n$ equations or $n-2$ unknowns, where $n$ = the number of slices. Therefore several assumptions have to be made before the method of slices can be used. For example the shear forces on the side of the slice cannot exceed the shear resistance of the soil and the side forces $E_i$ should act at a distance between one third and one half the height of the slice.

3.4.2. Ordinary, Swedish or Fellenius method of slices

This method is known by any of the above names and is probably the simplest of all the methods of slices. The forces considered for this method are shown in fig. 3.4. This method assumes that the side forces on any slice are zero or at least have a zero resultant effect on the base of the slice. Therefore the forces on the base of the slice become:
Fig. 3.3 Complete set of forces acting on a slice
(after Lambe & Whitman, 1969)
Fig. 3.4 Forces considered in Ordinary method of slices
Therefore the normal force on the base of the slice can be given by:

\[ N'_i = W_i \cos \alpha_i - U_i \]  

(3.5)

The factor of safety is therefore given by:

\[ F = \frac{\sum (C + \tan \phi (W_i \cos \theta_i - u_i l_i))}{\sum (W_i \sin \theta_i)} \]  

(3.6)

where \( i \) is the slice number. The factor of safety computed by this method is always in error but gives a lower bound for the factor of safety. Lambe and Whitman (1969) state that in some problems the factor of safety determined by this method may be 10-15% below the range of correct answers, but in other problems may be in error by as much as 60% (see Whitman & Bailey, 1967). The accuracy of this method decreases when, (a) there is a large variation in the angle of orientation of the slice base, angle \( \alpha_i \), since this can have both positive and negative values; and, (b) when pore-water pressures are high.

3.4.3. Simplified Bishop Method of Slices

This method was first introduced by Bishop (1955) who studied this problem in detail and considered the effects of interslice forces. However he showed that the method can be simplified without much loss in accuracy even if the side forces on any slice are ignored. The normal forces on the base of the slice \( N_i \) are found by considering the shear forces \( T_i \) expressed in terms...
of the factor of safety $F$ and these terms are the same as those in fig. 3.3. Therefore the normal force, $N_i$, is given by:

$$N_i = \frac{W_i - u_i x_i - (1/F) C' x_i \tan \theta_i}{\cos \theta_i (1 + (\tan \theta_i \tan \phi_i)/F)}$$  \hspace{1cm} (3.7)

The factor of safety is then given by:

$$F = \frac{\sum (C' x_i + (W_i - u_i x_i) \tan \phi') (1/M_i(\theta))}{\sum (W_i \sin \theta_i)}$$  \hspace{1cm} (3.8)

where:

$$M_i(\theta) = \cos \theta_i (1 + \tan \theta_i \tan \phi')/F$$  \hspace{1cm} (3.9)

In equation (3.8), $F$ appears on both sides of the equation and to solve this problem an initial assumption has to be made for the value of $F$. By a process of iteration the correct value of $F$ can be determined and generally convergence to the correct value is very rapid.

For most problems the Bishop Simplified method is fairly accurate, although Whitman and Bailey (1967) have pointed out that misleading results can be obtained when the failure circle is deep in the slope when $F$ is less than unity. The problem arises when $\alpha_i$, the angle of orientation of the failure surface, becomes negative near the toe of the slope, and this would occur with failure circles of relatively small radius which cut deep into the slope. If this happens the term $(1+(\tan \alpha_i \tan \phi')/F)$ in
equation (3.9) can become zero or negative resulting in the normal force approaching infinity for these slices thus overestimating the factor of safety. Whitman and Bailey (1967) state that this method should be used with caution if the term \( \cos \theta_1 \left( 1 + \left( \tan \theta_1 \tan \theta_1' \right) / F \right) \) is less than 0.2. Nevertheless the Bishop Simplified method is fairly accurate for most problems and has gained wide acceptance in engineering practice.

3.4.4. Janbu's Method of Slices

Janbu (1954) introduced a generalized procedure of slices for composite slip surfaces of any shape in which he considered the forces acting on the sides of a slice. The forces which are considered for each slice in Janbu's method are shown in fig. 3.5.

In this method an initial assumption is made for the position and magnitude of the interslice forces. Normally these forces are assumed to be zero on the first side of the first slice at the crest of the slope (although this may not always be the case), and the forces on successive sides of slices are calculated by the following:

\[
dE = (W + dT) \tan \alpha - (sx/F) \sec^2 \alpha \tag{3.10}
\]

and:

\[
T = -E \tan \alpha_t + h_t dE / x \tag{3.11}
\]

where \( s \) is the mobilized shear strength at failure and is given by:
Fig. 3.5 Forces considered in Janbu's method
\[ s = \frac{C + ((W + dT)/x - u) \tan \phi}{1 + (\tan \alpha \tan \phi)/F} \]  

By using equation (3.11) the side forces on all the slices can be calculated. The values of \((\alpha t)\) and \(ht\) which define the initial position of the line of thrust are assumed for the first slice and then determined for successive slices by the resultant of the forces \(E\) and \(T\). The factor of safety is then determined by:

\[ F = \frac{\Sigma (x \sec^2 \alpha)}{\Sigma (\nu + dT) \tan \alpha} \]  

However it can be seen that to determine \(dE\) initially using equation (3.11), or to determine \(s\) using equation (3.12), \(F\) needs to be known. Therefore this problem is overcome by an iterative solution. The value of \(dT\) in equation (3.13) is assumed to be zero and an initial estimate is made for \(F\). These values are then used in equations (3.11) and (3.12) and the values of \(s\) and \(T\) obtained from them are input into equation (3.13) to obtain a new value for \(F\). The process is repeated until convergence to a constant value of \(F\) is reached. This method uses the same term as the Bishop Simplified method, i.e. \((1 + (\tan \alpha \tan \phi)/F)\) and therefore similar problems can arise if the value is allowed to become very small or negative. Janbu (1957) later included earth pressures and bearing capacity calculations in his method and other terms for forces on the top of a slice were included. However the basic method remained unaltered.
3.4.5. Spencer's Method of Slices

Spencer (1967) introduced a method of slices for the analysis of circular failure surfaces which assumed parallel interslice forces. The forces on any slice in this method are shown in fig. 3.6. The shear force mobilized along the failure surface, $S_m$, is expressed as a proportion of the available shear strength, $S$, (after Bishop (1955)), and is given by:

$$S_m = S / F$$  \hspace{1cm} (3.14)

The normal force acting on the base of the slice, $P$, is determined from the normal component of the weight of the slice and the location of this force is unknown. Similarly the resultant forces acting on the side of the slice, act at an unknown orientation, have an unknown point of application, and have an unknown magnitude. Spencer (1967) pointed out that since every slice is in force and moment equilibrium, the sum of the forces and moments for each slice must be zero. Since the mass is in equilibrium, the sum of all forces and moments external to the mass must be zero. Also the sum of all the forces internal to the mass and the interslice forces, must also be zero. The assumptions which are therefore made in Spencer's method are, (a) all interslice forces are parallel, and, (b) for each slice the weight, the normal force on the base of the slice and the resultant of the two side forces act at the mid-point of the base of the slice. The five forces which act at the base of the slice are:

1. the weight $W$;
Fig. 3.6 Forces considered in Spencer's method
2. the reaction $P$ normal to the base of the slice and this has components $P'$, the effective stress, and the force $(ub \sec \alpha)$ due to the pore pressure $u$, and therefore:

$$P = P' + (ub \sec \alpha) \quad (3.15)$$

3. the mobilized shear force $S_m = S / F$, where:

$$S = c'b \sec \alpha + P' \tan \phi' \quad (3.16)$$

where $c = $ the cohesion and $b = $ the width of the slice, and therefore:

$$S_m = (c'b/F)\sec \alpha + (P' \tan \phi')/F \quad (3.17)$$

4. & 5. the interslice forces $Z_n$ and $Z_{n+1}$ and for equilibrium the resultant of these two forces, $Q$, is assumed to pass through the mid-point of the base of the slice.

Initially a factor of safety is obtained by ignoring interslice forces and is given by:

$$F = \frac{1}{\Sigma(W \sin \alpha)} \Sigma (C'b \sec \alpha + \tan \phi' (W \cos \alpha - ub \sec c)) \quad (3.18)$$

By resolving normal and parallel to the base of a slice, Spencer
(1967) obtained the following expression for the value of the resultant of the two interslice forces ($Q$):

$$
Q = \frac{C' b \sec \alpha + \tan \phi'}{F} \left( W \cos \alpha - u b \sec \alpha \right) - W \sin \alpha \cos (\alpha - \theta) \left( 1 + \frac{\tan \phi'}{F} \tan (\alpha - \theta) \right)
$$

(3.19)

If the external forces on the embankment are in equilibrium the vectorial sum of the interslice forces must be zero and if they are assumed to be parallel then:

$$
\Sigma Q = 0
$$

(3.20)

Similarly if the sum of the moments of the interslice forces about the centre of rotation are zero, then:

$$
\Sigma (QR \cos (\alpha - \theta)) = 0
$$

(3.21)

and for a circular failure surface where $R = \text{constant}$, this becomes:

$$
\Sigma (Q \cos (\alpha - \theta)) = 0
$$

(3.22)

The method now requires that values for $(\alpha)$, the angle of the interslice force, and $F$ be chosen which satisfy equations (3.20) and (3.22). In practice this is achieved by choosing values of $F$ which satisfy moment equilibrium (equation (3.22)) designated $F_m$, and values of $F$ which satisfy force equilibrium designated $F_f$. $F_m$ and $F_f$ are then plotted against values of $(\alpha)$ and their point of intersection is the true value of $F$. Obviously this has to be repeated for each slice, and then a new value of $F$ ob-
tained. An iterative procedure is then followed similar to the other methods to obtain a final factor of safety.

Spencer's method was later refined by Wright (1969) who extended the method to include external loads and non-circular failure surfaces. This method satisfies both force and moment equilibrium and has a wide range of applications from embankments (as it was originally intended) to rock slopes (see Major et al., 1977).

3.4.6. Morgenstern Price Method of slices.

The Morgenstern Price method of analysis allows failure surfaces of arbitrary shape to be analysed and satisfies all the equilibrium and boundary conditions. This method divides the slope into a series of slices similar to all the other methods and the forces which are considered to act on any one slice are shown in fig. 3.7. In fig. 3.7 the forces acting on a slice of width dx are defined as:

- \( E' \) = the lateral thrust on the side of a slice in terms of effective stresses;
- \( X \) = is the vertical shear force on the side of the slice;
- \( dW \) = is the weight of the slice;
- \( dP_b \) = is the water pressure on the base of the slice;
- \( dN' \) = is the effective normal pressure on the base of the slice;
- \( dS \) = is the shear force acting on the base of the slice;

and,

...
Fig. 3.7 Forces considered in Morgenstern & Price method of slices
\( \alpha \) = is the inclination of the base of the slice with respect to the horizontal.

In order to use this method of analysis assumptions have to be made in regard to the side forces on a slice, \( E \) and \( X \). The assumption is that:

\[
X = \lambda f(x) E
\]  

(3.23)

where \( \lambda \) is a parameter which is defined at the outset. For equilibrium there must be no rotation of the slice and this is satisfied if the sum of the moments about the centre of the base of the slice is zero. Therefore it can be shown that as \( dx \) approaches zero:

\[
\frac{d}{dx} \left( E' y' \right) - y \frac{dE'}{dx} + \frac{d}{dx} \left( P_w h \right) - y \frac{dP_w}{dx}
\]  

(3.24)

If we consider the forces at the base of the slice it can be shown that for equilibrium in the \( N \) direction:

\[
dN' + dP_b = dW \cos \alpha - dX \cos \alpha - dE' \sin \alpha - dP_w \sin \alpha
\]  

(3.25)

and that for equilibrium in the \( S \) direction:

\[
dS = dE' \cos \alpha + dP_w \cos \alpha - dX \sin \alpha + dW \sin \alpha
\]  

(3.26)

However we can also express \( dS \) in terms of the shear strength by the Mohr-Coulomb failure criterion as:

\[
dS = \frac{1}{F} \left( C' dx \sec \alpha + (dN') \tan \phi' \right)
\]  

(3.27)

where \( C' = \) cohesion intercept, \( \phi' = \) angle of shearing resistance, and \( F = \) factor of safety. Therefore we can eliminate \( dS \) from
equations (3.26) and (3.27) and set them equal to each other (i.e., the right hand side of eq. (3.26) equals the right hand side of eq. (3.27)). We can also eliminate dN' from equation (3.27) by substituting equation (3.25) for the value of dN', and if we also divide by \((dx \cos \alpha)\) we get the following expression:

\[
\frac{C'}{F} \sec^2 \alpha + \frac{\tan \phi'}{F} \left( \frac{dW}{dx} - \frac{dX}{dx} - \frac{dE'}{dx} \tan \alpha - \frac{dP_w}{dx} \tan \alpha \frac{dP_b}{dx} \sec \alpha \right)
\]

\[
= \frac{dE'}{dx} + \frac{dP_w}{dx} - \frac{dX}{dx} \tan \alpha + \frac{dW}{dx} \tan \alpha \quad (3.28)
\]

Since \((\tan \alpha) = -(dy/dx)\) we can then substitute this in equation (3.28) and we get:

\[
\frac{C'}{F} \left( 1 + \frac{(dy)^2}{(dx)} \right) + \frac{\tan \phi'}{F} \left( \frac{dW}{dx} - \frac{dX}{dx} + \frac{dE'}{dx} \frac{dy}{dx} + \frac{dP_w}{dx} \frac{dy}{dx} - ru \frac{dW}{dx} \left( 1 + \frac{(dy)^2}{(dx)} \right) \right)
\]

\[
= \frac{dE'}{dx} + \frac{dP_w}{dx} + \frac{dX}{dx} \frac{dy}{dx} - \frac{dW}{dx} \frac{dy}{dx} \quad (3.29)
\]

Finally if we rearrange equation (3.29) and collect terms for \(dE'\) and \(dX\) we get:

\[
\frac{dE'}{dx} \left( 1 - \frac{\tan \phi'}{F} \frac{dy}{dx} \right) + \frac{dX}{dx} \left( \frac{\tan \phi'}{F} + \frac{dy}{dx} \right) = \frac{C'}{F} \left( 1 + \frac{(dy)^2}{(dx)} \right) + \frac{dP_w}{dx} \left( \frac{\tan \phi'}{F} \frac{dy}{dx} \right) - ru \left( 1 + \frac{(dy)^2}{(dx)} \right) \frac{\tan \phi'}{F}
\]

\[
+ \frac{dW}{dx} \left( \frac{\tan \phi'}{F} + \frac{dy}{dx} \right) - ru \left( 1 + \frac{(dy)^2}{(dx)} \right) \frac{\tan \phi'}{F} \quad (3.30)
\]

Equations (3.24) and (3.30) are therefore the two governing differential equations. To simplify the equations Morgenstern and Price (1965) used total horizontal stress \(E\) instead of the effective stress \(E'\), therefore:

\[
E = E' + P_w \quad (3.31)
\]
and the point of application $y_t$ of the total stress by:

$$E_y = E'y + P$$

therefore equation (3.24) becomes:

$$X = \frac{d}{dx} (E_y) - y \frac{dE}{dx}$$

and equation (3.30) becomes:

$$(Kx + L) \frac{dE}{dx} + KE = Nx + P$$

where:

$$K = \lambda k \left( \frac{\tan \phi'}{F} + A \right)$$

$$L = \lambda m \left( \frac{\tan \phi'}{F} + A \right) + 1 - A \frac{\tan \phi'}{F}$$

$$N = p \left( \frac{\tan \phi'}{F} + A - r_u (1 + A^2) \frac{\tan \phi'}{F} \right)$$

$$P = C' \left( 1 + A^2 \right) + q \left( \frac{\tan \phi'}{F} + A - r_u (1 + A^2) \frac{\tan \phi'}{F} \right)$$

In the analysis the slices are such that each portion of the failure surface is linear as well as the interface between material types and pore pressure zones. Also the function $f$ as defined in equation (3.23) depends linearly on $x$. Therefore for each slice:
\[ y = Ax + B \]  
\[ \frac{dw}{dx} = px + q \]  
\[ f = kx + m \]

Therefore the values of \( A, B, p, q, k \) and \( m \) are chosen to suit the particular problem. Equation (3.34) is now integrated over each slice starting with \( E = 0 \) for the first slice. This gives the expression for \( E \) as:

\[
E = \frac{1}{L + kx} \left( E_1 L + \frac{Nx^2}{2} + Px \right)
\]  
(3.42)

The value of \( E \) at the end of the slice is used as the initial value of \( E \) for the next slice and so on. To satisfy equilibrium equation (3.42) and equation (3.33) must both be satisfied. This is done by determining values of \( y_t \) and \( E \) from equations (3.42) and (3.23) providing they satisfy equilibrium of moments in the following equations. By integrating equation (3.33) over the first slice we get:

\[
M = E (y_t - y) = \int_{x_0}^{x} \left( x - E \frac{dy}{dx} \right) dx
\]  
(3.43)

at the end of the failure surface when \( x = xn \), \( M = 0 \), therefore:

\[
M_n = \int_{x_0}^{xn} \left( x - E \frac{dy}{dx} \right) dx = 0
\]  
(3.44)

Therefore equation (3.44) also has to be satisfied so that each slice is in moment equilibrium. In the Morgenstern and Price
method $\lambda$ and $F$ are then modified until all the equations of equilibrium are satisfied. Morgenstern and Price (1965) state that, "We start with guessed values of $\lambda$ and $F$ and then integrate across all the slices to obtain values of $E_n$ and $M_n$ which in general will not be zero. Then by a systematic iterative method of modifying $\lambda$ and $F$, values are finally obtained for which $E_n$ and $M_n$ are zero."

Other methods for the analysis of non-circular failure surfaces have been proposed by Nonveiller (1965), Bell (1968), and Sarma (1973), and have been compared by Chowdhury (1978). There are disadvantages with all three methods and other methods are generally used in preference.

3.5. DISADVANTAGES OF LIMIT EQUILIBRIUM METHODS

There are disadvantages with all methods of limit equilibrium analysis. Firstly, the choice of method is extremely important. The use of stability charts for a complex problem would give very inaccurate results, whereas the use of the Morgenstern Price method for a homogeneous clay slope would be unnecessary. Therefore the method must be chosen to suit the problem. Secondly, with the more complex methods, certain assumptions have to be made, particularly in regard to the side forces on slices (their magnitude and point of application). Care is therefore required in the choice of these assumptions since they can effect the factor of safety. Thirdly, the Bishop and Janbu methods overestimate the factor of safety when failure circles with short radius and large internal angles are used. If high pore-water pressures also exist then the inaccuracies become greater and other methods
should be used for such problems. Lastly, in all methods the weight of the slice is used to derive the normal force on the base of the slice. This could be the total weight or the effective weight. In either case the effect of in-situ stresses is not taken into account, and as shown in subsequent chapters, high horizontal in-situ stresses can have a very large adverse effect on stability. The magnitude and direction of the principal stresses acting along a potential failure surface will determine the ultimate slope stability, but limit equilibrium methods cannot determine these. To do this a stress analysis approach is required and this is discussed in subsequent chapters for the case of excavated clay slopes.
4. PROGRESSIVE FAILURE

4.1. INTRODUCTION

This chapter sets out to describe the concept of progressive failure in slopes and then outlines the various theories and approaches that have been adopted to incorporate progressive failure mechanisms in analyses. There are several different approaches to the problem and some authors place more emphasis on one parameter than another and for this reason progressive failure mechanisms are not clearly defined processes and vary from slope to slope and from material to material. Nevertheless progressive failure is a very important process in many slope failures in both natural and excavated slopes as well as in both soils and rocks. This chapter outlines the salient points of progressive failure mechanisms and some progressive failures are briefly described.

Progressive failure implies that there is a gradual reduction in the factor of safety of a slope by the progressive development of a failure surface along which ultimate failure will occur. There are many parameters which may cause progressive failure in a slope and they are:

1. the state of stress,
2. the extent of weathering,
3. slope geology,
4. changes in pore-water pressures,
5. discontinuities and weak zones,
6. breakdown of structural bonds within the material, and
7. non-uniform stress and strain conditions.

Different slopes are affected to a greater or lesser extent by some or all of these parameters and therefore a progressive failure mechanism at one locality may not be applicable elsewhere.

4.2. PROGRESSIVE FAILURE IN CLAYS

The first investigation of progressive failure was undertaken by Terzaghi (1936), who looked at progressive failure in stiff fissured clays. Terzaghi pointed out that slopes in stiff fissured clays failed at strengths much lower than the undrained shear strength of intact samples of the same clay. He attributed this to a process of softening due to water infiltration into cracks and fissures associated with secondary structure. However, this is only a time-dependent function and not a shear-strain function. A delayed failure may be due, for example, to a rise in pore-water pressures and not to any shear strain movement. A distinction should therefore be made between a strain-softening material which exhibits a progressive reduction in strength with strain, a perfectly plastic material that has infinite strain for any increase in stress, and a work-hardening material that has an increase in stress for an increase in strain.

Materials which exhibit a strain-softening behaviour have a distinct drop in their shear strength beyond their peak strength. These include most rocks, dense sands, stiff clays or overconsolidated clays.
Back analyses of slope failures in overconsolidated clays have been performed by many authors and generally give too high a factor of safety if only the peak strength values are used and too low a value if only the residual strength is used. Skempton and Hutchinson (1969) have summarised most of the analyses performed on slope failures in London clay and have shown this to be the case. Skempton (1964) proposed that at failure, a certain portion of the slip surface would be at the peak strength and the remaining portion would be at the residual strength. He introduced a Residual Factor which was defined as:

\[ R = \frac{(S_p - S_p)}{(S_p - S_r)} \]  

where \( S_p \) is the peak shear strength, \( S_r \) is the residual shear strength and \( S \) is the average shear strength around the failure surface. The Residual Factor is therefore the proportion of the slip surface which has fallen to the residual strength. However it is unlikely that along a potential failure surface there will simply be a portion of the failure surface at the residual strength and the remainder at the peak strength. Therefore Skempton later revised some of his ideas regarding progressive failure (Skempton 1970). He pointed out that first time slides in overconsolidated clays have a comparatively small initial displacement and residual strengths would only be reached after very large displacements. He introduced the concept of a softened strength and its relationship to peak and residual strength can be seen in fig. 4.1. The displacements required to reach the softened strength are not very large and he suggested that at this level of shear deformation it is possible that no principal
shear surface would have developed. Instead there could be a "complex of minor shears such as Riedal, Thrust, and Displacement shears which have not been linked into a smooth continuous surface." He suggests however that particle reorientation will have occurred along these surfaces. Skempton also pointed out that there are probably two successive stages in the development of progressive failure in overconsolidated clays:

1. a dilatancy and an opening of fissures leading to increases in water content and culminating in a drop in strength to the fully softened value, at which stage there is a softened shear zone with numerous discontinuous shears, and

2. the development of principal shears of appreciable length some of which eventually link together and form a continuous shear when the residual strength is reached along the entire slip surface.

He also pointed out that the residual strength will only be reached after the slide has occurred.

The drop from peak to softened strength is not applicable to all overconsolidated clays and Skempton and Hutchinson (1969) quote the example of the failure at Selset which had almost no progressive failure. For most overconsolidated clays without fissures the peak shear strength is appropriate for back analysis or design for first time slides. For stiff fissured overconsolidated clays there is a significant loss in strength at failure and the fully softened strength is appropriate for design
Fig. 4.1 SHEAR CHARACTERISTICS OF CLAYS (after Skempton 1970)
purposes. Therefore for stiff fissured overconsolidated clays there must be some progressive failure prior to ultimate failure. For all clay slopes the ultimate limiting value is the residual strength. After a slide has occurred involving large relative displacements, then only the residual strength is available on that slip surface.

Bishop (1967) introduced a Brittleness Index to try and quantify strain-softening behaviour in soils. The Brittleness Index was defined as:

$$I_b = \frac{(S_p - S_r)}{S_p}$$

where $S_p =$ peak strength, and $S_r =$ residual strength. Bishop (1971) later pointed out that there was a significant non-uniformity of shear stress both under undrained and drained conditions in slopes undergoing progressive failure, and that this was directly related to the Brittleness Index $I_b$. The closer the value of $I_b$ was to 1.0, then the greater was the probability of progressive failure. He also updated Skempton's (1964) Residual Factor by proposing a local Residual Factor $R_l$. If reasonable values were chosen for $R_l$ factors along the failure surface he postulated that failures in overconsolidated clays could be explained with more accuracy and that the distribution of local residual factors around the slip surface at failure would be a function of the following parameters:-

1. the relationship between post-peak drop off in strength and displacement,
2. the swelling characteristics of the soil,
3. the pre-peak stress deformation characteristics of the soil under the appropriate conditions of stress change,
4. the value of $K_q$ before the slope was formed,
5. the geometry and scale of the slope, and
6. the long-term ground water conditions.

The above factors will influence progressive failure in all clay slopes although stiff fissured clays are particularly susceptible to progressive failure and have a high Brittleness Index. Bishop (1971) stated that "It may be that the fissures themselves are a consequence of the high Brittleness Index and swelling characteristics of clays and clay-shales of high plasticity index."

Some soft silty clays also exhibit a strain softening behaviour and Bjerrum (1969) has shown that the difference between peak and residual strength for these clays increases with higher clay content which may suggest a post-failure breakdown in clay structure. Highly sensitive marine clays also exhibit a very large undrained strain-softening behaviour and this is generally referred to as the quick condition (Bjerrum, 1954). Bjerrum and Kenny (1967) have proposed that for quick clays the collapse of the structural arrangement of particles produces high enough pore pressures to offset the pre-peak increase in frictional strength with strain, and therefore in some cases the final stages of failure could be undrained. They quote the example of the Furre
Landslide in which back analysis indicated that the average stress at failure was less than both the undrained peak or residual strengths (assuming a residual strength similar to other Norwegian quick clays).

One problem associated with trying to analyse slope failures is in the choice of strength parameters. For materials of low permeability, one can generally decide if a total or effective stress analysis is appropriate depending on the time to failure. However there is a problem since the strength of the sample tested may be considerably greater than the in-situ strength.

Bishop (1966) investigated this problem in relation to stiff fissured clays, but the argument is just as appropriate for jointed rock masses (Bieniawski, 1969, 1975, Singh et al., 1972). Bishop attributed differences between the field strength and that of small samples to the following factors:

1. the time to failure. It may take many years for actual slope failures to occur yet testing procedures are relatively very short.

2. stress conditions. Stress conditions in-situ may be very different to those in a carefully prepared test specimen.

3. anisotropy and non-homogeneity of materials. Samples tested may only be those that form intact cores for example, and may neglect weaker material which is difficult to sample.

4. sample size. Test specimen may not encompass all
the in-situ properties such as widely spaced joints.

Biewiawski (1969) performed tests on large cubes of coal and found that there was a significant reduction in sample strength up to samples 1.5 metres cube. At this size and larger sizes, the strength did not decrease any further and he concluded that a 1.5 metres cube of coal adequately modelled a large in-situ coal pillar.

Singh (1975) performed some tests on granite from Minnesota and limestone from Indiana with sample sizes ranging from 2" diameter up to 36" diameter. He found that the reduction in strength for the specimens tested was up to 50% for the larger samples, and this can obviously lead to errors when analysing slope failures or performing design work.

4.3. ANISOTROPY AND SAMPLE ORIENTATION

Clays are generally anisotropic materials and in-situ undrained strength varies with orientation of the potential failure surface. Duncan and Seed (1966) conducted undrained shear strength tests on clay samples at different orientations with respect to the principal stresses. The tests were Vertical Plain Strain tests (VPS) and Horizontal Plain Strain tests (HPS). The VPS tests had the major principal stress in the direction of the axis of the sample both during consolidation and at failure, whereas the HPS tests had the major principal stress in the lateral direction during consolidation and in the axial direction during failure. The average strain at failure in the VPS tests was only
about one third of the average strain at failure in HPS tests and their results are presented in fig. 4.2.

Since the strains are different for the two different types of test and since the stress directions are different at the toe and the crest of an excavation as shown in fig. 4.2, then it is almost certain that the peak strength cannot be mobilised at both the upper and lower ends of a failure surface simultaneously. If the strains were uniform along the entire failure surface then the clay at the crest of the slope (the VPS sample) would fail before the clay at the toe of the slope (the HPS sample). Therefore anisotropy with respect to strain at failure may contribute to the occurrence of progressive failure in the field by preventing simultaneous development of peak strength along the entire failure surface.

4.4. DIFFERENTIAL (NON-UNIFORM) STRAINS

Patton (1966) conducted some experiments on synthetic rock and came to the conclusion that for a given material the strain at peak strength is dependent on the normal load. For low normal stresses the strain to peak strength is low, but as the normal stress increases so does the strain to failure, and at very high normal stresses the strain to peak strength decreases once more.

Peck (1967) reported some work done by R.J. Conlon on undisturbed samples of Agassiz clay near Winnipeg and these results support Patton's conclusions. Fig. 4.3 shows that the stress-strain curves exhibit peak strengths but the peaks are different for different normal loads. Plots can also be made of the total developed shear strength along any failure surface for a given
VARIATION OF AXIAL STRAIN AT FAILURE WITH CONSOLIDATION PRESSURE IN PLANE STRAIN TESTS

VARIATION OF AXIAL STRAIN AT FAILURE WITH CONSOLIDATION PRESSURE IN PLANE STRAIN TESTS

ORIENTATION OF STRESS DIRECTIONS AT FAILURE OF A SLOPE IN CLAY (after Duncan & Seed, 1966)
deformation. These plots show that the peak strength cannot be mobilised simultaneously along the entire failure surface. As strains occur along a failure surface the shear resistance does however increase at first, and Conlon produced a plot of total shearing resistance against shear deformation and this is shown in fig. 4.4. It can be seen that the average strength along the entire failure surface that has to be overcome before failure will occur lies between the peak and residual strengths and supports Skempton's (1970) proposal of a softened strength for first time slides in stiff fissured clays.

However the in-situ strains within soils are not uniform and will vary from one end of a failure surface to the other. Fissures may create anisotropic compressibility and hence differential strains parallel to the fissures. For these reasons Peck (1967) suggests that where the normal stresses are low (in the upper part of the failure surface) the shear stress will exceed the peak strength at small deformations, whereas near the centre of the failure surface where the normal stresses are much higher, the shear stress may not have even reached the peak strength. At the toe of the slope deformations will be controlled by those deformations along the central portion of the failure surface. Peck (1967) therefore suggests that failure in overconsolidated clays may start at the crest of a slope and progress down the failure surface.
Fig. 4.3 RELATION BETWEEN SHEAR STRESS AND DISPLACEMENT FOR UNDISTURBED SAMPLES OF LAKE AGASSIZ CLAY, WINNIPEG, TESTED IN DIRECT SHEAR AT VARIOUS NORMAL PRESSURES

(after Conlon, reproduced by Peck (1969))
Fig. 4.4 TOTAL SHEARING RESISTANCE AVAILABLE AT VARIOUS UNIFORM SHEARING DISPLACEMENTS ALONG POTENTIAL FAILURE ARC
(after Conlon, reproduced by Peck (1969))
4.5. DEVELOPMENT OF THE FAILURE SURFACE

This leads us to the question of where does the failure surface start in a slope? Theoretical arguments as well as field evidence can be put forward to show that failure could develop from either the toe or the crest of a slope or both, depending on the particular slope. It is generally agreed however, that failure does not start in the central portion of a potential failure surface because the normal stresses are high (Peck, 1967). There are many papers which show that there are zones of high shear stress developed at the toe of an excavated slope, and tensile stresses at the crest of an excavated slope and their extent is dependent upon $K_0$ (Duncan & Dunlop 1966, Desai & Abel 1972, Stacey, 1970, Lo & Lee, 1973). Therefore the toe of the slope may be undergoing shear failure while the crest of a slope may be undergoing tensile failure. There is no reason to assume that these two failure modes cannot be occurring simultaneously. De Beer (1969) presented data from inclinometers to indicate that rupture progresses from the toe of the slope upwards and many slope failures exhibit heave at the toe prior to ultimate failure (Skempton & Hutchinson, 1969). However the first indication of other slope failures is with the development of tension cracks at the crest of the slope. The well-known Abbotsford slide near Dunedin in New Zealand in August, 1979 developed a large graben structure at the top of the slide prior to ultimate failure. This graben eventually became 10 metres deep and 50 metres wide (Ministry of Works, Lower Hutt, New Zealand, Personal Communication, May, 1980).
Romani and Lovell (1972) have studied the effects of a failure surface in a clay embankment developing from the crest downwards and also developing from the toe upwards. They simulated the development of a failure surface by incorporating a slit in their model and they found that the overall factor of safety of the embankment varied depending on the direction in which the failure surface progressed. This would indicate that a failure surface would always develop in one particular direction (i.e., the least stable direction). However they did not take into account in-situ stresses in their model or the effects of pore-water pressures which may account for differences to actual slope failures.

Haefeli (1965) also investigated progressive failure in a clay embankment and concluded that the zone of maximum shear stress would be developed at the toe of the slope, and hence progressive failure would most likely start from there.

Saito and Uezawa (1961) and Saito (1965) have studied the development of failure surfaces in clay slopes and have forecasted the time of occurrence of slope failures by accurate monitoring techniques of slope displacements. They even used full scale models which were made to fail by sprinkling water on the slope and then monitoring the progressive failure. They also studied the creep-failure characteristics of soil in the laboratory, and a relationship between the strain rate and the time to failure was found experimentally. They reported that this relationship appears to be independent of soil type or testing method and the relationship was:

\[ \log_{10} t_r = 2.33 - 0.916 \log_{10} E + /- 0.59 \]  

(4.3)
where $t_r$ = creep rupture life in minutes and $E$ = constant strain rate in 0.001 %/min. Fig. 4.5 shows a typical set of displacement results obtained by them prior to a retaining wall failure and fig. 4.6 shows this failure plus other slope failures on which they had recordings of displacement with time. Fig. 4.6 shows a constant strain rate against creep rupture life and it can be seen that most of their results fall within the creep rupture life as predicted by equation 4.3.

4.6. CREEP

Creep is a phenomenon in which shear strain occurs under an applied stress system with time and may or may not be associated with ultimate slope failure. The strains associated with creep are therefore non-elastic irrecoverable and time-dependent, and both the material type and the duration of the loading have a significant effect on creep behaviour. Haefeli (1965) pointed out that ice behaves elastically under stresses of short duration but behaves as a viscous liquid under stresses of long duration. He conducted experiments on cubes of snow, ice and soil and found that there were similarities in their creep characteristics. The major difference being that a yield stress has to be overcome in soil before creep will occur, whereas even the smallest shear stress causes a continuous deformation in snow and ice since their viscosities are dependent both upon the temperature and the shear level (see fig. 4.7).

Haefeli (1965) developed a relationship for creep which was given as:-
Different strain rates recorded on different instruments.

Fig. 4.5 RESULTS OF FIELD MEASUREMENTS OF COLLAPSE OF A LARGE RETAINING WALL ON OOIGAWA RAILROAD, JAPAN (after Saito 1965)
Fig. 4.6 COMPARISON OF RESULTS OF FIELD MEASUREMENTS WITH THE RELATIONSHIP BETWEEN STRAIN RATE AND CREEP RUPTURE LIFE (after Saito, 1965)
Fig. 4.7 RESULTS OF STRAIN RATE vs SHEAR STRESS FOR 50mm CUBES OF SNOW ICE AND SOIL (after HAEFELI 1965)
where $E = \text{creep rate or strain rate}$, $\tau = \text{the average applied stress}$, $\tau_r = \text{the average residual strength along the plane of failure}$, and $A$ and $n$ are material constants.

Nelson and Thompson (1977) modified this expression to determine the actual time to failure which was given by:

$$t_f = \left( \frac{E_f}{A} \right) \times \left( \frac{\tau}{\tau_r} \right)^n$$  \hspace{2cm} (4.5)

where $t_f = \text{time to failure}$. They performed analyses on three excavated slopes in London clay (Northolt, Kensal Green and Sudbury Hill). The relationship they derived for London clay was:

$$t_f = 0.00518 \left( \frac{\tau}{\tau_r} \right)^{-3.95}$$  \hspace{2cm} (4.6)

and the calculated times of failure are compared with the actual time to failure in Table 4.1.

The results are in good agreement and from this they concluded that creep was one of the major mechanisms causing progressive failure in overconsolidated clays. They postulated that creep would cause failure at a much lower peak strength than those obtained from conventional (constant strain) testing methods. Fig. 4.8 illustrates that if the shear stress is held constant at some level above the yield point ($\tau^*$), then a plastic strain, $E_p$, will result causing the sample to fail to a much lower peak strength. Nelson and Thompson used this theory of creep behaviour in a finite element model to demonstrate the propagation of zones of creep failure in an excavated slope. They selected parameters...
such that complete failure would occur in 100 years but with this model the creep failure zone only started to develop after 83 years. They also had computational problems with this model close to failure and had to stop the analysis just prior to failure to avoid very large displacements (a discussion of very large displacements associated with failure in excavated clay slopes is given in Chapter 8). However the major limitation with their analysis is that they did not include pore-water pressures. It has been demonstrated that the long-term stability of excavated slopes in overconsolidated clays is very dependent upon pore-water pressures (Vaughan and Walbancke 1973, Walbancke 1976, Skempton 1977), and the time-dependent creep function they used reflects to a significant extent the equilibration of pore-water pressures in the long-term condition. The significance of pore-water pressures will be examined more thoroughly in Chapter 8.
EFFECT OF CREEP ON STRENGTH (after Nelson & Thompson, 1977)

Fig. 4.8
The exact reasons for creep are not fully understood but studies have been performed to outline the major factors involved. Yen (1969) performed a theoretical stability analysis in terms of creep velocity of an infinite slope and compared the results of actual slope failures. His conclusions were that for creep to occur the shear stress must be equal to or greater than the residual strength and there must be either a pre-existing failure surface or a change in slope parameters to create a "trigger" mechanism. The trigger mechanism could be a change in slope geometry due to erosion or excavation, a rise in pore-water pressures or an earthquake loading etc. Yen indicates that these trigger mechanisms cause progressive failure to develop resulting in ultimate failure. However this may not necessarily be the case, and movement could be intermittent and related to such things as seasonal rises in the water table.

4.7. STRAIN ENERGY CONSIDERATIONS

The effect of clay structure and its dramatic breakdown in highly sensitive clays has been known for many years. However Bjerrum (1967) pointed out that clays may also have a stored strain energy due to deposition, erosion or uplift etc. The strain energy stored in a particular deposit may cause a different susceptibility to progressive failure than a clay with a lower stored strain energy. He developed a classification system for clay shales which listed their susceptibility to failure depending on their strain energy potential. The least stable materials were considered to be overconsolidated clays possessing strong bonds that had been subjected to gradual degradation by
weathering. Weathering released much of the stored strain energy and lateral stresses would therefore be developed and consequently the materials were likely to expand. Overconsolidated clays with relatively weak bonds were also considered fairly unstable either in a weathered or unweathered condition. Unweathered overconsolidated clays with strong bonds were considered to be the most stable materials. The bonds in this case being considered as being so strong as to prevent the release of stored strain energy. For failure to occur in a clay or clay-shale, Bjerrum (1967) proposed that the following three criteria had to be satisfied:-

1. there must be large internal stresses in the clay mass,
2. there must be a significant drop from peak to residual strength i.e., it must be strain-softening, and,
3. there must be sufficient strain energy such that the peak strength can be exceeded.

Stored strain energy is only one of the many factors influencing progressive failure. However clays which have a high degree of overconsolidation generally also have high horizontal stresses as well, which are remnants from a previous stress system and as such have "stored strain energy." The importance of in-situ stresses is emphasised later in this chapter.
4.8. SHEAR BAND APPROACH

The problems of the initiation and development of slip surfaces has been examined in the shear band approach by Palmer and Rice (1973) and Rice (1973). They have tried to resolve the problem of progressive failure occurring even though the mean shear stress on the observed failure surface is generally much less than the peak strength of the clay. This is particularly true for overconsolidated clays and clay-shales. Morgenstern and Tchalenko (1967) have studied the development of shear surfaces in consolidated kaolin and have shown that shear displacement is restricted to narrow bands which ultimately form a single failure surface. Palmer and Rice (1973) use concepts of fracture mechanics to derive the conditions for the propagation of the shear band (i.e., failure surface) and this propagation is obviously time-dependent in actual slopes.

They used a simple model as shown in fig. 4.9b where the slip surface had already developed and they investigated the further propagation of this slip surface. They defined a work integral for purely elastic materials as:

\[ W(\varepsilon_{pq}) = \int_{0}^{\varepsilon_{pq}} \sigma_{ij} \varepsilon_{ij} \]  

(4.7)

where \( \varepsilon_{pq} \) = the strain experienced for the applied stress, \( \sigma_{ij} \) = stress applied, and \( \varepsilon_{ij} \) = the increment of strain. From this they define a J-integral for the propagation of the fracture. In fig. 4.9b, \( X_1 \) and \( X_2 \) are cartesian coordinates and \( \Gamma \) is the curve in this coordinate system which is integrated to determine the J-integral. \( P^+ \) and \( P^- \) determine the end points of the curve \( \Gamma \),
(a) RELATIONSHIP BETWEEN SHEAR STRESS AND DISPLACEMENT

(b) SIMPLIFIED INTEGRATION PATH FOR THE J-INTEGRAL

(after Palmer & Rice, 1973)

Fig. 4.9
ni is the outward pointing normal vector to \( \Gamma \), \( U_i \) is the component of displacement and \( T_i \) represents the surface tractions across \( \Gamma \) which is related to the stress component by:

\[
T_i = \sigma_{ij} n_j
\]  

(4.8)

If the components of the body force per unit volume of material are \( f_i \), and \( ds \) is an increment of the arc described by \( \Gamma \), then the J-integral is given as:

\[
J_p = \int_\Gamma \left[ (W-f_1 u_1) dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds \right]
\]  

(4.9)

The J-integral is useful because its value is independent of the path of integration of \( \Gamma \) and is dependent only upon the end points \( P^+ \) and \( P^- \). This expression can be simplified since if the shear band develops as shown in fig. 4.9b, i.e., the path of \( \Gamma \) goes from \( P^- \) to \( T \) and then from \( T \) to \( P^+ \) then in the coordinate system \( X_2 \) and \( X_1 \), \( dx_2 \) is zero and so the first term in equation (4.8) disappears. The displacement across the band \( (u_2) \) is continuous therefore \( du_2/dx_1 \) is continuous. \( T_2 \) on the upper surface of the shear band is equal and opposite to \( T_2 \) on the lower surface therefore the term \( T_2 du_2/dx_1 \) can be ignored. From equation (4.8) we get:

\[
J_p = \int_\Gamma \sigma_{21} \frac{\partial u_1}{\partial x_1} dx_1
\]  

(4.10)

The stress \( \sigma_{21} \) over the length of the band is continuous and \( u_1^+ \) and \( u_1^- \) are the displacements on the upper and lower parts of the band. Let \( \delta \) be the relative displacement \( (u_1^+ - u_1^-) \) and \( t \) is the shear stress across the band, then:
\[ J_p = \int_0^p \sigma_{21} \frac{\partial}{\partial x_1} (\dot{u}_1 - \ddot{u}_1) \, dx_1 = \int_0^p \frac{\partial \delta}{\partial x_1} \, dx_1 \]  

(4.11)

Assuming the shear stress and the relative displacement have a constant relationship then \( \tau(\delta) \) is uniquely defined by \( \delta \) and then:

\[ J_p = \int_0^{\delta_p} \tau(\delta) \, d\delta \]  

(4.12)

In areas along the shear band away from the tip the displacement will already be considerable and hence may have reached the residual strength. Equation 4.12 can then be split into that section which is at the residual strength only (i.e., away from the tip) and that section which is above the residual strength (i.e., close to the tip). This becomes:

\[ J_p - \tau_r \delta_p = \int_{\tau_r}^{\tau} (\tau - \tau_r) \, d\delta \]  

(4.13)

The right hand side of equation 4.13 represents the cross-hatched area in fig. 4.9a. Therefore any displacement \( \delta' \) can be defined by:

\[ \int_{\tau_r}^{\tau} (\tau - \tau_r) \, d\delta = (\tau_p - \tau_r) \delta' \]  

(4.14)

Skempton (1964) and Skempton and Petley (1968) have shown that \( \delta' \) was between 2 and 10 mm for shear tests on overconsolidated clays. The value of \( J_p - \tau_r \delta_p \) in equation 4.13 is for an active shear band and Palmer and Rice state that it "can be interpreted as the energy surplus made available per unit area of advance of"
the band, this surplus being the excess of the work input of the applied forces over the sum of the net energy absorbed in deforming material outside the band and the frictional dissipation against the residual part $\tau_r$ to the slip resistance within the band. For propagation to occur this net energy surplus must just balance the additional dissipation in the end region against shear strengths in excess of the residual."

Palmer and Rice (1973) went on to consider a slip surface in a long shear apparatus and used the $J$-integral to find the criterion for continued propagation of the shear band. They calculated that there was a significant size effect and that the greater the thickness or height of the soil layer, the smaller the excess stress $(\tau_0 - \tau_r)$ required for propagation. They found that there was a critical height above which this did not occur and this was found to be approximately 1 metre for London clay. Finally Palmer and Rice considered a slip surface in a long slope starting from a cut or a step in the slope and they concluded that there were three main reasons for the time dependence of the growth of the shear band and these were:-

1. pore-water diffusion into the dilating tip of the band which controls the rate at which local strength reductions can occur,

2. visco-elastic deformations in the clay which allow a gradual increase in strain concentration at the tip of the band, and,

3. weathering breakdown of soil bonds or clay struc-
ture which may increase the amount of energy made available (Bjerrum, 1967).

Christian and Whitman (1969) also considered the propagation of a shear band using a perfectly brittle material but only over a finite distance. They found that yield occurs if:

\[ \frac{p}{\tau_p} = \frac{E^{0.5}}{kh} \]  \hspace{1cm} (4.15)

where \( p \) = shear forces acting parallel to the failure surface, \( \tau_p \) = peak shear strength, \( E \) = pre-peak modulus, \( k \) = slope of pre-peak modulus and \( h \) = layer thickness. The length of the failure surface was given as:

\[ \frac{L}{h} = \left( \frac{-p}{\tau_p} + \frac{E^{0.5}}{kh} \right) \frac{\tau_p^*}{\tau_r^*} \]  \hspace{1cm} (4.16)

where \( \tau_p^* \) and \( \tau_r^* \) are equal to \( (\tau_p - h \sin \alpha) \) and \( (\tau_r - h \sin \alpha) \) respectively, and where \( \alpha \) is the slope angle. A detailed description of the shear band approach has been given by Chowdhury (1977 and 1978).

