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Abstract
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Stability and Phase Space Analysis of Fluidized-Dense Phase Pneumatic Transport System

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Abstract

Fluidized dense-phase pneumatic conveying of powders is gaining popularity in several industries, such as power, chemical, cement, refinery, alumina, pharmaceutical, limestone etc., because of reduced gas flow rate and power consumption, decreased conveying velocities, improved product quality control, reduced pipeline sizing and wear rate, increased workplace safety etc. However, understanding the fundamental transport mechanism of fluidized dense-phase transport has only made limited progress because of the highly concentrated and turbulent nature of the gas-solids mixture. In the present work, pneumatic conveying trials were conducted with power plant fly ash (median particle diameter: 30 μm; particle density: 2300 kg/m\textsuperscript{3}; loose-poured bulk density: 700 kg/m\textsuperscript{3}) through 69 mm I.D. x 168 m long pipeline. The mass momentum and energy balance of the system lead to the formulation of governing equations of flow for the dense-phase pneumatic conveying, which were solved using Runge-Kutta-Fehlberg (RKF45) method for different mass flow rates of fly ash and air. The stability of the system has been established corresponding to the four critical conveying parameters: pressure drop, particle
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**Keywords:** pneumatic conveying, dense-phase, fine powders, linear stability analysis, solids friction, phase space and bistability

1. **Introduction**

Fluidized dense-phase pneumatic transport is the preferred method for conveying fine powders in several industries, such as coal fired thermal power plants, chemical, pharmaceutical, petrochemical plants, cement, food industries, etc. over the conventional dilute-phase mode due to the reasons of high solids to gas mass ratio, low gas flow requirement (i.e. smaller compressor and savings in operating power requirement), smaller size of pipes and fittings, reduced conveying velocity resulting in lowering of wear rate of pipelines and bends, decreased rate of product attrition, reduced size of solids-gas separation unit etc. [1, 2]. Typically fine powders (Geldart Group A material) that have good air retention capabilities, such as fly ash, cement,
pulverized coal etc., are able to support fluidized dense-phase mode of conveying [2, 3]. In this regime of transport, the gas velocities are kept lower than the saltation velocities of particles, so that the conveying can take place in a non-suspension mode [1, 2]. In spite of having several merits, fundamentally understanding the flow mechanisms and reliably designing fluidized dense-phase systems have made only limited progress so far. This is because in fluidized dense-phase flows, the aerated fine powders form dunes and move along the bottom of pipe [2]. Such turbulent natured dunes are very difficult to be fundamentally modelled due to the complexities to link the particle and bulk properties during actual flow conditions. Comparatively, relatively more success based on fundamental modelling methods using powder mechanics have been developed for low-velocity slug-flow of granular products [4] or the dilute-phase flow of bulk solids flow mechanics can be considered to be applicable with a fair accuracy [5]. Because of the limited progress achieved so far in fundamentally understanding the transport mechanisms of fluidized bulk solids, empirical power function based models have been popularly developed over the years by several investigators for important design parameters, such as minimum transport (to predict flow blockage condition) and solids friction factor (to determine pressure drop in pipes) [2, 6-10]. These models [2, 6-10] have shown good predictions when applied to researchers’ own data, but previous investigation [2] has demonstrated that the empirical models can unexpectedly result in significant inaccuracy under scale-up conditions of pipeline length and diameter. The existing models are based on dimensionless numbers, such as particle and gas based Froude number, volumetric loading ratio etc. [2, 6-10], but these have not taken into consideration the important flow attributes, such as, the flow characteristics of the dunes. The dunes are formed when the fluidized powders are pushed out of blow tanks with the help of a combination of top, fluidization and conveying air (depending on blow tank design and
operation) [1]. Most of the existing models for solids friction factor or minimum transport criteria include mass or volumetric flow ratios, but these do not consider the nature of the dunes. In the experience of the authors, the nature of the dunes is a critically important parameter that strongly influences the pressure drop in the pipeline; it has been experimentally observed that there could be considerably different pressure drop due to the varying nature of the moving dunes (dunes filling up the pipe cross section) even for the similar mass flow rates of solids and air. Hence, in order to model fluidized dense-phase flow reliably and from a more fundamental standpoint, it is necessary to analyze the important dune characteristics. Fundamentally and practically, it is of considerable importance to learn whether the dunes change their characteristics along the length of flow. This can happen in a long pipeline, such as in the case of fluidized dense-phase pneumatic conveying of fly ash in power stations (the distance could be as long as 1 km). It is intuitive to think that if the nature of the dunes are changing along the length of the flow (due to increasing flow velocity caused by gas expanse in the direction of flow and due to the impact of the dunes on the wall of pipes and bends), the pressure drop per unit length of pipes should vary from the feed point to the exit of pipeline. Therefore, stability of the dunes are to be examined along the length of flow. Stability analysis is a mathematical technique to determine the behavior of the system w.r.t to time/space and this can be extended to pneumatic transport [1]. Number of methods for stability analysis exist in literature, such as Lyapunov exponent’s method [11], linear stability analysis [12], perturbation method [13] and von Neumann Stability analysis [14]. There have been some cases where the researchers have used stability analysis to study the particle-fluid systems, such as Chen et al. [15] to Fullmer and Hrenya [21], Masnadi et al. [22] and Zhang et al. [23, 24]. These work were carried out either with granular products or coarser particles in standpipes or with fluidized beds and very little
research has been conducted in the area of stability analysis for fluidized dense-phase pneumatic conveying of fine powders through pipes. This paper investigates into the stability of moving dunes of fine aerated powders through pipelines. An unstable dune would indicate change in flow mechanism along the length of pipe and the need to incorporate such change in flow condition in the model describing the transport mechanism.

