Abstract
A simple method is given for estimating the continuous PQ disturbance level at a site. It is based on determining a "Voltage Distortion Increment" (VDI) for each segment of the network and then adding the VDIs from a given site back to an upstream site where levels are taken to be zero. The VDI is conveniently expressible in the form of S(MVA) times length (km), where "length" correspond to the physical length in the case of MV overhead lines and an equivalent length for other components. The method is verified by a comparison with field survey data. The voltage distortion figures assist in the choice of PQ monitor placement and for estimating levels at unmonitored sites.

1. INTRODUCTION

1.1 General objectives
Power systems analysis can be classified into two main types, deterministic and stochastic. In the first, a few critical loads are represented by detailed models with much data having to be obtained from tests or design details. In the second, individual loads are combined to give a few lumped loads with average characteristics. This later study is the type required to estimate overall PQ disturbance levels.

Nothing is known of a general method of predicting the general levels of continuous PQ disturbances but there have been several attempts to predict individual disturbance types. The predicting of voltage is an essential part of distribution voltage control [1], and is a stochastic study. Flicker has been studied in a few well-defined situations [2] by deterministic studies. Harmonics has mainly been studied where there has been one critical load, but there has been one stochastic study to estimate the general growth of harmonics [3].

PQ (Power Quality) disturbance types can be classified into continuous (or variation) and discrete (or event) types [4, 5]. The first type is present in every cycle of the voltage waveform and is due to the impact of customer load current on supply impedance. The second arises for a short time after some external event, such as impulsive transients and voltage sags following a fault. The first is more predictable and will be the emphasis of this paper, in particular voltage, unbalance, flicker and harmonics.

Since the cause of continuous disturbances is a disturbing component in customer load current, and these disturbances are becoming more distributed throughout the power system, it appears that a general approach to the estimation of continuous PQ levels might be possible. Our aim is to produce, for each site, a number (the voltage disturbance figure or VDF), which is related to the general level of continuous PQ disturbances. Such a number would be very convenient for the PQ management in power systems:-

(i) It would guide the placement of PQ monitors when only a few are available
(ii) It would allow the PQ levels of non-monitored sites to be estimated
(iii) It could be used as a figure of merit for system planning to keep PQ levels at a satisfactory level.

It should be obvious that the different PQ levels are not going to vary in exactly the same way throughout a power system, so the aim of any such general figure is to give an idea of overall levels. In particular, it would be useful if such a figure could be used to rank sites a priori from good to bad. As a high level of accuracy is not expected or required, there is no point in searching for a method which is complex or requires a great deal of data which is difficult to access. The emphasis, in seeking such a figure, should be on the most simple of calculations and data which is readily available. This data would normally be

- Design MVA figures, maybe some MDI figures for line flows
- Line lengths and possibly impedance figures
- Upstream Fault Levels
- Transformer per unit reactances

1.2 Typical distribution system layout and parameters
In this paper we shall consider the relative levels at sites all supplied by the one zone substation (33kV/11kV). A typical subsystem supplied from the one zone substation is shown in Figure 1. There are about 10 MV feeders, each up to 5km long. Some large customers are supplied directly at the MV level. Each feeder has a number of 11kV/415V distribution transformers teeing off at about 0.5km intervals. These may supply 50-100 houses by means of LV mains up to 500m long. Typical data is given in Table I.
The assumptions of Section 1.3 will give a pattern of PQ disturbance levels which is zero in the 33kV system (or small if upstream impedance is considered) and increases as one moves down any of the MV feeders to the extremities of any of the LV mains. The disturbance level at any site is the sum of that at the next upstream site plus the drop in the connecting feeder or mains. We thus find it necessary to define the Voltage Disturbance Increment (VDI) across a link (line, cable or transformer) as being proportional to the product of the MVA through the component times the fundamental reactance. The determination of the VDF thus has the following steps

(i) Define the VDF for the supply point as zero.
(ii) Find the VDI for each link
(iii) Find the VDF for a node by adding the sum of all the VDIs back to the supply point.