4.9. PROGRESSIVE FAILURE IN ROCKS

We have considered progressive failure in soft materials since the shear stresses required to cause failure are much lower than those required to cause failure in rocks. Rocks generally behave as brittle materials under relatively low normal stresses and therefore it may be thought that failure in rock slopes would not be gradually progressive but would be very rapid once the peak strength had been exceeded. However rock strength in-situ is controlled to a large degree by the extent, spacing and orientation
of discontinuities. It is therefore the shear strength along the discontinuities as well as the intact rock strength which has to be considered when assessing the stability of rock slopes. Hoek and Bray (1977) introduced a total friction angle to be used for discontinuities which included a roughness component \( i \), so that the total friction angle became \( (\phi + i) \). If sliding does occur along discontinuities, then this \( i \) component gradually decreases until the asperities along the discontinuity have been sheared through, and \( i \) reduces to zero. Hence rocks can exhibit a strain-softening behaviour.

Zienkiewicz et al. (1970) recognised that strain-softening is a characteristic of many rock slides, particularly on jointed rock masses and applied the finite element method to try and simulate rock mass behaviour. He used an iterative technique but pointed out that this could lead to non-unique solutions for strain-softening materials. This iterative method of stress release and transfer was first used by Zienkiewicz et al. (1968) and is described in more detail in Chapter 6.

Barton (1971) also studied the progressive failure in excavated rock slopes. He studied jointed rock masses by using a two-dimensional slope model which he gradually inclined until failure was induced. He found that since both intact rock and rock joints have a marked drop from peak to residual strength that progressive failure can develop if:

1. the slope is steepened due to excavation or erosion,
2. displacement occurs due to earthquakes, blasting, pore-water pressures etc., or
3. weathering reduces the rock strength.

Stress concentrations have been shown to exist in excavated slopes both by photo-elastic and by finite element studies and a more detailed discussion of this is given in Chapter 6.

The problem is to decide if these stress concentrations are sufficient to cause progressive failure in intact rock slopes. In many cases they are. In many large excavations in rocks (mainly mining operations but also road cuttings) blasting plays a very important role and is often ignored when considering its effect on rock slope stability. The giant Mt. Newman iron ore mine in Western Australia had slope stability problems due to blasting overbreak and has been reported by Gray and Hargraves (1978). Boyd et al. (1978) has also reported similar problems with highwall damage associated with careless blasting practice in the Bowen Basin of Central Queensland, and Hagan (1980) has proposed careful blasting practice to overcome this problem.

Blasting not only opens up existing fractures in the rock and creates new fractures, it also enables stress relief to take place by outward movement of the rock. Hence excessive overbreak by blasting can trigger progressive failure.

Progressive movements recorded on rock slopes can be quite large. Muller (1964) referred to a rock slope bulging in its lower half by about 7-8% of the slope height before failure. Ross-Brown and Barton (1969) have recorded vertical and horizontal
displacements of 5 and 3 metres respectively at the crest of a 100 metre high rock slope prior to failure. These movements are by no means uncommon and tension cracks and floor heave can be seen at the crest and toe of many large open cut mines in Australia. A well documented progressive rock slope failure was at the Chuquicamata Mine in Chile. Kennedy and Niermeyer (1970) reported this slide and recorded both horizontal and vertical movements prior to ultimate failure. Fig. 4.10 shows the resultant movement of this slide was approximately 7 metres before ultimate slope failure. Hamel (1971) reported a large slide at the Kennecott Copper Corporation's Kimberley Pit near Ruth, Nevada, which occurred progressively and involved both rotation and downhill sliding.

It is also quite probable that many rock slides in natural slopes are progressive. Two of the most famous large rock slides, the Frank Slide and the Vajont Slide were both progressive failures and have been studied in detail in the literature (see McConnell and Brock (1904), Daly et al. (1912), Cruden and Krahn (1973), and Krahn and Morgenstern (1976), for the Frank Slide, and Muller (1964 and 1968), Mencl (1966), Kenney (1969) Broili (1967), and Chowdhury (1978a) for the Vajont Slide).

The Frank Slide rocks had a large drop from peak to residual strengths and it is probable that the slide had been developing for many years and the actual slip surface was controlled by flexural slip features that originated during folding (Krahn and Morgenstern, 1976). Movements had been noticed above the Vajont Slide for some time before failure occurred and therefore there
Fig 4.10 PRE-FAILURE MOVEMENTS CHUQUICAMATA MINE, CHILE
(after Kennedy & Niermeyer, 1970)
must have been some progressive failure. Chowdhury (1978a) has summarised the literature on this slide and explained this failure in terms of in-situ stresses.

4.10. CONCLUSIONS

The conclusions that can be drawn from this are that it is not the strength of the material that governs whether or not progressive failure will develop in a slope, but the shape of the stress-strain curve and the shear stress level in the particular material. In many slopes, either natural or excavated, the shear stress level increases with time due to many factors as outlined previously and therefore progressive failure can develop. Skempton and Hutchinson (1969) point out that "progressive failure is most unlikely with non-brittle, almost flat-topped stress strain curves," and have analysed the failure at Selset that had almost no progressive failure. However progressive failure is an important mechanism in many slope failures, especially in stiff-fissured clays such as London clay. The stability of excavations in London clay is investigated in detail in Chapter 8.
5. THE PRESENCE OF HIGH IN-SITU HORIZONTAL STRESSES IN SOILS AND ROCKS

5.1. INTRODUCTION

The initial stresses in soils or rocks are extremely important and have a major effect on the stability of excavated tunnels, underground openings, canals and slopes etc. Generally, the greater the horizontal stress then the more adverse are the conditions for stability although the material properties are of course extremely important. A slope excavated in granite with high horizontal stresses would obviously be more stable than the same slope excavated in clay with high horizontal stresses. However in some deep mines horizontal stresses are high enough to cause rock bursts in very strong igneous and metamorphic rocks. Therefore stability is controlled both by the material properties and the stress system imposed on them.

The initial stresses in either soils or rocks have often been considered simply as those due to the weight of the overlying strata, \( \gamma H \). The vertical and horizontal stresses are then defined as:

\[
\sigma_v = \sigma_y = \gamma H, \quad \text{and} \quad \sigma_h = \sigma_x = K \gamma H \text{ or } K_0 \gamma H \tag{5.1}
\]

where \( K \) and \( K_0 \) are parameters which define the ratio of in-situ stresses, \( K \) in terms of total stress and \( K_0 \) in terms of effective stress. \( K_0 \) is therefore generally used for soils. In most analyses it is generally assumed that \( \sigma_x \) and \( \sigma_y \) correspond to principal stress directions so that if \( K_0 \) is greater than 1.0, \( \sigma_1 = \sigma_x \) and \( \sigma_3 = \sigma_y \) or if \( K_0 \) is less than 1.0, \( \sigma_1 = \sigma_y \) and \( \sigma_3 = \sigma_x \). In practice this is probably an oversimplification but it is
a reasonable assumption if no other data is available. The ratio between the principal stresses is very important since this determines the magnitude of the shear stresses. In the case of clays with relatively low cohesion values, increasing values of $K_q$ cause the shear stress to be greater than the shear strength and fig. 8.13 in chapter 8 shows a plot of shear strength vs depth and shear stress vs depth for different values of $K_q$. It can be seen that failure occurs at shallow depths for values of $K_q$ of 2.0 or greater. Therefore the value of $K_q$ needs to be known with considerable accuracy for any analysis considering in-situ stresses.

5.2. THE COEFFICIENT OF EARTH PRESSURE AT REST IN SOILS

The parameter $K_q$ is referred to as the coefficient of earth pressure at rest and was originally introduced by Donath (1891), and he defined it as the ratio of the vertical and horizontal earth pressure resulting in a soil from the application of vertical load under a condition of zero lateral deformation. Subsequently Terzaghi (1920) measured $K_q$ in sand and clay and reported values of 0.42 and 0.70 respectively. Jaky (1948) conducted some experiments on normally consolidated samples in the laboratory and found that $K_q$ could be given by:

$$K_q = 1 - \sin \phi$$

and therefore generally has a range of 0.3 to 0.8. For clays which have an approximate angle of shearing resistance of 20 degrees, $K_q$ would therefore be 0.65. However a soil or rock in-situ may have undergone a complex stress history and may have been
subjected to such things as deposition, uplift, erosion and tectonic forces (perhaps resulting in folding and faulting). For the case of clays, the extent of deposition and subsequent erosion is very important since this determines the overconsolidation ratio. From simple consolidation tests on sand in the laboratory, Kjellman (1936) established that $K_o$ was related to the overconsolidation ratio. He found that $K_o$ increased the more the sample was unloaded, increasing to a maximum value of 1.5 when the sample was almost completely unloaded.

Brooker and Ireland (1965) also found the same relationship for clays and quoted a maximum value for $K_o$ of 3.0 for remoulded clays that were heavily overconsolidated. They conducted some experiments on five cohesive soils to determine values of $K_o$ both during loading and unloading. The clays they studied were Chicago Clay, Goose Lake Flour, Weald Clay, London clay and Bearpaw Shale which had a range of plasticities and angles of shearing resistance. Their results showed that $K_o$ was related to both material properties and stress history. The effective angle of shearing resistance and the plasticity index were the most important material properties and the stress history was defined as the maximum pre-consolidation load given by the overconsolidation ratio. For normally consolidated samples they found that $K_o = 0.95 - \sin \alpha$ for cohesive soils and that $K_o = 1 - \sin \alpha$ for cohesionless soils which is almost identical to Jaky (1948).

Bishop et al. (1965) have quoted values of $K_o$ as high as 3.4 for London clay at the Ashford Common site, whereas Skempton (1961) quoted a maximum $K_o$ value of 2.8 for London clay at the
Bradwell site. Windle and Wroth (1977) have also quoted $K_o$ values in the range 2.0 to 3.5 for the top 10 metres of London clay. Burland and Hancock (1977) quote $K_o$ values ranging from 0.5 near the surface to 2.5 at a depth of 74 metres at the New Palace Yard site, whereas Sills et al. (1977) used constant values of $K_o$ for different depth ranges for the Neasden site. There is therefore a wide range of $K_o$ values quoted for London clay depending on location. The London clay is a heavily overconsolidated deposit but differences in $K_o$ values are attributable to different pre-consolidation loads over the London Basin. Chandler (1974) has indicated that the Lias clay is very heavily overconsolidated having a maximum overburden of 1,000 metres removed although no $K_o$ values were quoted. Parry (1972) has also indicated that the Oxford clay is very heavily overconsolidated and scanning electron microscope pictures have shown it to have a very strong particle orientation although again no $K_o$ values were quoted. Peterson (1978) has stated that the Bearpaw Shale in Saskatchewan must have had a pre-consolidation load of 10 MPa (1,500 psi) and Smith (1953) indicated a similar pre-consolidation load for the Fort Union Shale. There is therefore considerable evidence to suggest that high $K_o$ values in clays are by no means uncommon and that overconsolidated clays almost certainly have high $K_o$ values unless they have undergone stress relief for example by weathering and these high lateral stresses can cause considerable stability problems.
5.3. IN-SITU STRESSES IN ROCKS

High horizontal in-situ stresses have also been reported in many rocks and have a significant effect on such things as tunnels and deep excavations. Rock bursts in deep mines and floor heave in excavations are a result of high in-situ stresses. Hast (1967) measured rock stresses in Scandinavia and concluded that the horizontal stress field was usually several times the dead-weight of the overlying rock and was highly directional. (However Scandinavia has and still is undergoing isostatic compensation due to melting of the ice caps from the last glaciation and may therefore be a special case). Palmer & Lo (1976) conducted stress measurements on dolomite and limestone at Thorold in south-western Ontario and found that the maximum principal stress was horizontal and approximately 8 MPa at depths down to 25 metres. Simple overburden considerations would only give a vertical stress of approximately one tenth of this figure. Lo and Morton (1976) went on to point out how these high horizontal stresses adversely affected tunnelling. Lo (1978) summarised most of his previous work and stated that most of the Silurian and Ordovician rocks in southern Ontario exhibit high horizontal stresses. He quoted examples of heaving at the base of quarries, buckling of the concrete lining of canal floors, cracking of the concrete of tunnels and movement of the walls of unsupported excavations as examples of high horizontal in-situ stresses. Excavations in rock with high horizontal stresses cause a stress relief as well as the development of zones of high shear and tensile stress, and consequently possible failure (Stacey, 1970).
There is therefore considerable evidence to show that high horizontal stresses exist in rocks and that these can cause stability problems (Hooker and Johnson, 1969; Dodd and Anderson, 1972; Haimson, 1975; Jaeger and Cook, 1976 and Blackwood et. al, 1976).
6. Finite Element Techniques

6.1. Finite Element Technique in Soil and Rock Mechanics

The Finite Element Method was first used by Clough (1960) and he was the first person to refer to this numerical technique as the finite element method. Subsequently the method has been developed to a very high level and there are a large number of publications on the subject. The most commonly used books are by Zienkiewicz (1971) and by Desai and Abel (1972) who give a general introduction to the subject. The purpose of this thesis is not to review the finite element method in detail, but rather to discuss previous finite element work applied to excavations, to discuss the application of the method to excavations in this present study, and to discuss techniques which have been employed to solve specific problems in the application of the method.

Soils and rocks are highly anisotropic and their behaviour is influenced by such factors as physical structure, porosity, density, stress history, loading characteristics, existence and amount of fluid in the pores, and time-dependent or viscous effects on the soil skeleton and the pore fluid. Rocks have the added disadvantage that they are structurally highly anisotropic and such features as joints, bedding planes, faults, cleats, veins, fissures, and weak or weathered zones make it extremely difficult to model a soil or rock as an isotropic continuum. The finite element method cannot model all of these features, and in many cases complete information is not available (since exposures of a soil or rock may be limited and information at depth non-existent). However various techniques have been employed to
overcome some of these problems and reasonable results can be obtained. For example joints can be modelled in the finite element method in three ways, either by assuming the material to be isotropic and homogeneous and then studying a large number of hypothetical joint orientations to see their effect on stability; or by assuming the material to be anisotropic and layered, and applying the corresponding stress/strain relationships; or by assigning weak zones in the mesh to major structural features such as faults. In this way some of the problems associated with modelling soils and rocks can be overcome. Some previous work on the finite element analysis of excavations and associated problems will now be examined.

6.2. PREVIOUS FINITE ELEMENT WORK ON EXCAVATIONS.

Many authors have applied the finite element method to problems associated with excavations in both soils and rocks and a brief review of them is given here.

Duncan and Dunlop (1969) studied the effect of high horizontal stresses on excavated clay slopes and later (1970) studied the development of failure zones around excavated clay slopes by using a multi-stage excavation process with the finite element method, and they encountered several significant problems that are worth examination. Their stress/strain model was represented by two straight lines, being perfectly elastic up until failure and then being perfectly plastic post-failure. They studied the short-term failure of saturated clay slopes on the basis of the well known "\( \phi = 0 \)" assumption. They modelled both normally consolidated and overconsolidated clays by assuming different values
for increase in strength with depth. For normally consolidated clays the shear strength increased approximately linearly with depth below the water table, whereas for overconsolidated clays the strength was relatively high near the surface where the effect of overconsolidation would be greatest, and gradually decreased to strength values approaching those for normally consolidated clay at the base of the clay layer. The modulus values selected were proportional to the undrained shear strength and typically the value of modulus was taken as 100 times the undrained shear strength. Modulus values for failed elements were 1/10,000 of the non-failed modulus and the excavated modulus was set as 0.5 Pa (0.01 psf). However once failure had occurred, any excess shear stress on failed elements was not removed or redistributed. They simulated the excavation process by having 3 metre lifts and the base of the mesh was set fairly close to the final excavated level to simulate an underlying hard layer such as rock. The number of lifts varied from eight to ten. They simulated both normally consolidated and overconsolidated clays by using values of $K_0$ of 0.75 and 1.25 respectively.

The problems they encountered were firstly, that elements which had failed in shear had a certain amount of excess shear stress which had not been removed after failure, in some cases being as much as 20% of the undrained shear strength. The only way they reduced this excess shear stress was to make the excavation lifts small. Secondly, they had serious problems with displacements. In order to prevent the stresses changing on elements which had already failed they set the post-failure modulus
at a very low value which was at least two orders of magnitude less than the peak value. They state that, "although the values of modulus used in the analysis influence the magnitudes of the calculated displacements and strains, it has been found that they have no appreciable effect on the calculated values of stress, provided the modulus values used before and after failure differ by more than two orders of magnitude." By using low post-failure modulus values the failed elements deform around non-failed elements rather than undergoing significant stress change. However if the failure zone becomes large, this approach is not successful since the application of further excavation forces causes the displacements to be unrealistically large. In one example quoted by Dunlop and Duncan (1970), excavation could not proceed beyond the second stage because the failure zone was already very large, and some other examples only went to the fourth excavation stage. However in their examples which did proceed to the full excavation level they analysed the stability of the excavation by putting a series of slip circles through the slope. They assigned the normal undrained shear strength to the non-failed zone and the mobilized shear strength to the failure zone where the mobilized values of shear strength are the calculated values of shear stress for each element. Therefore on failed elements which had excess shear stress, the shear strength was greater than on non-failed elements which would not be the case with actual failures. It is obvious, therefore, that Dunlop and Duncan (1970) were not completely successful in developing a valid model for multi-stage excavation analysis even for "$\phi=0$" assumptions.
Dunlop and Duncan (1970) also studied the effect of Poisson's ratio on their results and initially used a value of 0.475 for undrained behaviour of clay. However they quote a formulation developed by Hermann and Toms (1964) which allows a value of 0.5 to be used. This formulation included in a finite element method takes approximately 70% more time than a conventional finite element method. Dunlop and Duncan (1970) demonstrated that the difference between using 0.475 and using 0.5 for Poisson's ratio was very minor and concluded that there was no benefit in using the more detailed method. A value of 0.475 has therefore been used in this present study as well.

Snithban and Chen (1976) studied deformations in an excavation with a vertical cut using the finite element method. However they did not apply excavation forces in the conventional manner, but induced failure by increasing the gravitational load and for this they increased the density up to 2.7 tonnes/m³ for clay. They found that the largest deformations were just above the toe of the cut, but their analysis was stopped before the deformations became excessive.

Chang and Duncan (1970) used the finite element method to analyse the extent of floor heave at the base of a large excavated pit, the Buena Vista Pumping Plant at Bakersfield, California. This excavation was 60 metres deep and approximately 460 metres in diameter at the crest of the pit. The pit was initially excavated down to 50 metres and floor heave up to 73 cms. was experienced. It was feared that further excavation might lead to
major slope failure and so a detailed finite element study was undertaken.

Chang and Duncan (1970) conducted detailed tests on the various horizons of sands and clays to obtain their physical properties especially modulus values both during loading and unloading. Values for $K_0$ were obtained from the relationship between $K_0$ and overconsolidation ratio as given by Brooker and Ireland (1965) and these were $K_0 = 0.9$ for clay and $K_0 = 0.5$ for sand. Values of the tangent modulus for all elements in the mesh were initially calculated and the excavation forces were applied. Three excavation stages were used and on the second and third stages a check was made to see if elements were loading or unloading so that the correct value of modulus could be determined. Instrumentation within the pit had shown the uplift to be time-dependent corresponding to only partial drainage during the period of excavation with complete uplift occurring some time later corresponding to full drainage conditions. To simulate this, Chang and Duncan (1970) used both no drainage and complete drainage models, the former simulating short-term conditions and the later simulating long-term conditions.

The results they obtained agreed very well with the results from instrumentation within the pit. At a location in the centre of the excavated pit the fast and slow analyses gave uplifts of 14 cms. and 68 cms. respectively, compared to an actual final uplift of 74 cms., which is within 10% of the calculated values. Other locations within the pit had a similar correspondence between calculated and measured uplifts. Chang and Duncan (1970)
also plotted zones of shear stress level and showed a small shallow failure zone at the base of the pit and they said that from this it could be concluded that the overall pit slopes were stable since the shear stress level in most of the pit slopes was approximately 0.5. However no stability analyses were performed.

Simpson et al. (1979) developed a computer model for the simulation of large excavations in London clay specifically to predict ground movements using the finite element method. They used a non-linear stress/strain function which was in three parts; elastic, pseudo-elastic, and plastic. They found that the modulus values measured in the laboratory for undrained conditions were often much less than those from field measurements. They quoted results of Butler (1975) who gave values of the ratio of undrained modulus ($E_u$) to undrained shear strength ($c_u$) of 100 for laboratory results, but $E_u/c_u = 500-1000$ for field results. To model this high stiffness at small strains they introduced a kinematic yield surface (KYS) in which straining was purely elastic irrespective of direction, but the range of this linear elasticity was only 0.02% strain. Above this value the KYS moved and straining was a combination of both plastic and elastic deformations since it was not wholly recoverable. At strains greater than 1%, deformations were assumed to be purely plastic. Simpson et al. performed back analyses on the New Palace Yard underground car park and the Neasden underpass to check calculated movements against actual movements. Their model of the New Palace Yard site was a finite element model with six excavation stages and they carried out a parametric study to determine calculated
curves which best fitted the known displacements. Their calculated curves were a good fit to the known data but it is interesting to note that Ward and Burland (1973), who tried to predict the movements prior to excavation, assumed linear elasticity and plane strain conditions and Simpson et al. (1979) state that, "their (Ward & Burland's) predictions were in many respects close to measured values." The Neasden underpass was modelled using three excavation stages, and the calculated results predicted up to 15 mm. more movement at the base of the retaining wall compared to actual movement. For both the New Palace Yard site and the Neasden site, large negative pore pressures were predicted at the base of the excavation, being -100 metres head (-980 kPa) and -50 metres head (-490 kPa) respectively.

This model was only attempting to predict movements and no attempt was made to predict shear stress levels in relation to ultimate failure. However, Simpson et al. (1979) adopted very different values of \( K_0 \) for the two sites investigated. For the New Palace Yard site they used \( K_0 \) values which varied from 0.5 at the surface increasing asymptotically with depth to a value of 2.5 at a depth of 74 metres after data from Burland and Hancock (1977). At the Neasden site they used constant values of \( K_0 \) for different depth ranges, being \( K_0 = 1.0 \) between 0-3 metres, \( K_0 = 2.0 \) between 3-8.5 metres, \( K_0 = 1.5 \) between 8.5-28 metres, and \( K_0 = 1.3 \) between 28-40 metres, after data from Sills et al. (1977). The two relationships adopted for \( K_0 \) with depth for the two sites studied are completely different. However, it is known that \( K_0 \) does vary across the London basin due mainly to variations in
overburden removal. But the value of $K_0$ used in a finite element stress analysis is crucial to the overall stability of excavated slopes as will be shown in subsequent Chapters.

Lo and Lee (1973a & 1973b) used a finite element analysis to explain slope failures in London clay using a strain-softening model. They analysed excavated clay slopes using different values of $K_0$ and from this they found areas of the mesh that had failed in shear or tension. By using Skempton's residual factor they assigned sections of potential failure surfaces as being at the peak or residual strength. From this they could then calculate a factor of safety and hence find the critical failure circle. However in comparison to actual slope failures in London clay, the factors of safety they obtained were still too high. Consequently they incorporated a strength decrease with time in their analysis (this being 6% per log cycle of time) and the results they obtained from this were in better agreement with actual failures as shown in Table 6.1. It should be noted that calculations of factor of safety were made using a limit equilibrium approach and not on the basis of actual stress distribution. Moreover, different strength parameters were used for portions of the slip surface passing through failed and unfailed zones, being peak and residual (not softened) strength parameters.

This model was the starting point for this present work and the actual method is described in more detail in Chapter 7.
Table 6-1.
Stability Analyses of first-time slides on excavated slopes in London clay
(after Lo & Lee, 1973)

<table>
<thead>
<tr>
<th>site</th>
<th>FOS wrt peak strength</th>
<th>FOS wrt residual strength</th>
<th>FOS with strain softening</th>
<th>FOS with strain softening &amp; time effects</th>
<th>Time to failure (years) actual</th>
<th>predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northolt</td>
<td>1.63</td>
<td>0.54</td>
<td>1.38</td>
<td>0.96</td>
<td>19</td>
<td>14</td>
</tr>
<tr>
<td>Sudbury Hill</td>
<td>2.27</td>
<td>0.74</td>
<td>1.98</td>
<td>0.97</td>
<td>49</td>
<td>42</td>
</tr>
<tr>
<td>Upper Holloway</td>
<td>1.62</td>
<td>0.55</td>
<td>1.44</td>
<td>0.95</td>
<td>81</td>
<td>55</td>
</tr>
</tbody>
</table>

NOTE: the above analysis was based on the following shear strength parameters:

\[
c_r' = 15 \text{ KPa}, \phi_r' = 20^\circ, c_p' = 1 \text{ KPa}, \phi_p' = 13^\circ, K_r = 2.5 \text{ (constant with depth)}, \mu_0 = 0.35, E_p = 13800 \text{ KPa}, E_r = 0
\]

A rate of decrease in drained strength with time was also applied and was equivalent to 6% per log cycle of time.

open pit mines to determine the extent of floor heave and its effect on stability. Yu and Coates (1979) simulated several Canadian open pit mines and concluded that the in-situ stresses and stiffness must be known before any meaningful results can be obtained. Dolezalova et. al. (1977) successfully used the finite element method to predict floor heave at the base of a 190 metre
open cut coal mine. Heave problems at the base of excavations for large civil engineering works have been studied in detail by many researchers (e.g. Kylm et al. (1977), Stroh and Breth (1976), Izumi et al. (1976), etc.).

6.3. THE DEVELOPMENT OF PORE PRESSURES

Consideration of developed pore-water pressures in slope analysis is extremely important and the total pore pressure is in many cases, the most important parameter controlling stability. The development of pore-water pressures is controlled by the rate of construction or excavation in relation to the mass permeability of the slope material. The choice of either undrained or drained analysis techniques is therefore also dependent upon the type of material and the rate of construction. Several authors have developed methods to simulate the development of pore pressures during either construction or excavation using the finite element method and a brief review is given here.

Naylor (1973) described a way of calculating excess pore pressures by using the finite element method. The basic assumption in the finite element method is that the stress matrix \( \sigma \) = the stiffness matrix \([D]\) times the displacement matrix \(\mathbf{E}\) and this can be written as:

\[
\sigma = [D] \mathbf{E} \quad (6.1)
\]

For undrained conditions Naylor (1973) assumed that the strains would be the same in the soil skeleton as in the pore fluid and therefore:
\[(\sigma') = [D'] (E)\]  \hspace{1cm} (6.2)

and

\[u = Ku (Ex + Ey + Ez)\]  \hspace{1cm} (6.3)

where the prime indicates effective, \(u\) = the excess pore-water pressure, and \(Ku\) = the apparent pore fluid bulk modulus. Equation (6.3) can be expressed as:

\[(\sigma_u) = [D_u] (E)\]  \hspace{1cm} (6.4)

Therefore the following relationships can be obtained for total and effective stress:

\[\sigma = (\sigma') + (\sigma_u)\]  \hspace{1cm} (6.5)

and

\[[D] = [D'] + [D_u]\]  \hspace{1cm} (6.6)

Naylor (1973) then input his material properties as effective stress components in \([D']\) and also input the pore fluid stiffness \(Ku\), which were combined by using equation (6.6). The total stresses were then calculated by using equation (6.1) and the excess pore pressures obtained by using equation (6.3). However Naylor (1973) and Simpson (1973) have pointed out that there are problems in using this method. Naylor (1973) used a bulk modulus for the pore fluid that was ten times the effective stress stiffness of the soil skeleton and this can lead to very high pore pressures on one element and very low pressures on an adjacent element. Simpson (1973) has pointed out that low permeability
clays only have a very high bulk modulus in the undrained state for a very short period of time, since only a very small migration of water is necessary to relieve the development of very high pore-water pressures. Both Naylor (1973) and Simpson (1973) have tried to overcome these problems by smoothing out pore pressures over a series of elements, but this is still not entirely satisfactory.

Osaimi and Clough (1979) studied the effects of pore pressure dissipation during excavations and were concerned with the fact that rapid dissipation of pore pressures during excavation would mean that undrained behaviour would not be applicable for some clays. They developed a finite element program to model heave and consolidation of excavations incorporating a flow matrix $[H]$ in terms of the permeability of the material in their formulation of the stiffness matrix. They also used a time-dependent factor $t$, so that pore pressures were calculated at the end of each time period (i.e. $t$ and $t+dt$) as well as the displacements.

Their results showed that the development of negative pore pressures during excavation was dependent upon the permeability and negative pore pressures up to 240 kPa (5000 psf) were calculated for a 9 metre high excavation in low permeability clay, and also that 50% pore pressure dissipation would take 2000 days. However for clays of higher permeability, pore pressure dissipation could occur fairly rapidly (60 days in the example quoted). These calculated pore pressures were compared with tests on clays performed in the laboratory and found to agree closely.
The generation and dissipation of pore-water pressures using the finite element method have also been studied by Verruijt (1977). He assumed that the soil would be an isotropic, linear, elastic, porous material and assumed values for the compression and shear modulus of the soil mass. By assuming that the stress tensor plus the volume force must equal zero for equilibrium, he split the stress tensor into that due to the pore pressure and that due to particle contact stress. At time $t = 0$ all variables were set to zero and by using a step time function, the pore pressures and stresses for each timed interval were determined.

Raymond (1972) used a finite element analysis to predict the undrained deformations and pore pressures below two embankments in different clays. However he did not calculate pore pressures directly from the finite element method, but used laboratory triaxial experiments to determine the pore pressure parameter $A$ and then combined this with principal stresses obtained from the finite element analysis to predict changes in pore pressures. He then compared the results with pore pressures obtained from instrumentation in the two embankments. Although there were variations, Raymond (1973) states that, "It seems reasonable to conclude that the measured maximum pore pressures fall within the range of likely values predicted from the triaxial test results and the use of principal stresses in Skempton's (1954) pore pressure equation." These results support the approach adopted later in this thesis (see Chapters 7 & 8) of determining pore-water pressures by using Skempton's pore pressure parameters $A$ and $B$. 
7. DEVELOPMENT OF A COMPREHENSIVE STRESS ANALYSIS APPROACH FOR EXCAVATIONS

7.1. METHOD OF ANALYSIS FOR PROGRESSIVE FAILURE

As outlined in Chapter 4, progressive failure is a characteristic of many slope failures and can be due to various factors. Therefore quantitative methods of slope analysis incorporating progressive failure mechanisms are complex, since the normal stress and shear stress will vary along any potential failure surface. It follows therefore, that the peak strength will be reached at some points before others, and it is possible that some portions of the failure surface will have exceeded the peak strength and will be approaching the residual strength, while other portions of the failure surface will not have even attained their peak strength. Analyses are therefore necessarily complex, and as indicated by Bishop (1971), would require both a finite element analysis to obtain the complete stress distribution incorporating a strain-softening material, as well as a limit equilibrium approach to define the critical failure surface. In this Chapter, the development of a suitable approach to model progressive development of failure within a slope is described. The approach is based on the finite element method. Factors of safety on trial failure surfaces are determined on the basis of stress analysis calculations, and a large number of failure surfaces are examined (over 100 for most cases) in order to locate the critical slip surface.
A detailed discussion of applying the finite element method to a strain-softening material with its associated stress release and transfer has been discussed by Zienkiewicz et al. (1968) and Zienkiewicz (1970). The method of reducing the excess shear stress on an overstressed element is shown in fig. 7.1a. The shear stress is reduced to the required value, which will depend on the shape of the stress-strain curve. Assuming the stress-strain curve used is that shown in fig. 7.2, then the shear stress is brought down to equal the lower or reduced strength. This is achieved by modifying the principal stresses such that the mean normal stress, and hence the strength, remain constant during iterations (Chowdhury & Gray, 1976). If the excess shear stress is reduced by using the method shown in fig. 7.1b, then the strength would reduce, and the new value of shear stress would still be greater than this new value of residual strength.

As outlined in Chapter 6, Lo & Lee (1973a & 1973b) developed a method of analysis adopting both a finite element analysis using a strain-softening material, as well as a limit equilibrium solution for the critical failure surface. They used the finite element method to define areas of the mesh where the shear stress was greater than the peak strength. The shear stress on these areas was then reduced to the residual strength, and excess shear stresses were redistributed throughout the continuum. This in turn may cause other elements to fail, and so the process was repeated until any remaining excess shear stresses were very small and therefore negligible. The finite element analysis also defines areas of the mesh which have failed in tension, and these
Fig. 7.1 MODIFICATION OF SHEAR STRESSES IN AN OVER-STRESSED ELEMENT

(a) correct method
(b) incorrect method

envelope of residual strength

$\Delta \tau$

Fig. 7.1 MODIFICATION OF SHEAR STRESSES IN AN OVER-STRESSED ELEMENT
(after Chowdhury & Gray 1976)
Fig. 7.2 IDEALISED STRESS STRAIN CURVE WITH ABRUPT DECREASE IN POST-Peak STRENGTH

(adopted, amongst others, by Lo & Lee, 1973)
are similarly reduced to their tensile strength (in the Lo & Lee analysis the tensile strength was set to zero, however no redistribution of tensile stresses was performed). If a limit equilibrium analysis is then applied to the mesh, potential slip surfaces may pass through shear failure zones, tensile failure zones and the remaining non-failed zone. These zones are then assigned the residual, tensile and peak strength parameters respectively.

Lo & Lee (1973b) applied their method of analysis to London clay and the factors of safety they obtained for known slope failures were generally too high, even using a strain-softening material. To try and reduce their calculated factors of safety they also included in their analysis a 6% decrease in drained strength per log cycle of time as mentioned in Chapter 6. This decrease in strength value is in agreement with Skempton & Hutchinson (1969), who quote values of the decrease in the drained strength with time as 6% per log cycle of time for Cambridge Clay, slightly more for Brown London clay and up to 14% for Fornebu clay in Oslo. The three slope failures in London clay analysed by Lo & Lee (1973b), Northolt, Sudbury Hill and Upper Holloway, incorporated this strength decrease with time. The final factors of safety they obtained were 0.96, 0.97 and 0.95 respectively (as shown in Table 6.1, Chapter 6). Since these results were very close to 1.00, this method of analysis was considered successful, and it was decided to adopt their computer program as a method of analysing the the stability of excavations. A copy of their program was obtained, but it was discov-
7-6

ered that this must have been a very old version of the program, since extensive modifications had to be made before it would run successfully.

Initially the same mesh as that used by Lo & Lee (1973b) was adopted, since it allowed for a larger number of elements near the slope surface where more detailed information is required, and a smaller number of elements further away from the slope, thus economising on the total number of elements. This mesh is shown in fig. 7.3, and initially the input data used was the same as that used in an example problem described by Lo & Lee (1973b), i.e.

\[ c_p = 14.4 \text{ KN/m}^2, \quad \phi_p = 30^\circ \text{ for peak strength}, \]
\[ c_r = 0, \quad \phi_r = 15^\circ \text{ for residual strength}, \]
\[ u = 0.35 \text{ for Poisson's ratio}, \]
\[ \gamma = 2.0 \text{ tonnes/m}^3 \quad \text{for bulk density}, \]
\[ E_1 = 13800 \text{ KN/m}^2 \text{ for pre-peak modulus}, \]
\[ E_2 = 0.07 \text{ KN/m}^2 \text{ for post-peak modulus}. \]

NOTE: modulus kept constant with depth

The water table was assumed to be 1.5m. below the ground surface, and the coefficient of earth pressure at rest in terms of effective stress, was taken to be 1.0 and 2.0.

The Mohr-Coulomb failure criterion can be expressed in terms of peak and residual strength for a strain-softening material by two straight lines by using the parameters \( c_p \phi_p, c_r \phi_r \) (where \( p \) equals the peak strength value and \( r \) equals the residual strength value). These strength values correspond to the idealisation represented by the stress/strain curve shown in fig. 7.2. This
Fig. 7.3  FINITE ELEMENT MESH AFTER LO & LEE (1973)
curve is the one adopted by Lo & Lee (1973b) to represent the behavior of London Clay, and exhibits an abrupt drop from peak to residual strength, which is probably more gradual in practice. However this simple model enables simplification of the stress release and transfer process, and any errors involved with assuming an abrupt decrease from peak to residual strengths will be on the conservative side. This idealization was, therefore, used in the present analysis.

The forces applied to the surface nodal points to simulate excavation were calculated after Chowdhury & King (1970).

7.2. EFFECTIVE STRESS ANALYSIS: SINGLE STAGE EXCAVATION

Once the results from the initial runs were obtained it was apparent that too many elements were failing compared with the results of Lo & Lee (1973b) even when using their input data. Several modifications were again made to the program, the most significant being to change the value of Young's Modulus on elements that fail in tension to a low value.

Typical results for a single-stage analysis for $K_q = 1.0$ are shown in fig. 7.4, and the results obtained by Lo & Lee (1973b) for an identical problem are shown in fig. 7.5. The results for $K_q = 2.0$ both for this present analysis and for Lo & Lee's analysis are shown in figs. 7.6 and 7.7 respectively. As can be seen, the results are similar, especially for $K_q = 1.0$. For $K_q = 2.0$ differences occur along the floor of the excavation where more elements fail using the present analysis. It was initially thought that the large failed areas in the mesh were due to slight differences in mesh configurations, but in fact the reason
for such large failed areas is the assumed value of $K_0$ used for the analysis. This problem is discussed in detail later, but it should be pointed out that the large failed areas of the mesh in fig. 7.6 are not due to errors in the method itself, but to the assumptions made in regard to the in-situ stresses. However the area of the mesh where a potential failure surface would pass through has a similar failed zone both for fig. 7.6 and 7.7. Therefore calculated factors of safety would correspond with those of Lo & Lee (1973b).

7.3. EFFECTIVE STRESS ANALYSIS: INCREMENTAL EXCAVATION

The initial analyses were performed using a single-stage excavation which is probably the same procedure adopted by Lo & Lee (1973b). The only mention of a stage excavation by Lo & Lee is when they state that they analysed a 21 m. high 25 degree slope by first performing the analysis on a 9 m. high 25 degree slope, and then completing the analysis by "stage excavation down to a depth of 21 m." However a single stage finite element excavation procedure is invalid except for the case of linear elasticity. For embankments a single stage procedure is invalid even for linear elasticity. Also for a single stage analysis the pore-water pressures are set to their long-term values at the same time as the applied excavation forces. This is obviously not the case in practice, especially with analyses on materials of low permeability. It was decided therefore to simulate the process of excavation by using a multi-stage excavation analysis. To simplify the excavation process, a different mesh was chosen as shown in fig. 7.8. This mesh enabled a sequential numbering of elements to be
Fig. 7.4 CONVENTIONAL EFFECTIVE ANALYSIS $K_0 = 1.0$ SINGLE STAGE EXCAVATION
shear failure

Fig. 7.5 RESULTS OBTAINED BY LO & LEE
FOR A SINGLE STAGE ANALYSIS WITH $K_o = 1.0$
shear failure

Fig. 7.6 CONVENTIONAL EFFECTIVE ANALYSIS
SINGLE STAGE EXCAVATION

K₀ = 2.0

tensile failure
Fig. 7.7 RESULTS OBTAINED BY LO & LEE FOR A SINGLE STAGE ANALYSIS $K_0 = 2.0$
adopted for adjacent excavated and non-excavated elements and hence the half-band stiffness matrix was kept to reasonable dimensions. As can be seen, this mesh required a larger number of elements than the initial mesh to maintain the same degree of fineness near the toe of the slope. This necessitated some elements having elongated shapes but only in areas of the mesh remote from the critical zone (toe) of the slope area. This mesh is similar to those adopted by other workers for multi-stage excavations (e.g. Dolezalova et.al. (1977), Kylm et.al. (1977)).

Chandrasekaran and King (1974) developed a computational process to reduce the numerical errors involved with an incremental analysis, and their procedure was incorporated in the computer program.

7.4. ADJUSTMENT OF WATER TABLE LEVELS

The method of lowering the water table in a six stage analysis was initially as shown in fig. 7.9. From fig. 7.9 it can be seen that the water table was not lowered to its final position until after the final excavation stage had been completed. Consequently the water pressures were highest just after the completion of the first excavation stage. This led to the calculated result that after the first stage of excavation the slope was in its least stable condition in terms of effective stress, which is not the case in practice. Figs. 7.10 and 7.11 show the results for six stage analyses with $K_o = 1.0$ and $K_o = 2.0$ respectively. For analysis with $K_o = 1.0$ the results are almost identical to that shown in fig. 7.4 since the extent of the failure zone is very small, therefore most of the mesh behaves as a linear elas-
Fig. 7.8 MESH FOR MULTI-STAGE EXCAVATION
METHOD OF LOWERING WATER TABLE DURING INCREMENTAL EXCAVATION (INCORRECT)

Fig. 7.9
This is based on the gradual adjustment of pore pressures according to the water table levels shown in Fig. 7.9.
This is based on the gradual adjustment of pore pressures according to the water table levels shown in Fig. 7.9
tic material and consequently the results are almost identical. However for analysis with \( K_0 = 2.0 \) the results for single and six stage analysis are quite different.

Comparison of fig. 7.6 with 7.11 reveals that the extent of the failure zones in the incremental analysis is much larger than in the single-stage analysis. A consideration of the pore-water pressures used in the incremental analysis revealed that the most stable condition was in fact in the final long-term situation, which should be the least stable condition. A detailed explanation of this is given in the following section, but its effects were to cause a large number of elements to fail, in this case, on the sixth stage of excavation, and then make them more stable by lowering the water table. However the process of failing an element and redistributing excess shear stress is irreversible, and consequently the failure zones for the incremental analysis were much larger than for the single-stage analysis.

The stages of analysis for six stage excavation are shown in fig. 7.9. As can be seen from this figure, the water table is maintained along the surface of the slope throughout stages 1 to 6 and then finally lowered to its long-term position. This appeared to be the most reasonable approach to the problem since the long-term water table condition would not be reached until a considerable time after excavation occurred, especially in London clay which has a low permeability. However, since the analysis is performed in terms of effective stress (i.e. \( \sigma' = \sigma - u \)), when the water table is lowered to its final position \( u \) decreases,
therefore the effective stresses $\sigma_1'$ and $\sigma_3'$ increase. Since the effective shear stress is given by:

$$\frac{(\sigma_1 - u - (\sigma_3 - u))}{2}$$

(7.1)

the effective shear stress remains constant irrespective of the value of $u$. The effective shear strength with respect to peak strength is given by:

$$C_p' \cos \phi_p' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right) \sin \phi_p'$$

(7.2)

and with respect to residual strength is given by:

$$C_r' \cos \phi_r' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right) \sin \phi_r'$$

(7.3)

Therefore using this method of analysis, both the peak and residual effective shear strengths will increase in the long-term condition, which is not the case in practice.

Bishop and Bjerrum (1960) have shown that the factor of safety decreased with time since the pore-water pressures immediately after excavation are lower than they are in the long-term condition. There are therefore difficulties involved in using an incremental analysis with a conventional effective stress approach and Skempton and Hutchinson (1969) state that this approach is usually only performed for "slopes in the long-term condition when the pore pressures are most easily determined." To obtain a realistic assessment of pore pressures and factors of safety during excavation, at the end of excavation and in the long-term, a different approach was developed.
7.5. TOTAL/EFFECTIVE STRESS APPROACH

On the basis of the principle of effective stress, pore pressure changes during excavation must be dependent on, and related to, the reductions in stress due to excavation. To determine excess pore pressures realistically, a knowledge of the total stresses and pore pressure parameters for the soil mass is, therefore, essential.

Fig. 7.12 shows the value of the pore-water pressure immediately after an excavation is dependent upon the value of the pore-water pressure parameter $A$, but that its the long-term value is independent of $A$. Skempton (1954) introduced the pore pressure parameters $A$ and $B$ where, in a standard undrained triaxial test, $B = \frac{\Delta u_1}{\Delta \sigma_3}$, $\Delta u_1$ being the pore pressure increase developed in the application of the all round stress $\Delta \sigma_3$; and, $A = \frac{\Delta u_2}{B(\Delta \sigma_1 - \Delta \sigma_3)}$, $\Delta u_2$ being the pore pressure increase developed on the application of deviator stress. Hence the change in pore pressure is given by:

$$
\Delta u = \Delta u_1 + \Delta u_2
$$

(7.4)

For saturated soils $B = 1.0$ and $A$ can range from about -0.5 for heavily overconsolidated clays to 3.0 for very loose fine sands (Lambe and Whitman, 1969). The value of the parameter $A$ is not a constant soil property and can vary with stress level, stress history, test conditions and sample disturbance. Simons and Som (1969) have shown that London clay typically has values of $A$ ranging from 0.2 to 0.6. For compression tests the value of $A$ is approximately 0.6 for low stress levels and decreases to 0.4 at failure. For extension tests, $A$ is approximately 0.2 for low
Fig. 7.12 CHANGES IN PORE PRESSURE AFTER AN EXCAVATION, DEPENDENCE UPON A 
(after Bishop & Bjerrum, 1960)
stress levels and increases to 0.4 at failure. Their test results are shown in fig. 7.13.

The parameters A and B can be used to find the change in pore-water pressure by the following expression:

\[ \Delta u = B(\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)) \]  \hspace{1cm} (7.5)

for saturated soils \( B = 1.0 \) and this simplifies the equation to:

\[ \Delta u = \Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3) \]  \hspace{1cm} (7.6)

where \( \Delta u \) = change in the value of pore-water pressure, \( \Delta \sigma_1 \) = change in the value of the major principal stress in terms of total stress, \( \Delta \sigma_3 \) = change in the value of the minor principal stress in terms of total stress.