2. Experimental work

Power station fly ash was conveyed from fluidized dense- to dilute-phase in 69 mm I.D. x 168 m long pipeline at the Bulk Materials Handling Laboratory of University of Wollongong, Australia. The fly ash was conveyed with different air and solids mass flow rates. Physical properties of the fly ash and pipeline conditions are given in Table 1. Particle size distributions were measured using laser diffraction method.

**Table 1:** Physical properties of the fly ash conveyed

Schematic diagram of the 69 mm I.D. x 168 m long pipeline is shown in Figure 1.

**Figure 1:** Layout of the 69 mm I.D. x 168 m test rig (for fly ash)

The test facility comprised of:
- Tandem 0.425 m$^3$ bottom-discharge blow tank feeding system;
- 69 mm I.D. $\times$ 168 m long mild steel pipeline, including 6.5 m vertical lift, five 1 m radius 90° bends and 150 mm N.B. tee-bend connecting the end of the pipeline to the feed bin;
- Static pressure measurements at two straight pipe tapping locations: P9-P10 (52.68 m apart) and P11-P12 (40.41 m apart), the tapping locations were selected such that they are out of the influence of any bend effect. Total pipeline pressure drop was measured using the P8 transducer. The static pressure transducers had the following specification: manufacturer: Endress and Hauser, model: Cerabar PMC133, pressure range: 0-6 and 0-2 bar-g, maximum pressure: 40 bar (absolute), current signal: 4 to 20 mA;
- Ingersoll Rand diesel-powered Model P375-WP, 10.6 m$^3$/min free air delivery screw compressor was used to supply compressed air at a maximum pressure of approximately 800 kPa-g;
- 6 m$^3$ receiving bin with insertable pulse-jet dust filter;
- All necessary instrumentation (e.g. pressure transmitters, load cells on feed bin and receiving bin, annubar with DP meter);
- Data acquisition unit for data recording and analysis.