It would seem from the above discussion that the VDI could be taken as the product of the fundamental current times the reactance. We have modified this approach for practical reasons. Rather than current, we shall use the apparent power in MVA. Because of the dominance of overhead lines with different points of connection to transformers and loads, we shall use line length instead of reactance. This approach is attractive as all overhead lines (MV and LV) have approximately the same reactance. We shall see that it is possible to express the reactance of other series devices by means of an equivalent length. Hence we define the VDI for a link as

\[ \text{VDI} = |S(\text{MVA})| \times |\text{equivalent length(km)}| \]  

1.4 Voltage Disturbance Figure (VDF) and Voltage Disturbance Increment (VDI)

2. THEORY

2.1 MV overhead line

Concentrating on magnitude, not phase angle, the voltage drop across a device with impedance \( Z \) and carrying a current \( I \) is

\[ \Delta V = |I|Z \]  

If voltage drops across different parts of the system are to be compared, it is imperative that \( \Delta V \) be expressed in per unit.

If \( V \) is about 1pu

\[ \Delta V(\text{pu}) = |S(\text{pu})|V(\text{pu})|Z(\text{pu})| \approx S(\text{pu})Z(\text{pu}) \]  

In all overhead systems, the impedance/km of MV lines is approximately constant, so that impedance is...
directly proportional to length. This suggests the use of the length of overhead MV lines in km as a direct measure of its impedance. In most cases, S is a variable throughout the day. Nevertheless, it is possible to specify a typical value for a given circuit from experience, design values or MDI readings. We shall suppose that this has the units of MVA. We now explore the concept of using the product of line length in km and S in MVA as a measure of voltage disturbance increase. We introduce the following symbols: $z_{MV}$ (impedance of MV lines/km), $l_{MV}$ (length of MV line), $V_{MV}$ (line-line value of nominal MV system voltage), $S_{base}$ (base MVA) and $Z_{base.MV}$ (MV base impedance, equals $V_{MV}^2/S_{base}$).

Re-expressing (3)

$$\Delta V(pu) \sim (S/S_{base}) \times (z_{MV}l_{MV}/Z_{base.MV})$$

$$= (z_{MV}/V_{MV}^2) \times S_{MV}$$

(4)

Since the LH part of the expression is constant for the overhead part of the MV system, we shall take the reminder of the expression as the measure of the voltage disturbance increase (VDI) in the circuit. Hence

$$VDI = S_{MV}$$

(5)

If $z_{MV}$ is in Ohms/km and $V_{MV}$ in kV, then suitable units for S and $l_{MV}$ are MVA and km respectively. The actual voltage drop in pu equals the VDI times the quantity $z_{MV}/V_{MV}^2$.

The voltage drop in several devices in series is the sum of the individual voltage drops. We note that the reactance of lines is more important than the resistance in most cases, especially for harmonic calculations. This suggests the approximation of neglecting phase difference and finding the total voltage drop in a system by adding the individual drops arithmetically. Thus the VDI for several lines in series is the sum of the individual VDIs.

Suppose we have a MV line which is loaded, so that the input MVA is $S_{in}$ and the output at the far end is $S_{out}$ (Figure 2). In principle, if one knew the lengths between intermediate take-off points and the corresponding loads, one could sum the VDI for individual section lengths. A suitably approximate approach is to assume the average line flow along the length of the line, that is

$$VDI = S_{average}l_{MV} = 0.5(S_{in} + S_{out})l_{MV}$$

(6)

2.2 MV feeder example

Figure 3 shows an MV feeder example to illustrate the method, with data given in Table II. The VDI for each line section is given in Table III. The VDF at any of the nodes A-D is found by adding the VDI for each feeder section from the reference point of zero disturbance, taken as A in this case and are given in Table IV.

In order of increasing voltage disturbance, the sites can be ranked A, B, D, C. C is significantly worse than any other site.

2.3 LV overhead line

Re-expressing (3)

$$\Delta V(pu) \sim (S/S_{base}) \times (z_{MV}l_{MV}/Z_{base.MV})$$

$$= (z_{MV}/V_{MV}^2) \times S_{MV}$$

(7)

With some manipulation to get the form of (4)

$$\Delta V = (z_{LV}l_{LV}/Z_{base.LV}) \times (S_{LV}/Z_{LV})$$

(8)

Hence, following (5)

$$VDI = S_{LV}l_{LV}$$

(9)