This presents the problem that changes in total stresses must be known to calculate the change in the pore-water pressure. However at every stage of the excavation and in the long-term, calculation of stability requires knowledge of effective stresses and effective strength parameters. If a purely effective stress approach were adopted, the variation in pore pressure with time after the excavation has taken place could not be determined on the basis of pore pressure parameters. If, on the other hand, a purely total stress approach were adopted, stability calculations could only be based on the short-term \( \Delta = 0 \) assumption. Such an analysis does not enable the calculation of stress distributions or pore-water pressures on a realistic basis even during excavation. Moreover it is invalid for the long-term stability anal-
Fig. 7.13 PORE PRESSURE PARAMETER $A$ vs STRESS RATIO FOR LONDON CLAY (SIMONS & SOM, 1969)
ysis. Accordingly, there are advantages in combining the two approaches.

This new total/effective stress approach involved setting up the initial stresses and applying the excavation forces (which are based on the initial in-situ stress field) in terms of total stress. This enabled the change in total stresses \( \Delta \sigma_1 \) and \( \Delta \sigma_3 \) and hence \( \Delta u \) to be calculated. The initial pore-water pressure on each element was simply set as equal to \( \nu w H \) where \( \nu w \) is the density of water and \( H \) is the depth below the original water table. The new pore-water pressure after a stage of excavation then became:

\[
u_n = \nu_o + \Delta u
\] (7.7)

where the suffix \( n \) represents the new value and the suffix \( o \) represents the old value. The effective stresses at any stage of excavation are therefore:

\[
\sigma_1' = \sigma_1 - u_n
\]
\[
\sigma_3' = \sigma_3 - u_n
\] (7.8)

The stability analysis in terms of effective stresses can then be conducted in the normal manner by comparing the shear stress with the peak shear strength. If the shear stress exceeds the peak shear strength, the shear strength is set to the softened or lower limiting strength value and the excess amount of shear stress \( \Delta \tau \) (shear stress - softened strength) is calculated (see also section 7.7.1).
7.6. REDISTRIBUTION OF EXCESS SHEAR STRESS ON FAILED ELEMENTS WITHIN THE CONTINUUM

The stress changes \( \Lambda \sigma \) on the XY plane which are necessary to reduce the excess shear stress on the plane of maximum shear are given by Zienkiewicz et al. (1968) and are:

\[
\Lambda \sigma = \begin{cases} 
\Lambda \sigma_x = -\Lambda \tau \cos 2\alpha \\
\Lambda \sigma_y = \Lambda \tau \cos 2\alpha \\
\Lambda \tau_{xy} = -\Lambda \tau \sin 2\alpha
\end{cases}
\]  

(7.9)

where \( \alpha \) is the inclination of \( \sigma_4 \) to \( \sigma_x \). From this, the excess stresses \( \Lambda \sigma \) on the elements which have failed, have to be distributed throughout the other elements of the mesh. The method of stress release and transfer involves creating a series of nodal forces to simulate the removal of the excess stresses \( \Lambda \sigma \). This is given by:

\[
(F) = \int [B]^T (-\Lambda \sigma) \, dV
\]  

(7.10)

where \([B]^T\) is the transpose of the position matrix \([B]\) in the finite element analysis, \((-\Lambda \sigma)\) are stress increments equal and opposite to \(\Lambda \sigma\) and \(\int dV\) is the volume of the element. The new effective stresses are then given by:

\[
\begin{align*}
\sigma_{xn}' &= \sigma_{xo} - \Lambda \tau \cos 2\alpha \\
\sigma_{yn}' &= \sigma_{yo} + \Lambda \tau \cos 2\alpha \\
\tau_{xyn}' &= \tau_{xyo}' - \Lambda \tau \sin 2\alpha
\end{align*}
\]  

(7.11)

from which the effective principal stresses are given by:

\[
\sigma_{ln}' = \left( \frac{\sigma_{xn} + \sigma_{yn}}{2} \right) + \sqrt{\left( \left( \frac{\sigma_{xn} - \sigma_{yn}}{2} \right)^2 /4 \right) + (\tau_{xy})^2}
\]
The new total stresses are then set as:

\[ \sigma'_{1n} = \sigma_{1n} + u_n \]
\[ \sigma'_{3n} = \sigma_{3n} + u_n \]  

(7.13)

and these stresses are then put back into the finite element analysis.

7.7. PORE PRESSURE DURING ITERATIONS CONCERNED WITH STRESS RELEASE AND TRANSFER

This method of stress release is illustrated on a Mohr diagram in fig. 7.14. It can be observed that the pore-water pressure remains constant throughout the removal of excess shear stress and that:

\[ \frac{\sigma_{10} + \sigma_{30}}{2} = \frac{\sigma_{1n} + \sigma_{3n}}{2} \]  

(7.14)

After much experimentation it was concluded that this was the most logical approach to adopt. However it must be noted that this applies only during iterations with each stage and not from stage to stage.

Originally it was thought that if the principal stresses were changed to reduce the excess shear stress, then this would also cause the pore-water pressure to change, and a method was developed to try and simulate this. For an element that fails in shear, the effective stresses are changed such that the shear stress equals the residual strength. The new effective stresses must also conform to the condition that:
Fig. 7.14 RELATIONSHIP BETWEEN OLD AND NEW TOTAL AND EFFECTIVE STRESSES
\[(\sigma_{1n}' + \sigma_{3n}')/2 \text{ must equal } (\sigma_{10}' + \sigma_{30}')/2 \quad (7.15)\]

and therefore \(\sigma_{1n}'\) and \(\sigma_{3n}'\) are uniquely defined as shown in fig. 7.14. It was assumed that the change in total stresses resulting from this change in effective stresses could be obtained by using equation (7.6).

The approach initially used was to assume new values of total stress which would have the same centroid on the Mohr diagram as the old total stresses. The change in the values of total stress \(\Delta \sigma_1\) and \(\Delta \sigma_3\) could then be obtained and substituted in equation (7.6) to give \(\Delta u\). Thus, the new pore-water pressure, \(U_n\) could be obtained. If this new value of \(U_n\) were added to the new values of effective stress, the new values of total stress could be obtained. However, it would be unlikely that these two calculated values of total stress would equal the two assumed values of total stress that were used initially. The process was then repeated until the assumed values of total stress equalled the calculated values of total stress. An iterative routine was written to perform this task. Various values of the pore-pressure parameter \(A\) were also used in the calculations and the results are shown in fig. 7.15. Clearly, at no single point do the assumed values equal the calculated values, and this iterative process was therefore non-convergent.

A reconsideration of the approach adopted gave the conclusion that, the assumption that the pore-water pressure changes for every change in total stress, does not apply when the total stress is artificially and drastically changed as it is during iteration on elements which fail in shear. The procedure adopted,
Fig. 7.15 Plot of Assumed vs Calculated Total Stress for various values of A (for a typical element during iteration for excess shear stress)
therefore, was to keep the pore-water pressure at the same value it had before failure took place so that the redistribution of excess shear stress during iteration cycles would not change the pore-water pressures.

7.7.1. Further considerations of the change in Pore Pressures

For a general three dimensional stress system the equation for excess pore pressure in a saturated soil may be written as follows:

\[ \Delta u = \frac{1}{3}(\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3) + \alpha \sqrt{\left(\frac{\Delta \sigma_1 - \Delta \sigma_2}{2}\right)^2 + \left(\frac{\Delta \sigma_2 - \Delta \sigma_3}{2}\right)^2 + \left(\frac{\Delta \sigma_3 - \Delta \sigma_1}{2}\right)^2} \]

\[ + \left(\frac{\Delta \sigma_2 - \Delta \sigma_3}{2}\right)^2 + \left(\frac{\Delta \sigma_3 - \Delta \sigma_1}{2}\right)^2 \]  

(7.16)

Comparing this to the simple Skempton equation (equation (7.6)) for \( \Delta \sigma_2 = \Delta \sigma_3 \) (axi-symmetrical case), it is easy to show that:

\[ \alpha = \frac{(A - 0.33)}{\sqrt{2}} \]  

(7.17)

For plane strain deformation \( \Delta \sigma_2 = \nu(\Delta \sigma_1 + \Delta \sigma_3) \). Furthermore, Poisson's ratio (\( \nu \)) for total stress equals 0.5 when considering undrained deformation of saturated clay. Therefore \( \Delta \sigma_2 = 0.5(\Delta \sigma_1 + \Delta \sigma_3) \) and the general equation given above (7.16) yields the following for plane strain conditions:

\[ \Delta u = \frac{(\Delta \sigma_1 + \Delta \sigma_3)}{2} + \frac{3}{2} \alpha (\Delta \sigma_1 - \Delta \sigma_3) \]  

(7.18)

in which \( \alpha \) is given as in equation (7.17).

All the analyses in this thesis have been based on Skempton's pore pressure equation (equation 7.6). However a check of the calculated pore pressures was also made on the basis of the plane strain equation given above (equation 7.18). On the whole, the differences in pore pressures calculated from the two
different equations were not significant, and therefore Skempton's equation (equation 7.6) was used to determine pore pressures in the stability analyses.

7.8. OTHER FEATURES OF THE COMPUTER PROGRAM

7.8.1. Modulus in zones of shear and tensile failure

One aspect of the program which caused a large number of elements to fail was the value of Young's modulus, $E$, given to elements that failed in tension. On the initial computer program, elements that failed in tension had the same value of Young's modulus both before and after failure. This enabled elements that had already failed in tension to remain relatively very stiff and give more concentrated loadings to adjacent non-failed elements. Consequently, a large number of elements were failing in shear. The value of $E$ was therefore reduced to a very low value for elements that failed in tension (the same as that used for shear failure elements i.e. 0.07 KN/m²) and this reduced the number of failures. This approach was later found to be unsatisfactory and refinements had to be developed based on a sensitivity analysis as discussed in Chapter 8.

7.8.2. Designation of failure zones

Another aspect of the analysis that became a problem was the designation of shear and tensile failures. The initial program simply had a test for tensile failures and then a test for shear failures. However, it was found, especially on analyses that had a large number of failed elements, (such as when $K_o$ was greater than 1.0) that there would be a large number of tensile failures
in areas of the mesh that were designated as shear failure elements in the original program. An examination of the actual run of the program revealed that these areas would have failed in shear if the test for shear failure had been put before the test for tensile failure. To overcome this problem, and to give more information about the type of failure on the element in question, a series of tests were included to differentiate between tensile failure, shear failure and tensile and shear failure. Elements that had both tensile and shear failures were those elements in which $\sigma_3'$ was negative (the tensile strength was assumed to be zero), and which at the same time had a shear stress higher than the peak strength. The basis for including these tests was that in natural slopes an element of soil would fail in tension or shear at a point when the tensile or shear strength is just exceeded. In the finite element analysis, the stress changes due to an excavation stage are large enough, such that in some cases, both the tensile and shear strength are exceeded simultaneously. If the stress changes were in smaller increments then either the tensile or shear strength would be exceeded first, and one could assign the element as a purely tensile or purely shear failure element. To increase the number of stages of excavation may help, but would not completely solve the problem since shear and tensile failures occurred when excess stresses were being redistributed from other shear failure elements. The errors involved in such an approach would, in any case, be small. One would not know with absolute certainty if an element would fail in tension or shear, but from previous finite element work (e.g. Dunlop 

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Duncan, 1969, Chang & Duncan, 1970, Stacey, 1970, Lo & Lee, 1973, Yu & Coates, 1979, etc.), and from practical observation of slope failures (e.g. Skempton & La Rochelle, 1965, Bjerrum, 1967, Skempton & Hutchinson, 1969), it is known that the zone of tensile failure is generally at the crest of the slope and the zone of shear failure is generally at the toe of the slope. Therefore in the current program the major difference between shear and tensile failure of an element is that elements which fail in shear are assigned a lower strength (softened or residual), whereas elements which fail in tension are assigned zero strength.

7.9. INITIAL TRIALS WITH TOTAL/EFFECTIVE STRESS PROGRAM

The initial runs of the total/effective stress program were made using the same data as that used by Lo & Lee (1973b) for their example problem (see Table 6.1) except that Poisson's ratio was increased to 0.48. The results of these runs can be seen in figs. 7.16 to 7.17 for $K_o = 1.0$ for both single and six stage excavations. As would be expected when using the same data as for an effective stress run, but using a total/effective stress analysis, the results are entirely different. Different values for the pore-water pressure parameter $A$ were also tried, being -0.5, 0, 0.5, and 1.0, but there were only slight differences in the final extent of the failure zone. Fig. 7.18 shows the results for $K_o = 2.0$ for a single-stage analysis which was typical of the results obtained. For a six stage analysis almost the entire mesh failed and the results became meaningless. For $K_o = 1.0$ the failure zones are almost identical for both single and six stage
Based on data used by Lo and Lee (1973).
Based on data used by Lo and Lee (1973).

Fig. 7.17 TOTAL/EFFECTIVE ANALYSIS SIX STAGE EXCAVATION $K_0 = 1.0$
Fig. 7.18 TOTAL/EFFECTIVE ANALYSIS SINGLE
Based on data used by Lo and Lee (1973).
excavations regardless of the value of $A$ used. However, the major tensile zones are at the crest of the slope and the major shear failure zones are around the toe and along the floor of the excavation. As explained in Chapter 8, it is not surprising that unrealistic results are obtained when $K_o = 2.0$ (constant with depth) is used, since the initial state of horizontal stress implied by $K_o = 2.0$ represents failure in significant areas even before excavation has begun (refer fig. 8.16). In short, $K_o = 2.0$ is shown to be an unrealistic assumption not only on this basis, but also on the basis of published field data.

The main differences between the Lo & Lee (1973b) analysis and the present total/effective stress approach are that: (a) Lo & Lee worked mainly on the basis of single stage excavation; (b) a multi-stage analysis was made for an example problem but the question of pore pressures during excavation was not resolved; and (c) Lo & Lee considered only effective stresses and this approach appears to be successful as long as the question of pore-water pressures during excavation is ignored. The present method (a) considers both total and effective stresses; (b) incorporates a comprehensive multi-stage approach; and (c) computes and monitors pore pressures, effective stresses and total stresses during excavation. Using the data of Lo & Lee (1973b) for the initial total/effective stress runs enabled a check to be made on the method itself and to allow for rectification of "bugs" within the program. However, it should be noted that more realistic data had to be used for analysis of case histories as discussed in Chapter 8. When the program had reached this stage, more realistic input
data was used. (Further refinement and development of the inte-
grated total/effective stress model is explained in the next
Chapter).

7.10. A REALISTIC PROCEDURE FOR STABILITY ANALYSES

Stability Analyses are generally performed on excavated slopes
using one of the limit equilibrium methods as outlined in Chapter
3. Some authors have also combined conventional limit equilibri-
um stability analyses with the finite element method and assigned
different strength properties to different portions of the fail-
ure surface. However this method only uses the finite element
method to define areas of the mesh that have failed. It does not
actually use the calculated stresses for each element. This is an
unrealistic approach since it does not take into consideration
the initial stress field and the change and reorientation of
principal stresses with excavation. It is especially unrealistic
for overconsolidated soils. Therefore errors may occur with this
type of stability analysis. In this study it was decided to use
the actual stresses as derived from the finite element method in
the stability analysis. The factor of safety on a slip surface
was taken as the ratio:

\[ F = \frac{\text{total resisting moment}}{\text{total disturbing moment}} \]

Since the slip surfaces were approximated as circular in cross
section, one may abbreviate with:

\[ F = \frac{\text{total resisting force}}{\text{total disturbing force}} = \frac{\bar{\tau}_{\text{shear strength}}}{\bar{\tau}_{\text{shear stress}}} \]
It may be noted that, contrary to the limit equilibrium approach, the local factor of safety is not assumed to be equal to the overall factor of safety. Such an assumption often leads to a stress distribution which is different from the one based on stress analysis and especially so in different stages of excavation in a soil with high initial stresses.

The finite element method adopted in this study was a multi-stage excavation technique which consisted of seven stages; these being six excavation stages and a final stage in which the pore-water pressures were adjusted to their final long-term levels. After each of these stages the stresses on each element were stored for later use. After the finite element program completed an analysis, a stability program retrieved the stored stress information and calculated a factor of safety using these in-situ stresses. This was done in a conventional manner by drawing a slip circle through the slope and dividing the slope up into a series of slices. For each slice, the mid-point at the base of the slice was calculated. This point is on the failure surface and so the stresses were interpolated for this point from the closest adjacent elements. The angle of orientation of the base of the slice as well as its length, were also calculated so that the shear and normal stresses operating on the base of the slice could be determined. The normal stress and pore-water pressure was simply used to calculate the shear strength using the Mohr-Coulomb failure criterion. These stresses were then converted to forces with respect to the length of the base of the slice and then summed for all slices. The factor of safety was then cal-
8. INITIAL ANALYSES OF CASE HISTORIES AND REFINEMENT OF THE PROPOSED MODEL

8.1. INTRODUCTION

It is often difficult to analyse a slope failure due to lack of quantitative data. For the finite element stability analysis model used in this study, accurate data was particularly important since a slight variation in strength parameters or in-situ stresses could seriously affect the results. Also for any study, it would be useful to have several case histories of slope failures to check the method. For these reasons it was decided to use this model to study case histories in London clay. The physical properties of London clay have been well documented over the years by Skempton, Bishop and co-workers at Imperial College and also by workers at the Building Research Station in England. Back analyses have been performed on many excavated slope failures to determine in-situ strength parameters and information is also available on in-situ stresses. Therefore this Chapter describes initial studies of case histories in London clay based on the finite element stability analysis model. As a consequence of this work, further refinements in the model were made before complete studies, described in Chapter 9, could be carried out.

8.2. THE GEOLOGY OF THE LONDON BASIN

The clay which underlies most of the London Basin is a stiff, fissured, overconsolidated, blue-grey clay and was called the London clay by William Smith in 1812. The sediments were deposited under marine conditions in the Eocene period, about 30 mil-
lions years ago, and subsequently the Claygate beds, followed by the Bagshot, Bracklesham and the Barton beds were deposited. These were all predominantly sandy beds with occasional clay layers. However, uplift and erosion in the late Tertiary and Pleistocene have removed most of the overlying beds and half to two-thirds of London clay itself; and only in a few areas do any of the overlying beds remain (e.g. the Bagshot beds at Hamstead, Bishop et al. (1965)).

Much of this erosion took place at intervals during the Pleistocene glaciations, and following each period of downcutting terrace gravels were deposited by the River Thames. These include the Taplow gravels which are thought to have been formed some 150,000 years ago, and the Flood Plain gravels which were formed about 50,000 years later. The alluvium which overlies the Flood Plain gravels is recent post-glacial material and contains Neolithic as well as Roman remains.

The amount of material removed by erosion varies from place to place. Skempton and Henkel (1957) estimated a pre-consolidation load of 2145 KN/m² for the central London area suggesting a removal of 150 to 210 metres of material. After further work Skempton (1961) suggested a removal of about 150 metres of material for the Bradwell area in Essex, 80 kilometres north-east of London. However tests by Bishop et al. (1965) on London clay at the Ashford Common shaft site at a depth of 35 metres suggests a pre-consolidation load of 4,190 KN/m² which would correspond to an overburden of 365 to 400 metres of submerged sediments (a removal of approximately 335 metres of material). This seems to
suggest that the thickness of overlying sediments was greatest in the western part of the basin.

At the present time London clay has a maximum thickness of 150 metres and overlies the Woolwich and Reading beds as shown in fig. 8.1. These are hard, mottled, red and brown clays and sands which were deposited under estuarine conditions. Below these beds lie the Thanet sands which rest unconformably on the Chalk. Little geotechnical data exists on the Reading beds since they are too deep for most foundation work, although according to Skempton and Henkel (1957) it is a stronger and less compressible material than London clay.

The London clay itself consists of a lower sandy clay; varying from 0 to 3 metres thick and is known as the Basement bed. This is overlain by a blue-grey clay varying from 30 to 150 metres thick which is London clay proper, and finally near the surface is the brown clay varying from 0 to 10 metres thick. The sands and gravels previously mentioned overlie London clay in many areas, as does alluvium near the River Thames, and soft Marsh clay and peat near the sea.

The upper layer of the so called Brown London clay is yellow brown near the surface to grey brown at depth due to oxidation of the iron salts in the blue London clay probably when the ground water level was low. This clay is strongly weathered to a depth of about 1.5 metres below ground surface and has a fragmented texture. Below this the clay becomes more homogeneous until about 3 to 5 metres when no obvious sign of weathering is present except the brown staining which continues to a depth of about 10
Fig. 8.1 GEOLOGICAL SECTION THROUGH CENTRAL LONDON
metres. However the brown clay is much thinner and often completely absent beneath the Terrace gravels, indicating that there was little delay between erosion of the clay and deposition of the Terrace gravels. This clay immediately beneath the gravels is often soft and shattered (Skempton and Henkel, 1957) due to intense physical weathering, but this layer is only about 1 metre thick, and then there is a sudden transition to the unweathered blue clay.

The structure of London clay on a regional scale has the form of a very gentle syncline with some minor folding in places (Ward et al., 1959), although dips of more than 3 degrees are rare.

8.3. PHYSICAL PROPERTIES

The physical properties of London clay have been studied in great detail over the last 30 years and reliable information is readily available in the literature. The major publications on the physical properties of London clay including pore pressure parameters and in-situ stresses, are as follows: Bishop et al. (1965), Bishop (1966), Bishop & Henkel (1962), Bishop et al. (1971), Bishop et al. (1973), Burland & Hancock (1977), Burnett & Fookes (1974), Chandler & Skempton (1974), DeLory (1957), Henkel (1957), Henkel (1959), Marsland (1971), Sandroni (1977), Simmons & Som (1969), Sills et al. (1977), Skempton (1959), Skempton (1961), Skempton (1964), Skempton (1970), Skempton (1977), Skempton & De Lory (1957), Skempton & Henkel (1960), Skempton & LaRochelle (1965), Skempton & Petley (1967), Skempton et al. (1969), Skempton & Hutchinson (1969), Vaughan & Walbancke (1973), Wal-
bancke (1976), Ward et.al. (1959), Ward et.al. (1965), Windle & Wroth (1977).

Skempton (1977) has summarized most of the physical property data for excavations in London clay and most of these excavations are in the top weathered zone known as the Brown London clay. The Brown London clay is generally regarded as being weaker than the Blue London clay since slope failures do not penetrate to any great depth into the Blue clay unless forced to do so by, for example, a retaining wall (Henkel, 1957). Table 8.1 shows the physical properties of the Brown London clay after Chandler & Skempton (1974) and Table 8.2 summarizes the mineralogy of the clay fraction (less than 2 microns).

Table 8-1.
Typical properties of Brown London clay (after Chandler & Skempton, 1974)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>water content</td>
<td>31%</td>
</tr>
<tr>
<td>liquid limit</td>
<td>82%</td>
</tr>
<tr>
<td>plastic limit</td>
<td>30%</td>
</tr>
<tr>
<td>plasticity index</td>
<td>52%</td>
</tr>
<tr>
<td>clay fraction</td>
<td>55%</td>
</tr>
<tr>
<td>unit weight</td>
<td>18.8 KN/m²</td>
</tr>
</tbody>
</table>

Table 8-2.
Mineralogy of the clay fraction of London clay (after Burnett & Fookes, 1974)

<table>
<thead>
<tr>
<th>Clay Type</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illite</td>
<td>47%</td>
</tr>
<tr>
<td>Montmorillonite</td>
<td>35%</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>15%</td>
</tr>
<tr>
<td>Chlorite</td>
<td>3%</td>
</tr>
</tbody>
</table>
8.3.1. Shear Strength Parameters

The shear strength parameters for the Brown London clay in terms of effective stress have been carefully measured in the laboratory by many authors. For conventional testing using either 38mm diameter triaxial tests or 60mm shear box tests, the shear strength parameters are: \( c' = 14 \text{ KN/m}^2 \) and \( \phi'_p = 20 \text{ degrees} \) (Skempton, 1977). Sandroni (1977) conducted some large, 250mm diameter triaxial tests on Brown London clay and found that the shear strength parameters were: \( c' = 7 \text{ KN/m}^2 \) and \( \phi'_p = 20 \text{ degrees} \). Although these tests show the strength to be smaller than that from standard tests, the results from back analyses of first time slides in the Brown London clay are still lower. Chandler and Skempton (1974) performed several back analyses on first time slides and their results are reproduced in fig. 8.2. The back analysis results are dependent on the choice of pore-water pressures used in the analysis, and Chandler and Skempton used a range of possible values for the pore pressure parameter \( r_u \) ranging from 0.25 to 0.35. Therefore a line of best fit for these pore pressures is: \( c' = 1 \text{ KN/m}^2 \) and \( \phi'_r = 20 \text{ degrees} \), with a lower bound being: \( c' = 0, \phi'_r = 20 \text{ degrees} \). Back analyses of post-slip movements for London clay at the Sudbury Hill site are also shown in fig. 8.2 and indicate residual strength parameters of: \( c'_r = 1 \text{ KN/m}^2 \) and \( \phi'_r = 13 \text{ degrees} \). These results are in agreement with laboratory results of Skempton and Petley (1967) who determined the residual strength on natural slip surfaces in the Brown London clay to be: \( c'_r = 1.4 \text{ KN/m}^2 \) and \( \phi'_r = 13 \text{ de-} \)
8-8

grees, although Bishop et al. (1971) has obtained \( r' \) values as low as 8 degrees using a ring shear apparatus.

8.3.2. Pore Pressure Equilibration

The reasons for the long delays in excavated slope failures have been of immense interest for decades. For first-time slides of slopes excavated in Brown London clay delays as long as 50 years have been noted between excavation and ultimate failure. Until about 1973 explanations proposed by research workers (e.g. Skempton 1964, 1970; Lo & Lee (1973)) were based on the supposition that pore pressure equilibration in London clay was relatively rapid, i.e. it would take only months rather than tens of years for long-term (steady state) pore pressure conditions to be established in an excavated slope. However work by Vaughan and Walbancke (1973) and Walbancke (1976) has shown that pore pressures in excavated slopes in Brown London clay take 50 to 60 years to reach their long-term equilibration values. This ultimate pore pressure value can be defined by Bishop's \( r_U \) factor as being approximately 0.30. Skempton (1977) conducted back analyses on excavated slope failures in Brown London clay assuming that the factor of safety = 1.0 and that \( c' = 1 \) KN/m² and \( \phi' = 20 \) degrees to determine the value of \( r_U \). He then plotted these \( r_U \) values against time and the results are shown in fig. 8.3. It can be seen that the field observations support the results from the back analyses and that the long-term pore pressures are not reached until 50 to 60 years after excavation. Consequently pore pressures equivalent to \( r_U = 0.30 \) were used for the long-term values in this study.
Fig. 8.2 SUMMARY OF SHEAR STRENGTHS FOR THE BROWN LONDON CLAY
(after Chandler & Skempton 1974, and Skempton 1977)
Fig. 8.3 VARIATION OF $\bar{u}$ WITH TIME FOR CUTTINGS IN BROWN LONDON CLAY (after Skempton 1977)
Excavations made for Great Central Railroad approx. 1903 failure on Section I in 1949

Fig. 8.4 SUDBURY HILL SECTIONS I & II IN BROWN LONDON CLAY (after Skempton, 1964)
8.4. INITIAL ANALYSES OF SUDBURY HILL SLIDE

At the Sudbury Hill site an excavation was made for the Great Central Railway in 1903 and a slip occurred on the southern side of this excavation in 1949, 46 years after excavation. Both the southern and northern sides of this excavation are shown in fig. 8.4 and have been described by Delory (1957) and Skempton and Henkel (1960). Piezometers were installed in both the southern and northern sides of this excavation and $r_u$ values found to be 0.26 and 0.28 respectively, despite trench drains at the base of the slope.

The Sudbury Hill failure was the first case history to be modelled on the basis of the approach developed in this thesis. Additional problems concerning the proposed method were encountered and solved in relation to this case history. Therefore, it is desirable to describe this work in detail here, although final analyses on the basis of the corrected and refined model, are described in the next Chapter.

The northern side of the Sudbury Hill site was modelled by a finite element mesh similar to the one shown in fig. 7.8, Chapter 7, since this is a slightly higher slope than the southern side and should therefore be more susceptible to failure. The strength parameters adopted for this site were: $c_p' = 10 \text{ KN/m}^2$ and $\phi_p' = 20$ degrees, and $c_r' = 1 \text{ KN/m}^2$ and $\phi_r' = 20$ degrees. Skempton (1977) has shown that the strength at the time of failure is: $c_f' = 1 \text{ KN/m}^2$ and $\phi_f' = 20$ degrees and so the final reduced values (in this case they are in fact the fully softened values) were set to these values. The peak values were initially
set as shown above since they are in agreement with known values (Skempton 1977).

8.4.1. Slope Geometry and Failure Circles

A problem which became evident in later analyses was the choice of slope geometry to be adopted in the analysis work for the location of the failure circles, and there are three possibilities which are described here. Firstly, the final excavated slope geometry can be used (stage 6 slope) for all excavation stages. This means that the failure circles chosen pass through the same slope geometry at all stages. This accurately simulates the end-of-excavation (stage 6) and long-term (stage 7) stability, but does not accurately simulate stability during the excavation sequence (referred hereafter as method 1).

Secondly, the excavated slope geometry at each stage can be used. In this case, the failure circles used check for stability of the "total exposed slope up to that stage". Therefore after stage 1 there is only a small amount of excavated slope exposed, and consequently the failure circles used generally increase in length as excavation proceeds. This method checks for stability of the slope as it is being excavated, but in the early excavation stages the slope exposed is so small that failure would only occur through a very small amount of material and therefore may not be very significant. Since this method searches for the circle with the minimum factor of safety for the total slope exposed at that stage, one cannot therefore compare the successive decrease in stability of any one failure circle. This method does, however, show the overall stability of the exposed slope at any
stage including the final excavated slope geometry (stages 6 & 7; referred hereafter as method 2).

Finally, the long-term (stages 6 & 7) critical failure circle can be examined throughout the excavation sequence. In this method the failure circle extends to meet the excavated slope at any stage of excavation. Therefore, before any excavations have taken place, the failure circle is a complete semi-circle and gradually decreases in length as excavation proceeds. (In contrast to the first method where the failure circle would remain the same length throughout the excavation procedure.) In this way, the gradual decrease in stability along the final critical circles for overall slope failure can be examined. The factor of safety obtained using this method is obviously very high for the initial excavation stages, but it rapidly decreases with excavation (referred hereafter as method 3).

There are advantages and limitations with all three methods of failure circle location. For the initial analysis work, the first method was used. Once many of the problems with the computational procedure had been overcome, the second and third methods were used and these are presented later in the analysis work. Therefore many of the subsequent figures refer to Method 1, Method 2 or Method 3 for the location of the failure circle, and these refer to the above explanations.

8.4.2. Determination of Pre-Failure Modulus

Undrained modulus values, $E_u$, were set according to data given by Wroth (1971). Wroth has shown that for vertical samples in London clay: $(E_u/E') = 1.62$, and that the effective modulus
also increases with depth. Therefore transforming Wroth's data we get the following relationship for $E_u$ at any depth:

$$E_u = \frac{27586 \times (\text{depth} + 14)}{26 \times 1.62}$$

where depth is in metres and $E_u$ is in KPa. The values for $E_u$ assigned to each element were therefore determined according to this relationship. Modulus values for failed elements were reduced from their peak values by different amounts to see what effect this had on stability, and a sensitivity study outlining this will be described in detail later.

8.4.3. Initial Results

This excavation was simulated by using six excavation stages as described in Chapter 7 and appropriate nodal forces to simulate excavation were applied at each excavation stage. The pore-water pressures were adjusted according to the total/effective stress approach as outlined in Chapter 7.

The program incorporating all the modifications was initially run using all the previously mentioned data and using values of $K_0 = 1.0 = \text{constant with depth}$ and $K_0 = 2.0 = \text{constant with depth}$. Values of the post-peak modulus were also varied, and were: $E_r = E_p \times \text{factor}$ where the factor was: 1.0, 0.75, 0.5, 0.25, 0.1, 0.01, 0.001 and 0.0001. The results for $K_0 = 1.0$ are shown in figs. 8.5, 8.6, 8.7 and 8.8. Figs. 8.5 and 8.6 show the failure zones for the two extreme post-peak modulus values used, where the numbers in circles represent the failure zone at the respective stage number. It can be seen that the two results are practically identical, with a shear failure zone at the toe and
along the base of the excavation and a tensile failure zone at the crest of the slope.

The vertical displacement for the corner node at the base of the slope was also recorded to check for excessive uplift, and is shown in fig. 8.7. For all values of the post-peak modulus the displacements are practically identical since the failure zone is only fairly small.

The stability analysis program was then used to calculate a factor of safety for a series of slip circles to define the most critical failure surface and the results are shown in fig. 8.8. It can be seen that the factor of safety decreases progressively with stage number, and then takes a dramatic drop in the factor of safety when the pore-water pressures are adjusted to their long-term values. However the minimum factor of safety obtained was just over 2.4 and therefore supports the evidence that for failure to occur, the value of $K_o$ must be greater than 1.0 for London clay.

Previous workers have used constant values of $K_o$ with depth and for London clay. Lo and Lee (1973b) have used $K_o = 2.0 = \textrm{constant with depth}$, and consequently this value was initially used in this present study. The results for $K_o = 2.0 = \textrm{constant with depth}$ are shown in figs. 8.9, 8.10, 8.11 and 8.12, with the results for the two extreme values of the post-peak modulus are shown in figs. 8.9 and 8.10. The failure zones are only slightly different for the two post-peak modulus values. However the major factor is the very large extent of the failure zone using $K_o = 2.0 = \textrm{constant with depth}$. The failure zones in both figs. 8.9
\( C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ \)

\( C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ \)

\( A = 0.40 \)

\( K_0 = 1.0 = \text{constant with depth} \)

Fig. 8.5 SUDBURY HILL MESH \( E_r = E_p \times 1.0 \)
$C_p = 10 \text{ KN/m}^2 \quad \phi = 20^\circ$

$C_r = 1 \text{ KN/m}^2 \quad \phi = 20^\circ$

$A = 0.40$

$K_0 = 1.0 = \text{constant with depth}$

Fig. 8.6 SUDBURY HILL MESH, $E_r = E_p \times 0.0001$
displacements identical (to 4th decimal place) for all post-peak modulus values used (see text)

Fig. 8.7 RELATIONSHIP BETWEEN VERTICAL DISPLACEMENT & EXCAVATION SEQUENCE FOR VARIOUS POST-PEAK MODULUS VALUES
Note: Method 1 used to locate failure circle (see text for details).

\[ Er = Ep \times \text{number} \]

\[ K_0 = 1.0 = \text{constant with depth} \]

\[ C_p = 10 \text{KN/m}^2 \quad \phi_p = 20^\circ \]

\[ C_r = 1 \text{KN/m}^2 \quad \phi_r = 20^\circ \]

--- circle close to actual failure surface

--- min. f.o.s.

Fig. 8.8 SUDBURY HILL, VARIATION IN FACTOR OF SAFETY WITH STAGE NUMBER
$C_p = 10 \text{ KN/m}^2\quad \phi_p = 20^\circ$

$C_r = 1 \text{ KN/m}^2\quad \phi_r = 20^\circ$

$A = 0.40$

Fig. 8.9 SUDBURY HILL MESH, $E_r = E_p \times 1.0$
C_p = 10 KN/m$^2$  \( \phi_p = 20^\circ \)

C_r = 1 KN/m$^2$  \( \phi_r = 20^\circ \)

A = 0.40

tensile failure

K_o = 2.0 = constant with depth

Fig. 8.10 SUDBURY HILL MESH, \( E_r = E_p \times 0.0001 \)
and 8.10 extend to the base of the mesh which appears to be unrealistic. The displacements associated with such an analysis are shown in fig. 8.11, again for the corner node at the base of the slope. It can be seen that for low post-peak modulus values the displacements are very large and also unrealistic. The variation in the factor of safety with excavation sequence as shown in fig. 8.12, is discussed shortly.

8.4.4. Adjustment of Post-Peak Modulus values

It should be mentioned here that problems had been experienced for some considerable time with large displacements associated with high values of $K_o$. The major problem was that for high values of $K_o$, large areas of the slope were failing before the final excavation forces were applied (i.e. before stage 6). On the next excavation stage, when further nodal forces were applied, the displacements would be very large. Fig. 8.9 shows that most of the mesh below the slope has failed by stage 4 and the modulus values for these failed elements was initially set very low to simulate no increase in resistance to further unloading. However a flat stress/strain curve (i.e. perfectly plastic medium), implies that any increase in stress causes infinite displacements. It may appear that the way to overcome the problem is to simply retain the original modulus or stiffness of the elements. However then the problem arises that large stress changes occur on failed elements which is unrealistic, because once elements have failed and excess shear stress has been removed and redistributed, further unloading should ideally produce no more excess shear stress on these failed elements. These failed ele-
Fig. 8.11 RELATIONSHIP BETWEEN VERTICAL DISPLACEMENT & EXCAVATION SEQUENCE FOR VARIOUS POST-PEAK MODULUS VALUES FOR SUDBURY HILL

Ko = 2.0 = constant with depth
Cp = 10 KN/m²   φp = 20°
Cr = 1 KN/m²   φr = 20°
Note: Method 1 used to locate failure circle (see text for details).

\[ Er = E_p \times \text{number} \]

\[ Ko = 2.0 \text{ constant with depth} \]

\[ C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ \]

\[ C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ \]

\[ A = 0.40 \]

**Sudbury Hill**

Fig. 8.12 VARIATION IN FACTOR OF SAFETY WITH STAGE NUMBER
ments are at their reduced (softened or residual) strength and are unable to accept an increase in stress. To sum up, the assumption of a low modulus value for failed elements causes unacceptable displacements and that of a high modulus value causes unacceptable stress changes when using a multi-stage excavation procedure.

The problem was overcome in the following way. Once an element failed, the modulus was set to a low value as previously described so that the stresses were kept constant during iteration cycles when the excess shear stresses from other elements were being redistributed. However in a subsequent stage of the analysis, if material were removed from above an element of failed soil in the ground, the confining pressure would reduce, and some heaving would occur. Therefore just prior to an excavation stage, failed elements had their modulus values increased to their pre-failure values. This process therefore allowed the confining pressure to change due to excavation but at the same time kept stresses constant during iteration cycles. The values of the post-peak modulus shown in figs. 8.7, 8.8, 8.11 and 8.12 are therefore the modulus values used during iteration cycles and not during the multi-stage excavation process. Fig. 8.11 shows that the stress distribution process causes unacceptably large displacements if a very low failure modulus, as previously described, is used. The deformation of the slope using these very low modulus values is so large that the stability program cannot work properly. Therefore results using the very low modulus values are omitted from fig. 8.12.
Fig. 8.12 shows the variation in the factor of safety for the same two failure circles as shown in fig. 8.8, for different stages of excavation and for different failure modulus values. It can be seen that despite the very large failed area of the mesh as shown by figs. 8.9 and 8.10, the factors of safety are still above 1.0 for all cases. However the minimum factor of safety obtained was very close to 1.0, being 1.03. Fig. 8.12 shows that the failure modulus value has little effect on the factor of safety except when \( E_r = E_p \times 1.0 \). For values of \( E_r = E_p \times 0.75 \) or less, the factors of safety are similar. Also the factor of safety does not progressively decrease with excavation sequence as would be expected in contrast to the \( K_o = 1.0 \) analysis (see fig. 8.8). However, fig. 8.12 shows that a \( K_o = 2.0 \) analysis produces a very large failure zone and that there are therefore major stress changes occurring within the slope during excavation (e.g. high excavation forces, large redistribution of excess shear stress etc.). This does not occur for a \( K_o = 1.0 \) analysis. Also the factors of safety for a \( K_o = 2.0 \) analysis (fig. 8.12) are already relatively low at stage 1 compared to the factors of safety at any stage for a \( K_o = 1.0 \) analysis (fig. 8.8), since for a \( K_o = 2.0 \) analysis there are high shear stresses already existing in the slope even before excavation commences. Fig. 8.12 shows that the factor of safety initially decreases for the first few excavation stages and then levels out or in some cases, increases until the pore-water pressures are finally adjusted at stage 7. The reason for this is the development of high negative pore-water pressures with excavation sequence. These
negative pore pressures cause the shear strength to increase while the shear stress remains constant, since pore pressure has zero effect on shear stress if the principal stresses do not rotate but only change in magnitude, and therefore the factor of safety increases. At stage 6 all the excavations have been completed and the water pressures are then adjusted to their long-term values which causes a very large drop in the factor of safety. The development of negative pore pressures in London clay has been demonstrated by Vaughan and Walshancke (1973) and Walshancke (1976) and shown to exist at least several years after excavation (e.g. the New Cross excavation, fig. 8.3). Whereas the increase in factor of safety due to the development of negative pore pressures is quite reasonable, the apparent sudden drop in the factor of safety between stage 6 and stage 7 may appear surprising. However, it is not sudden in reality. It must be remembered that stages 1 to 6 represent a time period in the order of months, whereas stages 6 to 7 represent a time period in the order of 40 to 60 years.

It was also realized later that the data presented in fig. 8.12 is for the same two circles and for the same final slope geometry for all the excavation stages shown, which is not strictly realistic. The failure circles should be used with the slope geometry for that particular stage of the excavation (see appropriate stage number) and these are shown later. However it does mean that the increase in the factor of safety shown in fig. 8.12 must be due to stress changes along the final failure surface such as high negative pore pressures. This is associated
with the whole problem of the location of the failure surface and this has been mentioned earlier in this Chapter, and will also be discussed in the next Chapter.

8.4.5. Validity of In-Situ Stress Assumptions

It was apparent from early in the investigations that large areas of the mesh were failing when using high values of \( K_0 \) and these results appeared to be unreasonable. Extensive checks were made to try and find any errors in the method or in the computer program. A check was made using a known linear elastic structural problem and the correct displacements and stresses were obtained and therefore the program was working correctly. A careful consideration of the problem led to the conclusion that for high values of \( K_0 \) large areas of the mesh would indeed fail and this is explained below.

Fig. 8.13 shows a plot of shear strength and shear stress vs depth for different values of \( K_0 \). As outlined in Chapter 5 the principal stresses are assumed to act vertically and horizontally if no other information is available. Therefore, any value of \( K_0 \) except 1.0, implies existence of shear stresses, since the shear stress is half the principal stress difference. For the case of \( K_0 \) values greater than 1.0, which is the case for overconsolidated clays, it can be seen from fig. 8.13 (based on typical London clay shear strength parameters), that the shear stress exceeds the shear strength at approximately 6 m. depth for \( K_0 = 2.5 \). For \( K_0 = 4.0 \) the shear strength is exceeded at about 1.2 m. depth. Therefore different depths can be calculated at which the local factor of safety equals 1.0 for different values of \( K_0 \). This
$\gamma = 2000 \text{ kg/m}^3$

$C = 10 \text{ KN/m}^2$

$\phi = 20^\circ$

water table 1.5m below ground surface

shear strength

shear stress

Fig. 8.13 PLOT OF STRENGTH & SHEAR STRESS ENVELOPES FOR VARIOUS VALUES OF $K_o$
data is presented in fig. 8.14. It can be seen that if a constant value of $K_q$ exists in the ground, then London clay has a local factor of safety which decreases with increasing depth. Fig. 8.14 assumes that no excavations have yet taken place. However if an excavation is then made, nodal forces are applied to simulate the removal of material and the greatest stress changes will obviously occur in areas of the mesh close to the excavated surface. However, at depth it only requires a very small stress change to cause failure for the case of $K_o = 2.0$, whereas it requires a very large stress change to cause failure near the surface.

The distributions of $K_o$ suggested by different authors for London clay are shown in fig. 8.15, and it can be seen that there is a wide scatter of values. Windle and Wroth (1977) quote a range of results obtained from pressuremeter tests and they attribute the scatter in their results to "variations in drilling technique such as rate of advance, cutter speed, weight of clay suspended in the flushing water, etc." However their results are only for a relatively small range of depths and no consistent pattern can be determined. Skempton (1961) determined $K_o$ values for the Bradwell site and his results show increasing $K_o$ with decreasing depth between 33 and 6 m. due to the greater effect of stress relief near the surface. However between 6 m. depth and the surface $K_o$ again decreases due to such things as weathering effects causing stress relief. Bishop et al. (1965) calculated a similar trend for the Bradwell site although his results were slightly higher than Skempton's and he did not have
Fig. 8.14 VARIATION IN FACTOR OF SAFETY WITH DEPTH FOR DIFFERENT VALUES OF $K_0$ FOR LONDON CLAY
Fig. 8.15 VARIATION OF $K_0$ WITH DEPTH FOR LONDON CLAY
any results close to the surface. Sills et al. (1977) have used a step-wise linear trend for $K_0$ with depth which also has the same general trend as Skempton's relationship although using slightly lower values. Burland and Hancock (1977) have used increasing $K_0$ with depth, increasing from 0.3 at the surface to 2.2 at 33 m. depth, although their data was primarily used to simulate deformations in excavations. Lo and Lee (1973b) have used a value of $K_0$ of 2.5 for the analysis of slope failures in London clay and it is presumed that this was a constant value with depth since no other details were given.

Consequently there is a large scatter of the available data for $K_0$ with depth, but the trend is clear that $K_0$ values greater than 1.0 almost certainly exist and that they may be as high as 3.5. It must however be remembered that the data presented in fig. 8.15 have been obtained from different locations within the London basin (Sills et al. (1977) from Neasden, Burland and Hancock (1977) from the New Palace Yard site at Westminster, Windle and Wroth (1977) from Hendon, Skempton (1961) from Bradwell 80 km. from central London, and Bishop et al. (1965) from the Ashford Common site). Since the degree of overburden removal and hence overconsolidation ratio vary across the basin, one would also expect that $K_0$ would vary across the basin. However there appears to be no detailed information in the literature on this aspect, and therefore the $K_0$ values used in this present study are those as defined by Skempton (1961) for the Bradwell site.