Using steady state straight-pipe pressure drop data for a range of solids and air flow rates through P9-P10 and P11-P12 tapping points for the 69 mm I.D. $\times$ 168 m long pipe, the following straight-pipe pneumatic conveying characteristics (PCC) have been obtained (Figures 2 and 3). Constant mean Froude number lines have been shown.
Figure 2: Straight-pipe Pneumatic Conveying Characteristics (PCC) for fly ash, P9-P10, 69 mm I.D. × 168 m long pipe

Figure 3: Straight-pipe Pneumatic Conveying Characteristics (PCC) for fly ash, P11-P12, 69 mm I.D. × 168 m long pipe

Comparing Figures 2 and 3, it becomes evident that the PCC for “straight pipe” sections (even for the same product, i.e. fly ash) differ depending upon the location of the tapping points, thus indicating a change in local flow mechanisms. Minimum transport boundary would be located to the left of the $Fr_m = 4$ line in Figure 2. Total pipeline PCC with the location of minimum transport boundary for this pipeline and product can be found in Mallick [2], where reliable conveying was found to occur with a minimum Froude number of 4 at the blow tank location (product feed point). For $m_a = 0.082 \text{ kg/s}$, $m_s = 14.2 \text{ t/h}$, mean velocity of within P9-P10 is calculated as 9 m/s, whereas for the same values of $m_a$ and $m_s$ (same experiment), mean velocity within P11-P12 is calculated as 11.2 m/s. The PCC obtained from the initial section of the pipe loop (i.e. P9-P10) show a significant portion where pressure drop decreases with an increase in air flow rate. On the other hand, the PCC obtained from the latter section of the pipe loops (i.e. P11-P12) show a more significant flat portion (i.e. pressure drop remains almost constant with increasing air flow rate). This indicates a possible change in flow characteristics along the pipeline.

3. Flow model for solids-gas transport
Considering the following assumptions, the governing equations for the flow have been developed:

- One-dimensional flow along the bulk flow direction (x-direction).
- In an infinitesimal control volume, the gas-solids composition is uniform with no concentration gradient of particles across the diameter of pipe.
- The gas-solids flow is in steady state.
- Ideal gas equation of state is obeyed by gas-phase in an isothermal system.

Considering mass, momentum and energy balance across a small control volume [22], the following governing equations have been obtained:

**Figure 4:** Energy exchanges across a control volume

Along x-direction, considering the velocities of air and solids phase and their respective frictional resistances, pressure and area of cross-section of the pipeline, net accumulation of momentum is:

\[
= APu_a - A \left[ \left( P_a + \frac{dp}{dx} \Delta x \right) \left( u_a + \frac{du_a}{dx} \Delta x \right) \right] - \left[ F_a \left( u_a + \frac{du_a}{dx} \Delta x^2 \right) + F_s \left( u_s + \frac{du_s}{dx} \Delta x^2 \right) \right]
\]  

(1)

To study the nature of gas-solids flow dynamics, Kuang et al. [26] studied the resultant energy dissipation generated from the inter-particles, particles and wall, particles and fluid, and between
fluid to fluid interactions. Computational analysis has been performed using CFD-DEM based technique to solve the following equations:

\[
\begin{align*}
    m_i \frac{dV_i}{dt} &= f_{pg,i} + f_{drag,i} + \sum_{j=1}^{k_i} (f_{c,ij} + f_{d,ij}) + m_i g \\
    I_i \frac{d\omega_i}{dt} &= \sum_{j=1}^{k_i} (T_{t,ij} + T_{r,ij}) \\
    \frac{\partial (\rho f \varepsilon_f)}{\partial t} + \nabla \cdot (\rho f \varepsilon_f u) &= 0 \\
    \frac{\partial (\rho f \varepsilon_f u)}{\partial t} + \nabla \cdot (\rho f \varepsilon_f u) &= -\nabla P - F_{pf} + \nabla \cdot (\varepsilon_f \tau) + (\rho_f \varepsilon_f) g
\end{align*}
\] (2-5)

Equations (2) and (3) correspond to particle or solids flow, and equations (4) and (5) correspond to gas flow. However, the results concluded that dissipation in energy occurred due to particle-fluid interaction, particle-wall friction and fluid-wall viscous energy at steady state pneumatic conveying. In an inclined flow, gravitational potential energy generates an additional energy loss in such systems. In view of the results presented in [26], the net accumulation of kinetic energy (KE) along x-direction is given by:

\[
\begin{align*}
    KE &= \frac{1}{2} m_a u_a^2 + \frac{1}{2} m_s u_s^2 - \frac{1}{2} m_a \left( u_a + \frac{du_a}{dx} \Delta x \right)^2 - \frac{1}{2} m_s \left( u_s + \frac{du_s}{dx} \Delta x \right)^2
\end{align*}
\] (6)