In many cases, $z_{LV}$ will be very close to $z_{MV}$. For the common situation of $V_{MV} = 11kV$, $V_{LV} = 415V$, (9) can be simplified to

$$VDI = 703 S_{LV}$$

(10)

$$\text{Figure 3 - MV feeder example}$$

![Figure 3 - MV feeder example](image)

**Table II - System data**

<table>
<thead>
<tr>
<th>Lengths</th>
<th>1 = 0.5km, 2 = 2km, 3 = 3km</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVA</td>
<td>S1 = 5MVA, S2 = 2MVA, S3 = 0.5MVA, S4 = 2MVA, S5 = 0MVA</td>
</tr>
</tbody>
</table>

**Table III - Calculated VDI**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.75</td>
<td>4.75</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Table IV - Calculated VDF**

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_{in}</td>
<td>5</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>S_{out}</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>S_{average}</td>
<td>1.5</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>VDI</td>
<td>0.75</td>
<td>4</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Figure 2 - MV feeder with several take-off points.**
This shows that each km of LV line has the same referred impedance as 703km of MV line.

### 2.4 Underground cable

(9) can be applied to MV cable by taking $V_{LV} = V_{MV}$. Cable reactance figures are generally about 1/6 of overhead line. Hence (9) becomes

$$VDI = 0.17 S_{MV,cable}^2$$

For LV cable, (10) is modified by multiplying the its coefficient by 0.17 to give

$$VDI = 121 S_{LV,cable}^2$$

### 2.5 Transformer

We shall find that a transformer can be defined to have an equivalent length for use in a VDI equation. This corresponds to the length of MV overhead line having the same impedance. We first need to distinguish between transformer base $S_t$ (taken as its rating) and the system base $S_{base}$. If $x_t(S_t)$ is the transformer reactance on its own rating as base, in the system base

$$x_t(S_{base}) = x_t(S_t)\times\frac{S_{base}}{S_t}$$

$$\Delta V_{pu} = S_{base} \times (S_{base} x_t(S_t)) \times \frac{S_{base}}{S_t}$$

Following (5) $VDI = S_t \times (V_{MV}^2 x_t(S_t)) / \sqrt{S_t S_{base}}$

By a comparison with (5), the "equivalent length" of a transformer is given by

$$l_t = V_{MV}^2 x_t(S_t) / z_{MV} S_t$$

Using typical values for $z_{MV}$, and $V_{MV} = 11kV$, (16) becomes

$$l_t = 346 x_t(S_t) / S_t$$

Note that this result applies irrespective of the voltage level at which the transformer is situated. For example, a 500 kVA transformer with 5% reactance has an equivalent length of 35 km of MV line (or 50m of LV overhead line).

### 2.6 Fault level

Suppose a fault level $FL(MVA)$ is given at a voltage level $V$. This corresponds to an absolute impedance $V^2/FL$ and a per unit impedance of $S_{base}/FL$. This can be treated as a transformer of the same pu reactance where $S_t$ is now $S_{base}$. Using result (16)

$$l_{FL} = (V_{MV}^2 / z_{MV}) / FL$$

Using typical values for $z_{MV}$, and $V_{MV} = 11kV$, (18) becomes

$$l_{FL} = 346 / FL$$

### 2.7 Complex example

In this example, we have a zone substation with a number of MV feeders connected to the 11kV busbar of which we have an interest in one. An 11kV/415V distribution transformer is connected 5km along the feeder and supplies 200m of overhead LV distributor. Typical data has been used to indicate the magnitude of the equivalent lengths and typical orders of VDI.

We see that the equivalent lengths of each component increases downstream. Of course the MVA carried also drops rapidly and we cannot conclude from this alone that the major voltage drops are downstream. Let us now put in the following figures (Table V)

<table>
<thead>
<tr>
<th>Component</th>
<th>Upstream</th>
<th>T1</th>
<th>MV line</th>
<th>T2</th>
<th>LV line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent length</td>
<td>0.35</td>
<td>1.73</td>
<td>5.00</td>
<td>34.57</td>
<td>210.77</td>
</tr>
<tr>
<td>MVA</td>
<td>15</td>
<td>15</td>
<td>3</td>
<td>0.4</td>
<td>0.25</td>
</tr>
<tr>
<td>VDF</td>
<td>2.6</td>
<td>25.9</td>
<td>15.0</td>
<td>13.8</td>
<td>52.7</td>
</tr>
</tbody>
</table>

Each link component contributes to the VDF as shown in the pie chart Figure 5. The LV line makes the major contribution in this case. Both transformers and the MV line make similar 2nd order contributions. The Upstream system has the smallest contribution.