Fig. 8.16 shows a curve for the relationship of $K_0$ with depth from Skempton's (1961) data, and three curves showing the upper
Fig. 8.16 VARIATION OF Ko WITH DEPTH, ASSUMING FOS = 1.0

- Depth - in metres

- Variation of Ko with depth from Skempton's Bradwell investigations

- Softened strength
  \[ Cs = 1 \text{ kN/m}^2 \quad \phi_s = 20^\circ \]

- Passive failure indicated

- \[ K_a = \frac{1 - \sin \phi}{1 + \sin \phi} + \frac{2c \cos \phi}{\delta z \sin \phi} \]

- \[ K_p = \frac{1 + \sin \phi}{1 - \sin \phi} + \frac{2c}{\delta z} \frac{\cos \phi}{1 - \sin \phi} \]

- Peak strength
  \[ C_p = 10 \text{ kN/m}^2 \quad \phi_p = 20^\circ \]
and lower bounds for failure depending on the strength parameters adopted. The three curves showing these upper and lower bounds assume that the shear stress = shear strength at any depth and therefore the factor of safety equals 1.0 at any depth. The lower bound for failure is defined by the active ($K_a$) failure curve assuming peak strength parameters ($c_p = 10 \text{ KN/m}^2$, $\phi_p = 20^\circ$). The upper bound is defined by two curves and these are passive ($K_p$) failure curves assuming both the softened strength ($c_s = 1 \text{ KN/m}^2$, $\phi_s = 20^\circ$), and the peak strength. The stresses used to derive these relationships assume that no excavations have yet taken place. Therefore the $K_a$ and $K_p$ curves shown give a range for the in-situ stress ratio, within which failure should not occur depending on strength parameters. It can be seen that failure occurs between 4 and 17 m. if Skempton's $K_0$ relationship with depth and the shear strength values shown in fig. 8.16 are assumed. Fig. 8.16 is also plotted on exactly the same scale as fig. 8.15 and therefore any of the $K_0$ relationships with depth shown in fig. 8.15 could be directly transcribed to fig. 8.16 (they have been omitted for clarity). For example Bishop's (1965) relationship would cause failure from a depth of 23 m. upwards.

Skempton's (1961) $K_0$ relationship was used in the Sudbury Hill analysis and the results are presented in fig. 8.17. It can be seen that the failure zone in fig. 8.17 corresponds to approximately the same depth as the failure zone shown in fig. 8.16, considering nodal forces and redistribution of excess stress have taken place in fig. 8.17. The failure zone at the base of the mesh is caused by assuming that $K_0$ continued to de-
C_p = 10 KN/m^2  \phi_p = 20^\circ
C_r = 1 KN/m^2  \phi_r = 20^\circ
A = 0.40

Ko varies with depth according to Skempton's Bradwell investigations

Fig. 8.17  SUDBURY HILL MESH, \( E_r = E_p \times 0.75 \)
crease below 33 m. depth (Skempton's last data point), which gave a value of $K_0$ of 0.5 at the base of the mesh. This low value of $K_0$ corresponds to the $K_a$ curve in fig. 8.16 and hence failure zones develop at the base of the mesh as shown. In reality the $K_0$ relationship with depth would probably decrease asymptotically to some value close to the normally consolidated value ($1 - \sin \phi$, approximately 0.66 when $\phi = 20^\circ$), and hence these failures at the base of the mesh would not occur.

8.4.6. Forces to simulate Excavation

Initially the program had the facility to read in excavation forces and then apply these sequentially to the mesh to simulate multi-stage excavation. These excavation forces were calculated using simply the weight of the overlying strata without regard to horizontal stresses, and since this was a total stress analysis, the forces were also in terms of total stress. This procedure is strictly only correct if $K_0 = 1.0$. For the case of London clay, $K_0$ values are considerably higher than 1.0 and excavation forces should be calculated from in-situ stresses. Consequently a subroutine was written which calculated the stresses and hence the forces at nodal points on the excavated surface. When these new forces were calculated some surprising results were obtained. Table 8.3 shows a comparison of the forces applied to an excavated surface calculated both using simple overburden considerations and also using in-situ stresses. The in-situ stress relationship for this particular example is after Skempton (1961) and the forces are listed for two excavation stages.
Table 8-3.
Comparision of excavation forces calculated both from overburden and in-situ stresses.

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<th>Direction</th>
<th>Node</th>
<th>O/B Force</th>
<th>in-situ Force</th>
<th>Node</th>
<th>O/B Force</th>
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NOTE: the X and Y forces to be applied to their respective nodal points are listed down the columns. The nodal points are listed from the top left hand side of the mesh across towards the excavated slope on the right hand side. As excavation proceeds, there are a smaller number of forces applied, hence there are a smaller number of forces listed for stage 6 than for stage 2. Units are in Newtons.

It can be seen that the forces calculated using only the overburden strata considerations remain constant, whereas the forces
calculated using in-situ stresses are significantly higher, particularly for the final excavation stage where the $K_0$ values are higher than those nearer to the original surface. Consequently this method of calculating the excavation forces was adopted.

A flow chart of the major steps in the total/effective finite element program is given in Table 8.4. It can be seen that although the finite element analysis is performed in terms of total stress, the failure criteria is determined in terms of effective stress. The long-term pore water pressures which are applied at the final stage are determined from a static water table level equivalent to the long-term equilibrium condition. A flow chart of the adjustment of modulus and stress values during failure and excavation is shown in Table 8.5.
Table 8-4. Flow chart of the major steps of the Finite Element Total/Effective Stress Program

1. Calculate initial total stress and initial pwp's based on original water table
2. Calculate and apply excavation forces based on total stresses
3. Calculate total stress changes
4. Use total stress changes and value of A to calculate change in pwp's
5. Calculate new total stresses pwp's and effective stresses
6. Check for failure using Mohr-Coulomb failure criterion in terms of effective stress
7. Redistribute excess shear stresses and check for additional failures
8. Are there additional excavation stages? (YES/NO)
9. Have long term pwp's been applied? (YES/NO)
10. Apply long term pwp's

End
Table 8-5  Flow Chart for the Adjustment of Modulus and Stresses

- check for failure using Mohr-Coulomb failure criterion in terms of effective success
- set appropriate shear strength parameters on failed elements, reduce modulus on failed elements to $E_r/E_p = 0.75$
- redistribute excess shear stresses (in terms of total stress) keep stresses constant on failed elements

- are there any more elements this cycle? Yes

- are there any more excavation stages? Yes

- restore modulus on "old" failed elements back to "failure" value
- apply excavation forces
- set modulus on failed elements to pre-failure values; allow stresses to change on failed elements

No
No
9. CASE STUDIES

9.1. SUDBURY HILL

9.1.1. Analysis of Sudbury Hill

The Sudbury Hill slope was analysed using excavation forces derived from in-situ stresses and using the $K_q$ relationship with depth after Skempton (1961). The Skempton relationship for in-situ stresses appears to be a reasonable approximation to the $K_q$ data presented in fig. 8.15, since the effect of overconsolidation would be greatest nearest the surface, except that stress relief would take place at very shallow depths. This type of relationship is evident in Skempton's relationship for $K_q$ with depth (see fig. 8.15).

Two sets of strength parameters were initially adopted and these were: $c_p = 10$ KN/m$^2$, $c_r = 1$ KN/m$^2$, $\phi_p = \phi_r = 20$ degrees, and as above but with $c_p = 7$ KN/m$^2$. The vertical displacements of the corner node and the factors of safety are presented in figs. 9.1 to 9.4. It can be seen that the vertical displacements and the factors of safety are similar for both sets of strength parameters. The vertical displacements are small for all values of the pore-water pressure parameter $A$ used, and for the different post-peak modulus values used. The curves for the factor of safety for the different parameters shown in figs. 9.2 and 9.4 are also very similar. However the minimum factors of safety are 1.07 and 1.13 for $c_p = 10$ KN/m$^2$ and $c_p = 7$ KN/m$^2$ respectively. The apparent anomaly in the two different values is not immediately apparent but may be due to different amounts of excess
shear stress being redistributed for the two different strength parameters. However since these two factors of safety are very close, these results were considered reasonable.

The difference between figs. 8.12 and 9.2 is that fig. 9.2 uses in-situ stresses to calculate excavation forces whereas fig. 8.12 does not. Also three values of the pore pressure parameter A were used, these being 0.20, 0.40 and 0.60.

As mentioned previously little attention had been given to the actual failure surface used except to use a series of circles which corresponded to the actual failure surface. Up until this stage in the investigations over 100 different failure surfaces had been used to determine the minimum factor of safety for each excavation stage but these were based on the final geometry of the excavated slope. Hence the factor of safety for such a failure surface is only dependent upon stress changes as a result of excavation forces and not due to a change in slope geometry (i.e. the failure surface remains the same length for each excavation stage). Figs. 9.2 and 9.4 therefore represent such an analysis. However this does not accurately represent the stability of the slope during the excavation procedure and so the failure surfaces were modified to correspond to each stage of the excavation process. Hence the factors of safety determined for each excavation stage were then based on a different set of failure surfaces each time.

At this stage it was decided to use a wider range of strength parameters by incorporating the residual friction angle of London clay (13 degrees) which corresponds to Skempton's (1964,1977)
Fig 9.1 RELATIONSHIP BETWEEN VERTICAL DISPLACEMENT AND EXCAVATION SEQUENCE FOR VARIOUS POST PEAK MODULUS VALUES
SUDBURY HILL

\[ C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ \]

\[ C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ \]

\[ E_r = E_p \times \text{number on curve} \]

Note: Critical circles used encompass the whole slope, although circles vary depending on physical properties. Method 1 used to locate failure circle (see text for details).

Fig. 9.2 VARIATION IN FACTOR OF SAFETY WITH STAGE NUMBER
SUDBURY HILL

$C_p = 7 \text{ KN/m}^2 \quad \phi_p = 20^\circ$

$C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ$

--- $A = 0.20$

--- $A = 0.40$

--- $A = 0.60$

$E_r = E_p \times \text{number on curve}$

**Fig. 9.3** RELATIONSHIP BETWEEN VERTICAL DISPLACEMENT AND EXCAVATION SEQUENCE FOR VARIOUS POST PEAK MODULUS VALUES
SUDBURY HILL

\[ C_p = 7 \text{ KN/m}^2 \quad \phi_p = 20^\circ \]

\[ C_r = 1 \text{ KN/m} \quad \phi_r = 20^\circ \]

\[ E_r = E_p \times \text{number on curve} \]

**Note:** Critical circles used encompass the whole slope, although circles vary depending on physical properties. Method 1 used to locate failure circle (see text for details).

**Fig. 9.4** VARIATION IN FACTOR OF SAFETY WITH STAGE NUMBER
residual strength rather than the fully softened strength. The justification for this is that, although back analyses of London clay failures gave strength parameters equal to the fully softened strength (i.e. cohesion equals 1 KN/m^2 and friction angle equals 20 degrees), this assumes that the entire failure surface is at this strength. For analyses using the present finite element method most, but not all, of the failure surface passed through failed elements. Hence some sections of the failure surface would have a strength greater than the fully softened strength. This would increase the factor of safety above 1.0 if a back analysis showed that the softened strength was applicable to the whole failure surface.

The results for the four sets of strength parameters are shown in figs. 9.5 to 9.8, and the location of the critical failure surfaces for each of the four strength parameters is shown in fig. 9.9 (these failure surfaces are located according to method 2 as described previously). Therefore, although the critical circles are similar for each set of strength parameters, they are not exactly the same, and vary not only due to strength parameters used but also due to the value of A. The results do not show distinct patterns except that the factor of safety for the total slope exposed at that stage safety increases at stage 3 or 4 and then decreases towards the final stages. However two factors play an important role and affect the factor of safety. Firstly, the difference between the size of the failure between stage 1 and stage 7 is very large. Failures on stage 1 are really only minor slumps of material and are close to the element size used.
Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

\[ C_p = 7 \text{ KN/m} \quad \phi_p = 20^\circ \]
\[ C_r = 1 \text{ KN/m} \quad \phi_r = 13^\circ \]
Fig. 9.6 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).
SUDBURY HILL

$C_p = 10 \text{ KN/m}^2$  $\phi p = 20^\circ$

$C_r = 1 \text{ KN/m}$  $\phi r = 13^\circ$

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

Fig. 9.7 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
SUDBURY HILL

$C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ$

$C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ$

Fig. 9.8 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).
failure circle applicable to respective excavation stage

Fig 9.9 LOCATION OF CRITICAL FAILURE SURFACE FOR EACH EXCAVATION STAGE FOR DIFFERENT STRENGTH PARAMETERS
(method 2 used to locate failure circle, see text)
In some cases there may be only three or four elements along a failure surface on stage 1. Hence if tensile failure occurs (as is very likely at the crest of a slope) and the element strength is set to zero, the overall factor of safety for this circle immediately becomes very low. However if tensile failure does not occur then the factor of safety remains relatively high. Consequently fig. 9.7, for example, shows both high and low factors of safety for stage 1. In all four figures (9.5 to 9.8) there is a much larger variation in the factor of safety for stage 1 than for stage 7. Secondly, although the factors of safety shown in figs. 9.5 to 9.8 are the minimum factors of safety for the total slope exposed up to that stage, the absolute minimum factor of safety for any failure surface in the slope would be different. For example, some of the curves in fig. 9.6 show very low factors of safety at stage 1. If these same failure surfaces were considered in subsequent excavation stages they would still show very low factors of safety. However as mentioned previously, these are really only small slumps of soil, and are not significant to the final overall slope failure.

Fig. 9.9 shows that in almost all cases the minimum failure surface as defined above cuts through the toe of the presently exposed slope at that stage. The consistent trend that emerges is that the higher the post-peak modulus the lower the final factor of safety. Figs. 9.5 to 9.8 show three post-peak modulus values and these are $E_r = E_p \times \text{number}$, where number equals 0.25, 0.50 and 0.75. It is also interesting to note that the final factors of safety obtained are not significantly different for
all four sets of strength parameters and that they are reasonably close to 1.0. One value in fig. 9.5 drops just below 1.0, although this uses residual strength parameters.

Although the factors of safety are not significantly different for the different strength parameters used, the zones of failure are. Figs. 9.10 and 9.11 show the zones of failure for the weakest and strongest strength parameters used. The pore pressure and modulus values shown are those which gave the minimum factor of safety for that particular set of strength parameters. It can be seen that almost any failure circle for the slope shown would cut through a large failure zone both in fig. 9.10 and 9.11, although it would be slightly larger in fig. 9.10. The only major difference would therefore be the residual or softened friction angle used (13 degrees in fig. 9.10 compared to 20 degrees in fig. 9.11). This explains partially why the final factors of safety in figs. 9.5 to 9.8 are similar. The major difference between figs. 9.10 and 9.11 is the much larger failure zone in fig. 9.10, and this corresponds to the previous explanation of zones of failure as demonstrated by fig. 8.16. In both figs. 9.10 and 9.11 $K_q$ was not permitted to fall below 1.0 and hence the absence of a failure failure zone at the base of the mesh in contrast to fig. 8.17.

All of the above data was then collated and the four circles with the minimum final factors of safety for the four sets of strength parameters for overall slope failure were plotted. These were plotted assuming sequential excavation geometry of the slope and the results are shown in fig. 9.12. The results show
Cp = 7 KN/m$^2$  \( \varphi_p = 20^\circ \)
Cr = 1 KN/m$^2$  \( \varphi_r = 13^\circ \)
A = 0.60
Er = Ep x 0.75

Fig. 9.10  SUDBURY HILL
Cp = 10 KN/m²
Cr = 1 KN/m²
A = 0.20
Er = Ep x 0.75

Fig. 9.11  SUDBURY HILL
Critical circles for each set of strength parameters are in general different for each excavation stage. Critical circle always has $E_r = E_p \times 0.75$, but value of $A$ varies. Method 3 used to locate failure circle (see text for details).

Fig. 9.12 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE FOR OVERALL SLOPE FAILURE
a trend exactly as would be expected. The factors of safety are very high initially (since the failure circle would be almost a complete semi-circle at stage 1), and are therefore plotted on a log scale. It can be seen that there is a steady progressive decrease in the factor of safety with excavation sequence. The relationship shown on this scale is very close to linear, and from this one could obtain a relationship for decrease in factor of safety with time, at least between stages 1 to 6. This would represent a time period in the order of months. However, the adjustment of pore pressures to their long-term values takes considerable time and the adjustment appears to follow an asymptotic curve as shown by fig. 8.3. Therefore it would be misleading to try and interpret values between stages 6 and 7 which represent a time period of 46 years. The factor of safety against overall slope failure (during the excavation process) could be obtained for any number of excavation stages simply by dividing the line up to stage 6 (see fig. 9.12) into the appropriate number of stages.

Fig. 9.12 shows the minimum factor of safety for overall slope failure with excavation sequence and as such the critical circle is different depending on the strength parameters and pore pressures used. In contrast, fig. 9.13 shows the same variations except for one circle only. The circle considered is shown on fig. 9.13 and is close to the observed failure surface. However the factor of safety obtained using this circle is not necessarily the absolute minimum factor of safety. The four different strength parameters used are shown, and in general the weakest
SUDBURY HILL

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Note: Critical circle always has $E_r = E_p \times 0.75$ but value of $A$ varies. Only one circle considered for all strength parameters. Method 3 used to locate failure circle (see text for details).

Fig. 9.13 VARIATION IN FACTOR OF SAFETY FOR ONE CIRCLE ONLY
Note: Pore pressure parameter
A = 0.40, Er/Ep = 0.75

Fig. 9.14 VARIATION IN PORE PRESSURE WITH EXCAVATION SEQUENCE
strength parameters give the lowest factors of safety. However at stage 7 the weakest strength parameters did not give the lowest factor of safety. The explanation for this is not clear, but may be due to a greater amount of stress redistribution occurring for weaker strength parameters than for higher strength parameters. This may increase normal stresses on adjacent elements, thus making them apparently stronger. Whatever the explanation, the variation in the factor of safety at stage 7 is only minor, and is similar to the trend shown in fig. 9.12.

As stated in Chapter 7, the stability calculations are based on undrained pore pressures predicted on the basis of Skempton's simple equation (equation 7.6). Pore pressures were also calculated on the basis of the plane strain assumption (equation 7.18), but on the whole the differences were not significant and therefore they were not used in these case histories.

The variation in the pore water pressure throughout the excavation procedure was also considered to be very important. Fig. 7.12 showed the theoretical development of pore pressures throughout an excavation procedure. This showed a decrease in pore pressure after excavation and then a gradual increase in pore pressure until the long-term condition was achieved. Fig. 9.14 shows the actual variation in pore pressure for the Sudbury Hill slope for three points under the slope. All three points show the same trend as fig. 7.12 in that there is a decrease in pore pressure as excavation takes place, and then a final increase to the long-term value. The stage number on the X-axis is not a linear time scale, and therefore the long-term increase in
pore pressure appears to be a sudden increase whereas it is in fact a gradual increase. The largest decrease in pore pressure occurred near the toe of the slope as would be expected since the largest stress changes occur at this point.

9.2. NORTHOLT

The slope failure at the Northolt site has been described by Henkel (1957), Skempton (1964) and Skempton and Hutchinson (1969). A cross section through this slide is shown in fig. 9.15, and it can be seen that the original excavation was made in 1903 and later extended in 1936 to accommodate additional railway tracks. The 1936 slope had an inclination of 2.5:1 and was 10 metres high. Failure occurred in 1955, 19 years after the second excavation. The failure surface was approximately circular and the toe of the failure surface was just above a small, 1 metre high retaining wall at the base of the slope. Initial movements occurred in January 1955 and piezometers were installed in November 1955 as well as trench drains. The slope was also reduced to an inclination of 3:1.

9.2.1. Analysis of Northolt

The slope at the Northolt site is very similar to the slope at the Sudbury Hill site. Northolt had a final slope of 2.5:1 and was 10 metres high and Sudbury Hill had a slope of 3:1 and was 10.7 metres high. Therefore one could expect similar failure zones and factors of safety for both slopes assuming the same strength parameters. The finite element mesh used for the Northolt analysis is shown in fig. 9.16 and the same analysis proce-
slip occurred after 19 years in 1955, piezometers installed Dec. 1955

**Fig. 9.15** SECTION THROUGH THE SLIDE IN THE NORTHOLT CUTTING
(after Henkel, 1957)
dures was adopted for Northolt as had been used for Sudbury Hill. However, many problems encountered with the method had been solved during the Sudbury Hill analysis and therefore will not be mentioned further.

The analysis used the same pore pressure parameters (0.20, 0.40 and 0.60), the same post-peak modulus values (0.25, 0.50 and 0.75), and the same sets of strength parameters as used previously. Figs. 9.17 to 9.20 show the minimum factor of safety for the overall slope exposed at that stage. The factor of safety shown at stage 7 therefore represents overall slope failure whereas that shown at stage 1 represents only a very small failure surface at the crest of the slope (the failure circles used in this analysis were similar to those shown in fig. 9.9 for the Sudbury Hill analysis). The results are similar in all four figures, showing a high initial factor of safety dropping very rapidly at stage 2. The factor of safety then increases slightly on subsequent stages and then drops at stage 7. Figs. 9.17 and 9.19 show a high factor of safety at stage 1, whereas figs. 9.18 and 9.20 show both high and low factors of safety at stage 1. High factors of safety are a result of "non-failure" of elements at the crest of the slope at stage 1 and hence only peak strength parameters are used. Low factors of safety are a result of failure of some elements at the crest of the slope, and hence softened or tensile strength parameters are used for these elements and this reduces the overall factor of safety. The increase in factor of safety on subsequent stages is due both to the development of high negative pore pressures, and to the failure surface incor-
Fig. 9.16  NORTHORT MESH
NORTHOLT

\[
\begin{align*}
C_p &= 7 \text{KN/m}^2 & \phi_p &= 20^\circ \\
C_r &= 1 \text{KN/m}^2 & \phi_r &= 13^\circ 
\end{align*}
\]

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

Fig. 9.17 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
NORTHOLT

\[ C_p = 7 \text{ KN/m}^2 \quad \phi_p = 20^\circ \]
\[ C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ \]

*Note:* These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

**Fig. 9.18** VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
Fig. 9.19 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).
Fig. 9.20 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
porating a much larger number of elements on subsequent stages and therefore localized element failure becomes less significant.

The range of final factors of safety for stage 7 is similar for all strength parameters used, and the total range is from 0.85 to 1.32. Figs. 9.17 and 9.19 which had the residual friction angle of 13 degrees, had a small range of final factors of safety, 1.05 to 1.20 and 0.97 to 1.25 respectively. Both these figures also show a dramatic drop in factor of safety at stage 7. Figs. 9.18 and 9.20, which had the softened friction angle of 20 degrees, had a wider range of final factors of safety, 0.90 to 1.32 and 0.85 to 1.30 respectively. These figures show both a slight and a dramatic drop in factor of safety at stage 7. It is also interesting to note that the minimum final factor of safety obtained was 0.85 in fig. 9.20 which has the strongest strength parameters. The reason for this is not immediately apparent but the results have the same order of magnitude for all four figures.

A more direct comparison can be made between the extent of the failure zones for the different strength parameters. Figs. 9.21 and 9.22 show the extent of the failure zones for the Northolt slope for the weakest and the strongest strength parameters used. The actual failure zones shown are those which gave the minimum factor of safety for that particular set of strength parameters. It can be seen that the extent of the failure zone is much greater in fig. 9.21 than in fig. 9.22. Most of the additional failure zone shown in fig. 9.21 occurs at stage 7 and is obviously unrealistic. The extent of the failure zones for Northolt is
similar to those for Sudbury Hill (figs. 9.10 and 9.11) as would be expected for a similar slope height and inclination. The extent of the Northolt failures also corresponds well with the failure zones shown in fig. 8.16.

Finally the minimum factor of safety for overall slope failure for each excavation stage was plotted and this data is shown in fig. 9.23. As would be expected the factor of safety is very high for stage 1 and progressively decreases until stage 7. The factor of safety is plotted on a log scale since factors of safety at stage 1 are very high. On this scale, there is almost a perfectly linear relationship for decrease in factor of safety with excavation sequence. A higher number of failure circles were tried for the Northolt analysis compared to the Sudbury Hill analysis and this probably explains why there is a greater linearity in fig. 9.23 compared to 9.12. Since the decrease in the factor of safety is almost perfectly linear on fig. 9.23, any number of excavation stages could be substituted on the X-axis and the factor of safety obtained. The rate of decrease in factor of safety with time between stages 6 and 7 (a period of 19 years) cannot be easily obtained since the adjustment of pore pressures to their long-term equilibration values, is not linear as explained for the Sudbury Hill case.

As with the Sudbury Hill case, the variation in factor of safety for one circle only was also plotted for Northolt and this is shown in fig. 9.24. Fig. 9.24 shows a similar trend to fig. 9.23 and in general, the weaker strength parameters give the lowest factors of safety. The circle considered is shown on fig.
\( \sigma_p = 7 \text{KN/m}^2 \)  \( \phi = 20^\circ \)
\( C_r = 1 \text{KN/m}^2 \)  \( \phi_r = 13^\circ \)
\( A = 0.20 \)
\( E_r = E_p \times 0.75 \)

**Fig. 9.21** NORTHOLT
Cp = 10 KN/m²  φp = 20°
Cr = 1 KN/m²  φr = 20°
A = 0.20
Er = Ep x 0.75

Fig. 9.22  NORTHOLT
Fig. 9.23 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE FOR OVERALL SLOPE FAILURE

Note: Critical circles for each set of strength parameters are in general different for each excavation stage. Critical circle always has $Er = Ep \times 0.75$, but value of $A$ varies. Method 3 used to locate failure circle (see text for details).
The diagram illustrates the variation in factor of safety for one circle only, with stage number on the x-axis and factor of safety (log scale) on the y-axis. The critical circle always has $Er = Ep \times 0.75$ but the value of $A$ varies. Only one circle is considered for all strength parameters. Method 3 is used to locate failure circle (see text for details).

### Table

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**Note:**

Critical circle always has $Er = Ep \times 0.75$ but value of $A$ varies. Only one circle is considered for all strength parameters. Method 3 is used to locate failure circle (see text for details).

---

*Fig. 9.24 VARIATION IN FACTOR OF SAFETY FOR ONE CIRCLE ONLY*
Fig. 9.25 VARIATION IN PORE PRESSURE WITH EXCAVATION SEQUENCE
9.24 and encompasses the whole slope as did the observed failure surface. However, the observed failure surface was not circular, and therefore some variations occur between the observed failure surface and that used in the present analysis. The actual failure circle with the minimum factor of safety at each stage, varies slightly depending on strength parameters and pore water pressures. Since the Northolt slope geometry is very similar to the Sudbury Hill slope geometry, the variation in factor of safety with excavation is also similar in both cases, and similar comments apply to both. The abrupt drop in the factor of safety for strength parameters $c_p = 10 \ \text{KN/m}^2$, $\phi_p = 20^\circ$, $c_r = 1 \ \text{KN/m}^2$, $\phi_r = 13^\circ$ at stage 7 may be due to redistribution of excess shear stresses as explained for the Sudbury Hill case.

The variation in the pore pressures for the Northolt case are shown in fig. 9.25 and is again similar to the Sudbury Hill example (see fig. 9.14). The decrease in pore pressures is greatest nearest the toe of the slope and all three points considered show the same trend. The points considered are element centroids and hence their exact locations vary slightly from mesh to mesh. All three points are below the final water table level and hence the final pore pressures are all positive values.

9.3. NEW CROSS

The excavation for the London and Croydon Railway at the New Cross site was made in 1838 and was one of the earliest deep excavations made in London clay. The slope failure at this site was initially described by Gregory (1844) and more recently by Skempton (1977). The cross-section of this excavation is shown
in fig. 9.26 and it can be seen that it was 23 metres deep and had slopes of 1.5:1. On the 2nd November 1841 40,000 cubic metres of clay slipped into the excavation as shown in fig. 9.26. Shortly afterwards a similar large slip occurred on the other side of the excavation. The slope was finally stabilized by putting benches in the slopes and reducing the batter faces to a slope of 2:1. The slip surface for this clay was described as having passed along the base of the Brown London clay indicating that the Blue London clay is probably slightly stronger than the Brown London Clay, although the failure surface was not surveyed in.

9.3.1. Analysis of New Cross

The failure at the New Cross site was both the largest failure, and the deepest excavation studied in these present investigations (23m.). However the mesh used for this analysis was similar to that used previously, except that the excavated section was obviously much larger. The mesh used for the New Cross analysis is shown in fig. 9.27 and it can be seen that only one symmetrical half of the cutting has been modelled.

The same range of strength parameters were adopted for New Cross as had been used previously, as well as using the same pore pressure parameters (0.20, 0.40 and 0.60), and the same post-peak modulus values (0.25, 0.50 and 0.75). This facilitated both a comparison to be made with other results, and variations in parameters from site to site. The stability analysis procedure was also the same as had been used previously.

Figs. 9.28 to 9.31 show the variation in factor of safety
excavated in 1838, failure occurred after 3 years

Fig. 9.26 EXCAVATION AT THE NEW CROSS SITE (after Gregory 1844) FOR THE LONDON & CROYDON RAILWAY
NEW CROSS

\[ \begin{align*}
\text{C}_p &= 7 \text{ KN/m}^2 & \phi_p &= 20^\circ \\
\text{C}_r &= 1 \text{ KN/m}^2 & \phi_r &= 13^\circ
\end{align*} \]

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

Fig. 9.28 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
NEW CROSS

\[ \begin{align*}
C_p &= 7 \text{ KN/m}^2 & \phi_p &= 20^\circ \\
C_r &= 1 \text{ KN/m}^2 & \phi_r &= 20^\circ 
\end{align*} \]

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

---

**Fig. 9.29 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE**
NEW CROSS

\[ C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ \]

\[ C_r = 1 \text{ KN/m}^2 \quad \phi_r = 13^\circ \]

\[ A = 0.20 \]

\[ A = 0.40 \]

\[ A = 0.60 \]

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

Fig. 9.30 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
NEW CROSS

\[ \text{Cp} = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ \]

\[ \text{Cr} = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ \]

--- \( A = 0.20 \)

--- \( A = 0.40 \)  \hspace{1cm} \text{Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).}

--- \( A = 0.60 \)

\text{Fig. 9.31 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE}
with excavation sequence for the slope exposed at that stage for the four different strength parameters used. The results are initially quite surprising. In all four figures the factor of safety is shown to increase with excavation sequence and only decreases between stages 6 and 7. The range in factors of safety is also very similar for all four figures, increasing from about 0.1 at stage 1, to about 1.0 at stage 6 and then decreasing to about 0.7 (minimum) at stage 7. The final minimum factor of safety (i.e. at stage 7) was obtained using the highest residual modulus value (0.75). The explanation for these results is given in figs. 9.32 and 9.33. These two figures show that a large section of the mesh fails at stage 1. Hence failure circles for stages 1 to 5 pass through mostly failed material and therefore the factors of safety are low. The failure circles for stage 1 only pass through a small section of material at the crest of the slope and hence are really only small slumps of material. However failure circles for subsequent stages pass through greater portions of the slope and hence the factor of safety increases.

At stage 6 the failure surface encompasses the whole slope and hence the factor of safety shown represents overall slope stability. It is therefore interesting to note that the factors of safety obtained for stage 6 in figs. 9.28 to 9.31 are very close to 1.0 or slightly below 1.0. This indicates that the slope was close to, or at, the limit of stability immediately after excavation. The adjustment of pore-water pressures to the long-term values causes an even greater decrease in stability. In reality, failure occurred at New Cross 3 years after excavation occurred,
which is a very short time span considering the rate of pore pressure dissipation in excavated London clay slopes (of the order of 40 years).

Finally the variation in the factor of safety with excavation sequence for overall slope failure is shown in fig. 9.34. The results shown in this figure are for the highest residual modulus value and the failure circles used encompass the whole slope and hence are very stable in the early excavation stages. Plotted on a log scale, the factor of safety shows an almost linear decrease in stability with excavation sequence, similar to both Sudbury Hill and Northolt. It can be clearly seen that, depending on the strength parameters adopted, the theoretical factor of safety at stage 6 is very close to the actual factor of safety at failure.

The variation in factor of safety for overall slope failure for one circle only is shown in fig. 9.35 along with the circle considered. This shows a more linear decrease in the factor of safety with excavation sequence than fig. 9.34, and in general the weakest strength parameters give the lowest factors of safety.

The variation in pore pressure with excavation sequence is shown in fig. 9.36. The three points considered for this case history are slightly different to the position of those considered for the three other case histories, but show trends exactly as would be expected. The point close to the toe of the slope has the greatest change in pore pressure. Points 2 and 3 show a similar trend to each other although the pore pressures for point 2 are higher. The trends are similar to the other case histories.
$C_p = 7 \text{ kN/m}^2$, $\phi_p = 20^\circ$

$C_r = 1 \text{ kN/m}^2$, $\phi_r = 13^\circ$

$A = 0.20$

$E_r = E_p \times 0.75$
\( C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ \)

\( C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ \)

\( A = 0.20 \)

\( E_r = E_p \times 0.75 \)

Fig. 9.33  NEW CROSS
Critical circles for each set of strength parameters are in general different for each excavation stage. Critical circle always has $E_r = E_p \times 0.75$, but value of $A$ varies. Method 3 used to locate failure circle (see text for details).

Fig. 9.34 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE FOR OVERALL SLOPE FAILURE
**NEW CROSS**

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**Note:** Critical circle always has \( E_r = E_p \times 0.75 \) but value of \( A \) varies. Only one circle considered for all strength parameters. Method 3 used to locate failure circle (see text for details).

*Fig. 9.35 VARIATION IN FACTOR OF SAFETY FOR ONE CIRCLE ONLY*
Fig. 9.36 VARIATION IN PORE PRESSURE WITH EXCAVATION SEQUENCE

Note: Pore pressure parameter A = 0.40, Er/Ep = 0.75
9.4. BRADWELL

The slope failures at the Bradwell site in Essex were a result of deep excavations in London clay for the foundations of a nuclear power station in 1957. The slope failures at this site have been described in detail by Skempton and La Rochelle (1965) and the horizontal stresses at this site have also been described in detail by Skempton (1961). Since the variation of $K_0$ with depth for London clay used in this study has been taken from Skempton's investigations at Bradwell, this site warrants particular attention. However this site differs from the other sites studied, since failure occurred at Bradwell very soon after excavation, whereas failures at each of the other sites occurred many years after excavation. Therefore the Bradwell failures are considered to be short-term ($\theta=0$) total stress failures, whereas failures at the other sites are considered to be effective stress failures.

Approximately half the power station site at Bradwell is covered by a soft post-glacial Marsh Clay, which increases in thickness towards the nearby River Blackwater which is approximately 150 metres away. This Marsh clay has a maximum thickness of 3.5 metres and is underlain by the Brown and the Blue London clay which has a total thickness of about 50 metres at this site. There were four major excavations at Bradwell and these were the Turbine House excavation, completed in April 1957; the Reactor No.1 excavation, completed in April 1957; the Reactor No.2 excavation, completed in July 1957; and the Pump House excavation completed in August 1957. The major failures at the Bradwell site
occurred in the Reactor No.1 excavation and a cross section of this slide is shown in fig. 9.37 after Skempton and La Rochelle (1965). It can be seen that the failure was progressive until it encompassed the whole slope.

9.4.1. Analysis of Bradwell

Bradwell was the most difficult excavation to analyse since a surcharge of clay was placed at the top of the slope during the excavation procedures, and therefore compromises have been made with the mesh adopted. A surcharge load could have been simulated by simply applying forces to the appropriate nodal points at the top of the Marsh clay layer (see fig. 9.37), but the large failure surface which developed on the 25th. April actually passed through this layer, and hence this layer is required for the stability analysis. Therefore it was decided to incorporate this clay layer within the initial mesh, even though it was appreciated that this may cause an unusually high decrease in stability in the early excavation stages since there would be a high surcharge load on this section of the mesh. This is correct for the final excavated slope, but not for the situation prior to excavation. However, stability is only crucial for the final excavated slope and it was considered that this was the best approach to adopt. Another problem arising with the development of this mesh was that the total number of elements available using the Univac 1106 was restricted to about 500 elements. Additional elements could be used but it would have resulted in much longer run-times. Hence the mesh used for Bradwell is as shown in fig. 9.38.

The strength parameters, pore pressures and modulus values
Fig. 9.37 BRADWELL REACTOR No.1 EXCAVATION (after Skempton & LaRochelle, 1965)
used for Bradwell were the same as those used in previous analyses. The relationship for $K_0$ with depth used was as determined by Skempton for Bradwell, and this has been explained previously. The failure circles used for this slope geometry were also a problem since the amount of slope exposed at any one stage does not increase with excavation sequence. In particular, stage 3 exposes a small ridge of material at a slope of 0.5:1, and therefore the stability of this is low. In contrast, stages 1 and 2 expose a slope of only 1:1 and hence is relatively stable. Although a failure circle at stage 3 could pass through the Brown London clay and the Marsh clay (see fig. 9.37), the failure circle with the minimum factor of safety only passes through the Brown London clay. Hence there is a sudden drop in stability at stage 3, as will be shown shortly, which can be explained simply by considering the slope geometry.

The variation in factor of safety with excavation sequence (for the slope exposed at that excavation stage), is shown in figs. 9.39 to 9.42. The results shown are in reasonably good agreement with the actual failure, although not as good as the other case histories. The sudden drop in stability at stage 3 has already been explained and apart from this, there is a gradual reduction in stability with increasing excavation sequence. The trends are very similar for all four figures and the factor of safety at stage 6 ranges from about 1.1 to 1.2. The factor of safety at stage 7 ranges from about 0.75 to 1.1. However actual failure occurred at stage 6.
Fig. 9.39 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
BRADWELL

C_p = 7 KN/m^2  \ \phi_p = 20°

C_r = 1 KN/m^2  \ \phi_r = 20°

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

Fig. 9.40 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
BRADWELL

$C_p = 10 \text{KN/m}^2 \quad \phi_p = 20^\circ$

$C_r = 1 \text{KN/m}^2 \quad \phi_r = 13^\circ$

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

Fig. 9.41 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
BRADWELL

$C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ$

$C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ$

Note: These are absolute minimum values of F.O.S. for the total slope exposed up to that stage. Method 2 used to locate failure circle (see text for details).

Fig. 9.42 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE
Differences between actual and theoretical results are probably explained by lower strength parameters in the clay fill and the Marsh clay than in the Brown London clay. However the ultimate factors of safety are not significantly different between the different strength parameters shown in figs. 9.39 and 9.42. Also the ultimate factors of safety shown are for overall slope failure (including the clay fill) as shown in fig. 9.37, and this is certainly more stable than the other two failure circles shown in fig. 9.37.

The extent of the failure zones for the weakest and strongest strength parameters used is shown in fig. 9.43 and 9.44 respectively. It can be seen that the extent of the failure zones appears to be very large compared to the other case histories. However the physical size of the Bradwell mesh is approximately one half that of the other case histories since the Bradwell mesh necessitated a large number of small elements to accurately represent its complex slope geometry, and there was a restriction on the total number of elements available. Stage 1 causes extensive failure and the failure zone gradually increases with excavation procedure. It can also be seen that a failure circle during stage 1 or stage 2 would not be 100% within a failed zone, whereas on stage 3 it would be. This partly explains the differences between Bradwell and the other case histories.

Finally, the variation in factor of safety with excavation sequence for overall slope failure is shown in Fig. 9.45. This figure is similar to those plotted for the other case histories and shows that the decrease in stability for the overall slope is
C_p = 7 KN/m^2 \quad \phi_p = 20^\circ

C_r = 1 KN/m^2 \quad \phi_r = 13^\circ

A = 0.40 \quad E_r = E_p \times 0.75

Fig. 9.43 - BRADWELL
\[ C_p = 10 \text{ KN/m}^2 \quad \phi_p = 20^\circ \]
\[ C_r = 1 \text{ KN/m}^2 \quad \phi_r = 20^\circ \]
\[ A = 0.40 \quad E_r = E_p \times 0.75 \]

Fig. 9.44  BRADWELL
almost linear plotted on a log-scale. The four different strength parameters only make minor variations in the factor of safety particularly towards the end of the excavation sequence. However, the Bradwell case history is different from the other case histories since failure occurred almost immediately (5 days) after excavation took place at Bradwell, whereas failure occurred many years after excavation for the other case histories. The factor of safety for stage 6, is slightly greater than 1.0 as shown in fig. 9.45, but the results are so close to 1.0 that they are considered reasonable. There is a progressive decrease in stability with excavation sequence for overall slope failure for all case histories studied, and this indicates that the decrease in stability is not related to the type of failure (short or long-term).

The variation in factor of safety for one circle only is shown in fig. 9.46. The relationship is very similar to that shown in fig. 9.45 except that the factors of safety are slightly higher.

The variation in pore pressure with excavation sequence for Bradwell is shown in fig. 9.47. This shows the pore pressure at three points beneath the slope, and all points generally show a decrease in pore pressure with excavation sequence. However there are differences between Bradwell and the other case histories. Firstly, for two points the pore pressures increase both at stage 6 and stage 7, and secondly, the minimum negative pore pressure was not recorded close to the toe of the slope. The reasons for this are not clear. Bradwell certainly had a much steeper slope
Fig. 9.45 VARIATION IN FACTOR OF SAFETY WITH EXCAVATION SEQUENCE FOR OVERALL SLOPE FAILURE
Fig. 9.46 VARIATION IN FACTOR OF SAFETY FOR ONE CIRCLE ONLY

Note: Critical circle always has $E_r = E_p \times 0.75$ but value of $A$ varies. Only one circle considered for all strength parameters. Method 3 used to locate failure circle (see text for details).
Note: Pore pressure parameter $A = 0.40$, $Er/Ep = 0.75$

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**BRADWELL**

![Diagram of pore pressure variation with excavation sequence]

**Fig. 9.47 VARIATION IN PORE PRESSURE WITH EXCAVATION SEQUENCE**
than any of the other case histories studied, and as such, failure occurs at stage 6 not at stage 7. This may cause a large redistribution of excess shear stress to occur at stage 6 which will obviously affect the pore pressures. The geometry of the Bradwell slope is also more complex than the other case histories and hence different forces are applied at different points to simulate excavation. Whatever the exact reasons for the differences in the pore pressures, the overall trends are as would be expected.
10. DISCUSSION

10.1. ASSUMPTIONS, LIMITATIONS, AND EVALUATION OF MODEL

For any computer simulation of a "real world" process, many assumptions have to be made which can make the model very restrictive. This is especially true where a complex problem is being studied and there are restrictions with this present model. However the assumptions and limitations with this model do not make it overly restrictive for its designed application.

The model is based on the widely used finite element method of stress analysis and the model has to start from some initial premise of the in-situ stress conditions. It has been recognised in the literature that in-situ stresses play an important role in slope stability analysis, but little attention has been paid to analysing slope failures using in-situ stresses. This model considers in-situ stresses and enables studies to be made of changes in these stresses when an excavation procedure is simulated. The excavation procedure was simulated in six stages but the number of stages can be increased with modifications to the program. Six stages was a compromise between excavating a thin enough layer of material to be realistic, and the computer time required to process a complete multiple stage excavation.

10.2. ELEMENT SHAPE

Each excavated layer was generally kept uniformly thick and the elements close to the excavated surface were also kept close to equilateral triangles. This necessarily involved elongated element shapes in areas of the mesh remote from the excavated
surface. However this is not considered to be a serious problem. It does present some difficulties when the boundary of failed elements extends to areas where there are large elongated elements because the boundary between failed and non-failed elements can then become erratic. This problem can be overcome by the type of mesh generated but this would involve a much larger number of elements.

One of the good features of this model is the mesh generation program. It requires a minimal amount of input data, has a convergence factor to increase fineness near critical areas and the lines which define the mesh need not be vertical. The lines which define the excavated and non-excavated sections of the mesh can be considered as two separate lines although they must join at the final excavated level. This mesh generation program allows meshes of various shapes and element configurations to be developed with ease.

10.3. MATERIAL TYPES AND WATER PRESSURES

One of the major problems with an excavation procedure is the material type and the generated water pressures both during and after the excavation process. The two are closely linked since material type obviously affects the permeability. In this study only one material type has been used since the slope failures studied have consisted of one material type only (some excavations studied have consisted of both the Brown and the Blue London clay). As described previously in Chapter 7, the water pressures can then be determined throughout the excavation procedure by a knowledge of the material properties. It is considered
that the restriction of only one material type is not a major drawback since such an analysis would only be strictly applicable to thick clay deposits. Layered deposits are in some ways less complex since equilibration of pore pressures will probably be much faster and failure could be prevented by individual hard bands (e.g., interbedded sandstone and shale). Any clay layers within such a sequence may not be so important.

The present model has the limitation that the final pore pressures are applied in one step. The pore pressures developed throughout the excavation procedure are generally negative, depending on the value of A and the location of the element within the mesh. In some cases these negative pore pressures can be quite high. These negative pore pressures are a result of rapid excavation and are not related to the position of the water table at that time. However the long-term pore pressures are a result of the long-term position of the water table. Therefore the pore pressures can be calculated both up to the end of excavation and for the long-term condition. However the change from end of excavation to the long-term condition can take up to 50 years. It would be useful to be able to gradually change the pore pressure to the long-term value and this could be a future refinement of the model. However the extent of the decrease in the factor of safety from the end of excavation to the long-term condition can be significant as shown in Chapter 8, and from this the rate of decrease in factor of safety with time can be calculated.
10.4. STRESS-STRAIN ASSUMPTIONS

One of the major assumptions with this and any model is the stress-strain criterion to be used. This model uses a fairly simple stress-strain model where there is an abrupt drop in the modulus after failure, and this may not accurately represent the actual stress-strain behaviour of the material. However this present model is not concerned primarily with displacements or strains within elements. The extent of floor heave, within limits, is not of primary concern to the model. The strains are important in as much as they affect the stresses and it is the overall stability of the slope and hence the factor of safety which is important.

During the development of this model it was also found that elements with a very low modulus cannot be used during a multi-stage excavation process. If elements have a very low modulus, and excavation forces are applied to those elements, the displacements become extremely large and the mesh becomes meaningless. This is not a problem restricted to the stress-strain curve used in this present study but would occur with any stress-strain curve with a low post-failure modulus. For a single-stage analysis this is not a problem since all elements have a high modulus to start with and only fail (and hence have a low modulus), after excavation forces are applied. If a high modulus is used for failed elements, the stress changes then become unacceptably large. The problem was solved by making failed elements stiff just prior to an excavation stage and keeping the stresses artificially constant. The modulus values were
then set back to their original values after the excavation forces were applied.