Assuming, at steady state there is no net accumulation of energy in the control volume and neglecting the higher order terms, we get:

\[
\begin{align*}
    P \frac{du_a}{dx} + u_a \frac{dp}{dx} + u_a f_a + u_s f_s + \rho_a \varepsilon_a u_a^2 \frac{du_a}{dx} + \rho_s \varepsilon_s u_s^2 \frac{du_s}{dx} = 0
\end{align*}
\] (7)
For gas-solids flow, the conservation of momentum is written as:

\[
\left(\rho_a \varepsilon_a A u_a\right) \left(u_a + \frac{du_a}{dx} \Delta x\right) - \left(\rho_a \varepsilon_a A u_a\right) u_a + \left(\rho_f \varepsilon_s A u_s\right) \left(u_s + \frac{du_s}{dx} \Delta x\right) - \left(\rho_f \varepsilon_s A u_s\right) u_s
\]

\[
= PA - \left(P + \frac{dP}{dx} \Delta x\right) A - F_a - F_s
\]

(8)

Dividing both sides of equation (8) by elemental volume \((A \Delta x)\),

\[
\varepsilon_a \rho_a u_a \frac{du_a}{dx} + \varepsilon_s \rho_f u_s \frac{du_s}{dx} + \frac{dP}{dx} = -f_a - f_s
\]

(9)

Mass conservation for gas-phase and solids-phase are given by the following equations, respectively:

\[
\frac{d}{dx} \left(\varepsilon_a \rho_a u_a\right) = 0
\]

(10)

\[
\frac{d}{dx} \left(\varepsilon_s \rho_f u_s\right) = 0
\]

(11)

Gas-phase and solids-phase are related by the following relation:

\[
\varepsilon_a + \varepsilon_s = 1.
\]

(12)

Following equations represent the evolution of parameters \(u_s, u_a, \varepsilon_s\) and \(P\) w.r.t the pipeline distance \(x\) and are known as evolution equations of the system. These equations show how the different parameters under the study are varying with the evolving flow.

\[
\frac{du_s}{dx} = \frac{u_s}{\varepsilon_s \left(\rho_f u_s^2 (u_s - u_a) - \frac{u_a P}{(1 - \varepsilon_s)}\right)} \left[-(u_s - u_a) f_s + u_a \frac{d \rho_a}{\rho_a} \frac{dx}{dx}\right]
\]

(13)
\[
\frac{du_a}{dx} = -\frac{u_a}{\rho_a} \frac{d\rho_a}{dx} - \frac{u_a}{\rho_a} \frac{u_a}{(1-\epsilon_s)(\rho_f u_s^2 (u_s - u_a) - \frac{u_a p}{(1-\epsilon_s)})} \left[ -(u_s - u_a) f_s + \frac{u_a p}{\rho_a} \frac{d\rho_a}{dx} \right]
\]

(14)

\[
\frac{d\epsilon_s}{dx} = -\frac{1}{(\rho_f u_s^2 (u_s - u_a) - \frac{u_a p}{(1-\epsilon_s)})} \left[ -(u_s - u_a) f_s + \frac{u_a p}{\rho_a} \frac{d\rho_a}{dx} \right]
\]

(15)

\[
\frac{dp}{dx} = -f_a - f_s + (1 - \epsilon_s) u_a \frac{d\rho_a}{dx} + \frac{(\rho_a u_s^2 - \rho_f u_s^2)}{(\rho_f u_s^2 (u_s - u_a) - \frac{u_a p}{(1-\epsilon_s)})} \left[ -(u_s - u_a) f_s + \frac{u_a p}{\rho_a} \frac{d\rho_a}{dx} \right]
\]

(16)