Another method for displaying this effect is that shown in Figure 6, which shows a plot of VDF versus distance from the source. Distance is not shown to scale – each link is given a similar width, so that its contribution is proportional to its slope. This figure has the virtue that it gives all the insights of the Pie chart for VDI and also shows the VDF figures so that sites can be compared.
3. FIELD TEST RESULTS

3.1 The system

The analysis method has been applied to field data obtained for the system shown in Figure 7. The system monitor positions were chosen to give a good coverage of both MV and LV disturbance levels. The only MV site with a voltage transducer fitted is the 11 kV busbar (Site 1). Monitors were also connected at LV sites 2, 6, 9. VDFs were calculated for all numbered sites, with the 11kV busbar taken as zero, and are shown in Table VI and Figure 8. The monitor used gave readings of voltage and harmonics but did not have a flicker capability. An attempt was made to calculate unbalance from the voltage readings. Only line-neutral values were available for LV sites, and the unbalance values calculated are thus only indicative. They should not be compared with the MV site unbalance which has been calculated correctly from measured line-line voltage values.

Table VI - VDF for numbered sites

<table>
<thead>
<tr>
<th>Site</th>
<th>VDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.1%</td>
</tr>
<tr>
<td>2</td>
<td>4.1%</td>
</tr>
<tr>
<td>3</td>
<td>5.0%</td>
</tr>
<tr>
<td>4</td>
<td>5.0%</td>
</tr>
<tr>
<td>5</td>
<td>5.0%</td>
</tr>
<tr>
<td>6</td>
<td>5.0%</td>
</tr>
<tr>
<td>7</td>
<td>5.0%</td>
</tr>
<tr>
<td>8</td>
<td>5.0%</td>
</tr>
<tr>
<td>9</td>
<td>5.0%</td>
</tr>
</tbody>
</table>

3.2 Results

Although the voltage drop in a series component is the product of impedance and current, the use of off-nominal voltage ratios for voltage regulation gives another component which is just as important. Hence it is not expected that VDF will correlate well with average voltage level. However, it is expected that voltage will be increasingly affected by upstream load current effects as sites are chosen further downstream. This spread in voltage can be measured by the standard deviation (normalised relative to the average voltage) and is plotted against VDF in Figure 9. Site ranking is shown to be given correctly.
Unbalance is caused by non-ideal effects which are normally not known. Nevertheless, VDF should be a good indicator of unbalance if the following assumptions hold:

1. the unbalance arises mainly from load imbalance, not impedance imbalance
2. the unbalance current in LV networks is in proportion to the fundamental current

Figure 10 shows that site ranking for unbalance is correctly given at LV by the VDF. The apparent anomaly for the MV Site 1 is most likely due to it being determined from a different set of voltage readings (line-line rather than line-neutral).

Figure 11 shows good agreement between site ranking for both the VDF and THD measurements. This is not surprising since most of the assumptions made about VDF calculations (Section 1.3) should hold well for harmonics. With more data, we may be able to go further and modify this for particular types of loads, e.g. residential, industrial, commercial and rural.

4. CONCLUSION

The paper has shown a simple method for determining a voltage disturbance figure for each site in a section of a power system. The method is oriented to determining the rank of a site for the overall levels of continuous PQ disturbances. It makes use of a number of simplifying assumptions to match the very limited data which will be available. It is based on determining the VDI for each segment of a circuit and then adding the VDI from a site back to an upstream site where levels are taken to be zero. The VDI has been shown to be expressible in the form of $S(MVA)$ times length (km). The lengths to be used correspond to the physical length in the case of overhead lines - equivalent lengths can be determined corresponding to fault level, transformers and underground cable.

The method has been tested by a comparison with the data from measurements at a MV/LV subsystem and has been shown to give the correct ranking of sites.

The development of this VDF opens up several lines of research. More PQ surveys need to be performed and the usefulness of the method confirmed for several types of systems. Possible uses of the method include assisting in determining which sites should be monitored and estimating levels at unmonitored sites.

5. REFERENCES