Excessive displacements can also be a problem during iteration cycles if extremely low post-failure modulus values are used and this aspect was also investigated in Chapter 8. As a result the minimum post-failure modulus value used was 25% of the peak value. Post-failure modulus values of this order give reasonable displacements and stresses. The actual field situation would be that there would occur at the base of the excavation but this would be small compared to the overall height of the slope and this present model achieves such a result. However it is considered that refinements to the program would have to be made before it could be used to accurately simulate such displacements in excavations. In this model the factor of safety is the important criterion, and the values obtained for it in this study compared to known slope failures are reasonable. Therefore the assumptions made within the programs currently used also must be reasonable.

10.5. DISCUSSION OF RESULTS

In order to develop the model as presented in this thesis, a large amount of computer time was necessary and it was difficult to determine overall trends until all the analysis work had been completed. However several things are now immediately apparent. It must be emphasised here that the four case histories studied are completely different excavated slopes. They are not the same slope with different heights and inclinations. In particular the magnitude of the in-situ stresses plays a vital role in both the
extent of the failure zones and the overall stability, and quantititative information in regard to this was only available from one location. Therefore it would be indeed fortuitous if all the theoretical results fitted the actual results.

One of the first things that is apparent is the completely different trend in results for the variation in factor of safety with excavation sequence for that particular excavation stage for all four case histories (these variations will not be described again, but more detailed information can be obtained by referring to figs. 9.5 to 9.8, 9.17 to 9.20, 9.28 to 9.31 and 9.39 to 9.42). Some explanation for the large variation in individual results has been given previously, but an examination of all the results leads to the conclusion that these variations are principally due to two factors. These are the failure circles used and the extent of the failure zones.

The location of the failure circle used in the stability analysis is crucial to the stability result obtained. It was mentioned previously that an automatic circle generator was tried but the results were very erratic and consequently this was not used again. An automatic circle generator can give a circle which only just cuts into the slope but which gives a very low factor of safety. Hence these results can be misleading. Even with a manual input of circles, which only allowed circles to cut through the whole slope at that stage, the stability result obtained was often very low for the initial excavation stages (see figs. 9.28 to 9.31). However these would only be small slumps of material if failure occurred. The question can then be asked,
"why didn't failure occur along these failure circles during the initial excavation stages?". The answer is in the extent of the failure zones.

The failure zones are controlled by the strengths and stresses assigned to those particular elements. The stresses are in turn controlled by the value of $K_0$, the pore pressures and the slope geometry. As soon as the excavation forces are applied to simulate excavation, the stresses change and failure can occur. If this happens during the early excavation stages the failure circle passes through failed material close to the excavated slope surface. Hence normal forces are low on the failure circle and the cohesion is also low since the material has failed. If elements are relatively large compared to the excavation stages and the failure circles used, then failure of only a few elements can cause a drastic reduction in the factor of safety although actual failure may not occur. As excavation proceeds, individual elements become much less significant in relation to the failure circle, and stability can apparently increase. Another factor to be considered is the development of high negative pore pressures which can also increase stability as excavation develops.

The abrupt change in the factor of safety between some excavation stages can also be explained by changes in slope geometry between stages (e.g. stage 3 for Bradwell). If all the failure circles used for all stages were used for each excavation stage, then there would be no abrupt changes in the calculated factor of safety but the minimum value obtained would always be very low. The minimum factor of safety would be obtained from a circle
which is small and shallow as just explained. In practice this is not the case and failure involves the whole excavated slope and hence failure circles were used which went to the toe of the slope exposed at that stage.

In support of the previous discussion are the results obtained for overall slope stability. These results are shown in figs. 9.12, 9.23, 9.34 and 9.45. These results were calculated using circles which passed through the toe of the final slope geometry. Hence at stage 1 these failure circles would be almost complete semi-circles and hence are very stable. As excavation proceeds, stability decreases rapidly and the range of factors of safety for stages 6 and 7 are shown in Table 10.1. The timing of these failures is important and should be considered here. Table 10.2 summarises the timing of the failures from the four case histories studied.

Sudbury Hill and Northolt are relatively long-term failures and figs. 9.12 and 9.23 show that the calculated factor of safety is above 1.0 at stage 6. The factor of safety just drops to 1.0 at stage 7 for Sudbury Hill but drops just below 1.0 for Northolt at stage 7. These results are consistent with the timings shown above in Table 10.2 since these slopes have similar slope heights and geometries. The factor of safety for New Cross (fig. 9.34) at is both above and below 1.0 at stage 6 depending on strength parameters. Assuming Skempton's strength parameters derived from back analyses (i.e. softened strength is \( c = 1 \text{ KN/m}^2, \phi = 20 \))
Table 10-1.
Actual factors of safety immediately after excavation and for the long-term condition based on the predicted critical surface in each case

<table>
<thead>
<tr>
<th>Location</th>
<th>After Excavation</th>
<th>Long-Term Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sudbury Hill</td>
<td>1.265-1.602</td>
<td>1.003-1.127</td>
</tr>
<tr>
<td>Northolt</td>
<td>1.264-1.395</td>
<td>0.853-1.051</td>
</tr>
<tr>
<td>New Cross</td>
<td>0.982-1.139</td>
<td>0.730-0.734</td>
</tr>
<tr>
<td>Bradwell</td>
<td>1.231-1.255</td>
<td>0.754-0.831</td>
</tr>
</tbody>
</table>

NOTE: factors of safety quoted cover the range for the four sets of strength parameters used

Table 10-2.
Summary of time to failure for different case histories

<table>
<thead>
<tr>
<th>Location</th>
<th>Date of Excavation</th>
<th>Date of Failure</th>
<th>Time to Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sudbury Hill</td>
<td>1903</td>
<td>1949</td>
<td>46 years</td>
</tr>
<tr>
<td>Northolt</td>
<td>1903 extended 1936</td>
<td>1955</td>
<td>19 years</td>
</tr>
<tr>
<td>New Cross</td>
<td>1838</td>
<td>1841</td>
<td>3 years</td>
</tr>
<tr>
<td>Bradwell</td>
<td>1957</td>
<td>1957</td>
<td>5-6 days</td>
</tr>
</tbody>
</table>
degrees), the factor of safety falls below 1.0 shortly after stage 6. This is again consistent with the timing shown in Table 10.2. Finally, the analysis of Bradwell (fig. 9.45) shows the factor of safety falling below 1.0 at about one-third the way between stage 6 and stage 7. At stage 6 the factor of safety is about 1.2, and this is probably the worst result obtained. Problems with the mesh as explained previously, may be partly responsible. It is also useful to note that the time period between stages 6 and 7 is between 30 and 50 years depending on the slope height and geometry, whereas between 1 and 6 is only in the order of months.

Table 10.1 gives the range of factors of safety for two stages, immediately after excavation (stage 6) and for the long-term condition (stage 7) for all the case histories studied. The actual factors of safety depend on many different parameters apart from strength, but the range of factors of safety shown is based on the four different strength parameters used in this analysis. It can be seen from Table 10.1 that the factors of safety are very close to 1.00 at stage 7 for Sudbury Hill and Northolt (both long-term failures), and both significantly below 1.00 for New Cross and Bradwell (New Cross was a relatively short-term failure, and Bradwell was a definite short-term failure). The results for stage 6 are also in good agreement with the actual failures except for Bradwell which has a higher factor of safety than would be expected.

The results obtained for the single critical circle shown in figs. 9.13, 9.24, 9.35 & 9.46 are also interesting to note. For
the four strength parameters used in each figure, the parameters with the same residual friction angle (either 20 or 13 degrees) tend to have very similar results. The peak cohesion value did not appear to have such a great effect since once failure has occurred, it is the residual strength parameters which are important.

The development of pore pressures throughout the excavation procedure is also important in terms of overall slope stability. The theoretical pore pressures developed in the four case histories studied are very similar to the classical development of pore pressures (compare figs. 9.14, 9.25, 9.36 & 9.47 with 7.12), and indicate that the method of simulating pore pressures must be reasonably accurate.

Considering the assumptions and limitations of the model and the different case histories studied, the results obtained are in excellent agreement with the actual slope failures.
11. SUMMARY AND CONCLUSIONS

The model is based on the well known finite element method and calculates stresses in terms of both "total" and "effective" values. A multi-stage excavation procedure has been incorporated into the model to simulate progressive excavation of material. It was considered that this was more realistic than a single excavation procedure. Throughout the excavation stage, the pore pressures were calculated using Skempton's pore pressure parameter \( A \), and the change in total stresses. Knowing the pore pressures, the effective stresses could be calculated. Failure, in terms of effective stress, could then be determined for each excavation stage by applying the appropriate strength parameters.

There were a total of six excavation stages, and a final stage where the water pressures were adjusted to their long-term values. This final stage was necessary to study long-term stability which is influenced by pore pressure equilibration.

The stresses after each excavation stage were written onto disc file, and a stability analysis was performed for each stage. The failure surfaces were circular and were split into two groups; those which intersected through the toe of the slope exposed at that stage, and those which intersected through the toe of the slope at the final stage.

Stability analyses were performed for a range of strength parameters, post-peak modulus values and pore pressure parameters. The results of the stability analyses indicate that stability is more sensitive to variations in strength parameters in the early excavation stages than in the later stages, since in the
later stages most of the failure surface passes through failed material. This study shows, however, that high initial stresses are of paramount importance in the development of failure zones and the reduction in overall factor of safety at every stage of excavation. An important consideration in this type of analysis is the adoption of post-peak modulus values for failed elements. Sensitivity analyses were made before adopting a suitable value. Application to actual case histories showed that adoption of a stiff post-peak modulus gives results closest to actual failures. The pore pressure parameter $A$ also influences the stability results, and for the range of values tried, $A = 0.40$ gives reasonable results.

A problem which appeared to be an anomaly when the initial analyses were conducted, was the almost identical factors of safety obtained at stage 7 even for different strength parameters used, in contrast to a conventional limit equilibrium analysis. However the explanation for this is now apparent. At stage 7, for all the four case histories, the failure zone encompasses almost all of the length of the critical failure circle. In these failure zones, the stresses have been modified such that the shear stress = the shear strength. Since the shear stress and shear strength used to calculate a factor of safety are derived from actual stresses within the slope, then the calculated shear stress and shear strength will be equal in the failure zone irrespective of the strength parameters used. Therefore the resulting factor of safety will be close to 1.0 for all cases. However the strength parameters do control the development and extent of the
failure zone, and as such have a major effect on stability throughout the excavation process.

It is considered that such a model is useful in determining both short-term and long-term stability of excavated clay slopes. However, care must be used in the selection of critical failure surfaces, since small failure surfaces (those which encompass a relatively small mass of slope material) that just intersect the slope can give very low stability results and be misleading.

The large differences in time to failure between the case histories studied, would suggest that the mechanism of failure would be different for each case history. This study has shown that this is not the case and that a stress analysis method, such as the one used (using shear strength parameters in terms of effective stress), can accurately model both short-term and long-term stability. A decrease in shear strength to residual values (on even a small part of the slip surface) is not necessary to explain long-term failures as has been used previously by some authors.

From the previous Chapter and from the results presented in this thesis, it can be concluded that a multi-stage finite element procedure with a stability analysis based on in-situ stresses, is useful in analysing the stability of excavated clay slopes. The previous Chapter has pointed out the assumptions and limitations of the model used for this work and these should not be overlooked. However the theoretical results obtained for the final factors of safety are so close to the actual results, that the method must be considered as being able to model actual
behaviour fairly accurately. This study highlights the importance of modelling the variation of both $K_0$ with depth and the increase of modulus with depth. It has been shown that assuming a constant value of $K_0$ with depth gives misleading results.
12. RECOMMENDATIONS AND SCOPE FOR FUTURE RESEARCH

The development of the model to its present state has taken a considerable amount of time since many of the problems of the method itself had to be resolved. Future developments would not take so long since most of them would be refinements of the basic model. However, there are several areas in which future research could be directed.

Firstly, the stress-strain assumptions used in this thesis are fairly simple, and do not accurately represent the post-failure deformation characteristics of clays. A more accurate stress-strain assumption would enable displacements as well as the overall factor of safety to be known with accuracy. This may be important for deep excavations for civil engineering works.

Secondly, the pore pressures are one of the major factors controlling stability in the long-term, and refinements could be made to the present method of adjusting them. For example, the pore pressures are adjusted in one step between stages 6 and 7 which represents a time period of about 30 to 50 years depending on the particular slope. Stage 7 pore pressures are calculated by using the final static water levels, whereas stage 6 values have been calculated by using the stress changes up until that stage. Therefore, the difference between the two values of pore pressure could be calculated, and this difference could be applied gradually in stages to represent increments of time. Providing the input data were sufficiently accurate, then the time to failure could be obtained more accurately than is available from this present model.
Thirdly, the model could be developed to incorporate multiple layers. However, as explained previously, interbedded deposits would not have the same pore pressure dissipation characteristics as uniform clay deposits. Hence it would not be feasible to incorporate a large number of layers.

Fourthly, refinements could be made to the meshes used. Restrictions were placed on the total number of elements used in this present study due to the computer used. This was not entirely satisfactory. If more elements were used then the failure boundary could be defined with greater precision as well as improving the accuracy of the stability calculations.

Fifthly, the application of the model to different case histories would be useful since it would enable additional checks to be made with the method. However, case histories would have to be chosen which had detailed information on the physical characteristics of the material.

Lastly, the development of computer graphics to present the results, would save considerable time in analysing and in interpreting the results. Graphics have been used to plot generated meshes but not the final results. Graphics packages are available to plot contours and vectors of various functions, and many finite element programs have this facility.

Refinements and additions can always be made to computer programs and these present programs are no exception. However, the programs have reached the stage where they are useful and can be applied to either actual case histories or proposed excavations. It is recommended that care be taken in the choice of excavation
to be studied, and that the physical properties of materials in such excavations be known accurately. If these points are considered, then the stability of such excavations could be determined accurately using this model.
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APPENDIX A. USER GUIDES TO COMPUTER PROGRAMS

The computer programs used in this thesis have been written in FORTRAN and modified or developed to run on the University of Wollongong Univac 1106 computer. There are three major computer programs used in the analysis with another used for plotting different mesh configurations either on the University's Calcomp 1039 plotter or on a Tektronix 4014 graphics terminal. The three major programs are: MESH, FEP, and FESS, and the plotting program is called MPLOT. The following section outlines a user guide for these programs.

A.1. MESH GENERATION PROGRAM: MESH

MESH has one subroutine called GEN, and as the name implies is used for generating a finite element mesh with a minimum of data input. MESH.GEN works by defining a series of straight lines in the mesh between which elements are generated. These lines need not be vertical as shown in fig. 7.3, although they are for the multi-stage analysis (as shown in fig. 7.8). A convergence factor is used to create a finer mesh close to the excavated surface and this generally is between 1.2 and 1.3. However, for the excavated portion of the mesh, the excavation stages are kept as constant thicknesses and the convergence factor does not apply.

The program also has the facility to calculate water pressures from a given water table line which is defined by points at the top and the bottom of the slope. A listing of the input data for the Sudbury Hill slope, as shown in fig. 7.8, is given in Appendix 3.
A.1.1. Input data for MESH.GEN

CARD A NY, DEN, OH, SH, AKO, XTOP, XBASE
Format(15, 6F10.3)

NY

is the number of segments in the mesh between defined lines. It is always one less than the number of defined lines, i.e. if a mesh consisted of only two vertical defined lines, there would only be one segment between them.

DEN

is the density of the material.

OH

is the height of the water table above the base of the mesh at the top of the slope.

SH

is the fall in the height of the water table from the top of the slope to the bottom of the slope.

AKO

is the $K_0$ factor. This was set simply equal to 1.0 at this stage and then an equation was used to define $K_0$ with depth in the FEP program.

XTOP

is the X coordinate at the top of the slope.

XBASE

is the X coordinate at the bottom of the slope.

CARD B AH, BH, CH
Format(3F10.3)

AH
is the height of the mesh at the top of the slope above the base of the mesh.

BH

is the height of the mesh at the bottom of the slope above the base of the mesh.

CH

is the height of the mesh at the last excavation stage above the base of the mesh, usually BH equals CH.

CARD C NX,NXX,XF,YF,XL,YL,XEX,YEX,CON

Format(2I5,6F10.3,F6.3)

This card is repeated NY+1 times, i.e. one card per line.

NX

is the number of "sides" of elements on a line below the excavated section. The CON factor only applies to this number of "sides" of elements.

NXX

this is the number of "sides" of elements in the excavated zone. Thus the total number of sides on a line is NX+NXX.

XF & YF

are the X and Y coordinates of the top of the line.

XL & YL

are the X and Y coordinates of the bottom of the line.

XEX & YEX

are the X and Y coordinates of the final excavated level.

CON

is a convergence factor to create a finer mesh near the excavated surface. This is usually between 1.2 and 1.3. This present study used 1.22.
MESH.GEN generates all nodal point and element information and then writes it onto two data files which have internal reference numbers 21 and 22. These data files are then subsequently used by the finite element program, FEP, which has the same internal reference numbers.

A.2. FINITE ELEMENT PROGRAM: FEP

The FEP program consists of five subroutines which are MAIN, SYM, ESM, AESM, and CALSTR. SYM solves the half-band stiffness equations, ESM and AESM are for the element stiffness matrix and anisotropic element stiffness matrix equations, and CALSTR calculates the stress at nodal points and then the respective forces to simulate excavation. The capacity of FEP to handle large meshes really depends on the size of computer used. In this present study the array limit size was set to the number of elements and nodal points as required for each mesh in order to keep core storage requirements small.

FEP is designed to use a single or six stage excavation procedure and it reads nodal point and element information generated by MESH and then writes the stress analysis information onto a separate file after each excavation stage. In the case of the six stage excavation procedure, there are six excavation stages and a final seventh stage in which the pore pressures are set to the final long-term values. These final long-term pore pressures are initially generated by MESH.
The value of $K_0$, the in-situ stress ratio, was found to be extremely important, and eventually the relationship as defined by Skempton (1961) for London clay was used. Since this was not a simple value to input, a simple algorithm was written to generate the appropriate $K_0$ value with depth. Hence if alternate $K_0$ values are required this section of FEP.MAIN would have to be altered.

The modulus value was calculated after Marsland (1971) and increased with depth. The algorithm to do this can also be skipped if required.

Although FEP can cater for six excavation stages in its present form, the mesh is generated in such a way that the first excavation stage has already been excavated. This saves both on the total number of elements and also on the computer time required to "excavate" elements. Hence there are only five subsequent stages during which further elements are excavated. Elements which are excavated have their modulus values set very low and stresses set to zero.

A complete listing of the input data required for FEP is given in Appendix 3.

A.2.1. Input data for FEP

CARD A

NODES,NDISP,NEL,NZERO,E1,E3,POIS,AK0,NEXCA,ISIM,APWP,IFLG

Format(4I5,F10.2,F10.5,2F10.2,2I5,F7.3,/,I5)

NODES
is the number of nodal points.

NDISP

is the number of displacements (2 x NODES).

NEL

is the number of elements.

NZERO

is the total number of nodes with prescribed displacements, i.e. the boundary conditions.

E1 & E3

are values of peak and residual Young's modulus as used in the original program.

POIS

is the value of Poisson's Ratio input as 0.48 for undrained conditions.

AKO

is the value of K, set equal to 1.0 here and modified to suit depth later in the program.

NEXCA

is the number of excavation stages, in this case 6.

ISIM

is a flag to see if the long-term water table level is required, set equal to 0 for 6 stage analysis, set equal to 1 for single stage analysis.

APWP

is the pore-water pressure parameter A.

IFLG

is a print option to restrict output if not required according to the following:

IFLG = 0 minimum of output
    = 1 + nodal point input information
    = 2 + element input information
    = 3 + initial modulus values
    = 4 + nodal point displacement information
= 5 + stress information for each iteration stage.
Note: IFLG is on a separate line from card A although it is read by the same READ statement.

CARD B EFFC,EFFPHI,CR,PHIR,DIFF
Format(4F15.8,F8.3,I2)

EFFC
is peak effective cohesion.

EFFPHI
is peak effective friction angle in radians.

CR
is residual or softened cohesion.

PHIR
is residual or softened friction angle in radians.

DIFF
is the difference between the actual strength and the calculated strength and is therefore the accuracy required.

CARD C NF,NB(I,1),NB(I,2),BV(I,1),BV(I,2)
Format(3I5,2F10.3) This card is repeated NZERO times

NF
is the nodal point number for the prescribed displacement.

NB(I,1) & NB(I,2)
are non-zero for no prescribed displacement and zero for prescribed displacement in the X and Y directions respectively.
$BV(I,1) \& BV(I,2)$

are the magnitudes of the $X$ and $Y$ displacements.

**CARD D** **ISTEL**

*Format*(I5)

**ISTEL**

is the number of elements to be excavated at each stage.

**CARD E** **IELEM**

*Format*(I5)

**IELEM**

is the element number to be excavated. **CARD E** is repeated **ISTEL** times. **CARD D** is then repeated **NEXCA-1** times in this case 5 times. **CARD E** is then repeated the appropriate number of times after each **CARD D**.

**CARD F** **ISTNOD**

*Format*(I5)

**ISTNOD**

is the number of nodal points on an excavated surface.

**CARD G** **INODE**

*Format*(I5)

**INODE**

is the nodal point number on the excavated surface. **CARD G** is repeated **ISTNOD** times. **CARDS F \& G** are required so that the forces to simulate excavation can be calculated in FEP.CALSTR. Since there are 6 excavation
stages CARDS F and the appropriate number of CARD G's, are repeated 6 times.

FEP writes all of the stress information onto eight data files which are then subsequently used by the FESS program.

A.3. FINITE ELEMENT SLOPE STABILITY PROGRAM: FESS

FESS has one subroutine called STRBIS, since the method is a stress analysis which uses circular failure surfaces similar to the Bishop Simplified method of slope stability analysis. This program takes all the stress analysis information from FEP and then calculates the stability of various failure surfaces based on these in-situ stresses.

The method divides the slope up into 100 slices and then determines the intersections between each failure surface and these slices. Stresses are then determined for each point at the base of each slice by interpreting stresses from the closest adjacent element centroids. These stresses are then resolved into shear strength and shear stress with respect to the orientation of the base of the slice and summed for all the slices on that particular failure surface. In this way a factor of safety based on in-situ stresses is determined.

Many programs of this sort use an automatic circle generator to locate the circle with the minimum factor of safety. It was found that such an automatic circle generator was not sufficiently accurate for this type of multi-stage analysis and so a manual procedure of inputting failure circles was preferred.
A.3.1. Input data for FESS

CARD A NC, NL, NT, TITLE
Format(3I5, 15A4)
NC
is the number of specified circles
NL
is the number of different layers
NT
is greater than 1 if a second set of data is to be given.
TITLE
any suitable title.

CARD B NP(L)
Format(10I2)
NP
is the number of points defining the coordinates of each layer within the slope. In this present study there was only one material type hence only one value of NP.

CARD C X, Z
Format(12F6.1)
X & Z
are the X and Z (or Y) coordinates of the slope. For a simple slope only four sets of coordinates are required.
CARD D X0, Z0, RO

Format(3F10.2)

X0

X coordinate of circle

Z0

Z coordinate of circle

RO

radius of circle  This card is repeated NC times (limit 600)

A.4. MESH PLOTTING PROGRAM: MPLOT

MPLOT has 14 subroutines which enable a generated mesh to be plotted in various ways. This program was written by the Mining Research Laboratories of CANMET, Ottawa, Canada, and has been adapted by C. Bertoldi to run on the University of Wollongong Univac 1106.

The program has the following features: (a) a rectangular region can be specified and only elements inside this region plotted, thus allowing magnification of particular areas; (b) a rectangular region can be specified and only elements external to this region can be plotted; (c) nodal point numbers can be plotted and the user can specify a skip constant so that not all numbers need be plotted; (d) element numbers can be plotted and a skip constant can also be specified; (e) node numbers are plotted horizontally and the element numbers vertically; (f) the coordinate scale factor if not specified is automatically calculated by MPLOT.
A listing of the input data for MPLOT is given in Appendix 3.

A.4.1. Input data for MPLOT

CARD A Identification card
Format(20A1,10X,20A1)
NAME and ADDRESS usually input here.

CARD B YPLTIN,XPLTIN,NBOUND,NSKPNP,NSKPEL,SCALE,AROT
Format(2F10.0,3I5,2F10.0)

YPLTIN
is the plot width limit in inches (default value = 10). (see NOTE 1)

XPLTIN
is the plot length limit in inches (default value = 50).

NBOUND
if NBOUND = 0 the whole model is plotted. If NBOUND = 1 only the interior of the specified rectangle is plotted. If NBOUND = 2 only the exterior of the specified rectangle is plotted.

NSKPNP
is the nodal point number skip constant. Every NSKPNP nodal point number is plotted. If NSKPNP equals 0 no nodal point numbers are plotted.

NSKPEL
is the element number skip constant. Every NSKPEL element number is plotted. If NSKPEL equals 0 no element numbers are plotted.

SCALE
is the user specified coordinate scale factor in logical units per inch. If zero or blank, then the program will calculate scale using the value of YPLTIN and XPLTIN.
AROT

is the rotation indicator. If AROT equals 0 then MPLOT x-axis and z-axis must correspond to the plotter X and Z axes. If AROT = 90 then the axes are rotated by 90 degrees and so on.

CARD C follows CARD B ONLY if NBOUND equals 1 or 2

CARD C  ALPHA,XSPMN,XSPMX,YSPMN,YSPMX

Format(A3,7x,4F10.0)

ALPHA

the first 3 columns must contain the letters BND.

XSPMN

the left side of the specified rectangle in logical coordinates.

XSPMX

the right side of the specified rectangle in logical coordinates.

YSPMN

the lower side of the specified rectangle in logical coordinates.

YSPMX

the upper side of the specified rectangle in logical coordinates. The program recognizes the end of the data by an end-of-file card.

NOTE: MPLOT selects a plotter origin 0.5 inches above the lower edge of the plot paper. The minimum X value and minimum Z value of the region to be plotted are taken as logical coordinates of the plotter origin. Thus the whole plot will be above and to the right of the plotter origin. If
SCALE is not specified, then SCALE is calculated so that the region to be plotted lies within the plotter size limits XPLTIN and YPLTIN. Normally YPLTIN will be the constraint on the plotter size (if SCALE is specified the plotted region must be within the plotter size limits or the job will abort). If only the finite element mesh (with no numbering) is required the YPLTIN will have a maximum value of 0.5 inches less than the plotter paper width. Element numbers if required are plotted vertically upward from the centre of the element. If the elements are small, the plotted numbers extend into adjacent elements and the plot becomes confusing and this should be modified in future. If element numbers are requested, the maximum YPLTIN possible is the paper width minus 1 inch (the paper width - 1.5 inches will give a 0.5 inch border above as well as below).

A.5. TYPICAL RUNSTREAM OF FEP AND FESS PROGRAMS

@ASG,AZ FEP1.
@ASG,AZ FESS1.
@ASG,AZ DATAT.
@ASG,AZ FILE21.
@ASG,AZ FILE22.
@ASG,T FILE13.
@ASG,T FILE14.
@ASG,T FILE15.
@ASG,T FILE16.
@ASG,T FILE17.
@ASG,T FILE18.
@ASG,T FILE19.
@ASG,T FILE20.
@USE 21,FILE21.
@USE 22,FILE22.
@USE 13,FILE13.
@USE 14,FILE14.
USE 15, FILE15.
USE 16, FILE16.
USE 17, FILE17.
USE 18, FILE18.
USE 19, FILE19.
USE 20, FILE20.
XQT FEP1.ABS
ADD DATAT.SUD262
XQT FESS1.ABS
ADD DATAT.SUDSTAB1
USE 14, FILE15.
XQT FESS1.ABS
ADD DATAT.SUDSTAB2
USE 14, FILE16.
XQT FESS1.ABS
ADD DATAT.SUDSTAB3
USE 14, FILE17.
XQT FESS1.ABS
ADD DATAT.SUDSTAB4
USE 14, FILE18.
XQT FESS1.ABS
ADD DATAT.SUDSTAB5
USE 14, FILE19.
XQT FESS1.ABS
ADD DATAT.SUDSTAB6
USE 14, FILE20.
XQT FESS1.ABS
ADD DATAT.SUDSTAB6
APPENDIX B. LISTINGS OF COMPUTER PROGRAMS

NOTE:
Columns 1 to 6 have been compressed to fit within the constraints of
the daisy wheel printer used for this output. Every effort has been
made to ensure that the following computer listings are correct but no
guarantee is given that some errors may not be present. Tape versions
of these programs are available on request.

B.1. PROGRAM MESH

C AUTOMATIC MESH GENERATOR

C*** PROGRAM MODIFIED TO RUN ON UNIVAC 1106 AND REWRITTEN TO
C*** INCORPORATE EXCAVATED SECTION OF MESH BY P.GRAY, JAN 1980
C
DIMENSION NX(50), XL(50), YL(50), XF(50), YF(50), NOD(4),
1 SUM1(20), SUM2(20), X(800), Y(800), NXX(50), XEX(50), YEX(50),
2 CON(50)
C
INUM=0
C
C***'NY' IS THE NO. OF SEGMENTS IN THE MESH
C***'CON' IS A FACTOR FOR INCREASING THE LENGTH OF THE LINES ON THE
C*** SIDE OF THE ELEMENTS AS YOU GO AWAY FROM THE MESH SURFACE,
C*** APPROXIMATELY EQUAL TO 1.22 FOR AVERAGE MESH
C***'DEN' IS THE DENSITY OF THE MATERIAL
C***'OH' IS THE HEIGHT OF THE W.T. ABOVE THE BASE AT THE TOP OF
C*** THE SLOPE
C***'SH' IS THE FALL IN THE HEIGHT OF THE W.T. FROM THE TOP OF
C*** THE SLOPE TO THE BASE OF THE SLOPE
C***'AKO' IS THE 'KQ' FACTOR
C***'XTOP' IS THE X VALUE AT THE TOP OF THE SLOPE
C***'XBASE' IS THE X VALUE AT THE BOTTOM OF THE SLOPE
C***'AH' IS THE HEIGHT OF THE MESH AT THE TOP OF THE SLOPE
C***'BH' IS THE HEIGHT OF THE MESH AT THE BOTTOM OF THE SLOPE
C***'CH' IS THE HEIGHT OF THE MESH AT THE LAST EXCA. STAGE
C
READ (5,730) NY,DEN,OH,SH,AKO,XTOP,XBASE
WRITE (6,730) NY,DEN,OH,SH,AKO,XTOP,XBASE
READ (5,740) AH,BH,CH
WRITE (6,740) AH,BH,CH
NY1=NY+1
ALINE1=XTOP-XBASE
ALINE2=AH-BH
ANGLE=ATAN(ALINE2/ALINE1)
C
C***'NX(I)' IS THE NUMBER OF "SIDES" OF ELEMENTS ON A LINE
C***'XF(I)' & 'YF(I)' ARE THE "X" & "Y" CO-ORDINATES OF THE TOP OF
C*** THE LINE
C***'XEX(I)' & 'YEX(I)' ARE THE "X" & "Y" CO-ORDINATES OF THE
C*** FINAL EXCAVATED LEVEL
B-2

C***'NXX(I)' IS THE NO. OF STAGES OF EXCAVATION
C*** N.B. EXCAVATED MESH ASSUMED TO HAVE VERTICAL LINES, IF NO
C*** EXCAVATION, LINES MAY BE NON-VERTICAL IF XEX=XF,YEX=YF & NXX=0
C***'XL(I)' & 'YL(I)' ARE THE "X" & "Y" CO-ORDINATES OF THE BOTTOM
C*** OF THE LINE

DO 10 I=1,NY1
   READ (5,750) NX(I),NXX(I),XF(I),YF(I),XL(I),YL(I),XEX(I),
                 1YEX(I),CON(I)
10    WRITE (6,750) NX(I),NXX(I),XF(I),YF(I),XL(I),YL(I),XEX(I),
                 1YEX(I),CON(I)
    WRITE (6,760)
    N=0
    DO 70 J=1,NXX1
       NXI=NX(I)+1
       NXX1=NXX(I)+1
       NXX2=NXX1+1
       NXX3=NXX2+1
       SUM1(NXX1)=0.0
       SUM1(NXX2)=1.0
       SUM2(NXX1)=0.0
       SUM2(NXX2)=1.0
       SUM=1.0
       NPTS=NX(I)+NXX(I)+1
       EXCA=YF(I)-YEX(I)
       SLICE=EXCA/NXX(I)
       SLICE1=0.0
       DO 20 J=1,NXX1
          N=N+1
          X(N)=XEX(I)
          Y(N)=YF(I)-SLICE
          ND1=N*2-1
          ND2=N*2
          WRITE (6,770) N,X(N),Y(N),ND1,ND2
          WRITE (21,770) N,X(N),Y(N),ND1,ND2
          SLICE1=SLICE1+SLICE
          CONTINUE
20 IF (NXI-2) 30,50,30
30    DO 40 K=NXX3,NPTS
           SUM1(K)=SUM1(K-1)*CON(I)
           SUM2(K)=SUM+SUM1(K)
40      SUM=SUM+SUM1(K)
50    CONTINUE
    XX=XEX(I)
    YY=YEX(I)
    DO 60 J=NXX2,NPTS
       N=N+1
       X(N)=(XL(I)-XEX(I))*SUM2(J)/SUM+XX
       Y(N)=(YL(I)-YEX(I))*SUM2(J)/SUM+YY
       ND1=N*2-1
       ND2=N*2
       WRITE (6,770) N,X(N),Y(N),ND1,ND2
       WRITE (21,770) N,X(N),Y(N),ND1,ND2
60    CONTINUE
CONTINUE
N=0
NSUM=0
NYI=NY-1
WRITE (6,780)
DO 720 I=1,NY
   NXI=NX(I)+NXX(I)
   DO 710 J=1,NXI
      IF (J-NXI) 90,80,90
      80 NDIV1=NX(I+1)+NXX(I+1)
      NDIV2=NX(I)+NXX(I)
      IF (NDIV1-NDIV2) 100,90,110
      90 NOD(1)=J+NSUM
      NOD(2)=NOD(1)+1
      NOD(4)=NOD(2)+NXI+1
      NOD(3)=NOD(4)-1
      GO TO 400
   100 NOD(1)=NOD(1)
   NOD(3)=NOD(2)
   NOD(2)=NOD(1)+1
   NOD(4)=0
   GO TO 400
   110 NOD(1)=NOD(1)
   NOD(3)=NOD(2)
   NOD(2)=NOD(1)+1
   N=N+1
   NDEL1=NOD(1)*2-1
   NDEL2=NOD(1)*2
   NDEL3=NOD(2)*2-1
   NDEL4=NOD(2)*2
   NDEL5=NOD(3)*2-1
   NDEL6=NOD(3)*2
   NOD1=NOD(1)
   NOD2=NOD(2)
   NOD3=NOD(3)
   XC=(X(NOD1)+X(NOD2)+X(NOD3))/3.0
   YC=(Y(NOD1)+Y(NOD2)+Y(NOD3))/3.0
   ST1=DEN*(AH-YC)*AKO
   ST2=DEN*(AH-YC)
   ST3=0.0
   IF (OH-YC) 120,130,130
   120 PWP=0.0
   GO TO 140
   130 PWP=(OH-YC)*62.4
   140 CONTINUE
   IF (XC-XTOP) 180,150,150
   150 STX=DEN*(AH-YC)*AKO
   STY=DEN*(AH-YC)
   STXY=0.0
   IF (YC-OH) 160,160,170
   160 PWP1=62.4*(OH-YC)
   GO TO 250
   170 PWP1=0.0
   GO TO 250
IF (XC-XBASE) 190, 190, 220
IF (YC-CH) 200, 210, 210

PWP1=62.4*(CH-YC)
STX=DEN*(CH-YC)*AKO
STY=DEN*(CH-YC)
STXY=0.0
GO TO 250

PWP1=0.0
STX=0.0
STY=0.0
STXY=0.0
GO TO 250

ALINE3=XC-XBASE
ALINE4=ALINE3*TAN(ANGLE)
ALINE5=ALINE4+CH
STX=DEN*(ALINE5-YC)*AKO
STY=DEN*(ALINE5-YC)
STXY=0.0
IF (OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC) 230, 230, 240

PWP1=0.0
GO TO 250

PWP1=62.4*(OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC)

WRITE (6,790) N, NOD(1), NOD(2), NOD(3), NDEL1, NDEL2, NDEL3, NDEL4, NDEL5, NDEL6, XC, YC, ST1, ST2, ST3, PWP, STX, STY, PWP1
INUM=INUM+1
WRITE (22,790) N, NOD(1), NOD(2), NOD(3), NDEL1, NDEL2, NDEL3, NDEL4, NDEL5, NDEL6, XC, YC, ST1, ST2, ST3, PWP, STX, STY, PWP1
N=N+1
NOD(1)=NOD(1)+1
NOD(2)=NOD(3)+1
NOD(3)=NOD(3)
NDEL1=NOD(1)*2-1
NDEL2=NOD(1)*2
NDEL3=NOD(2)*2-1
NDEL4=NOD(2)*2
NDEL5=NOD(3)*2-1
NDEL6=NOD(3)*2
NOD1=NOD(1)
NOD2=NOD(2)
NOD3=NOD(3)
XC=(X(NOD1)+X(NOD2)+X(NOD3))/3.0
YC=(Y(NOD1)+Y(NOD2)+Y(NOD3))/3.0
ST1=DEN*(AH-YC)*AKO
ST2=DEN*(AH-YC)
ST3=0.0
IF (OH-YC) 260, 270, 270

PWP=0.0
GO TO 280

PWP=(OH-YC)*62.4
CONTINUE

IF (XC-XTOP) 320, 290, 290

STX=DEN*(AH-YC)*AKO
STY=DEN*(AH-YC)
STXY=0.0
IF (YC-OH) 300,300,310
300
PWP1=62.4*(OH-YC)
GO TO 390
310
PWP1=0.0
GO TO 390
320
IF (XC-XBASE) 330,330,360
330
IF (YC-CH) 340,350,350
340
PWP1=62.4*(CH-YC)
STX=DEN*(CH-YC)*AK0
STY=DEN*(CH-YC)
STXY=0.0
GO TO 390
350
PWP1=0.0
STX=0.0
STY=0.0
STXY=0.0
GO TO 390
360
ALINE3=XC-XBASE
ALINE4=ALINE3*TAN(ANGLE)
ALINE5=ALINE4+CH
STX=DEN*(ALINE5-YC)*AK0
STY=DEN*(ALINE5-YC)
STXY=0.0
IF (OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC) 370,370,380
370
PWP1=0.0
GO TO 390
380
PWP=62.4*(OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC)
390
WRITE (6,790) N,NOD(1),NOD(2),NOD(3),NDE11,NDE12,NDE13,NDE14,
NDE5,NDE6,JC,YC,ST1,ST2,ST3,PWP,STX,STY,PWP1
INUM=INUM+1
WRITE (22,790) N,NOD(1),NOD(2),NOD(3),NDE11,NDE12,NDE13,NDE14,
NDE5,NDE6,JC,YC,ST1,ST2,ST3,PWP,STX,STY,PWP1
NOD(1)=NOD(1)
NOD(3)=NOD(2)
NOD(2)=NOD(2)+1
NOD(4)=0
400
N=N+1
NDE11=NOD(1)*2-1
NDE12=NOD(1)*2
NDE13=NOD(2)*2-1
NDE14=NOD(2)*2
NDE5=NOD(3)*2-1
NDE6=NOD(3)*2
NOD1=NOD(1)
NOD2=NOD(2)
NOD3=NOD(3)
JC=(X(NOD1)+X(NOD2)+X(NOD3))/3.0
YC=(Y(NOD1)+Y(NOD2)+Y(NOD3))/3.0
ST1=DEN*(AH-YC)*AK0
ST2=DEN*(AH-YC)
ST3=0.0
IF (OH-YC) 410,420,420
410
PWP=0.0
GO TO 430
420 PWP=(OH-YC)*62.4
430 CONTINUE
440 IF (XC-XTOP) 470,440,440
440 STX=DEN*(AH-YC)*AKO
440 STY=DEN*(AH-YC)
440 STXY=0.0
440 IF (YC-OH) 450,450,460
450 PWP1=62.4*(OH-YC)
460 GOTO 540
460 PWP1=0.0
470 GOTO 540
470 IF (XC-XBASE) 480,480,510
480 IF (YC-CH) 490,500,500
490 PWP1=62.4*(CH-YC)
490 STX=DEN*(CH-YC)*AKO
490 STY=DEN*(CH-YC)
490 STXY=0.0
500 GOTO 540
500 PWP1=0.0
500 STX=0.0
500 STY=0.0
500 STXY=0.0
510 GOTO 540
510 ALINE3=Xc-XBASE
510 ALINE4=ALINE3*TAN(ANGLE)
510 ALINE5=ALINE4+CH
510 STX=DEN*(ALINE5-YC)*AKO
510 STY=DEN*(ALINE5-YC)
510 STXY=0.0
520 IF (OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC) 520,520,530
520 PWP1=0.0
520 GOTO 540
530 PWP1=62.4*(OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC)
540 WRITE (6,790) N,NOD(1),NOD(2),NOD(3),NDEL1,NDEL2,NDEL3,NDEL4
540, NDEL5,NDEL6,XC,YC,ST1,ST2,ST3,PWP,STX,STY,PWP1
540 INUM=INUM+1
540 WRITE (22,790) N,NOD(1),NOD(2),NOD(3),NDEL1,NDEL2,NDEL3,NDEL4
540, NDEL5,NDEL6,XC,YC,ST1,ST2,ST3,PWP,STX,STY,PWP1
550 IF (NOD(4)) 550,700,550
550 N=N+1
550 NOD(1)=NOD(2)
550 NOD(2)=NOD(4)
550 NOD(3)=NOD(3)
550 NDEL1=NOD(1)*2-1
550 NDEL2=NOD(1)*2
550 NDEL3=NOD(2)*2-1
550 NDEL4=NOD(2)*2
550 NDEL5=NOD(3)*2-1
550 NDEL6=NOD(3)*2
550 NOD1=NOD(1)
550 NOD2=NOD(2)
550 NOD3=NOD(3)
550 XC=(X(NOD1)+X(NOD2)+X(NOD3))/3.0
550 YC=(Y(NOD1)+Y(NOD2)+Y(NOD3))/3.0
ST1 = DEN*(AH-YC)*AK0
ST2 = DEN*(AH-YC)
ST3 = 0.0
IF (OH-YC) 560, 570, 570
560 PWP = 0.0
GO TO 580
570 PWP = (OH-YC)*62.4
580 CONTINUE
IF (XC-XTOP) 620, 590, 590
590 STX = DEN*(AH-YC)*AK0
STY = DEN*(AH-YC)
STXY = 0.0
IF (YC-OH) 600, 600, 610
600 PWP1 = 62.4*(OH-YC)
GO TO 690
610 PWP1 = 0.0
GO TO 690
620 IF (XC-XBASE) 630, 630, 660
630 IF (YC-CH) 640, 650, 650
640 PWP1 = 62.4*(CH-YC)
STX = DEN*(CH-YC)*AK0
STY = DEN*(CH-YC)
STXY = 0.0
GO TO 690
650 PWP1 = 0.0
STX = 0.0
STY = 0.0
STXY = 0.0
GO TO 690
660 ALINE3 = XC-XBASE
ALINE4 = ALINE3*TAN(ANGLE)
ALINE5 = ALINE4+CH
STX = DEN*(ALINE5-YC)*AK0
STY = DEN*(ALINE5-YC)
STXY = 0.0
IF (OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC) 670, 670, 680
670 PWP1 = 0.0
GO TO 690
680 PWP1 = 62.4*(OH-SH*(XTOP-XC)/(XTOP-XBASE)-YC)
690 WRITE (6, 790) N, NOD(1), NOD(2), NOD(3), NDEL1, NDEL2, NDEL3, NDEL4, NDEL5, NDEL6, XC, YC, ST1, ST2, ST3, PWP, STX, STY, PWP1
INUM = INUM+1
WRITE (22, 790) N, NOD(1), NOD(2), NOD(3), NDEL1, NDEL2, NDEL3, NDEL4, NDEL5, NDEL6, XC, YC, ST1, ST2, ST3, PWP, STX, STY, PWP1
700 CONTINUE
710 CONTINUE
NSUM = NSUM+NXI+1
720 CONTINUE
STOP
C
730 FORMAT (I5, 6F10.3)
740 FORMAT (3F10.3)
750 FORMAT (2I5, 6F10.3, F6.3)
760 FORMAT (1H1, 6H NODE, 14H X-COORDINATE, 1X, 14H Y-COORDINATE, 1X,
B.2. PROGRAM FEP

C PROGRAM FEP

C FEP CALCULATES STABILITY OF SLOPES IN STRAIN-SOFTENING SOILS
C FINITE ELEMENT ANALYSIS OF STRESSES AROUND AN EXCAVATION
C CONSTANT STRAIN TRIANGLES USED
C PROGRAM USES A MULTI-STAGE EXCAVATION PROCEDURE
C TO SIMULATE EXCAVATION SEQUENCE OF A SLOPE.
C WRITTEN BY P.GRAY FEB. 1981.