Coupled first order differential equations (13) to (16) are explicitly dependent on the air and solids phase velocity, volume fraction of solids, pressure, rate of change of air density, and friction force per unit volume due to air and solids phase. To obtain solutions of the \(u_s\), \(u_a\), \(\epsilon_s\) and \(P\), equations (13) to (16) have been solved numerically. The numerical procedure to solve the equations along the pipeline length is being discussed in the following. The entire horizontal pipeline length has been divided into a number of different sections consisting of straight pipe sections and bends. The segments of the pipeline has been marked at following distant points starting from the origin or the inlet of the pipe: 3.26 m, 24.62 m, 60.84 m, 81.31 m, 98.78 m and 129 m (Figure 1). A set of coupled ordinary differential equations (equations (13) to (16)) have been solved section by section with numerical technique based on the fourth-fifth-order Runge-Kutta-Fehlberg (RKF45) method \([27, 28]\). The values used for parameters \(f_s\), \(f_a\), \(\rho_a\) and \(\frac{d\rho_a}{dx}\) have been obtained from the experiments conducted through 69 mm I.D. x 168 m long test rig. It is assumed that the well aerated mixture of solid particles occupy the full cross-section of the pipeline when the product is discharged from the blow tank. Subsequently, the occupancy of
solid particles decreases along the direction of flow. Following relation has been used to obtain
the initial condition for particle velocity:

\[ u_s = \frac{m_s}{A \rho_{fl}} \]  

(17)

The initial values for the succeeding section are the numerical values obtained from the exit
point of the preceding section. This practice has been followed throughout the pipeline till the
exit point. System parameters, i.e. \( u_s, u_a, \varepsilon_s, u_s/u_a \) and \( P_s \) are calculated at each small interval
of the pipeline section. Fluidized bulk density for this fly ash was experimentally determined as
300 kg/m\(^3\) [25].

4. Stability analysis

Stability analysis is an important aspect of studying a system to identify its feasible operational
zone during the installation of new system or implying modifications to the existing design of the
system. Stability analysis considering linearization, Taylor’s series expansion and Lyapunov
analysis have been discussed in details for gas-solids flow mechanism [1]. In the present
investigation, the linearization approach has been adopted to study the stability of the system. A
practical physical system, such as the gas-solid flow, can be modeled by differential equations.
Consequently, prediction of the system behavior right away requires integration of those system-
defining differential equations. Now, majority of the differential equations are non-integrable,
i.e., does not possess a closed form solution. A quick and somewhat popular alternate option is to
carry out numerical analysis, which in turn requires a set of parametric values corresponds to the stable condition of the system. A numerical analysis itself cannot generate it, in general. In such cases Lyapunov’s stability analysis is in rescue. It allows determining the stability of a system without integrating the differential equation explicitly. The consequence is far reaching. This stability analysis method can be applied to any system that involves “measure of energy” as it can study the rate of change of system energy to determine the stability. In the field of stability analysis, Lyapunov technique can handle not only the non-linear terms, but also regions of stability can be predicted without much of computational complexities. However, by linearizing the dynamical equations eliminate the valuable information that describes the nonlinear nature of the flow dynamics, but in order to solve the nonlinear equations and gather information about stability, linearization is necessary. The linearized equations, thus obtained, have been used to find the associated eigenvalues. Actual gas and solids velocity, volume fraction of solids and pressure have been studied through eigenvalue and phase space analysis. In order to proceed with the stability analysis of the system, it is necessary to linearize the nonlinear evolution equations (13) to (16).

\[
\frac{du_s}{dx} = A_{11}u_s + A_{12}u_a + A_{13}\epsilon_s + A_{14}P
\] (18)

\[
\frac{du_a}{dx} = A_{21}u_s + A_{22}u_a + A_{23}\epsilon_s + A_{24}P
\] (19)

\[
\frac{d\epsilon_s}{dx} = A_{31}u_s + A_{32}u_a + A_{33}\epsilon_s + A_{34}P
\] (20)

\[
\frac{dp}{dx} = A_{41}u_s + A_{42}u_a + A_{43}\epsilon_s + A_{44}P
\] (21)
Equations (18) to (21) are the linearized form of evolution equations (13) to (16). From linearized equations (18) to (21), the following Jacobi determinant can be constructed from the derivatives w.r.t solids velocity, actual air velocity, volume fraction for solids and pressure at an equilibrium state.