DIMENSION ND(260,2), NDEL(455,6), ST(520,32), F(520), X(520),
1 B(3,6), D(3,3), E(455), ERES(455), EE(455), EDUM(455), S(6,6),
2 STRAIN(3)
DIMENSION NF(50), NB(50,2), BV(50,2)
DIMENSION DST(3), BT(6,3)
DIMENSION SDMAX(455), SDPEAK(455), SDRES(455), ZLAHDA(455)
DIMENSION IELEM(6,115), ISTEL(6), CORX(3), CORY(3), SMOVX(3),
1 SMOVY(3)
DIMENSION IAV(455)
COMMON ST,F
COMMON NEL,NODES,IEXCA,GND(260,2),NFEL(455,3)
COMMON STRESS(455,3),PRINST(455,2), XC(455), YC(455), STRESO(455,3)
COMMON ISTNOD(8),INODE(8,25), NFOR(8), NNF(8,50), F1(8,50)
COMMON SNOIDX(260), SNODY(260), SNODY(260)
COMMON ZZRESS(455,3), ESTRES(455,3), TSTRES(455,2), PWP(455)
COMMON ESTREX(455), ESTREY(455), PWP1(455), KAN(455)
KRASH=0
IEXCA=1
ISIMM=1
READ (5,1160) NODES,NDISP,NEL, NZERO, E1, E3, POIS, AKO, NEXCA, ISIM,
1 APWPP, IFLG
WRITE (6,1170) NODES,NDISP,NEL, NZERO, E1, E3, POIS, AKO, NEXCA, ISIM,
1 APWPP, IFLG

C
C***NODES IS NO. OF NODES,
C***NDISP IS NO. OF DISPLACEMENTS(2*NO. OF NODES)
C***NEL IS NO. OF ELEMENTS
C***NZERO IS THE TOTAL NO. OF NODES WITH PRESCRIBED DISPLACEMENTS
C***'E1 AND E3 ARE VALUES OF YOUNG'S MODULUS
C***'POIS' IS THE VALUE OF POISSON'S RATIO
C
C***THE FOLLOWING VALUES ARE OBTAINED FROM THE A.M.G. PROGRAM
WRITE (6,1180)
DO 10 I=1,NODES
    READ (21,1190) KJI,GND(I,1),GND(I,2),ND(I,1),ND(I,2)
    IF (IFLG.LT.1) GO TO 10
    WRITE (6,1190) KJI,GND(I,1),GND(I,2),ND(I,1),ND(I,2)
10 CONTINUE

CORX(1)=GND(66,1)
CORY(1)=GND(66,2)
CORX(2)=GND(111,1)
CORY(2)=GND(111,2)
CORX(3)=GND(138,1)
CORY(3)=GND(138,2)
SMOVX(1)=0.
SMOVY(2)=0.
SMOVY(3)=0.
KFLAG=0

C***GND(N,1) AND GND(N,2) ARE X-Y CO-ORDINATES OF NODE N
C***ND(N,1) AND ND(N,2) ARE THE DISPLACEMENT NOS. ASSOCIATED WITH
C***NODE
C
C***THE FOLLOWING VALUES ARE OBTAINED FROM THE A.M.G. PROGRAM
C
WRITE (6,1200)
MM=0
DO 70 I=1,NEL
    READ (22,1210) JKI,NFEL(I,1),NFEL(I,2),NFEL(I,3),NDEL(I,1),NDEL(1,2)
    STRESS(I,1),STRESS(I,2),STRESS(I,3),PWP(I),STX,STY,PWP1(I)
C
C***SET IN-SITU STRESSES (KO) AS REQUIRED
C
C***FOLLOWING SECTION VARIES KO ACCORDING TO SKEMPTON'S BRADWELL
C***INVESTIGATIONS (1961). N.B. KO KEPT > 1.0 AT DEPTH.
C
ADEPTH=285.0-YC(I)
IF (ADEPTH.GT.15.0) GO TO 20
AKO=1.23+0.094*ADEPTH
GO TO 50
20 IF (ADEPTH.GT.20.0) GO TO 30
AKO=2.16+0.032*ADEPTH
GO TO 50
30 IF (ADEPTH.GT.30.0) GO TO 40
AKO=3.00-0.01*ADEPTH
GO TO 50
40 AKO=3.435685*2.718282*(-0.007998*ADEPTH)
IF (AKO.LT.1.0) AKO=1.0
50 CONTINUE
IF (IFLG.LT.2) GO TO 60
WRITE (6,1220) JKI,NFEL(I,1),NFEL(I,2),NFEL(I,3),NDEL(I,1),NDEL(I,2),NDEL(I,3),NDEL(I,4),NDEL(I,5),NDEL(I,6),XC(I),YC(I),STRESS(I,1),STRESS(I,2),STRESS(I,3),PWP(I),STX,STY,PWP1(I),AKO,ADEPHT

CONTINUE

TX1=STRESS(I,1)*AKO+PWP(I)*(1-AKO)
STRESS(I,1)=TX1
TY1=STRESS(I,2)
TXY1=STRESS(I,3)
STRES0(I,1)=STRESS(I,1)
STRES0(I,2)=STRESS(I,2)
STRES0(I,3)=STRESS(I,3)
TSTRES(I,1)=(TX1+TY1)/2+SQRT((TX1-TY1)**2/4+TXY1**2)
TSTRES(I,2)=(TX1+TY1)/2-SQRT((TX1-TY1)**2/4+TXY1**2)
PRINST(I,1)=TSTRES(I,1)
PRINST(I,2)=TSTRES(I,2)
ESTRES(I,1)=TSTRES(I,1)-PWP(I)
ESTRES(I,2)=TSTRES(I,2)-PWP(I)
ESTREX(I)=STRESS(I,1)-PWP(I)
ESTREY(I)=STRESS(I,2)-PWP(I)
MAX=HAX0(NFEL(I,1),NFEL(I,2),NFEL(I,3))
MIN=MIN0(NFEL(I,1),NFEL(I,2),NFEL(I,3))
MDIFF=(MAX-MIN+1)*2
IF (MM.LT.MDIFF) MM=MDIFF

CONTINUE

WRITE (6,1230) MM,AKO

C
C*** NFEL ARE NODE NUMBERS WHICH TOGETHER FORM ONE ELEMENT
C*** NDEL ARE DISPLACEMENT NUMBERS ASSOCIATED WITH AN ELEMENT
C*** 'PWP(I)' IS THE PURE WATER PRESSURE
C
READ (5,1240) EFFC,EFFPHI,CR,PHIR,DFF

C
C*** 'EFFC' IS THE EFFECTIVE COHESION IN P.S.F.
C*** 'EFFPHI' IS EFFECTIVE ANGLE OF FRICTION IN RADIANS
C*** 'CR' IS RESIDUAL COHESION IN P.S.F.
C*** 'PHIR' IS RESIDUAL ANGLE OF FRICTION IN RADIANS
C*** 'NFOR' IS NUMBER OF NODAL FORCES
C*** 'MN' IS WIDTH OF HALF BAND STIFFNESS MATRIX
C*** 'DFF' IS THE DIFFERENCE BETWEEN THE ACTUAL STRENGTH AND THE
C*** CALCULATED STRENGTH, (I.E. THE ACCURACY REQUIRED)
C
IF (NZERO.EQ.0) GO TO 90
WRITE (6,1250)
DO 80 I=1,NZERO
READ (5,1260) NF(I),NB(I,1),NB(I,2),BV(I,1),BV(I,2)
WRITE (6,1350) NF(I),NB(I,1),NB(I,2),BV(I,1),BV(I,2)
80 CONTINUE
90 CONTINUE

C
C***'NF(I)' IS THE NODAL POINT NO. FOR THE PRESCRIBED DISPLACEMENT
C***'NB(I,1)' & 'NB(I,2)' ARE NON-ZERO FOR NO PRESCRIBED DISPLACEMENT
C*** AND ZERO FOR PRESCRIBED DISPLACEMENT IN THE 'X' & 'Y'
C*** DIRECTIONS RESPECTIVELY
C***'BV(I,1)' & 'BV(I,2)' ARE THE MAGNETUES OF THE 'X' & 'Y'
C*** DISPLACEMENTS
C*** 'NNF(I)' IS A NO. BETWEEN 1 & 458(2*NO. OF NODES) DEFINING
C*** FORCE ON THE NODAL POINT
C*** 'F1(I)' IS THE FORCE ON THE NODAL POINT, ODD NOS. REFER TO THE
C*** X-DIRECTION EVEN NOS. TO THE Y-DIRECTION
C
WRITE (6,1270) POIS
EPHI=EFFPHI*180.0/3.1416
RPHI=PHIR*180.0/3.1416
WRITE (6,1280) EFFC
WRITE (6,1290) EPHI
WRITE (6,1300) CR
WRITE (6,1310) RPHI
WRITE (6,1320) E3
WRITE (6,1330) E1
DO 120 I=1,NEL
  E(I)=E1
  C
  C
  IF (I.GT.1000) GO TO 110
  C
  C***SET MODULUS VALUES AS REQUIRED
  C
  YZ=285.0-YC(I)
  XZ=4000.0*(YZ+45)/85
  EE(I)=XZ*144*1.62
  IF (IFLG.LT.3) GO TO 100
  WRITE (6,1340) I,EE(I)
  100 CONTINUE
  E(I)=EE(I)
  ERES(I)=E(I)*E3

  C
  C
  SDMAX(I)=0.0
  SDPEAK(I)=0.0
  SDRES(I)=0.0
  ZLAMDA(I)=0.0
  KAN(I)=0

  C
  C
  ISTEL(I) IS THE NO. OF ELEMENTS TO BE EXCAVATED IN (I)TH. STAGE
  C***IELEM(I,J) IS THE ELEMENT TO BE EXCAVATED
  C***ISTNOD(I) IS THE NO. OF NODES ON AN EXCAVATED SURFACE
  C***INODE(I,J) IS THE NODE ON AN EXCAVATED SURFACE
  C***NFOR(I) IS THE NO. OF FORCES TO BE APPLIED TO AN EXCAVATED
  C***SURFACE
  C
  ISTEML(NEXCA)=1
  ISTANCE=NEXCA-1
  IF (NEXCA.EQ.1) ISTANCE=1
  DO 150 I=1,ISTAGE
READ (5,1350) ISTEL(I)
IF (ISTEL(I).EQ.0) GO TO 140
ISTE=ISTEL(I)
DO 130 J=1,ISTE
   READ (5,1350) IELEM(I,J)
130 CONTINUE
140 CONTINUE
150 CONTINUE
DO 170 K=1,NEXCA
   READ (5,1350) ISTNOD(K)
   ISTN=ISTNOD(K)
   NFOR(K)=ISTN*2
   DO 160 L=1,ISTN
      READ (5,1350) INODE(K,L)
      J=L*2
      NNF(K,J-1)=INODE(K,L)*2-1
      NNF(K,J)=INODE(K,L)*2
160 CONTINUE
170 CONTINUE
C
CALL CALSTR
C
C***GENERATION OF ELEMENT STIFFNESS MATRIX
C
IF (NEXCA.EQ.1.AND.ISTEL(1).EQ.0) GO TO 190
IF (NEXCA.EQ.1) GO TO 880
C
C***THE FOLLOWING IS REPEATED FOR EACH EXCAVATION STAGE
C
190 CONTINUE
DO 200 I=1,NDISP
   DO 200 J=1,MM
      ST(I,J)=0.0
200 CONTINUE
DO 250 K=1,NEL
   DX=E(K)*(1.0-POIS)/((1.0+POIS)*(1.0-2.0*POIS))
   DY=DX
   DXY=DX*(1.0-2.0*POIS)/(2.0*(1.0-POIS))
   D1=DX*POIS/(1.0-POIS)
   I1=NFEL(K,1)
   I2=NFEL(K,2)
   I3=NFEL(K,3)
   XI=GND(I1,1)
   XJ=GND(I2,1)
   XM=GND(I3,1)
   YI=GND(I1,2)
   YJ=GND(I2,2)
   YM=GND(I3,2)
   CALL ESMT (S,DX,DY,DXY,D1,XI,XJ,XM,YI,YJ,YM,DE4)
   IF (DE4.GT.0.) GO TO 210
   WRITE (6,1370) K
   KRASH=9
210 CONTINUE
***ASSEMBLAGE OF THE HALF BAND STIFFNESS MATRIX

DO 240 LL=1,6
   DO 240 KK=1,6
      M=NDEL(K,KK)
      N=NDEL(K,LL)
      IF (N-M) 230,220,220
   220 NNJ=N-M+1
      ST(M,NNJ)=ST(M,NNJ)+S(KK,LL)
      CONTINUE
   230 CONTINUE
   240 CONTINUE
   250 CONTINUE
   IF (KRASH.EQ.0.) GO TO 260
      WRITE (6,1380)
      GO TO 1120
   260 CONTINUE

***INCORPORATION OF PRESCRIBED DISPLACEMENTS AT BOUNDARY

IF (NZERO.EQ.0) GO TO 300
   DO 290 I=1,NZERO
      M=NF(I)-1
      DO 280 J=1,2
         IF (NB(I,J)> 280,270,280
            NMI=2*M+J
            ST(NMI,1)=ST(NMI,1)*.1E+12
            F(NMI)=ST(NMI,1)*BV(I,J)
         280 CONTINUE
   290 CONTINUE
   300 CONTINUE

***APPLICATION OF NODAL FORCES

IF (NEXCA.LT.IEXCA) GO TO 320
   WRITE (6,1390)
   JFORM=NFOR(IEXCA)
   WRITE (6,1350) IEXCA
   WRITE (6,1350) JFORM
   DO 310 I = 1, JFORM
      WRITE (6,1360) NNF(IEXCA,I),F1(IEXCA,I)
   310 CONTINUE
   320 CONTINUE
   330 I=I+1
   340 I=NFOR(IEXCA)-1
   IF (I-NFOR(IEXCA)) 340,350,340
   350 IER=1
   GO TO 330

***SET MODULUS VALUES BACK TO THOSE PRIOR TO EXCAVATION

IF (IEXCA.EQ.1) GO TO 370
DO 360 I=1, NEL
E(I)=EDUM(I)
360 CONTINUE
370 CONTINUE

C
C
C*** SOLUTION OF STIFFNESS EQUATIONS
C
NL=1
C
C*** THE FOLLOWING IS REPEATED FOR EACH ITERATION STAGE
C
380 CALL SYMT (NDISP, MM, NL)
C WRITE (6,1490)
DO 390 I=1, NDISP
  X(I)=F(I)
  F(I)=0.0
390 CONTINUE
DO 410 I=1, NODES
  IX=2*I-1
  IY=2*I
  IF (IFLG.LT.4) GO TO 400
  CORX(I)=CORX(I)-X(IX)
  CORY(I)=CORY(I)-X(IY)
  SMOVX(I)=SMOVX(I)-X(IX)
  SMOVy(I)=SMOVy(I)-X(IY)
  WRITE (6,1400) I, X(IX), X(IY), SMOVX(I), SMOVy(I), CORX(I), CORY(I)
1, GND(I,1), GND(I,2)
400 CONTINUE
410 CONTINUE
DO 420 I=1, NDISP
  DO 420 J=1, MM
    ST(I,J)=0.0
420 CONTINUE
C
C*** THIS SECTION CALCULATES THE CORRECTION FACTOR TO BE APPLIED
C*** TO EXCAVATION FORCES, RELEVANT ONLY FOR MULTISTAGE ANALYSIS
C
IF (NEXCA.LT.9) GO TO 510
IF (NEXCA.LT.IEXCA) GO TO 510
KJLM=IEXCA+1
DO 500 IJ=KJLM, NEXCA
  ISTN=ISTNOD(IJ)
  DO 490 JK=1, ISTN
    DO 480 KL=1, NEL
      IF (INODE(IJ, JK).EQ.NFEL(KL,1)) GO TO 430
      IF (INODE(IJ, JK).EQ.NFEL(KL,2)) GO TO 440
      IF (INODE(IJ, JK).EQ.NFEL(KL,3)) GO TO 450
    GO TO 480
 430  I4=2
  GO TO 460
 440  I4=4
  GO TO 460
 450  I4=6
IZESTL1=ISTEL(IJ-1)
DO 470 MN=1,ISTEL1
   LMNX=IJ-1
   IF (IELEM(LMNX,MN).EQ.KL) GO TO 480
   CONTINUE
   DX=E(KL)*(1.0-POIS)/((1.0+POIS)*(1.0-2.0*POIS))
   DY=DX
   DXY=DX*(1.0-2.0*POIS)/(2.0*(1.0-POIS))
   D1 = DX*POIS/(1.0-POIS)
   I1=NFEL(KL,1)
   I2=NFEL(KL,2)
   I3=NFEL(KL,3)
   XI=GND(I1,1)
   XJ=GND(I2,1)
   XM=GND(I3,1)
   YI=GND(I1,2)
   YJ=GND(I2,2)
   YM=GND(I3,2)
   CALL ESMT(S,DX,DY,DXY,D1,XI,XJ,XM,YI,YJ,YM,DE4)
   NFEL1=NFEL(KL,1)*2-1
   NFEL2=NFEL(KL,1)*2
   NFEL3=NFEL(KL,2)*2-1
   NFEL4=NFEL(KL,2)*2
   NFEL5=NFEL(KL,3)*2-1
   NFEL6=NFEL(KL,3)*2
   FORCEX=0.0
   FORCEY=0.0
   K3=I4-1
   FORCE1=S(1,1)*X(NFEL1)+S(1,2)*X(NFEL2)+S(1,3)*X(NFEL3)+S(1,4)*X(NFEL4)+S(1,5)*X(NFEL5)+S(1,6)*X(NFEL6)
   FORCE2=S(2,1)*X(NFEL1)+S(2,2)*X(NFEL2)+S(2,3)*X(NFEL3)+S(2,4)*X(NFEL4)+S(2,5)*X(NFEL5)+S(2,6)*X(NFEL6)
   FORCE3=S(3,1)*X(NFEL1)+S(3,2)*X(NFEL2)+S(3,3)*X(NFEL3)+S(3,4)*X(NFEL4)+S(3,5)*X(NFEL5)+S(3,6)*X(NFEL6)
   FORCE4=S(4,1)*X(NFEL1)+S(4,2)*X(NFEL2)+S(4,3)*X(NFEL3)+S(4,4)*X(NFEL4)+S(4,5)*X(NFEL5)+S(4,6)*X(NFEL6)
   FORCE5=S(5,1)*X(NFEL1)+S(5,2)*X(NFEL2)+S(5,3)*X(NFEL3)+S(5,4)*X(NFEL4)+S(5,5)*X(NFEL5)+S(5,6)*X(NFEL6)
   FORCE6=S(6,1)*X(NFEL1)+S(6,2)*X(NFEL2)+S(6,3)*X(NFEL3)+S(6,4)*X(NFEL4)+S(6,5)*X(NFEL5)+S(6,6)*X(NFEL6)
   FORCEX=FORCE1+FORCE3+FORCE5
   FORCEY=FORCE2+FORCE4+FORCE6
   MNX=JK*2-1
   MNY=JK*2
   F1(IJ,MNX)=F1(IJ,MNX)-FORCEX
   F1(IJ,MNY)=F1(IJ,MNY)-FORCEY
480 CONTINUE
490 CONTINUE
500 CONTINUE
510 CONTINUE
   IF (ISIM.EQ.0) GO TO 520
   IF (IEXCA.GT.NEXCA) IEXCA=NEXCA
   WRITE (6,1410) IEXCA
   GO TO 530
**EVALUATION OF STRAINS AND STRESSES IN THE ELEMENTS**

```plaintext
WRITE (6,1440)
K = 1
MD = 0
EXCESS = 0.0
NI = 0

THE FOLLOWING IS REPEATED FOR EACH ELEMENT

540 DO 550 I = 1, 3
   DO 550 J = 1, 3
     D(I,J) = 0.0
   550 CONTINUE

560 CONTINUE
I1 = NFEL(K,1)
I2 = NFEL(K,2)
I3 = NFEL(K,3)
XI = GND(I1,1)
XJ = GND(I2,1)
XM = GND(I3,1)
YI = GND(I1,2)
YJ = GND(I2,2)
YM = GND(I3,2)
A = (XJ*YM - XM*YJ + XI*(YJ - YM) + YI*(XM - XJ))
B(1,1) = (YJ - YH)/A
B(1,3) = (YM - YI)/A
B(1,5) = (YI - YJ)/A
B(2,2) = (XM - XJ)/A
B(2,4) = (XI - XM)/A
B(2,6) = (XJ - XI)/A
B(3,1) = B(2,2)
B(3,2) = B(1,1)
B(3,3) = B(2,4)
B(3,4) = B(1,3)
B(3,5) = B(2,6)
B(3,6) = B(1,5)
J1 = NDEL(K,1)
J2 = NDEL(K,2)
J3 = NDEL(K,3)
```
J4 = NDEL(K, 4)
J5 = NDEL(K, 5)
J6 = NDEL(K, 6)
IF (ISIMM .EQ. 2) GO TO 600
DO 570 I = 1, 3
  STRAIN(I) = B(I, 1) * X(J1) + B(I, 2) * X(J2) + B(I, 3) * X(J3) + B(I, 4) * X(J4) +
             B(I, 5) * X(J5) + B(I, 6) * X(J6)
570 CONTINUE
DO 580 I = 1, 3
  DO 580 J = 1, 3
    STRESS(K, I) = STRESS(K, I) + D(I, J) * STRAIN(J)
 580 CONTINUE
PRINST(K, 1) = (STRESS(K, 1) + STRESS(K, 2)) * 0.5 + SQRT((STRESS(K, 1) - STRESS(K, 2))**2 / 4.0 + STRESS(K, 3)**2)
PRINST(K, 2) = (STRESS(K, 1) + STRESS(K, 2)) * 0.5 - SQRT((STRESS(K, 1) - STRESS(K, 2))**2 / 4.0 + STRESS(K, 3)**2)

C***KEEP STRESSES CONSTANT ON FAILED ELEMENTS DURING ITERATION CYCLES
C
IF (KAN(K) .EQ. 0) GO TO 590
IF (KAN(K) .GT. 0 .AND. ITER .EQ. 1) GO TO 590
PRINST(K, 1) = TSTRES(K, 1)
PRINST(K, 2) = TSTRES(K, 2)
STRESS(K, 1) = STRESS(K, 1)
STRESS(K, 2) = STRESS(K, 2)
STRESS(K, 3) = STRESS(K, 3)
590 CONTINUE

C***ADJUSTMENT OF WATER PRESSURES DUE TO TOTAL STRESS CHANGES
C
IF (KAN(K) .EQ. 2) GO TO 600
IF (KAN(K) .GT. 0 .AND. ITER .GT. 1) GO TO 600
IF (ISIM .EQ. 1) GO TO 600
IF (YC(K) .GT. 240) GO TO 600
DSIG1 = TSTRES(K, 1) - PRINST(K, 1)
DSIG3 = TSTRES(K, 2) - PRINST(K, 2)
DPWP = DSIG3 + APWPP * (DSIG1 - DSIG3)
PWP(K) = PWP(K) - DPWP
600 CONTINUE

C***SET NEW TOTAL AND EFFECTIVE STRESSES
C
ESTRES(K, 1) = PRINST(K, 1) - PWP(K)
ESTRES(K, 2) = PRINST(K, 2) - PWP(K)
ESTREX(K) = STRESS(K, 1) - PWP(K)
ESTREY(K) = STRESS(K, 2) - PWP(K)
TSTRES(K, 1) = PRINST(K, 1)
TSTRES(K, 2) = PRINST(K, 2)
STRESO(K, 1) = STRESS(K, 1)
STRESO(K, 2) = STRESS(K, 2)
STRESO(K, 3) = STRESS(K, 3)

C
SDMAX(K) = (ESTRES(K, 1) - ESTRES(K, 2)) / 2.0
THETA=2.0*ATAN(STRESS(K,3)/(PRINST(K,1)-STRESS(K,2)))
ATEHT=ESTREX(K)-ESTRES(K,2)
IF (ATEHT.LT.0.001) THETA=3.1415926
IF (KAN(K).NE.2) GO TO 610
MD=MD+1
GO TO 710
610 SDPEAK(K)=EFFC*COS(EFFPHI)+(((ESTRES(K,1)+ESTRES(K,2))/2.0)*SIN(EFFPHI))
SDRES(K)=CR*COS(PHIR)+(((ESTRES(K,1)+ESTRES(K,2))/2.0)*SIN(PHIR))

C
C***TESTS FOR FAILURE
C
IF (KAN(K).EQ.2) GO TO 620
IF (ESTRES(K,2).LT.0.0.OR.KAN(K).EQ.3) GO TO 630
IF (SDMAX(K).GT.SDPEAK(K)) GO TO 640
IF (KAN(K).EQ.1.AND.SDRES(K).GT.SDRES(K)) GO TO 640
IF (KAN(K).EQ.1.AND.(SDMAX(K)+0.1).LT.SDRES(K)) GO TO 660
620 EXCESS=0.0
MD=MD+1
SI=THETA*0.5
GO TO 710
630 KAN(K)=3
EXCESS=0.0
SDPEAK(K)=0.0
SDRES(K)=0.0
E(K)=ERES(K)
MD=MD+1
GO TO 700
C
C***OVERSHOOTING CASE (1) PRE-PEAK OVERSHOOTING
C
640 EXCESS=SDMAX(K)-SDRES(K)
C
SDMAX(K)=SDRES(K)
C
E(K)=ERES(K)
C
KAN(K)=1
SI=THETA*0.5
IF (ABS(EXCESS)-DIFF) 650,650,670
650 EXCESS=0.0
MD=MD+1
GO TO 700
660 EXCESS=(SDRES(K)-SDMAX(K))*(-1.0)
KFLAG=1
670 CONTINUE
DST(1)=EXCESS*COS(THETA)
DST(2)=-(EXCESS*COS(THETA))
DST(3)=EXCESS*SIN(THETA)
C
IF (KFLAG.EQ.1) GO TO 690
DO 680 I=1,6
DO 680 J=1,3

\[ BT(1, J) = 0.0 \]

\[ BT(1, 1) = (YJ - YM) \]
\[ BT(3, 1) = (YM - YI) \]
\[ BT(5, 1) = (YI - YJ) \]
\[ BT(2, 2) = (XM - XJ) \]
\[ BT(4, 2) = (XI - XM) \]
\[ BT(6, 2) = (XJ - XI) \]
\[ BT(1, 3) = (XH - XJ) \]
\[ BT(2, 3) = (YJ - YM) \]
\[ BT(3, 3) = (XI - XM) \]
\[ BT(4, 3) = (YM - YI) \]
\[ BT(5, 3) = (XJ - XI) \]
\[ BT(6, 3) = (YI - YJ) \]
\[ L1 = ND(11, 1) \]
\[ L2 = ND(11, 2) \]
\[ L3 = ND(12, 1) \]
\[ L4 = ND(12, 2) \]
\[ L5 = ND(13, 1) \]
\[ L6 = ND(13, 2) \]

\[ F(L1) = F(L1) + BT(1, 1) \cdot DST(1) + BT(1, 3) \cdot DST(3) \]
\[ F(L2) = F(L2) + BT(2, 2) \cdot DST(2) + BT(2, 3) \cdot DST(3) \]
\[ F(L3) = F(L3) + BT(3, 1) \cdot DST(1) + BT(3, 3) \cdot DST(3) \]
\[ F(L4) = F(L4) + BT(4, 2) \cdot DST(2) + BT(4, 3) \cdot DST(3) \]
\[ F(L5) = F(L5) + BT(5, 1) \cdot DST(1) + BT(5, 3) \cdot DST(3) \]
\[ F(L6) = F(L6) + BT(6, 2) \cdot DST(2) + BT(6, 3) \cdot DST(3) \]

\[ KFLAG = 0 \]
\[ ESTREX(K) = ESTREX(K) - DST(1) \]
\[ ESTREY(K) = ESTREY(K) - DST(2) \]
\[ STRESS(K, 3) = STRESS(K, 3) - DST(3) \]
\[ SIG1 = (ESTREX(K) + ESTREY(K)) / 2 + SQRT((ESTREX(K) - ESTREY(K))**2/4.0 + STRESS(K, 3)**2) \]
\[ SIG3 = (ESTREX(K) + ESTREY(K)) / 2 - SQRT((ESTREX(K) - ESTREY(K))**2/4.0 + STRESS(K, 3)**2) \]
\[ ESTRES(K, 1) = SIG1 \]
\[ ESTRES(K, 2) = SIG3 \]
\[ TSTRES(K, 1) = ESTRES(K, 1) + PWP(K) \]
\[ TSTRES(K, 2) = ESTRES(K, 2) + PWP(K) \]
\[ PRINST(K, 1) = TSTRES(K, 1) \]
\[ PRINST(K, 2) = TSTRES(K, 2) \]
\[ STRESS(K, 1) = ESTREX(K) + PWP(K) \]
\[ STRESS(K, 2) = ESTREY(K) + PWP(K) \]
\[ STRESO(K, 1) = STRESS(K, 1) \]
\[ STRESO(K, 2) = STRESS(K, 2) \]
\[ STRESO(K, 3) = STRESS(K, 3) \]
\[ THETA = 2.0 \cdot ATAN(STRESS(K, 3) / (PRINST(K, 1) - STRESS(K, 2))) \]
\[ ATEHT = ESTREX(K) - ESTRES(K, 2) \]
\[ IF (ATEHT .LT. 0.001) THETA = 3.1415926 \]
\[ SI = THETA \cdot 0.5 \]

\[ KAN(K) .EQ. 0.0 .OR. KAN(K) .EQ. 2) GO TO 710 \]
\[ ANM = 0.5 / (1.0 + POIS) \]
\[ ANN = ABS(E(K) / EE(K)) \]
\[
\begin{align*}
EAN &= \frac{E1}{(1.0 + ANN \cdot POIS) \cdot (1.0 - ANN \cdot POIS - 2.0 \cdot ANN \cdot POIS \cdot POIS)} \\
T1 &= \cos(SI) \cdot \cos(SI) \cdot \cos(SI) \cdot \cos(SI) \\
T2 &= \sin(SI) \cdot \sin(SI) \cdot \sin(SI) \cdot \sin(SI) \\
T3 &= \sin(SI) \cdot \sin(SI) \cdot \cos(SI) \cdot \cos(SI) \\
T4 &= \sin(SI) \cdot \cos(SI) \\
T5 &= \cos(SI) \cdot \cos(SI) - \sin(SI) \cdot \sin(SI) \\
T6 &= \cos(SI) \cdot \cos(SI) \cdot \cos(SI) \cdot \sin(SI) \\
T7 &= \sin(SI) \cdot \sin(SI) \cdot \sin(SI) \cdot \cos(SI) \\
TT1 &= ANN \cdot (1.0 - ANN \cdot POIS \cdot POIS) \\
TT2 &= (1.0 + ANN \cdot POIS) \cdot ANN \cdot POIS \cdot 2.0 \\
TT3 &= 1.0 - POIS \cdot POIS \cdot ANN \cdot ANN \\
TT4 &= ANN \cdot (1.0 + ANN \cdot POIS) \cdot (1.0 - ANN \cdot POIS - 2.0 \cdot POIS \cdot POIS \cdot ANN) \\
\end{align*}
\[
\begin{align*}
DX &= (TT1 \cdot T1 + TT2 \cdot T3 + TT3 \cdot T2 + 4.0 \cdot TT4 \cdot T3) \cdot EAN \\
DY &= (TT1 \cdot T2 + TT2 \cdot T3 + TT3 \cdot T1 + 4.0 \cdot TT4 \cdot T3) \cdot EAN \\
D1 &= (TT1 \cdot T3 + TT2 \cdot 0.5 \cdot (T1 + T2) + TT3 \cdot T3 - 4.0 \cdot TT4 \cdot T3) \cdot EAN \\
D2 &= (TT1 \cdot T6 + TT2 \cdot 0.5 \cdot T7 - TT2 \cdot 0.5 \cdot T6 - TT3 \cdot T7 - 2.0 \cdot TT4 \cdot T4 \cdot T5) \cdot EAN \\
D3 &= (TT1 \cdot T7 + TT2 \cdot 0.5 \cdot T6 - TT2 \cdot 0.5 \cdot T7 - TT3 \cdot T6 + 2.0 \cdot TT4 \cdot T4 \cdot T5) \cdot EAN \\
DXY &= (TT1 \cdot T3 + TT2 \cdot T3 + TT3 \cdot T3 + TT4 \cdot T5 \cdot T5) \cdot EAN \\
\end{align*}
\]
C***ASSEMBLAGE OF THE HALF BAND STIFFNESS MATRIX

DO 790 LL=1,6
   DO 790 KK=1,6
      M=NDEL(K,KK)
      N=NDEL(K,LL)
      IF (N-M) 780,770,770
         NNJ=N-M+1
      ST(M,NNJ)=ST(M,NNJ)+S(KK,LL)
   CONTINUE
790 CONTINUE
    IF (K-NEL) 800,810,800
    K=K+1
   EXCESS=0.0
GO TO 540

810 NMD=NEL-MD
WRITE (6,1130)
WRITE (6,1140) (IAV(J),J=1,NI)
WRITE (6,1470) NMD

C     IF (IEXCA.EQ.6.AND.IMER.GT.999) STOP
C
C     IF (MD-NEL) 820,870,820
   IF (ITER-50) 830,870,870
830 ITER=ITER+1
C
C***INCORPORATION OF PRESCRIBED DISPLACEMENTS AT BOUNDARY
C
DO 860 I=1,NZERO
   M=NF(I)-1
   DO 850 J=1,2
      IF (NB(I,J)) 850,840,850
      NM1=2*M+J
      ST(NM1,1)=ST(NM1,1)*1E+12
      F(NM1)=ST(NM1,1)*BV(I,J)
   CONTINUE
850 CONTINUE
860 CONTINUE
   IF (ISIMM.EQ.2) ISIMM=1
C
GO TO 380

870 IF (IEXCA.GE.NEXCA) GO TO 1000
880 NEXEL=ISTEL(IEXCA)
   DO 890 LMN=1,NEL
      ZZRESS(LMN,1)=STRESS(LMN,1)
      ZZRESS(LMN,2)=STRESS(LMN,2)
      ZZRESS(LMN,3)=STRESS(LMN,3)
   CONTINUE
   IF (NEXCA.EQ.1) GO TO 900

IEXCA=IEXCA+1
WRITE (6,1500) IEXCA
CALL CALSTR
IEXCA=IEXCA-1

900 CONTINUE

WRITE (6,1480)

DO 910 IJ=1,NEXEL
NELOUT=IELEM(IEXCA,IJ)
WRITE (6,1490) NELOUT
PRINST(NELOUT,1)=0.0
PRINST(NELOUT,2)=0.0
STRESS(NELOUT,1)=0.0
STRESS(NELOUT,2)=0.0
STRESS(NELOUT,3)=0.0
KAN(NELOUT)=2
SDMAX(NELOUT)=0.0
SDRES(NELOUT)=0.0
SDPEAK(NELOUT)=0.0
TSTRES(NELOUT,1)=0.0
TSTRES(NELOUT,2)=0.0
ESTRES(NELOUT,1)=0.0
ESTRES(NELOUT,2)=0.0
PWP(NELOUT)=0.0
E(NELOUT)=.1E-15
EE(NELOUT)=.1E-15
910 CONTINUE

***MAKE ELEMENTS STIFF PRIOR TO EXCAVATION

IF (NEXCA.EQ.1) GO TO 930
DO 920 J=1,NEL
EDUM(J)=E(J)
E(J)=EE(J)
920 CONTINUE
930 CONTINUE

WRITE STRESSES ONTO DISC FILE AFTER EACH EXCAVATION STAGE

WRITE (6,1150) IEXCA
DO 990 I=1,NEL
GO TO (940,950,960,970,980), IEXCA
940 WRITE (14) XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),KAN(I)
GO TO 990
950 WRITE (15) XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(1,1),ESTRES(1,2),STRESS(1,3),E(I),KAN(I)
GO TO 990
960 WRITE (16) XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(1,1),ESTRES(1,2),STRESS(1,3),E(I),KAN(I)
GO TO 990
990 CONTINUE
970 WRITE (17) XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),KAN(I)
   GO TO 990
980 WRITE (18) XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),KAN(I)
990 CONTINUE

C

EXCESS=0.0
IEXCA=IEXCA+1
ITER=1
IF (NEXCA.EQ.1) IEXCA=1

C
GO TO 190

C
1000 CONTINUE
ISIM=ISIM+1
ISIMM=2

C
READ IN PWP'S FOR VARIOUS STAGES OF EXCAVATION
C
IF (ISIM.GT.1) GO TO 1040
WRITE (6,1150) IEXCA

C
***WRITE STRESSES ONTO DISC AFTER 6TH. EX. STAGE
C
DO 1010 J=1,NEL
   WRITE (19) XC(J),YC(J),ESTREX(J),ESTREY(J),ESTRES(J,1),ESTRES(J,2),STRESS(J,3),E(J),KAN(J)

C
***CHANGE PWP'S TO LONG TERM VALUES
C
1010 CONTINUE
IEXCA=IEXCA+1
ITER=1
K=1
MD=0
DO 1030 IL=1,NDISP
   F(IL)=0.0
   DO 1020 IM=1,MM
      ST(IL,IM)=0.0
1020 CONTINUE
1030 CONTINUE
WRITE (6,1510) ISIM
WRITE (6,1520)
GO TO 540

1040 WRITE (6,1530)
1050 WRITE (6,1540)
DO 1110 I=1,NEL
   IF (KAN(I).EQ.1) GO TO 1050
   ZLAMDA(I)=SDMAX(I)/SDPEAK(I)
   GO TO 1060
1050 ZLAMDA(I)=SDMAX(I)/SDRES(I)
1060 CONTINUE
IF (KAN(I).GE.2) ZLAMDA(I)=1.0
IF (KAN(I).EQ.3) GO TO 1080
IF (KAN(I).EQ.1) GO TO 1070
IF (KAN(I).EQ.4) GO TO 1090
WRITE (6,1550) I,KAN(I),SDPEAK(I),SDRES(I),SDMAX(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),ZLAMDA(I)
GO TO 1100
1070 WRITE (6,1560) I,KAN(I),SDPEAK(I),SDRES(I),SDMAX(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),ZLAMDA(I)
GO TO 1100
1080 WRITE (6,1570) I,KAN(I),SDPEAK(I),SDRES(I),SDMAX(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),ZLAMDA(I)
GO TO 1100
1090 WRITE (6,1580) I,KAN(I),SDPEAK(I),SDRES(I),SDMAX(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),ZLAMDA(I)
GO TO 1100
1100 CONTINUE
C***WRITE STRESSES ONTO DISC FILE AFTER 7TH. EX. STAGE
C
WRITE (20) XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),KAN(I)
1110 CONTINUE
1120 CONTINUE
C
I=1
C**WRITE STRENGTH PROPERTIES ONTO DISC FILE
C
WRITE (13) NEL,EFFC,CR,EFFPHI,PHIR,E3
C
STOP
C
1130 FORMAT (/,'THE FOLLOWING ELEMENTS HAVE FAILED IN SHEAR')
1140 FORMAT (25(I5))
1150 FORMAT (/,'STRESSES AFTER THE',5x,'EXCAVATION STAGE HAVE BEEN WRITTEN ONTO DISC FILE')
1160 FORMAT (4I5,F10.2,F10.5,2F10.2,2I5,F7.3,/,I5)
1170 FORMAT (/,'I5,2X,'TWICE NO. OF NODES =','I5,2X,'NO. OF BOUNDARY CONDITIONS =','I5,2X,E PEAK =','F10.2,10X,'E RESIDUAL =','F10.6,2X,'POISSONS RATIOS =','F10.2,2X,'THE KO FACTOR =','F10.2,2X,'NO. OF EXCAVATIONS =','I5,2X,'ISIM =','I5,2X,'THE PORE WATER PRESSURE PARAMETER A =','F7.3,2X,'PRINT OPTION SET AS',I5)
1180 FORMAT (1H1,6H NODE ,14H X-COORDINATE ,1X,14H Y-COORDINATE ,1X,10H DISP.NQ. )
1190 FORMAT (1014,2F8.2,2F10.2,F4.2,4F10.2)
1200 FORMAT (1014,2F8.2,2F10.2,F4.2,4F10.2,F5.2,F7.2)
1210 FORMAT (1014,2F8.2,2F10.2,F4.2,4F10.2,F5.2,F7.2)
1220 FORMAT (1014,2F8.2,2F10.2,F4.2,4F10.2,F5.2,F7.2)
1230 FORMAT (/,'THE HALF BAND STIFFNESS MATRIX EQUALS ',I5,2X,'THE KO FACTOR EQUALS',F8.3,/)
THE FOLLOWING BOUNDARY CONDITIONS ARE APPLIED

POISSON'S RATIO = 0.3

EFFECTIVE COHESION IN PSF = 12.3

EFFECTIVE FRICTION ANGLE IN DEG = 8.3

RESIDUAL COHESION IN PSF = 8.3

RESIDUAL FRICTION ANGLE IN DEG = 8.3

E RESIDUAL =

E PEAK =

PROGRAM ABORTED DUE TO ZERO OR NEGATIVE AREA

THE FOLLOWING FORCES ARE APPLIED

EXCAVATION STAGE

NUMBER OF ITERATION

EVALUATION OF STRESSES AND STRENGTHS

NO. OF ELEMENTS WITH EXCESS STRESS

THE FOLLOWING ELEMENTS ARE EXCAVATED

ADJUSTMENT OF PORE WATER PRESSURES TO SIMULATE THE LONG TERM WATER TABLE CONDITION STAGE

THE STRESS DISTRIBUTION IS AS SHOWN ABOVE

**************** WARNING!!! ******************

INCOMPATIBILITY BETWEEN STRESSES FOR ELEMENT NO.

******************** WARNING!!! ******************

INCOMPATIBILITY BETWEEN STRESSES FOR ELEMENT NO.

******************** WARNING!!! ******************

THE STRESSES FOR THIS ELEMENT ARE AS LISTED BELOW

END
0.2.1. SUBROUTINE ESM

SUBROUTINE FOR ELEMENTAL STIFFNESS FORMATION

SUBROUTINE ESM (A, DX, DY, DXY, D1, XI, XJ, XM, YM, YI, YJ, DE4)

DIMENSION A(6, 6)

AI=XJ*YM-XM*YJ
BI=YJ-YI
CI=XM-XJ
AJ=XM*YI-XI*YM
BJ=YM-YI
CM=XJ-XI

DET=(AI+XI*BI+YI*CI)/2.0

A(1, 1)=DX*BI*BI+DXY*CI*CI
A(2, 1)=(D1+DXY)*CI*BI
A(2, 2)=DY*CI*CI+DXY*BI*BI
A(3, 1)=DX*BI*BJ+DXY*CI*CJ
A(3, 2)=D1*BJ*CI+DXY*BI*CJ
A(3, 3)=DX*BJ*BJ+DXY*CJ*CJ
A(4, 1)=D1*BI*BJ+DXY*BJ*CI
A(4, 2)=DY*CI*CJ+DXY*BJ*BJ
A(4, 3)=(D1+DXY)*CJ*BJ
A(4, 4)=DY*CJ*CJ+DXY*BJ*BJ
A(5, 1)=DX*BI*BM+DXY*CI*CM
A(5, 2)=D1*CI*BM+DXY*BI*CM
A(5, 3)=DX*BJ*BM+DXY*CJ*CM
A(5, 4)=D1*CJ*BM+DXY*CM*BJ
A(5, 5)=DX*BM*BM+DXY*CM*CM
A(6, 1)=D1*B1*CM+DXY*BM*CI
A(6, 2)=DY*CI*CM+DXY*BI*BM
A(6, 3)=D1*BJ*CM+DXY*BM*CJ
A(6, 4)=DY*CJ*CM+DXY*BJ*BM
A(6, 5)=(D1+DXY)*BM*CM
A(6, 6)=DY*CM*CM+DXY*BM*BM

DE4=DET*4

DO 10 K=1, 6
   DO 10 J=1, K
   10 A(K, J)=A(K, J)/DE4

DO 20 K=1, 5
   K1=K+1
   DO 20 J=K1, 6
   20 A(K, J)=A(J, K)

RETURN

END
**B.2.2. SUBROUTINE AESM**

**SUBROUTINE FOR ANISOTROPIC STIFFNESS MATRIX**

**SUBROUTINE AESMT (A, DX, DY, DXY, D1, D2, D3, XI, XJ, XM, YI, YJ, YM)**

**DIMENSION A(6,6)**

AI = XJ * YM - XM * YJ
BI = YJ - YM
CI = XM - XJ
AJ = XM * YI - XI * YM
BJ = YM - YI
CJ = XI - XM
AM = XI * YJ - XJ * YI
BM = YI - YJ
CM = XJ - XI

DET = (AI + XI * BI + YI * CI) / 2.0

A(1,1) = DX * BI * BI + DXY * CI * CI + BI * CI * D2 * 2.0
A(2,1) = (D1 + DXY) * CI * BI + BI * BI * D2 * CI * CI * D3
A(2,2) = DY * CI * CI + DXY * BI * BI + BI * CI * D3 * 2.0
A(3,1) = DX * BI * BJ + DXY * CJ * CI + BJ * CI * D2 * 2.0
A(3,2) = D1 * BJ * CI + DXY * BI * CJ + CI * CJ * D3 + BI * BJ * D2
A(3,3) = DX * BJ * BJ + DXY * CJ * CJ + BJ * CJ * D2 * 2.0
A(4,1) = D1 * BI * CJ + DXY * BJ * CI + BI * BJ * D2 + CI * CJ * D3
A(4,2) = DY * CI * CJ + DXY * BI * BJ + BJ * CI * D3 + BI * CJ * D3
A(4,3) = (D1 + DXY) * CJ * BJ + BJ * BJ * D2 + CJ * CJ * D3
A(4,4) = DY * CJ * CJ + DXY * BJ * BJ + BJ * CJ * D3 * 2.0
A(5,1) = DX * BI * BM + DXY * CI * CM + BI * CM * D2 + BM * CI * D2
A(5,2) = D1 * CI * BM + DXY * BI * CM + CI * CM * D3 + BI * BM * D2
A(5,3) = DX * BJ * BM + DXY * CJ * CM + BJ * CM * D2 + BM * CJ * D2
A(5,4) = D1 * CJ * BM + DXY * CM * BJ + CJ * CM * D3 + BJ * BM * D2
A(5,5) = DX * BM * BM + DXY * CM * CM + BM * CM * D2 * 2.0
A(6,1) = D1 * BI * CM + DXY * BM * CI + BI * BM * D2 + CI * CM * D3
A(6,2) = DY * CI * CM + DXY * BI * BM + BM * CI * D3 + BI * CM * D3
A(6,3) = D1 * BJ * CM + DXY * BM * CJ + BJ * BM * D2 + CJ * CM * D3
A(6,4) = DY * CJ * CM + DXY * BJ * BM + BM * CJ * D3 + BJ * CM * D3
A(6,5) = (D1 + DXY) * BM * CM + BM * BM * D2 + CM * CM * D3
A(6,6) = DY * CM * CM + DXY * BM * BM + BM * BM * D2 * 2.0

DE4 = DET * 4

DO 10 K = 1, 6
   DO 10 J = 1, K
10   A(K, J) = A(K, J) / DE4

DO 20 K = 1, 5
   K1 = K + 1
   DO 20 J = K1, 6
20      A(K, J) = A(J, K)

RETURN

END
B.2.3. SUBROUTINE SYM

SUBROUTINE FOR SOLUTION OF HALF BAND STIFFNESS EQUATIONS

SUBROUTINE SYMT (NN, MM, NL)
DIMENSION A(520,32), B(520,1), C(32)
COMMON A,B
N=0
10 N=N+1
   DO 20 L=1,NL
   20 B(N,L)=B(N,L)/A(N,1)
   IF (N-NN) 30,90,30
30 DO 40 K=2,MM
   C(K)=A(N,K)
   40 A(N,K)=A(N,K)/A(N,1)
   DO 80 L=2,MM
      I=N+L-1
      IF (NN-I) 80,50,50
50 J=0
   DO 60 K=L,MM
      J=J+1
      60 A(I,J)=A(I,J)-C(L)*A(N,K)
   DO 70 LA=1,NL
      70 B(I,LA)=B(I,LA)-C(L)*B(N,LA)
80 CONTINUE
   GO TO 10
90 N=N-1
   IF (N) 100,140,100
100 DO 130 K=2,MM
      L=N+K-1
      IF (NN-L) 130,110,110
110 CONTINUE
   DO 120 IA=1,NL
      120 B(N,IA)=B(N,IA)-A(N,K)*B(L,IA)
130 CONTINUE
   GO TO 90
140 RETURN

END

B.2.4. SUBROUTINE CALSTR

SUBROUTINE FOR CALCULATING THE STRESS AT NODAL POINTS
AND THEN THE RESPECTIVE FORCES TO SIMULATE EXCAVATION

SUBROUTINE CALSTR
COMMON ST(520,32), F(520)
COMMON NEL, NODES, INEXCA, GND(260,2), NFEL(455,3)
COMMON STRESS(455,3), PRINST(455,2), XC(455), YC(455), STRES0(455,3)
COMMON ISTNOD(8), INODE(8,25), NFOR(8), NNF(8,50), F1(8,50)
COMMON SNODX(260), SNODY(260), SNODXY(260)
COMMON ZZRESS(455,3), ESTRES(455,3), TSTRES(455,2), PWP(455)
COMMON ESTREX(455), ESTREY(455), PWP1(455), KAN(455)

JSTNOD=ISTNOD(IEXCA)
WRITE (6,210)
DO 150 NS=1, JSTNOD
   HDIS1=100.0
   HDIS1A=100.0
   HDIS1B=100.0
   HDIS1C=100.0
   HDIS1D=100.0
   HDIS1E=100.0
   ICH1=1
   ICH1A=1
   ICH1B=1
   ICH1C=1
   ICH1D=1
   ICH1E=1
   ICHECK=0
   NK=INODE(IEXCA,NS)
   IF (NS.EQ.JSTNOD) GO TO 130
   DO 20 I=1, NEL
C
C***CHECK THE FOR EXCAVATED ELEMENTS!!
C
   IF (KAN(I).EQ.2) GO TO 20
C
C OBTAIN DISTANCE BETWEEN BASE OF SLICE AND ELEMENT CENTROIDS
C
   HYPDIS=SQRT((YC(I)-GND(NK,2))**2+(XC(I)-GND(NK,1))**2)
C
FIND NEAREST CENTROID IN THE 4 QUADRANTS, QUADRANTS ARE
C 1=LOWER LEFT, THEN SEQUENTIALLY ANTICLOCKWISE
C ORDER FOR DETERMINATION OF CLOSEST CENTROIDS IS 1,1A,1B,1C,1D,1E,
C CLOSEST TO FURTHEST
C
   IF (HYPDIS.GT.HDIS1E) GO TO 10
   HDIS1E=HYPDIS
   X1E=XG(I)
   Y1E=YC(I)
   ST11E=TSTRES(I,1)
   ST31E=TSTRES(I,2)
   BXX1E=STRESS(I,1)
   BYY1E=STRESS(I,2)
   STXY1E=STRESS(I,3)
   ICH1E=1
   IF (KAN(I).NE.0) ICH1E=0
   IF (HYPDIS.GT.HDIS1D) GO TO 10
   HDIS1E=HDIS1D
   HDIS1D=HYPDIS
   X1E=X1D
   X1D=XG(I)
   Y1E=Y1D
Y1D = YC(I)  
ST11E = ST11D  
ST11D = TSTRES(I,1)  
ST31E = ST31D  
ST31D = TSTRES(I,2)  
BXX1E = BXX1D  
BXX1D = STRESS(I,1)  
BYY1E = BYY1D  
BYY1D = STRESS(I,2)  
STXY1E = STXY1D  
STXY1D = STRESS(I,3)  
ICH1E = ICH1D  
ICH1D = 1  
IF (KAN(I).NE.0) ICH1D = 0  
IF (HYPDIS.GT.HDIS1C) GO TO 10  
HDIS1D = HDIS1C  
HDIS1C = HYPDIS  
X1D = X1C  
X1C = XC(I)  
Y1D = Y1C  
Y1C = YC(I)  
ST11D = ST11C  
ST11C = TSTRES(I,1)  
ST31D = ST31C  
ST31C = TSTRES(I,2)  
BXX1D = BXX1C  
BXX1C = STRESS(I,1)  
BYY1D = BYY1C  
BYY1C = STRESS(I,2)  
STXY1D = STXY1C  
STXY1C = STRESS(I,3)  
ICH1D = ICH1C  
ICH1C = 1  
IF (KAN(I).NE.0) ICH1C = 0  
IF (HYPDIS.GT.HDIS1B) GO TO 10  
HDIS1C = HDIS1B  
HDIS1B = HYPDIS  
X1C = X1B  
X1B = XC(I)  
Y1C = Y1B  
Y1B = YC(I)  
ST11C = ST11B  
ST11B = TSTRES(I,1)  
ST31C = ST31B  
ST31B = TSTRES(I,2)  
BXX1C = BXX1B  
BXX1B = STRESS(I,1)  
BYY1C = BYY1B  
BYY1B = STRESS(I,2)  
STXY1C = STXY1B  
STXY1B = STRESS(I,3)  
ICH1C = ICH1B  
ICH1B = 1  
IF (KAN(I).NE.0) ICH1B = 0
IF (HYPDIS.GT.HDIS1A) GO TO 10
HDIS1B=HDIS1A
HDIS1A=HYPDIS
X1B=X1A
X1A=XC(I)
Y1B=Y1A
Y1A=YC(I)
ST11B=ST11A
ST11A=TSTRES(I,1)
ST31B=ST31A
ST31A=TSTRES(I,2)
STXY1B=STXY1A
STXY1A=STRESS(I,3)
BXX1B=BXX1A
BXX1A=STRESS(I,1)
BYY1B=BYY1A
BYY1A=STRESS(I,2)
ICH1B=ICH1A
ICH1A=1
IF (KAN(I).NE.0) ICH1A=0
IF (HYPDIS.GT.HDIS1) GO TO 10
HDIS1A=HDIS1
HDIS1=HYPDIS
X1A=X1
X1=XC(I)
Y1A=Y1
Y1=YC(I)
ST11A=ST11
ST11=TSTRES(I,1)
ST31A=ST31
ST31=TSTRES(I,2)
STXY1A=STXY1
STXY1=STRESS(I,3)
BXX1A=BXX1
BXX1=STRESS(I,1)
BYY1A=BYY1
BYY1=STRESS(I,2)
ICH1A=ICH1
ICH1=1
IF (KAN(I).NE.0) ICH1=0
10 CONTINUE
20 CONTINUE

C

IF (X1.EQ.X1A.AND.X1.EQ.X1B) GO TO 30
IF (Y1.EQ.Y1A.AND.Y1.EQ.Y1B) GO TO 30
GO TO 70
30 X1B=X1C
Y1B=Y1C
ST11B=ST11C
ST31B=ST31C
BXX1B=BXX1C
BYY1B=BYY1C
STXY1B=STXY1C
ICH1B=ICH1C
HDIS1B=HDIS1C
IF (X1.EQ.X1B.OR.Y1.EQ.Y1B) GO TO 40
GO TO 70

C

40 X1B=X1D
Y1B=Y1D
ST11B=ST11D
ST31B=ST31D
BXX1B=BXX1D
BYY1B=BYY1D
STXY1B=STXY1D
ICH1B=ICH1D
HDIS1B=HDIS1D
IF (X1.EQ.X1B.OR.Y1.EQ.Y1B) GO TO 50
GO TO 70

C

50 X1B=X1E
Y1B=Y1E
ST11B=ST11E
ST31B=ST31E
BXX1B=BXX1E
BYY1B=BYY1E
STXY1B=STXY1E
ICH1B=ICH1E
HDIS1B=HDIS1E
IF (X1.EQ.X1B.OR.Y1.EQ.Y1B) GO TO 60
GO TO 70

C

60 WRITE (6,230) X1,X1A,X1B,X1C,X1D,X1E,Y1,Y1A,Y1B,Y1C,Y1D,Y1E,H
       1DIS1,HDIS1A,HDIS1B,HDIS1C,HDIS1D,HDIS1E,XC(I),YC(I)
GO TO 150

C

70 CONTINUE

C

AHDIS=HDIS1+HDIS1A+HDIS1B
IF (AHDIS.LT.100.1) GO TO 80
WRITE (6,240) GND(NK,1),GND(NK,2),HDIS1,HDIS1A,HDIS1B
GO TO 150

C

80 CONTINUE

C

C CALCULATE STRESSES AT THE NODAL POINT CONCERNED
C

SLOPE1=(Y1-Y1B)/(X1-X1B)
SLOPE2=(GND(NK,2)-Y1A)/(GND(NK,1)-X1A)
BEE1=Y1B-SLOPE1*X1B
BEE2=Y1A-SLOPE2*X1A
XINT=(BEE2-BEE1)/(SLOPE1-SLOPE2)
YINT=SLOPE1*XINT+BEE1
SLIN1=SQRT((Y1-Y1B)**2+(X1-X1B)**2)
SLIN2=SQRT((GND(NK,2)-Y1A)**2+(GND(NK,1)-X1A)**2)
SINT1=SQRT((Y1-YINT)**2+(X1-XINT)**2)
SINT2=SQRT((Y1A-YINT)**2+(X1A-XINT)**2)

IF (Y1.LT.YINT) GO TO 90
ST1=ST11+(ABS(ST11-ST11B)/SLIN1)*SINT1
ST3=ST31+(ABS(ST31-ST31B)/SLIN1)*SINT1
STXX=BXX1+(ABS(BXX1-BXX1B)/SLIN1)*SINT1
STYY=BYY1+(ABS(BYY1-BYY1B)/SLIN1)*SINT1
STXY=STXY1+(ABS(STXY1-STXY1B)/SLIN1)*SINT1
GO TO 100
90 CONTINUE
ST1=ST11-(ABS(ST11-ST11B)/SLIN1)*SINT1
ST3=ST31-(ABS(ST31-ST31B)/SLIN1)*SINT1
STXX=BXX1-(ABS(BXX1-BXX1B)/SLIN1)*SINT1
STYY=BYY1-(ABS(BYY1-BYY1B)/SLIN1)*SINT1
STXY=STXY1-(ABS(STXY1-STXY1B)/SLIN1)*SINT1
GO TO 100
100 CONTINUE
IF (Y1A.LT.GND(NK,2)) GO TO 110
AST1=ST11A+(ABS(ST11A-ST1)/SINT2)*SLIN2
AST3=ST31A+(ABS(ST31A-ST3)/SINT2)*SLIN2
SNODX(NK)=BXX1A+(ABS(BXX1A-STXX)/SINT2)*SLIN2
SNODY(NK)=BYY1A+(ABS(BYY1A-STYY)/SINT2)*SLIN2
SNODXY(NK)=STXY1A+(ABS(STXY1A-STXY)/SINT2)*SLIN2
GO TO 120
110 CONTINUE
AST1=ST11A-(ABS(ST11A-ST1)/SINT2)*SLIN2
AST3=ST31A-(ABS(ST31A-ST3)/SINT2)*SLIN2
SNODX(NK)=BXX1A-(ABS(BXX1A-STXX)/SINT2)*SLIN2
SNODY(NK)=BYY1A-(ABS(BYY1A-STYY)/SINT2)*SLIN2
SNODXY(NK)=STXY1A-(ABS(STXY1A-STXY)/SINT2)*SLIN2
GO TO 120
120 CONTINUE
SNODX(NK)=0.0
SNODY(NK)=0.0
SNODXY(NK)=0.0
130 WRITE (6,220) NS,NK,SNODX(NK),SNODY(NK),SNODXY(NK)
140 CONTINUE

WRITE (6,250)
LSTNOD=ISTNOD(IEXCA)
DO 200 JKL=1,LSTNOD
   IF (JKL.EQ.1) GO TO 160
   KSTNOD=ISTNOD(IEXCA)
   IF (JKL.EQ.(KSTNOD-1)) GO TO 170
   IF (JKL.EQ.KSTNOD) GO TO 180
200 CONTINUE

C***THIS SECTION CALCULATES FORCES FOR AN ORDINARY POINT
C***ON THE EXCAVATED SURFACE
C
L1=INODE(IEXCA,JKL-1)
L2=INODE(IEXCA,JKL)
L3=INODE(IEXCA,JKL+1)
STNOR1=SNODY(L1)
STTAN1 = SN0DXY(L1)
STN0R2 = SN0DY(L2)
STTAN2 = SN0DXY(L2)
STN0R3 = SN0DY(L3)
STTAN3 = SN0DXY(L3)
DIST1 = SQRT((GND(L2,2) - GND(L1,2))**2 + (GND(L2,1) - GND(L1,1))**2)
DIST2 = SQRT((GND(L3,2) - GND(L2,2))**2 + (GND(L3,1) - GND(L2,1))**2)
F0RN = (STN0R1 + 2.0*STN0R2)*DIST1/6 + (STN0R3 + 2.0*STN0R2)*DIST2/6
F0RT = (STTAN1 + 2.0*STTAN2)*DIST1/6 + (STTAN3 + 2.0*STTAN2)*DIST2/6
IX = JKL*2 - 1
IY = JKL*2
NNF(IEXCA, IX) = IN0DE(IEXCA, JKL)*2 - 1
NNF(IEXCA, IY) = IN0DE(IEXCA, JKL)*2
F1(IEXCA, IX) = F0RT*(-1.0)
F1(IEXCA, IY) = F0RN*(-1.0)
WRITE (6,260) IX, F1(IEXCA, IX), IY, F1(IEXCA, IY)
GO TO 190

C
C*** THIS SECTION CALCULATES FORCES FOR THE 1ST. POINT (TOP LEFT CORNER ON THE EXCAVATED SURFACE
C
160 CONTINUE
L1 = IN0DE(IEXCA, JKL)
L2 = IN0DE(IEXCA, JKL + 1)
STN0R1 = SN0DY(L2)
STTAN1 = SN0DXY(L2)
STN0R2 = SN0DY(L2)
STTAN2 = SN0DXY(L2)
DIST1 = SQRT((GND(L2,2) - GND(L1,2))**2 + (GND(L2,1) - GND(L1,1))**2)
F0RN = (STN0R2 + 2.0*STN0R1)*DIST1/6
F0RT = (STTAN2 + 2.0*STTAN1)*DIST1/6
IX = JKL*2 - 1
IY = JKL*2
NNF(IEXCA, IX) = IN0DE(IEXCA, JKL)*2 - 1
NNF(IEXCA, IY) = IN0DE(IEXCA, JKL)*2
F1(IEXCA, IX) = F0RT*(-1.0)
F1(IEXCA, IY) = F0RN*(-1.0)
WRITE (6,260) IX, F1(IEXCA, IX), IY, F1(IEXCA, IY)
GO TO 190

C
C*** THIS SECTION CALCULATES FORCES FOR THE CORNER POINT (LAST BUT 1)
C
170 CONTINUE
L1 = IN0DE(IEXCA, JKL-1)
L2 = IN0DE(IEXCA, JKL)
L3 = IN0DE(IEXCA, JKL + 1)
STN0R1 = SN0DY(L1)
STTAN1 = SN0DXY(L1)
STN0R2 = SN0DY(L2)
STTAN2 = SN0DXY(L2)
ANG = ATAN((GND(L3,2) - GND(L2,2))/(GND(L3,1) - GND(L2,1)))
CAN1 = COS(ANG)
SAN1 = SIN(ANG)
CAN2 = (COS(ANG))**2
SAN2 = (SIN(ANG))**2
STNOR3 = SN0DY(L2) * CAN2 + SN0DX(L2) * SAN2 - 2.0 * SN0DXY(L2) * SAN1 * CAN1
STTAN3 = (SN0DY(L2) - SN0DX(L2)) * SAN1 * CAN1 + SN0DXY(L2) * (CAN2 - SAN2)
STNOR4 = 0.0
STTAN4 = 0.0
DIST1 = SQRT((GND(L2, 2) - GND(L1, 2))**2 + (GND(L2, 1) - GND(L1, 1))**2)
DIST2 = SQRT((GND(L3, 2) - GND(L2, 2))**2 + (GND(L3, 1) - GND(L2, 1))**2)
F0RN1 = (STNOR1 + 2.0 * STNOR2) * DIST1 / 6
F0RT1 = (STTAN1 + 2.0 * STTAN2) * DIST1 / 6
F0RN2 = (STNOR4 + 2.0 * STNOR3) * DIST2 / 6
F0RT2 = (STTAN4 + 2.0 * STTAN3) * DIST2 / 6
F0RX = F0RT2 * CAN1 - F0RN2 * SAN1
F0RY = F0RT2 * SAN1 + F0RN2 * CAN1
IX = JKL * 2 - 1
IY = JKL * 2
NNF(IEXCA, IX) = IN0DE(IEXCA, JKL) * 2 - 1
NNF(IEXCA, IY) = IN0DE(IEXCA, JKL) * 2
F1(IEXCA, IX) = (FORX + F0RX) * (-1.0)
F1(IEXCA, IY) = (FORY + F0RY) * (-1.0)
WRITE (6, 260) IX, F1(IEXCA, IX), IY, F1(IEXCA, IY)
GO TO 190

C***THIS SECTION CALCULATES FORCES FOR THE LAST POINT
C
180 CONTINUE
L1 = IN0DE(IEXCA, JKL - 1)
L2 = IN0DE(IEXCA, JKL)
WRITE (6, 182) L1, L2, GND(L1, 1), GND(L1, 2), GND(L2, 1), GND(L2, 2)
ANG = ATAN((GND(L2, 2) - GND(L1, 2)) / (GND(L2, 1) - GND(L1, 1)))
CAN1 = COS(ANG)
SAN1 = SIN(ANG)
CAN2 = (COS(ANG))**2
SAN2 = (SIN(ANG))**2
C
STNOR1 = SN0DY(L1) * CAN2 + SN0DX(L1) * SAN2 - 2.0 * SN0DXY(L1) * SAN1 * CAN1
STTAN1 = (SN0DY(L1) - SN0DX(L1)) * SAN1 * CAN1 + SN0DXY(L1) * (CAN2 - SAN2)
STNOR2 = SN0DY(L2) * CAN2 + SN0DX(L2) * SAN2 - 2.0 * SN0DXY(L2) * SAN1 * CAN1
STTAN2 = (SN0DY(L2) - SN0DX(L2)) * SAN1 * CAN1 + SN0DXY(L2) * (CAN2 - SAN2)
DIST1 = SQRT((GND(L2, 2) - GND(L1, 2))**2 + (GND(L2, 1) - GND(L1, 1))**2)
F0RN = (STNOR1 + 2.0 * STNOR2) * DIST1 / 6
F0RT = (STTAN1 + 2.0 * STTAN2) * DIST1 / 6
F0RX = F0RT * CAN1 - F0RN * SAN1
F0RY = F0RT * SAN1 + F0RN * CAN1
IX = JKL * 2.0 - 1
IY = JKL * 2.0
NNF(IEXCA, IX) = IN0DE(IEXCA, JKL) * 2 - 1
NNF(IEXCA, IY) = IN0DE(IEXCA, JKL) * 2
F1(IEXCA, IX) = FORX * (-1.0)
F1(IEXCA, IY) = FORY * (-1.0)
WRITE (6, 260) IX, F1(IEXCA, IX), IY, F1(IEXCA, IY)
190 CONTINUE
200 CONTINUE
RETURN
B.3. PROGRAM FESS

PROGRAM FESS: PROGRAM USES STRESSES DERIVED FROM FINITE ELEMENT PROGRAM TO CALCULATE SLOPE STABILITY USING CIRCULAR FAILURE SURFACES

WRITTEN BY P. GRAY, NOV. 1980.

DIMENSION SLOPE(80), NP(10), X(10,10), Z(10,10)
DIMENSION XS(100), ZS(100,11), GRAD(10,10), CON(10,10)
DIMENSION X0(600), Z0(600), R0(600), ZC(100)
DIMENSION STRESS(555,3), XC(555), YC(555), ESTRES(555,3)
DIMENSION ESTREX(555), ESTREY(555), E(555), KAN(555)

I=1
AFOS=999.9
READ (13) NEL,EFFC,CR,EFFPHI,PHIR,E3
WRITE (6,800) NEL,EFFC,CR,EFFPHI,PHIR,E3
DO 10 I=1,NEL
   READ (14) XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),KAN(I)
   WRITE (6,810) I,XC(I),YC(I),ESTREX(I),ESTREY(I),ESTRES(I,1),ESTRES(I,2),STRESS(I,3),E(I),KAN(I)
10 CONTINUE
WRITE (6,980)
NUMBER=1
INP=1
GO TO 20
20 WRITE (6,990)
WRITE (6,1000)
C
READ SLOPE CHARACTERISTICS AND GEOMETRY

WRITE (6,1010)
READ (5,1020) NC,NL,NT,(SLOPE(J),J=1,80)
WRITE (6,1020) NC,NL,NT,(SLOPE(J),J=1,80)
\begin{verbatim}
NO=NC
WRITE (6,1030) (SLOPE(J),J=1,80)
WRITE (6,1040)
READ (5,1050) (NP(L),L=1,NL)
WRITE (6,1060)
DO 30 L=1,NL
   NPL=NP(L)
   READ (5,1070) (X(I,L),Z(I,L),I=1,NPL)
   WRITE (12,1150)
   WRITE (6,1070) (X(I,L),Z(I,L),I = 1,NPL)
30 CONTINUE
WRITE (6,1080) NL
40 CONTINUE
C
CDIVIDE SLOPE INTO 100 SLICES, FIND ABSICISSAE OF CENTRE LINE OF SLICES
CCALCULATE THEIR INTERSECTION WITH SEPARATION BETWEEN SOIL LAYERS
C
NP1=NP(1)
WSL=X(NP1,1)/100.
XS(1)=WSL/2.
DO 50 NS=2,100
50 XS(NS)=XS(NS-1)+WSL
DO 110 L=1,NL
   NPL=NP(L)
   DO 80 I=2,NPL
      DIFFER=X(I,L)-X(I-1,L)
      IF (DIFFER) 70,60,70
      60 DIFFER=0.1
      70 GRAD(I,L)=(Z(I,L)-Z(I-1,L))/(DIFFER)
      80 CON(I,L)=(X(I,L)*Z(I-1,L)-X(I-1,L)*Z(I,L))/(DIFFER)
   I=2
   DO 110 NS=1,100
90 IF (XS(NS)-X(I,L)) 110,110,100
100 I=I+1
110 GO TO 90
110 ZS(NS,L)=GRAD(I,L)*XS(NS)+CON(I,L)
NL1=NL+1
DO 120 NS=1,100
120 ZS(NS,NL1)=0.
C
C
CCOORDINATES OF CENTRES AND RADII OF SELECTED CIRCLES
C
IPN=9
IF (NC) 200,200,130
130 IF (NC-20) 140,140,190
C
CGENERATE GRID FOR SELECTED NUMBER OF CIRCLES WITH SEVERAL RADII
C
140 WRITE (6,1090)
   READ (5,1100) XIN,ZIN,DIX,DIZ,DIR,RID,NOC,NOX,NOZ
   IPN=10
   XO(1)=XIN
   DX=DIX
\end{verbatim}
ZO(1) = ZIN
DZ = DIZ
RO(1) = ZO(1) - RID
DR = DIR
NX = 1
NZ = 1
N1 = 2
N2 = NOC
150 DO 160 N = N1, N2
     XO(N) = XO(N-1)
     ZO(N) = ZO(N-1)
     RO(N) = RO(N-1) + DR
160     NAT = N
     NX = NX + 1
     N1 = N1 + NOC
     N2 = N2 + NOC
     N = NAT
     IF (NX.GT.NOX) GO TO 170
     XO(N+1) = XO(N) + DX
     ZO(N+1) = ZO(N)
     RO(N+1) = ZO(N+1) - RID
     GO TO 150
170 NZ = NZ + 1
     IF (NZ.GT.NOZ) GO TO 180
     NX = 1
     XO(N+1) = XO(1)
     ZO(N+1) = ZO(N) + DZ
     RO(N+1) = ZO(N+1) - RID
     GO TO 150
180 NC = N2 - NOC
     GO TO 260
C
C READ COORDINATES OF LIMITED NUMBER OF CENTRES FOR SELECTED CIRCLES
C
190 WRITE (6, 1110)
   INP = 10
   READ (5, 1120) (XO(N), ZO(N), RO(N), N = 1, NC)
   GO TO 260
C
C GENERATE GRID FOR 560 CIRCLES AUTOMATICALLY
C
200 ZMAX = 0
   WRITE (6, 820)
   DO 210 I = 1, NP1
210   ZMAX = AMAX1(ZMAX, Z(I, 1))
   EM = ZMAX/X(NP1, 1)
   EMP1 = SQRT(1 + EM**2)
   DR1 = ZMAX/(6. * EMP1)
   XO(1) = 0.200*X(NP1, 1)
   DX = 0.080*X(NP1, 1)
   ZO(1) = 0.8*ZMAX
   RO(1) = (ZO(1) - EM*XO(1))/EMP1 - DR1
   DR = (ZO(1) - RO(1))/9.
   DZ = EM*DX*0.9
NX = 1
NZ = 1
DXZ = 0.25 * EM * ZMAX
N1 = 2
N2 = 10

220 DO 230 N=N1,N2
   X0(N) = X0(N-1)
   Z0(N) = Z0(N-1)
   RO(N) = RO(N-1) + DR
230 NCIRC = N
   NX = NX + 1
   N1 = N1 + 10
   N2 = N2 + 10
   N = NCIRC
   IF (NX.GT.7) GO TO 240
   X0(N+1) = X0(N) + DX
   Z0(N+1) = Z0(N) + DZ
   RO(N+1) = RO(N-1)
   DR = (Z0(N+1) - RO(N+1)) / 9.
   GO TO 220
240 NZ = NZ + 1
   IF (NZ.GT.8) GO TO 250
   NX = 1
   X0(N+1) = X0(N-1) - DXZ
   Z0(N+1) = Z0(N-1) + 0.2 * ZMAX
   RO(N+1) = 0.8 * (Z0(N+1) - EM * X0(N+1) / EMP1 - DR1
   DR = (Z0(N+1) - RO(N+1)) / 9.
   GO TO 220
250 NC = N2 - 10
C
C WRITE HEADING FOR TABLE OF RESULTS
C
260 WRITE (6,1130)
   N=0
   XOM = X0(1)
   ZOM = Z0(1)
   FM = 1.0E6
   WRITE (6,1140)
   WRITE (6,830) NC
   WRITE (6,840) WSL
   DO 270 N=1,NC
      WRITE (6,850) N, X0(N), Z0(N), RO(N)
270 CONTINUE
C
C DO 770 N=1,NC
C
C DETERMINE INTERCEPTS OF CIRCLE WITH SLOPE
C
DO 340 I=2,NP1
   B = X0(N) + GRAD(I,1) * (Z0(N) - CON(I,1))
   A = 1.0 + GRAD(I,1)**2
   C = X0(N)**2 + (Z0(N) - CON(I,1))**2 - RO(N)**2
   DISC = B**2 - A*C
NPOINT=I
IF (DISC.LE.0.) GO TO 280
IF (XMIN.LT.X(I-1,1)) GO TO 310
IF (XMIN.LE.X(I,1)) GO TO 350
GO TO 340

280 IF (I-NP1) 340,290,290
290 IF (NO+1) 750,770,300
300 WRITE (6,1150)
WRITE (6,1160) N,XO(N),ZO(N),RO(N)

310 IF (I-2) 320,320,280
320 IF (NO+1) 750,770,330
330 WRITE (6,1150)
WRITE (6,1170) N,XO(N),ZO(N),RO(N)

340 CONTINUE

350 DO 360 J=1,100
IF (XMIN-XS(J)) 370,370,360
360 CONTINUE

370 XMAX=(B+SQRT(DIS)) / A
I=NPOINT
IF (XMAX-X(1,1)) 450,450,380
380 IF (I.LT.NP1) GO TO 400
IF (NO+1) 750,770,390
390 WRITE (6,1150)
WRITE (6,1180) N,XO(N),ZO(N),RO(N)

400 IP1=I+1
DO 440 I=IP1,NP1
B=X0(N)+GRAD(I,1)*(Z0(N)-CON(I,1))
A=1.+6RAD(I,1)**2
C=X0(N)**2+(Z0(N)-CON(I,1))**2-RO(N)**2
DISC=B**2-A*C
IF (DISC.LE.0.) GO TO 760
XMAX=(B+SQRT(DIS)) / A
IF (XMAX-X(I,1)) 450,450,410

410 IF (I-NP1) 440,420,420
420 IF (NO+1) 750,770,430
430 WRITE (6,1150)
WRITE (6,1180) N,XO(N),ZO(N),RO(N)
GO TO 770

440 CONTINUE

450 JP1=J+1
DO 460 K=JP1,100
IF (XMAX-XS(K)) 470,470,460
460 CONTINUE
GO TO 480

470 K=K-1

C
CCCALCULATE INTERCEPTS OF CIRCLE WITH EACH SLICE, EFFECTIVE WEIGHT OF EACH SLICE, SUM OF MOMENTS AND F0
C
480 ESTRM=0.0
ESTRSS=0.0
DO 490 I = J, K
   QUEST = RO(N)**2 - (XS(I) - XO(N))**2
   IF (QUEST .LE. 0) GO TO 770
   ZC(I) = ZO(N) - SQRT(QUEST)
   CONTINUE

490

DO 720 NS = J, K
   HDIS1 = 100.0
   HDIS1A = 100.0
   HDIS1B = 100.0
   HDIS1C = 100.0
   HDIS1D = 100.0
   HDIS1E = 100.0
   ICH1 = 1
   ICH1A = 1
   ICH1B = 1
   ICH1C = 1
   ICH1D = 1
   ICH1E = 1
   ICHECK = 0
   DO 510 I = 1, NEL

C***CHECK THE FOR EXCAVATED ELEMENTS!!
C
   IF (KAN(I) .EQ. 2) GO TO 510
C
   C OBTAIN DISTANCE BETWEEN BASE OF SLICE AND ELEMENT CENTROIDS
   HYPDIS = SQRT((YC(I) - ZC(NS))**2 + (XC(I) - XS(NS))**2)
C
   FIND NEAREST CENTROID IN THE 4 QUADRANTS, QUADRANTS ARE
   1 = LOWER LEFT, THEN SEQUENTIALLY ANTICLOCKWISE
   ORDER FOR DETERMINATION OF CLOSEST CENTROIDS IS 1, 1A, 1B, 1C, 1D, 1E,
   CLOSEST TO FURTHEST
C
   IF (HYPDIS .GT. HDIS1E) GO TO 500
   HDIS1E = HYPDIS
   X1E = XC(I)
   Y1E = YC(I)
   ST1E = ESTRES(I, 1)
   ST31E = ESTRES(I, 2)
   BXX1E = ESTREX(I)
   BYY1E = ESTREY(I)
   STXY1E = STRESS(I, 3)
   ICH1E = 1
   IF (KAN(I) .NE. 0) ICH1E = 0
   IF (HYPDIS .GT. HDIS1D) GO TO 500
   HDIS1D = HDIS1D
   HDIS1E = HYPDIS
   X1E = X1D
   X1D = XC(I)
Y1E=Y1D
Y1D=YC(I)
ST11E=ST11D
ST11D=ESTRES(I,1)
ST31E=ST31D
ST31D=ESTRES(I,2)
BXX1E=BXX1D
BXX1D=ESTREX(I)
BYY1E=BYY1D
BYY1D=ESTREY(I)
STXY1E=STXY1D
STXY1D=STRESS(I,3)
ICH1E=ICH1D
ICH1D=1
IF (KAN(I) .NE.0) ICH1D=0
IF (HYPDIS.GT.HDIS1C) GO TO 500
HDIS1D=HDIS1C
HDIS1C=HYPDIS
X1D=X1C
X1C=XC(I)
Y1D=Y1C
Y1C=YC(I)
ST11D=ST11C
ST11C=ESTRES(I,1)
ST31D=ST31C
ST31C=ESTRES(I,2)
BXX1D=BXX1C
BXX1C=ESTREX(I)
BYY1D=BYY1C
BYY1C=ESTREY(I)
STXY1D=STXY1C
STXY1C=STRESS(I,3)
ICH1D=ICH1C
ICH1C=1
IF (KAN(I) .NE.0) ICH1C=0
IF (HYPDIS.GT.HDIS1B) GO TO 500
HDIS1C=HDIS1B
HDIS1B=HYPDIS
X1C=X1B
X1B=XC(I)
Y1C=Y1B
Y1B=YC(I)
ST11C=ST11B
ST11B=ESTRES(I,1)
ST31C=ST31B
ST31B=ESTRES(I,2)
BXX1C=BXX1B
BXX1B=ESTREX(I)
BYY1C=BYY1B
BYY1B=ESTREY(I)
STXY1C=STXY1D
STXY1D=STRESS(I,3)
ICH1C=ICH1B
ICH1B=1
IF (KAN(I).NE.0) ICH1B=0
IF (HYPDIS.GT.HDIS1A) GO TO 500
HDIS1B=HDIS1A
HDIS1A=HYPDIS
X1B=X1A
X1A=XC(I)
Y1B=Y1A
Y1A=YC(I)
ST11B=ST11A
ST11A=ESTRES(I,1)
ST31B=ST31A
ST31A=ESTRES(I,2)
STXY1B=STXY1A
STXY1A=STRESS(I,3)
BXX1B=BXX1A
BXX1A=ESTREX(I)
BYY1B=BYY1A
BYY1A=ESTREY(I)
ICH1B=ICH1A
ICH1A=1
IF (KAN(I).NE.0) ICH1A=0
IF (HYPDIS.GT.HDIS1) GO TO 500
HDIS1A=HDIS1
HDIS1=HYPDIS
X1A=X1
X1=XC(I)
Y1A=Y1
Y1=YC(I)
ST11A=ST11
ST11=ESTRES(I,1)
ST31A=ST31
ST31=ESTRES(I,2)
STXY1A=STXY1
STXY1=STRESS(I,3)
BXX1A=BXX1
BXX1=ESTREX(I)
BYY1A=BYY1
BYY1=ESTREY(I)
ICH1A=ICH1
ICH1=1
IF (KAN(I).NE.0) ICH1=0
CONTINUE
CONTINUE

IF (N.GT.0) GO TO 520
WRITE (6,860) HDIS1,HDIS1A,HDIS1B,HDIS1C
WRITE (6,870) X1,Y1,ST11,ST31,BXX1,BYY1,STXY1
WRITE (6,870) X1A,Y1A,ST11A,ST31A,BXX1A,BYY1A,STXY1A
WRITE (6,870) X1B,Y1B,ST11B,ST31B,BXX1B,BYY1B,STXY1B
WRITE (6,870) X1C,Y1C,ST11C,ST31C,BXX1C,BYY1C,STXY1C
CONTINUE

IF (X1.EQ.X1A.AND.X1.EQ.X1B) GO TO 530
IF (Y1.EQ.Y1A.AND.Y1.EQ.Y1B) GO TO 550

CONTINUE
GO TO 630
530 IF (X1.EQ.X1C) GO TO 560
540 X1B=X1C
   Y1B=Y1C
   ST11B=ST11C
   ST31B=ST31C
   BXX1B=BXX1C
   BYY1B=BYY1C
   STXY1B=STXY1C
   ICH1B=ICH1C
   GO TO 630
   C
550 IF (Y1.EQ.Y1C) GO TO 580
   GO TO 540
   C
560 IF (X1.EQ.X1D) GO TO 590
570 X1B=X1D
   Y1B=Y1D
   ST11B=ST11D
   ST31B=ST31D
   BXX1B=BXX1D
   BYY1B=BYY1D
   STXY1B=STXY1D
   ICH1B=ICH1D
   GO TO 630
   C
580 IF (Y1.EQ.Y1D) GO TO 610
   GO TO 570
   C
590 IF (X1.EQ.X1E) GO TO 620
600 X1B=X1E
   Y1B=Y1E
   ST11B=ST11E
   ST31B=ST31E
   BXX1B=BXX1E
   BYY1B=BYY1E
   STXY1B=STXY1E
   ICH1B=ICH1E
   GO TO 630
   C
610 IF (Y1.EQ.Y1E) GO TO 620
   GO TO 600
   C
620 WRITE (6,880) X1,X1A,X1B,X1C,X1D,X1E,Y1,Y1A,Y1B,Y1C,Y1D,Y1E,
   1HDIS1,HDIS1A,HDIS1B,HDIS1C,HDIS1D,HDIS1E,XC(I),YC(I)
   GO TO 770
   C
630 CONTINUE
   C
AHDIS=HDIS1+HDIS1A+HDIS1B
IF (AHDIS.LT.100.1) GO TO 640
WRITE (6,890) XS(NS),ZC(NS),HDIS1,HDIS1A,HDIS1B
WRITE (6,900) N,X0(N),Z0(N),RO(N)
GO TO 770
CONTINUE

CALCULATE STRESSES AT THE BASE OF THE SLICE

SLOPE1=(Y1-Y1B)/(X1-X1B)
SLOPE2=(ZC(NS)-Y1A)/(XS(NS)-X1A)
BEE1=Y1B-SLOPE1*X1B
BEE2=Y1A-SLOPE2*X1A
XINT=(BEE2-BEE1)/(SLOPE1-SLOPE2)
YINT=SLOPE1*XINT+BEE1
SLIN1=SQRT((Y1-Y1B)**2+(X1-X1B)**2)
SLIN2=SQRT((ZC(NS)-Y1A)**2+(XS(NS)-X1A)**2)
SINT1=SQRT((Y1-YINT)**2+(X1-XINT)**2)
SINT2=SQRT((Y1A-YINT)**2+(X1A-XINT)**2)
ST1=ST11-((ST11-ST11B)/SLIN1)*SINT1
ST3=ST31-((ST31-ST31B)/SLIN1)*SINT1
STXX=BXX1-((BXX1-BXX1B)/SLIN1)*SINT1
STYY=BYY1-((BYY1-BYY1B)/SLIN1)*SINT1
STXY=STXY1-((STXY1-STXY1B)/SLIN1)*SINT1
AST1=ST11A-((ST11A-ST1)/SINT2)*SLIN2
AST3=ST31A-((ST31A-ST3)/SINT2)*SLIN2
ASTXX=BXX1A-((BXX1A-STXX)/SINT2)*SLIN2
ASTYY=BYY1A-((BYY1A-STYY)/SINT2)*SLIN2
ASTXY=STXY1A-((STXY1A-STXY)/SINT2)*SLIN2
ICHECK=ICH1+ICH1A+ICH1B

IF (N.GT.0) GO TO 650
WRITE (6,910) XS(NS),ZC(NS),AST1,AST3,ASTXX,ASTYY,ASTXY
CONTINUE

CALCULATE STRESSES, STRENGTH & FOS

STRCEN=(AST1+AST3)/2
STRRAD=(AST1-AST3)/2
IF (NS.NE.K) GO TO 660
XZ=XS(NS)+((WSL/2.0)
ANGXP=ATAN((ZC(NS)-ZC(NS))/(XZ-XS(NS)))
GO TO 670
ANGXP=ATAN((ZC(NS+1)-ZC(NS))/(XZ-XS(NS)))

ANGX1=(ATAN(ASTXY/(ASTXX-STRCEN)))/2
DIFANG=3.141593-ABS((ANGXP-(ANGX1)**2)
TXY=STRRAD*SIN(DIFANG)
IF (ANGXP.LT.0.OR.DIFANG.LT.0) TXY=TXY**-1
IF (ASTXX.GT.ASTYY) GO TO 680
SIGN=STRCEN-STRRAD*COS(DIFANG)
GO TO 690
SIGN=STRCEN+STRRAD*COS(DIFANG)

WIDBAS=ABS(WSL/COS(ANGXP))
COHES=CR
PHIANG=PHIR
IF (ICHECK.LT.2) GO TO 700
COHES=EFFC
PHIAN6=EFFPHI
700 CONTINUE
STREN=(COHES+SIGN*TAN(PHIAN6))*WIDBAS
IF (AST3.LT.0) STREN=0.0
STRSS=TXY*WIDBAS
ESTREN=ESTREN+STREN
ESTRSS=ESTRSS+STRSS
720 CONTINUE
C
FOS=ESTREN/ESTRSS
C
IF (FOS) 740,730,730
730 IF (FOS.LT.AFOS) AFOS=FOS
C
740 WRITE (6,950) N,XO(N),ZO(N),RO(N),ESTREN,ESTRSS,FOS
750 CONTINUE
760 CONTINUE
C
NUMBER=NUMBER+1
IF (NUMBER.GT.NT) GO TO 780
GO TO 40
780 CONTINUE
C
NUMBER=NUMBER+1
IF (NUMBER.GT.NT) GO TO 780
GO TO 40
780 CONTINUE
C
NUMBER=NUMBER+1
IF (NUMBER.GT.NT) GO TO 780
GO TO 40
780 CONTINUE

C
790 FORMAT (//,5X,'THE MINIMUM FACTOR OF SAFETY IS',F10.4)
800 FORMAT (2X,'NEL =',I5,5X,'EFFC =',F10.2,5X,'CR =',F10.2,5X,'EFFPHI =',F10.2,5X,'PHIR =',F10.2,5X,'E3 =',E15.6)
810 FORMAT (15,8E12.6,15)
820 FORMAT (/,2X,'ABOUT TO GENERATE 560 CIRCLES AUTOMATICALLY')
830 FORMAT ('AT THIS STAGE NC = ',I5)
840 FORMAT (5X,/,' WSL = ',F15.2)
850 FORMAT (15,2X,3F15.2)
860 FORMAT (/,2X,'HDIS1 = ',F15.4,5X,'HDIS1A = ',F15.4,5X,'HDIS1B = ',F15.4,5X,'HDIS1C = ',F15.4)
870 FORMAT (//,5X,'AT COORDINATE POINTS X = ',F10.4,5X,'Y = ',F10.4,5X,'X = ',F10.4,5X,'Y = ',F10.4,5X,'X = ',F10.4,5X,'Y = ',F10.4,5X)
880 FORMAT (5X,'THE STRESSES ARE ',5F15.2)
890 FORMAT (5X,'ERROR!! CLOSEST SIX CENTROIDS ARE IN A STRAIGHT LINE, INTERPOLATION OF STRESSES WILL BE WRONG.';'POINTS ARE 2:',12F9.2,'AND THE DISTANCES ARE:',6F12.4,'THE COORDINATES OF THE POINT IN QUESTION ARE',2F12.4,'CIRCLE ABANDONED')
900 FORMAT (//,5X,'ERROR, NO CLOSE ELEMENTS FOUND NEXT TO POINT WITH COORDINATES X = ',F8.2,'Y = ',F8.2,5X,'3F15.2,'2X,'CIRCLE ABANDONED')
910 FORMAT (5X,'AT THE SLICE BASE WITH COORDINATES X = ',F10.4,5X,'Y = ',F10.4,5X,'THE STRESSES WERE ',5F15.2)
950 FORMAT (/,5X,I5,5X,5F10.2,F12.4)
980 FORMAT (5X,'STABILITY ANALYSIS')
990 FORMAT (1H1,T40,'FACTORS OF SAFETY FOR MULTILAYERED SLOPES'/T40,
141(1H-)/T43,'METHOD OF SLICES FOR CIRCULAR SLIP')/T43,36(1H-)/)
1000 FORMAT (12G13.5)
1010 FORMAT (5X,'TYPE IN CIRCLE CODE (NC) - (I3)'' NO. OF LAYERS (NL)
1- (I2)'' NO. OF SOIL DATA SETS (NT) - (I2)'' AND ANY TITLE -
2(19A2) --- ALL ON THE SAME LINE'' NOTE'' IF NC = 1 - 20 IT
3GIVES THE NUMBER OF CIRCLES YOU MUST SUPPLY'' IF NC > 20 IT
4GIVES THE NUMBER OF CIRCLES GENERATED IN A GRID'' THAT YOU MUST
5SPECIFY LATER. MAXIMUM OF 600'' IF NC = 0 THEN 560 CIRCLES AUTO
6GENERATED'' NC=0 ALL PRINTED OUT'' NC=-1 ONLY
7INTERSECTING CIRCLES PRINTED OUT'' NC=-2 ONLY CRITICAL
8CIRCLE AT EACH CENTRE PRINTED')
1020 FORMAT (3I5,80A1)
1030 FORMAT ('..','TITLE :  *,80A1,//)
1040 FORMAT (5X,'TYPE IN NUMBER OF POINTS IN EACH LAYER-NP (L)-(1012) '  )
1050 FORMAT (1012)
1060 FORMAT (/5X,'NOW X AND Z COORDINATES FOR EACH LAYER'' DO LAYER
1BY LAYER'' FIRST X THEN Z , GIVING A MAX. OF 6 PAIRS ON EACH LIN
2E'' START A NEW LAYER ON A NEW LINE. - (12F6.1)')
1070 FORMAT (12F6.1)
1080 FORMAT ('..','T2,I2,T5,'LAYERS',T20,'COHESION',T35,'ANGLE FRICT',
1T52,'RU',T65,'DENSITY'/T5,6(1H-),T20,8(1H-),T35,11(1H-),T51,
23(1H-),T65,7(1H-))
1090 FORMAT (2X,'DETAILS REQUIRED OF YOUR CIRCLE GENERATION GRID''
1 ALL ON THE SAME LINE'' X COORD OF INITIAL POINT - (F6.1)''
2 Z COORD OF INITIAL POINT - (F6.1)'' INCREMENT IN X - (F
3 36.1)'' INCREMENT IN Z - (F6.1)'' RADIUS INCREMENT - (F
4 46.1)'' LEVEL BELOW WHICH CIRCLES ARE GENERATED(Z COORD) - (F
5 5.1)'' NUMBER OF CIRCLES AT EACH CENTRE - (I3)'' NUMBER
6 OF COLUMNS OF X - (I3)'' NUMBER OF ROWS OF Z - (I3)'' ---
7 NOTE --- THE LAST THREE NUMBERS ARE INTEGERS')
1100 FORMAT (6F6.1,313)
1110 FORMAT (2X,'INPUT X,Z,R VALUES FOR CIRCLE CENTRES -4 SETS PER LINE
1- (12F6.1)')
1120 FORMAT (3F10.2)
1130 FORMAT (///)
1140 FORMAT (12X,'CIRCLES NOW BEING ANALYSED - BE PATIENT')
1150 FORMAT (///)
1160 FORMAT ('..','T2,I3,T8,3(F8.2,8X),T61,'CIRCLE DOES NOT INTERSECT
1 SLOPE',T106,1H-,'T115,1H-)
1170 FORMAT ('..','T2,I3,T8,3(F8.2,8X),T61,'INTERCEPT BELOW BOTTOM OF
1 SLOPE',T106,1H-,'T115,1H-)
1180 FORMAT ('..','T2,I3,T9,3(F7.2,9X),T61,'INTERCEPT ABOVE TOP OF SLO
1 PE',T106,1H-,'T115,1H-)
1190 FORMAT (//5X,'ANALYSIS COMPLETED')
C
END
B.4. PROGRAM MPLOT

C PROGRAM MPLOT(INPUT,OUTPUT,TAPE1,TAPE5=INPUT)

C***PURPOSE-
C THIS ROUTINE DOES A PLOT OF THE 2-D FINITE ELEMENT MESH
C GENERATED BY THE PROGRAM PACKAGE MSHGEN
C CALCOMP SOFTWARE IS USED

C***SUBROUTINES INCLUDED
C PLTHED,MSHINP,SCALEF,ELPLOT,PLTNOD,INTCPT,PLTRCT

C***CALCOMP ROUTINES NEEDED
C PLT,SYMBOL,NUMBER,DASHP
C (DASHP(X,Y,DH) DRAWS A DASHED LINE WITH DASH LENGTH DL FROM
C THE CURRENT POSITION OF THE PLOTTER PEN TO THE POINT WITH
C PLOTTER CO-ORDINATES (X,Y))

C***CDCROUTINES
C EOF(LUN) - USED TO TEST FOR END OF FILE ON UNIT LUN
C TIME - ROUTINE GIVES THE CURRENT CLOCK TIME
C DATE - ROUTINE GIVES CURRENT DATE

C***NON-EXISTENTROUTINE
C ABORT - THIS NON-EXISTENT ROUTINE IS CALLED TO FORCE A JOB
C ABORT

C***MPLOTCARD INPUT
C
C ***CARD 1 IDENTIFICATION CARD
C **READ , ANAME(20),ADRESS(20)
C FORMAT(20A1,10X,20A1)
C *ANAME - USER NAME - UP TO 20 CHARACTERS
C *ADRESS - USER ADDRESS - UP TO 20 CHARACTERS
C
C FOR EACH PLOT THE FOLLOWING CARD(S) MUST BE INPUT
C ***PLOT PARAMETER CARD
C **READ ,YPLTIN,XPLTIN,NBOUND,NSKPNP,NSKPEL,SCALE,AROT
C FORMAT(2F10.0,3I5,2F10.0)
C *YPLTIN - PLOT WIDTH IN INCHES (NOTE- PLOT ORIGIN IS SET .5
C INCH ABOVE LOWER EDGE I.E. PAPER WIDTH .GE. (YPLTIN+.5))
C DEFAULT VALUE IS 10 INCHES
C *XPLTIN - UPPER LIMIT OF PLOT LENGTH IN INCHES
C DEFAULT IS 50 INCHES
C *NBOUND - PLOT TYPE INDICATOR
C IT IS POSSIBLE TO MAGNIFY A PART OF THE TOTAL MESH BY
C SPECIFYING A RECTANGULAR REGION AND INDICATING ALL POINTS
C EXTERIOR TO THE SPECIFIED RECTANGLE ARE TO BE EXCLUDED
C SIMILARLY IT IS POSSIBLE TO EXCLUDE A RECTANGULAR REGION
C THAT IS TOO REFINED FOR PLOTTING WITH CURRENT SCALE
C NBOUND =0 THE WHOLE REGION IS PLOTTED
C NBOUND =1 ONLY INTERIOR OF SPECIFIED RECTANGLE IS PLOTTED
C NBOUND =2 ONLY EXTERIOR OF SPECIFIED RECTANGLE IS PLOTTED
C *NSKPNP - NODAL POINT NUMBER SKIP CONSTANT
EVERY NSKPNP NODAL POINT NUMBER IS PLOTTED

IF NSKPNP=0 NO NODAL POINT NUMBERS ARE PLOTTED

*NSKPEL - ELEMENT NUMBER SKIP CONSTANT
EVERY NSKPEL ELEMENT NUMBER IS PLOTTED
IF NSKPEL=0 NO ELEMENT NUMBERS ARE PLOTTED

*SSCALE - USER SPECIFIED CO-ORDINATE SCALE FACTOR IN UNITS/INCH
IF ZERO OR BLANK SCALE WILL BE CALCULATED USING YPLTIN AND XPLTIN

*AROT - ROTATION ANGLE IN DEGREES. IF ZERO Y AND X CO-ORDINATES ARE PLOTTED IN THE +VE Y- PLOT AND +VE X-PLOT DIRECTIONS RESPECTIVELY. IF AROT=90 Y AND X CO-ORDINATES ARE PLOTTED IN +VE X-PLOT AND -VE Y-PLOT DIRECTIONS RESPECTIVELY.

I.E.
XPLT= X*COS(AROT) + Y*SIN(AROT)
YPLT=-X*SIN(AROT) + Y*COS(AROT)

NOTE. ROTATION OF SPECIFIED RECTANGLE OF ANGLES OTHER THAN 0 OR +90 OR -90 DEGREES WILL NOT WORK PROPERLY.

THE FOLLOWING CARD OCCURS ONLY IF NB0UND=1 OR 2
***SPECIFIED RECTANGLE CARD
** READ ,BND,XSPMN,XSPMX,YSPMN,YSPMX
FORMAT(A3,7X,4F10.0)
*BND - COLUMNS 1 TO 3 MUST CONTAIN THE LETTERS BND
*XSPMN - THE LEFT SIDE OF SPECIFIED RECTANGLE IN LOGICAL UNITS
*XSPMX - THE RIGHT SIDE OF SPECIFIED RECTANGLE IN LOGICAL UNITS
*YSPMN - THE LOWER SIDE OF SPECIFIED RECTANGLE IN LOGICAL UNITS
*YSPMX - THE UPPER SIDE OF SPECIFIED RECTANGLE IN LOGICAL UNITS

***ADAPTED FOR USE ON UNIVAC 1106 AT UNIVERSITY OF WOLLONGONG USING FORTRAN V. APRIL, 1979 BY C. DERTOLDI

ROUTINES PLOT, DASHP, SYMBOL, NUMBER WERE WRITTEN TO ENABLE USE OF PLOT PACKAGE AVAILABLE ON UNIVAC AT WOLLONGONG.

ROUTINE DAYTIM CALLED TO GIVE DATE AND TIME ON UNIVAC.

COMMON /IDENT/ HED(80), ANAME(20), ADDRESS(20)
COMMON /PLTDAT/ LNPLT, NUMNP, NUMEL, SCALE, XSPMN, XSPMX, YSPMN, YSPMX, 1ORGX, ORGY, NB0UND, NSKPEL, NSKPNP, HT, XPLTIN, YPLTIN, XMIN, XMAX, YMIN, 2YMAX, PLTXXN, PLTXXM, PLTTYMN, PLTTYMX, XINCH, YINCH, COEF, IEXP, AROT, AROTON, XPLTIN, YPLTIN
COMMON /PLTT/ XLAST, YLAST
DIMENSION PLTBUF(1024)
REPLACE CARDS BELOW BY
COMMON IP( MDIM )
MDIM = MDIM
WHERE MDIM .GE. 2*NUNMNP+4*NUMEL
COMMON IP(5000)
MDIM = 5000

HT IS THE HEIGHT OF PLOTTED NODAL AND ELEMENT NUMBERS IN INCHES
HT = .07
LNPLT=21
REWIND LNPLT
C INITIALISE PLOTTER
C CALL PLOTS(PLTBUF,1024)
CALL PPBGN (15)
CALL PLINIT
C SET PLOTTER ORIGIN .5 INCH ABOVE LOWER EDGE
C CALL PLOT(0.0,-30.0,-3)
C CALL PLOT(10.0,.5,-3)
READ (5,130) NUNP,NHED
C COMMON STORAGE ALLOCATION
N1=1
N2=N1+NUNP
N3=N2+NUNP
N4=N3+4*NUNP-1
C PRINT 140, HED,NUNP,NUNP,NUNP,N4,MXDIM
IF (N4.GT.MXDIM) STOP
C INPUT MSHGEN DATA
CALL MSHINP (IP(N1),IP(N2),IP(N3))
PRINT 150, XMIN,XMAX,YMIN,YMAX
C C READ IDENTIFICATION CARD
READ 160, ANAME,ADDRESS
PRINT 170, ANAME,ADDRESS
C C BEGIN MAJOR PLOT LOOP
NPLT=0
AROT=0.0
C READ PLOT PARAMETER CARD
10 READ (5,180,ERR=20,END=20) YPLTIN,XPLTIN,NBOUND,NSKPNP,NSKPEL,
1 SCALE,AROT
GO TO 40
20 IF (NPLT.EQ.0) GO TO 30
CALL PLOT (0.0,0.0,999)
STOP
30 PRINT 190
STOP
40 NPLT=NPLT+1
IF (NBOUND.NE.1.AND.NBOUND.NE.2) NBOUND=0
IF (YPLTIN.EQ.0.) YPLTIN=10.
IF (NPLT.EQ.1) CALL PLTHED
IF (XPLTIN.EQ.0.0) XPLTIN=50.
PRINT 200, NPLT
PRINT 210, YPLTIN,XPLTIN
IF (SCALE.EQ.0.0) PRINT 220
IF (SCALE.NE.0.0) PRINT 230, SCALE
IF (AROT.NE.0.0) PRINT 240, AROT
IF (NSKPNP.EQ.0) PRINT 250
IF (NSKPNP.NE.0) PRINT 260, NSKPNP
IF (NSKPEL.EQ.0) PRINT 270
IF (NSKPEL.NE.0) PRINT 280, NSKPEL
IF (NBOUND.NE.0) GO TO 50
PRINT 290
GO TO 70
50 IF (NBQUND.EQ.1) PRINT 300
   IF (NBQUND.EQ.2) PRINT 310
C READ IN SPECIFIED RECTANGLE
READ 320, ALPHA,XSPMN,XSPNX,YSPMN,YSPNX
   IF (ALPHA.EQ.3HBND) GO TO 60
PRINT 330
GO TO 120
60 PRINT 340, XSPMN,XSPNX,YSPMN,YSPNX
C ROTATE MESH IF NECESSARY
70 CALL ROTATE (IP(N1),IP(N2))
C CALL SCALE ROUTINE
SCALEI=SCALE
   CALL SCALEF (IP(N1),IP(N2))
   IF (SCALEI.EQ.0.) PRINT 350, SCALE
   IF (SCALE.GT.0.0) GO TO 80
PRINT 360
GO TO 120
80 CONTINUE
PRINT 370, YINCH,XINCH
   RND=1.+1.E-8
   IF (YINCH.LE.YPLTIN*RND.AND.XINCH.LE.XPLTIN*RND) GO TO 90
PRINT 380
GO TO 120
90 CONTINUE
C PLOT PLOT NUMBER AND SCALE FACTOR
HITE=YPLTIN/13.
   IF (HITE.LT.(.07)) GO TO 100
   IF (HITE.GT.(.21)) HITE=.21
IHITE=100.*HITE
IHITE=IHITE/7
HITE=.07*IHITE/3.*2.
   CALL SYMBOL (-1.5,0.0,HITE,'CO-ORDINATE SCALE FACTOR ',90.,225)
   CALL NUMBER (999.,999.,HITE,COEF,90.,2)
   CALL SYMBOL (999.,999.,HITE,1HE,90.,1)
   CALL NUMBER (999.,999.,HITE,FLOAT(IEXP),90.,-1)
   CALL SYMBOL (999.,999.,HITE,' UNITS/INCH',90.,211)
C PLOT ELEMENTS IN SPECIFIED REGION
100 CONTINUE
   CALL ELPLOT (IP(N1),IP(N2),IP(N3))
C IF (NSKPNP.EQ.0) GO TO 110
C PLOT NODAL POINT NUMBERS
   CALL PLTNOD (IP(N1),IP(N2))
C ADVANCE PLOT
110 CALL PLOT (XINCH+5.,0.,-3)
   PRINT 390, NPLT
   AROTO=AROT
   GO TO 10
C END MAJOR PLOT LOOP
C ERROR PROCESSING
120 IF (NPLT.GT.1) CALL PLOT (0.0,0.0,999)
STOP
C
C
130 FORMAT (2I5,15X,80A1)
140 FORMAT (1H1,10X,'M S H P L T E X E C U T I O N '/12X,80A1/12
1X,'NUMBER OF NODAL POINTS------------',I5/12X,'NUMBER OF ELEMENTS-
2----------',I5/12X,'BLANK COMMON STORAGE NEEDED------',I5/12X,
3'BLANK COMMON STORAGE ASSIGNED---',I5)
150 FORMAT (1H0,10X,'DATA COORDINATE LIMITS'/12X,'MINIMUM X-VALUE----
1-----------',E15.7,' UNITS'/12X,'MAXIMUM X-VALUE---------------
2E15.7,' UNITS'/12X,'MINIMUM Y-VALUE----------',E15.7,' UNITS'/
312X,'MAXIMUM Y-VALUE-----------------',E15.7,' UNITS')
160 FORMAT (20A1,1 OX,20A1)
170 FORMAT (1 H0,10X,'IDENTIFICATION CARD'/12X,20A1,10X,20A1)
180 FORMAT (2F10.0,3I5,2F10.0)
190 FORMAT (1H0,10X,'UNEXPECTED END OF FILE ON CARD INPUT - ABORT')
200 FORMAT (1H1,10H********* ,'PLOT NUMBER',13)
210 FORMAT (1H0,10X,'PLOT SIZE LIMITS'/12X,'PLOT WIDTH LIMIT----------
1---',F6.2,' INCHES'/12X,'PLOT LENGTH LIMIT----------',F6.2,' INC
2HES')
220 FORMAT (1H0,10X,'CO-ORDINATE SCALE FACTOR IS TO BE CALCULATED',
11X'USING PLOT SIZE LIMITS')
230 FORMAT (1H0,10X,'CO-ORDINATE SCALE FACTOR SPECIFIED---',1PE9.2,
1'UNITS/INCH')
240 FORMAT (1H0,10X,'MESH ROTATED ',F6.2,' DEGREES PRIOR TO PLOTTING
1:')
250 FORMAT (1H0,10X,'NODAL POINT NUMBERS ARE NOT TO BE PLOTTED')
260 FORMAT (1H0,10X,'THE NODAL POINT NUMBER PLOT SKIP CONSTANT IS ',
112)
270 FORMAT (1H0,10X,'ELEMENT NUMBERS ARE NOT TO BE PLOTTED')
280 FORMAT (1H0,10X,'THE ELEMENT NUMBER PLOT SKIP CONSTANT IS ',I2)
290 FORMAT (1H0,10X,'THE WHOLE REGION IS TO BE PLOTTED')
300 FORMAT (1H0,10X,'ONLY INTERIOR OF SPECIFIED RECTANGLE IS TO BE',
1PLOTTED')
310 FORMAT (1H0,10X,'ONLY EXTERIOR OF SPECIFIED RECTANGLE IS TO BE'
1,'PLOTTED')
320 FORMAT (A3,7X,4F10.0)
330 FORMAT (1H0,'****EXPECTED SPECIFIED RECTANGLE NOT FOUND--ABORT')
340 FORMAT (1H0,'SPECIFIED RECTANGLE'/12X,'MINIMUM XVALUE-------
1-----',F10.2,' UNITS'/12X,'MAXIMUM X-VALUE--------------',F10.2,
2UNITS'/12X,'MINIMUM Y-VALUE----------',F10.2,' UNITS'/12X,
3MAXIMUM Y-VALUE----------------',F10.2,' UNITS')
350 FORMAT (1H0,'CALCULATED CO-ORDINATE SCALE FACTOR------',1PE9.2,
1'UNITS/INCH')
360 FORMAT (1H0,'****BAD SCALE - ABORT')
370 FORMAT (1H0,'ACTUAL PLOT SIZE'/12X,'ACTUAL PLOT WIDTH ------
1------',F6.2,' INCHES'/12X,'ACTUAL PLOT LENGTH ------',F6.2,
2'INCHES')
380 FORMAT (1H0,'ACTUAL PLOT SIZE EXCEEDS LIMIT SIZE - ABORT')
390 FORMAT (1H0,10H********* ,'PLOT NUMBER ',I2,' IS FINISHED')
C
END
B.4.1. SUBROUTINE PLTHED

SUBROUTINE PLTHED
C THIS ROUTINE PlOTS PLOTTER IDENTIFICATION
C
COMMON /IDENT/ HED(80),ANAME(20),ADRESS(20)
COMMON /PLTDAT/ LNPLT,NUMNP,NUMEL,SCALE,XSPMN,XSPMX,YSFHN,YSFMX,
1 ORX, ORY,NBOUND,HSKPEL,NSKPEN,HT,XPLTN,YPLTN,XMIN,XMAX,YMIN,
2 YMAX,PLTXMN,PLTXMX,PLTYMN,PLTYMX,XINCH,YINCH,COEF,IEXP,AROT,AROTO
DOUBLE PRECISION TM
DOUBLE PRECISION DTE
DATA BLANK /1H /
C
C PLOT IDENTIFICATION
HITE=YPLTIN/50.
IF (HITE.LT.(.07)) GO TO 40
IF (HITE.GT. (.21)) HITE=.21
IHITE=10Q.*HITE
IHITE=IHITE/7
HITE=.Q7*IHITE/3.*2.
CALL SYMBOL (-9.,0.,HITE,ANAME(1),90.,1)
DO 10 1=2,20
CALL SYMBOL (999.,999.,HITE,ANAME(I),90.,1)
10 CONTINUE
DO 20 1=1,10
CALL SYMBOL (999.,999.,HITE,BLANK,90.,1)
20 CONTINUE
DO 30 1=1,20
CALL SYMBOL (999.,999.,HITE,ADRESS(I),90.,1)
30 CONTINUE
C
C PLOT TIME AND DATE
CALL DAYTIM (I,J)
DECODE (90,I) I1,I2,I3
DECODE (90,J) J1,J2,J3
ENCODE (100,TH) J1,J2,J3
ENCODE (110,DTE) I2,I1,I3
CALL SYMBOL (-7.25,0.,HITE,'TIME',90.,204)
CALL SYMBOL (999.,999.,HITE,TH,90.,1)
CALL SYMBOL (999.,999.,HITE,BLANK,90.,6)
CALL SYMBOL (999.,999.,HITE,BLANK,90.,6)
CALL SYMBOL (999.,999.,HITE,BLANK,90.,2)
CALL SYMBOL (999.,999.,HITE,'DATE',90.,204)
CALL SYMBOL (999.,999.,HITE,DTE,90.,1)
C
40 CONTINUE
C
C FIND NUMBER OF CHARACTERS IN HED
ICHARF=0
ICHARL=-1
DO 60 IC=1,80
IF (ICHARF.NE.0) GO TO 50
IF (HED(IC).EQ.BLANK) GO TO 60
ICHARF=IC
ICHARL=IC
DO 60 CONTINUE
50 IF (HED(IC).EQ.BLANK) GO TO 60
   ICHARL=IC
60 CONTINUE
   NC=ICHARL-ICHARF+1
   IF (NC.EQ.0) GO TO 80
   HITE=YPLTIN/FLOAT(NC)
   IF (HITE.LT.(.07)) GO TO 80
   IF (HITE.GT.(.21)) HITE=.21
   IHITE=100.*HITE
   HITE=IHITE/7
   HITE=.07*IHITE/3.*2.

C PLOT HEADING
   CALL SYMBOL (-5.5,0.,HITE,HED(ICHARF),90.,1)
   ICHARF=ICHARF+1
   IF (ICHARF.GT.ICHARL) GO TO 80
   DO 70 IC=ICHARF,ICHARL
   CALL SYMBOL (999.,999.,HITE,HED(IC),90.,1)
70 CONTINUE
80 RETURN
C
C END

B.4.2. SUBROUTINE MSHINP

SUBROUTINE MSHINP (X,Y,IX)
C
C ROUTINE INPUTS NODAL COORDINATES AND ELEMENT DATA
C ALSO FINDS MAXIMUM AND MINIMUM VALUES XMIN,XMAX,YMIN,YMAX
C
DIMENSION X(1), Y(1), IX(4,1), ND3(6), AND4(9)
COMMON /PLTDAT/ LNPLT,NUMNP,NUMEL,SSCALE,XSPMN,XSPMX,YSPMN,YSPMX,
1 ORGX,ORGY,NBOUND,NSKPEL,NSKPNP,HT,XPLTIN,YPLTIN,XMIN,XMAX,YMIN,
2 YMAX,PLTXMN,PLTXMX,PLTYMN,PLTYMX,XINCH,YINCH,COEF,IEXP,AROT,AROTO
C
LNPLT=21
C
INPUT NODAL POINTS
C
XMIN=1.E10
XMAX=-1.E10
YMIN=1.E10
YMAX=-1.E10
DO 10 NP=1,NUMNP
   READ (LNPLT,30) KJI,X(NP),Y(NP),ND1,ND2
   IF (X(NP).LT.XMIN) XMIN=X(NP)
   IF (X(NP).GT.XMAX) XMAX=X(NP)
   IF (Y(NP).LT.YMIN) YMIN=Y(NP)
   IF (Y(NP).GT.YMAX) YMAX=Y(NP)
10 CONTINUE
C
LNPLT=22
B.4.3. SUBROUTINE SCALEF

SUBROUTINE SCALEF (X,Y)
C
C ROUTINE FINDS MAXIMUM AND MINIMUM COORDINATE VALUES OF NODAL POINTS IN SPECIFIED PLOT REGION - PLTXMN,PLTXMX,PLTYMN,PLTYMX
C IF SCALE IS UNSPECIFIED IT IS CALCULATED
C THE ACTUAL PLOT SIZE XINCH BY YINCH IS CALCULATED
C
DIMENSION X(1), Y(1)
COMMON /PLTDAT/ LNPLT,NUMNP,NUMEL,SCALE,XSPMN,XSPMX,YSPMN,YSPMX,
ORGX, ORGY, NBOUND, NSKPEL, NSKPNP, HT, XPLTN, YPLTN, XMIN, XMAX, YMIN,
YMAX, PLTXMN, PLTXMX, PLTYMN, PLTYMX, XINCH, YINCH, COEF, IEXP, AROT, AROTO

IF (NBOUND.NE.0) GO TO 10
NBOUND=0
ORGX=XMIN
ORY=YM
PLTXMN=XMIN
PLTYMN=YMIN
PLTXMX=XMAX
PLTYMX=YMAX
GO TO 40
10 PLTXMN=1.E10
PLTXMX=-1.E10
PLTYMN=1.E10
PLTYMX=-1.E10
IF (NBOUND.EQ.2) GO TO 20

C NBOUND=1
IF (XSPMN.LE.XMIN) PLTXMN=XSPMN
IF (XSPMN.GT.XMIN) PLTXMN=XMIN
IF (XSPMX.GE.XMAX) PLTXMX=XSPMX
IF (XSPMX.LT.XMAX) PLTXMX=XMAX
IF (YSPMN.LE.YMIN) PLTYMN=YSPMN
IF (YSPMN.GT.YMIN) PLTYMN=YMIN
IF (YSPMX.GE.YMAX) PLTYMX=YSPMX
IF (YSPMX.LT.YMAX) PLTYMX=YMAX
ORGX=PLTXMN
ORGY=PLTYMN
GO TO 40

20 CONTINUE
C NBOUND=2
DO 30 I=1,NUMNP
  IF (X(I).LT.XSPMX.AND.X(I) .GT.XSPM11.AND.Y(I) .LT.YSPMX.AND.Y(I)
    1.GT.YSPMN) GO TO 30
    IF (X(I).LT.XPLTXMN) XPLTXMN=X(I)
    IF (X(I).GT.XPLTXMX) XPLTXMX=X(I)
    IF (Y(I).LT.PLTYMN) PLTYMN=Y(I)
    IF (Y(I).GT.PLTYMX) PLTYMX = Y(I)
30 CONTINUE
C
ORGX=PLTXMN
ORGY=PLTYMN
C
40 IF (SCALE.NE.0.) GO TO 70
C CALCULATE SCALE
SCALE=(PLTYMX-PLTYMN)/YPLTIN
XSCALE=(PLTXMX-PLTXMN)/XPLTIN
IF (SCALE.LT.XSCALE) SCALE=XSCALE
IF (SCALE.LE.0.0) RETURN
EXP=ALOG10(SCALE)
IEXP=EXP
COEF=10.*(EXP-IEXP)
ICOF=100.*COEF
ICOEF=ICOEF/5
COEFN=ICOEF*.05
50 IF (COEFN.GE.COEF) GO TO 60
COEFN=COEFN+.05
GO TO 50
60 COEF=COEFN
SCALE=COEF*10.*IEXP
GO TO 80
70 EXP=ALOG10(SCALE)
IEXP=EXP
COEF=10.*(EXP-IEXP)
80 XINCH=(PLTXMX-PLTXMN)/SCALE
YINCH=(PLTYMX-PLTYMN)/SCALE
RETURN
C
C END

B.4.4. SUBROUTINE EL PLOT

SUBROUTINE EL PLOT (X,Y,IX)
C THIS ROUTINE PLOTS THE ELEMENTS AND EVERY NSKPEL ELEMENT NUMBER
C
DIMENSION X(1), Y(1), IX(4,1), IXTMP(5)
COMMON /PLTDAT/ LNPLT,NUMNP,NUMEL,SCALE,XSPMN,XSPMX,YSPMN,YSPMX,
ORGX,ORGY,NBOUND,NSKPEL,NSKPNP,HT,XPLTIN,YPLTIN,XMIN,XMAX,YMIN,
2YMAX,PLTXMN,PLTXMX,PLTYMN,PLTYMX,XINCH,YINCH,COEF,IEXP,AROT,AROTO
COMMON /PLOTT/ XLAST,YLAST
DATA DIR /90 .  /
C
SCALI=1./SCALE
NDISP=NSKPEL-1
IF (NBOUND.NE.0) GO TO 50
C
NBOUND=0
ASSIGN 40 TO LOC1
IF (NSKPEL.GT.0) ASSIGN 30 TO LOC1
DO 40 NEL=1,NUMEL
DO 10 I=1,4
   IXTMP(I)=IX(I,NEL)
10  CONTINUE
   IXTMP(5)=IXTMP(1)
   L=IXTMP(1)
   CALL PLOT (SCALI*(X(L)-ORGX),SCALI*(Y(L)-ORGY),3)
   XC=0.0
   YC=0.0
   DO 20 I=1,4
      L=IXTMP(I+1)
      XP=SCALI*(X(L)-ORGX)
      YP=SCALI*(Y(L)-ORGY)
      CALL PLOT (XP,YP,2)
      XC=XC+XP
      YC=YC+YP
20  CONTINUE
   GO TO LOC1, (30,40)
30 IF (MOD(NEL+NDISP,NSKPEL).NE.0) GO TO 40
   XC=.25*XC
   YC=.25*YC
   CALL NUMBER (XC,YC,HT,FLOAT(NEL),DIR,-1)
40 CONTINUE
RETURN
C
C NBOUND NOT EQUAL TO ZERO
50 IF (NBOUND.NE.1) GO TO 180
C
NBOUND=1
ASSIGN 170 TO LOC2
IF (NSKPEL.GT.0) ASSIGN 150 TO LOC2
DO 170 NEL=1,NUMEL
DO 60 I=1,4
   IXTMP(I)=IX(I,NEL)
60  CONTINUE
   IXTMP(5)=IXTMP(1)
   NLST=-1
   IEINP=0
   DO 140 I=1,5
      L=IXTMP(I)
      IF (X(L).LT.XSPMN.OR.X(L).GT.XSPMX.OR.Y(L).LT.YSPMN.OR.Y(L).GT.YSPMX) GO TO 100
   100 IF (NLST) 70,80,90
   CURRENT POINT IS IN
   NLSTC=0
   IF (NLST) 70,80,90
CALL PLOT (SCALI*(X(L)-ORGX),SCALI*(Y(L)-ORGY),3)  
GO TO 130
CALL PLOT (SCALI*(X(L)-ORGX),SCALI*(Y(L)-ORGY),2)  
GO TO 130
CALL INTCPT (XI,YI,X(L),Y(L),XL,YL)  
CALL PLOT (SCALI*(XI-ORGX),SCALI*(YI-ORGY),3)  
CALL PLOT (SCALI*(X(L)-ORGX),SCALI*(Y(L)-ORGY),2)  
GO TO 130
C CURRENT POINT IS OUT
100  NLSTC=1
    IELNP=1
    IF (NLST) 110,120,130
GO TO 130
110  GO TO 130
120  CALL INTCPT (XI,YI,XL,YL,X(L),Y(L))  
    CALL PLOT (SCALI*(XI-ORGX),SCALI*(YI-ORGY),2)  
130  XL=X(L)
    YL=Y(L)
    NLST=NLSTC
CONTINUE
GO TO LOC2, (150,170)
150  IF (IELNP.GT.0) GO TO 170
    IF (MOD(NEL+NDISP,NSKPEL).NE.0) GO TO 170
    XC=.25*XC
    YC=.25*YC
CALL NUMBER (SCALI*(XC-ORGX),SCALI*(YC-ORGY),HT,FLOAT(NEL),  
DIR,-1)
170  CONTINUE
C PLOT SPECIFIED RECTANGLE
CALL PLTRCT
C
RETURN
C NBOUND=2
180  ASSIGN 300 TO LOC3  
    IF (NSKPEL.GT.0) ASSIGN 280 TO LOC3
    NPOUT=0
    DO 300 NEL=1,NUHLEL
        DO 190 1=1,4
            IXTMP(I)=IX(I,NEL)
190  CONTINUE
        IXTMP(5)=IXTMP(1)
        NLST=-1
        IELNP=0
        DO 270 1=1,5
            L=IXTMP(I)
            IF (X(L).GT.XSPMN.AND.X(L).LT.XSPMX.AND.Y(L).GT.YSPMN.AND.Y  
1(L).LT.YSPHX) GO TO 230
C CURRENT POINT IS IN EXTERIOR PLOT REGION
NLSTC=0
IF (NLST) 200,210,220
200 CALL PLOT (SCALI*(X(L)-ORGX),SCALI*(Y(L)-ORGY),3)
   GO TO 260
210 CALL PLOT (SCALI*(X(L)-ORGX),SCALI*(Y(L)-ORGY),2)
   GO TO 260
220 CALL INTCPT (XI,YI,XL,YL,X(L),Y(L))
   CALL PLOT (SCALI*(XI-ORGX),SCALI*(YI-ORGY),3)
   CALL PLOT (SCALI*(X(L)-ORGX),SCALI*(Y(L)-ORGY),2)
   GO TO 260
C CURRENT POINT NOT IN EXTERIOR REGION
230 NLSTC=1
   IELNP=1
   NPOUT=1
   IF (NLST) 240,250,260
   GO TO 260
240 CALL INTCPT (XI,YI,X(L),Y(L),XL,YL)
   CALL PLOT (SCALI*(XI-ORGX),SCALI*(YI-ORGY),2)
250 XL=X(L)
   YL=Y(L)
   NLST=NLSTC
260 CONTINUE
   GO TO L0C3, (280,300)
270 IF (IELNP.GT.0) GO TO 300
280 IF (MOD(NL+NDISP,NSKPEL).NE.0) GO TO 300
290 XC=0.0
   YC=0.0
   DO 290 I=1,4
      L=IXTHP(I)
      XC=XC+X(L)
      YC=YC+Y(L)
   CONTINUE
   XC=.25*XC
   YC=.25*YC
   CALL NUMBER (SCALI*(XC-ORGX),SCALI*(YC-ORGY),HT,FLOAT(NEL),
   1DIR,-1)
   CONTINUE
300 IF (NPOUT.EQ.0) RETURN
C PLOT SPECIFIED RECTANGLE
   CALL PLTRCT
C RETURN
C
END

B.4.5. SUBROUTINE PLTNOD

SUBROUTINE PLTNOD (X,Y)
C THIS ROUTINE PLOTS EVERY NSKPNP NODAL POINT NUMBER IN THE
C SPECIFIED REGION TO BE PLOTTED
C DIMENSION X(1), Y(1)
COMMON /PLDTAT/ LNPLT, NUMNP, NUMEL, SCALE, XSPMN, XSPMX, YSPMN, YSPMX,
1ORGX, ORGY, NBOUND, NSKPEL, NSKPNP, HT, XPLTIN, YPLTIN, XMIN, XMAX, YMIN,
2YMAX, PLTXMN, PLTXMX, PLTYMN, PLTYMX, XINCH, YINCH, COEF, IEXP, AROT, AROTO
DATA DIR /O.O/

C SCALI=1./SCALE
IDISP=NSKPNP-1
ASSIGN 30 TO LOC
IF (NBOUND.EQ.1) ASSIGN 10 TO LOC
IF (NBOUND.EQ.2) ASSIGN 20 TO LOC
DO 40 I=1, NUMNP
   IF (MOD(I+IDISP,NSKPNP)-NE.0) GO TO 40
   GO TO LOC, (10, 20, 30)
C NBOUND=1
  10 IF (X(I).GT.XSPMX.OR.X(I) .LT.XSPMN.OR.Y(I) .GT.YSPMX.OR.Y(I).
       1LT.YSPMN) GO TO 40
       GO TO 30
C NBOUND=2
  20 IF (X(I) .LT.XSPMX.AND.X(I)GT.XSPMN.AND.Y(I).LT.YSPMX.AND.Y(I).
       1GT.YSPMN) GO TO 40
  30 CALL NUMBER (SCALI*(X(I)-ORGX),SCALI*(Y(I)-ORGY),HT,FLOAT(I),
       1DIR,-1)
  40 CONTINUE
RETURN
C C
C END

B.4.6. SUBROUTINE INTCPT

SUBROUTINE INTCPT (XINT,YINT,XIN,YIN,XOUT,YOUT)
C THIS ROUTINE CALCULATES THE INTERCEPT OF ELEMENT SIDES WITH
C SPECIFIED RECTANGLE
C
COMMON /PLDTAT/ LNPLT, NUMNP, NUMEL, SCALE, XSPMN, XSPMX, YSPMN, YSPMX,
1ORGX, ORGY, NBOUND, NSKPEL, NSKPNP, HT, XPLTIN, YPLTIN, XMIN, XMAX, YMIN,
2YMAX, PLTXMN, PLTXMX, PLTYMN, PLTYMX, XINCH, YINCH, COEF, IEXP, AROT, AROTO
C
IF (XOUT.LE.XSPMN) GO TO 10
IF (XOUT.GE.XSPMX) GO TO 20
GO TO 40
10 XINT=XSPMN
GO TO 30
20 XINT=XSPMX
30 IF (XIN.EQ.XOUT) GO TO 40
   YINT=(XINT-XOUT)*(YIN-YOUT)/(XIN-XOUT)+YOUT
   IF (YINT.LT.YSPMN) GO TO 60
   IF (YINT.GT.YSPMX) GO TO 50
   RETURN
40 IF (YOUT.LE.YSPMN) GO TO 60
50 YINT=YSPMX
GO TO 70
60 YINT=YSPMN
SUBROUTINE PLTRCT

THIS ROUTINE PLOTS THE SPECIFIED RECTANGLE USING DASHED LINES

COMMON /PLTDAT/ LNPLT, NUMWP, NUMEL, SCALE, XSPMN, XSPMX, YSPMN, YSPMX, XORGX, ORGY, NBOUND, NSKPEL, NSKPNP, HT, XPLTIN, YPLTIN, XMIN, XMAX, YMIN, YMAX, PLTXMN, PLTXMX, PLTYMN, PLTYMX, XINCH, YINCH, COEF, IEXP, AROT, AROTO

COMMON /PLOTT/ XLAST, YLAST

DASH = .05

CALL PLOT (0.0, 0.0, 3)

CALL PLOT (XINCH, 0.0, 3)

CALL PLOT (0.0, YINCH, 3)

CALL PLOT (XINCH, YINCH, 3)

RETURN

NBOUND = 1

SCALI = 1.0 / SCALE

CALL PLOT (0.0, 0.0, 3)

CALL PLOT (XSPMN, XSPMN - ORGX)

CALL PLOT (0.0, XSPMN, ORGX)

CALL PLOT (XSPMX, YSPMN - ORGY)

CALL PLOT (YSPMN, YSPMN, ORGY)

CALL PLOT (XR, YT, YCH)

CALL PLOT (XL, YB, 3)

CALL DASHP (XL, YT, DASH)

CALL DASHP (XL, YB, DASH)

RETURN

NBOUND = 2

CALL PLOT (0.0, YINCH, DASH)

CALL PLOT (XINCH, 0.0, DASH)

CALL PLOT (XINCH, YINCH, DASH)

CALL PLOT (XINCH, YINCH, DASH)

RETURN
B.4.8. SUBROUTINE ROTATE

SUBROUTINE ROTATE (X,Y)

This routine rotates the mesh through \( \text{ANG} = (\text{AROT} - \text{AROTO}) \) degrees prior to plotting.

\[ \text{XPLT} = X \cos(\text{ANG}) + Y \sin(\text{ANG}) \]
\[ \text{YPLT} = -X \sin(\text{ANG}) + Y \cos(\text{ANG}) \]

DIMENSION X(1), Y(1)
COMMON /PLTDAT/ LNPLT, NUNNP, NUNEL, SCALE, XSPMN, XSPMX, YSPMN, YSPMX, 1ORGX, ORGY, NBOUND, NSKPEL, NSKPNP, HT, XPTIN, YPTIN, XMIN, XMAX, YMIN, 2YMAX, PLTXMN, PLTXMX, PLTYMN, PLTYMX, XINCH, YINCH, COEF, IEXP, AROT, AROTO

PIC = 3.141592654 / 180.
ANG = (AROT - AROTO) * PIC

IF (ANG.EQ.0.0) GO TO 20

CS = \cos(\text{ANG})
SN = \sin(\text{ANG})
XMIN = 1.E25
XMAX = -1.E25
YMIN = 1.E25
YMAX = -1.E25

DO 10 N = 1, NUNNP
  XSAVE = X(N)
  X(N) = XSAVE * CS + Y(N) * SN
  Y(N) = -XSAVE * SN + Y(N) * CS
  XMIN = MIN1 (XMIN, X(N))
  XMAX = MAX1 (XMAX, X(N))
  YMIN = MIN1 (YMIN, Y(N))
  YMAX = MAX1 (YMAX, Y(N))

10 CONTINUE

ROTATE SPECIFIED RECTANGLE THROUGH AROT DEGREES

IF (AROT.EQ.0.0) OR ( NBOUND.EQ.0) GO TO 40

CS = \cos(\text{ANG})
SN = \sin(\text{ANG})

XSAVE = XSPMN
XSPMN = XSAVE * CS + YSPMN * SN
YSPMN = -XSAVE * SN + YSPMN * CS
XSAVE = XSPMX
XSPMX = XSAVE * CS + YSPMX * SN
YSPMX = -XSAVE * SN + YSPMX * CS

IF (XSPMX.GE.XSPMN) GO TO 30

XSAVE = XSPMN
XSPMN = XSPMX
XSPMX = XSAVE

30 IF (YSPMX.GE.YSPMN) GO TO 40

YSAVE = YSPRN
YSPMN = YSPNX
YSPNX = YSAVE
SUBROUTINE PLOT

SUBROUTINE PLOT (XN, YN, NN)
COMMON /PLOTT/ XLAST, YLAST
DIMENSION XX(2), YY(2)
DATA XLAST /0./, YLAST /0./
N2 = ABS(NN)
IF (NN.EQ.999) CALL PPEND
XX(1) = XLAST
XX(2) = XN
YY(1) = YLAST
YY(2) = YN
IF (N2.EQ.2) CALL PPDRAW (XX, YY, 2, '')
IF (NN.GT.0) GO TO 10
XLAST = 0.
YLAST = 0.
CALL PPNEXT
CALL PLINIT
RETURN
10 XLAST = XX(2)
YLAST = YY(2)
RETURN

SUBROUTINE DASHP

SUBROUTINE DASHP (XN, YN, DDL)
COMMON /PLOTT/ XLAST, YLAST
DIMENSION XX(2), YY(2)
DATA XLAST /0./, YLAST /0./
XX(1) = XLAST
XX(2) = XN
YY(1) = YLAST
YY(2) = YN
LENG = 20.*DDL
CALL PPDDRW (XX, YY, 2, LENG, 0, 0)
XLAST = XX(2)
YLAST = YY(2)
RETURN

END

END
B.4.11. SUBROUTINE SYMBOL

SUBROUTINE SYMBOL (XP, YP, HH, TEXT, ANG, NCHAR)
DIMENSION XX(1), YY(1), TEXT(5)
IF (NCHAR.LT.0) GO TO 20
ISIZ=HH*100./7.
IANG=ANG
CALL PPANGL (IANG)
CALL PPSIZE (ISIZ)
IF (XP.GT.998. OR. YP.GT.998.) GO TO 10
CALL PPTEXT (XP, YP, TEXT, NCHAR)
RETURN
10 CALL PPTEXA (TEXT, NCHAR)
RETURN
20 ISIZ=HH*100./7.
IANG=ANG
CALL PPANGL (IANG)
CALL PPSIZE (ISIZ)
INT=TEXT
XX(1)=XP
YY(1)=YP
CALL PPPLLOT (XX, YY, 1, INT)
RETURN
C
C
END

B.4.12. SUBROUTINE NUMBER

SUBROUTINE NUMBER (XP, YP, HT, FPN, ANG, NDEC)
DOUBLE PRECISION FMT
DO 10 II=0,10
   NNUN=FPN/10.*II
10 IF (NNUN.EQ.0) GO TO 20
20 NNII=II
   IF (NNII.EQ.0) NNII=1
   IF (NDEC.LE.-1) NDEC=0
   NINT=NNII+NDEC+1
   ISIZ=HT*100./7.
   IANG=ANG
   ENCODE (40, FMT) NINT, NDEC
   CALL PPSIZE (ISIZ)
   CALL PPANGL (IANG)
   IF (XP.GT.998. OR. YP.GT.998.) GO TO 30
   CALL PPNUMB (XP, YP, FPN, FMT, NINT)
RETURN
30 CALL PPNUMA (FPN, FMT, NINT)
RETURN
C
C
C
END
B.4.13. SUBROUTINE PLINIT

SUBROUTINE PLINIT
CALL PPALL ('DISPLAY',0.,0.,900.,900.)
CALL PPALL ('USER',-10.,-10.,50.,50.)
RETURN

C

C

END
### APPENDIX C. INPUT DATA FOR COMPUTER PROGRAMS

#### C.1. MESH INPUT DATA

| x  | y  | z  | x  | y  | z  | x  | y  | z  | x  | y  | z  |
|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 18 | 125.0 | 240.0 | 30.0 | 1.0 | 315.315 | 245.105 | 210.0 | 125.0 | 240.0 | 30.0 | 1.0 | 315.315 | 245.105 | 210.0 |
| 9  | 5  | 0.0 | 240.0 | 0.0 | 0.0 | 0.0 | 210.0 | 1.22 |
| 9  | 5  | 40.0 | 240.0 | 40.0 | 0.0 | 40.0 | 210.0 | 1.22 |
| 9  | 5  | 80.0 | 240.0 | 80.0 | 0.0 | 80.0 | 210.0 | 1.22 |
| 9  | 5  | 110.0 | 240.0 | 110.0 | 0.0 | 110.0 | 210.0 | 1.22 |
| 9  | 5  | 140.0 | 240.0 | 140.0 | 0.0 | 140.0 | 210.0 | 1.22 |
| 9  | 5  | 170.0 | 240.0 | 170.0 | 0.0 | 170.0 | 210.0 | 1.22 |
| 9  | 5  | 190.0 | 240.0 | 190.0 | 0.0 | 190.0 | 210.0 | 1.22 |
| 9  | 5  | 210.0 | 240.0 | 210.0 | 0.0 | 210.0 | 210.0 | 1.22 |
| 9  | 4  | 228.0 | 240.0 | 228.0 | 0.0 | 228.0 | 216.0 | 1.22 |
| 9  | 3  | 246.0 | 240.0 | 246.0 | 0.0 | 246.0 | 222.0 | 1.22 |
| 9  | 2  | 264.0 | 240.0 | 264.0 | 0.0 | 264.0 | 228.0 | 1.22 |
| 9  | 1  | 282.0 | 240.0 | 282.0 | 0.0 | 282.0 | 234.0 | 1.22 |
| 9  | 0  | 300.0 | 240.0 | 300.0 | 0.0 | 300.0 | 240.0 | 1.22 |
| 9  | 0  | 315.315 | 245.105 | 315.315 | 0.0 | 315.315 | 245.105 | 1.22 |
| 9  | 0  | 330.0 | 245.105 | 330.0 | 0.0 | 330.0 | 245.105 | 1.22 |
| 9  | 0  | 350.0 | 245.105 | 350.0 | 0.0 | 350.0 | 245.105 | 1.22 |
| 9  | 0  | 380.0 | 245.105 | 380.0 | 0.0 | 380.0 | 245.105 | 1.22 |
| 9  | 0  | 410.0 | 245.105 | 410.0 | 0.0 | 410.0 | 245.105 | 1.22 |
| 9  | 0  | 450.0 | 245.105 | 450.0 | 0.0 | 450.0 | 245.105 | 1.22 |

#### C.2. FEP INPUT DATA

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### C.3. FESS INPUT DATA

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C.4. M P L O T I N P U T D A T A

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P E T E R  G R A Y          C I V I L  E N G I N E E R I N G
      50.  50.  0  0  1  0.  270.