\[
\begin{vmatrix}
A_{11} - \lambda & A_{12} & A_{13} & A_{14} \\
A_{21} & A_{22} - \lambda & A_{23} & A_{24} \\
A_{31} & A_{32} & A_{33} - \lambda & A_{34} \\
A_{41} & A_{42} & A_{43} & A_{44} - \lambda
\end{vmatrix} = 0
\]  

(22)

Where, \( \lambda \) is the eigenvalue of the Jacobi determinant. The steady state solutions of the linearized evolution equations are stable, if the coefficients of the resultant eigenvalue equation assume positive real values. The resultant eigenvalue in its general form can be represented as:

\[
\lambda^4 + c_1 \lambda^3 + c_2 \lambda^2 + c_3 \lambda + c_4 = 0
\]  

(23)

Where, the coefficients \( c_i \)'s (where \( i = 1 \) to 4) are the functions of \( A_{kl} \)'s (where, \( k,l = 1 \) to 4). In order to achieve stability, according to Hurwitz’s criteria, the following conditions should be satisfied. It is an algebraic method which provides information on the absolute stability of a linear time-invariant system having characteristic equation with real coefficients.

\[
|c_1| > 0, \quad \begin{vmatrix} c_1 & c_3 \\ 1 & c_2 \end{vmatrix} > 0 \quad \text{and} \quad \begin{vmatrix} c_1 & c_3 & 0 \\ 1 & c_2 & c_4 \\ 0 & c_1 & c_3 \end{vmatrix} > 0.
\]

(24)
The solutions of eigenvalue equation (23) provide the information about the stability, quasi-stability and instability of the system at an equilibrium state. The system is stable if the real part of its eigenvalue solution is a negative real number, and is unstable if the real part of the eigenvalue solution is positive [29]. For this system, the eigenvalues obtained are the combination of negative and positive real numbers, thus indicating that the system behaves like saddle point, it is neither stable nor unstable. The phase portraits obtained converge in every case under study, thus demonstrating that the system is operating under the stable conditions. However, on the contrary, it is indicating that the dunes formed at the initial point of the pipe (blow tank discharge) are deforming gradually as the flow moves towards the exit point of the pipeline. As the gas-solids flow develops along the pipeline, the solids and actual gas velocity increases and the pressure drops down with the reduction in volume fraction of solids [25]. The flow changes from fluidized-dense to dilute-phase. To determine the stability behavior of the system near equilibrium, the associated phase portraits demonstrate the system behavior very intricately. Phase portraits of gas and solids velocity are shown Figure 5.

**Figure 5:** Change in particle velocity with increasing actual gas velocity along the pipeline.

Figure 5 shows that the velocity of gas and solids increase as the flow develops. The rate of increase of particle velocity is steeper as compared to that of gas velocity. This analytical study has been conducted for the certain operating conditions with four different combination of mass flow rates of solids and gas, i.e., \( m_s = 19 \ t/h \) and \( m_a = 0.08 \ kg/s \); \( m_s = 19 \ t/h \) and \( m_a = 0.04 \ kg/s \); \( m_s = 09 \ t/h \) and \( m_a = 0.08 \ kg/s \); and \( m_s = 9 \ t/h \) and \( m_a = 0.04 \ kg/s \). In every plot (Figures 5 to 10), the red line corresponds to \( m_s = 19 \ t/h \) and \( m_a = 0.08 \ kg/s \); black line
corresponds to \( m_s = 19 \text{ t/h} \) and \( m_a = 0.04 \text{ kg/s} \); green line corresponds to \( m_s = 9 \text{ t/h} \) and \( m_a = 0.08 \text{ kg/s} \) and blue line corresponds to \( m_s = 9 \text{ t/h} \) and \( m_a = 0.04 \text{ kg/s} \). Multiple steady states can be predicted under certain operating conditions. A representation of such condition has been shown in Figure 5. The overlapping region of two different mass flow rates, the region between the pairs of dashed lines (\( DB \) and \( EC; D'B' \) and \( E'C' \)) defines the bi-stability for the different combination of mass flow rate of solids under the same mass flow rate conditions for air. As the range of operating steady state characteristics for mass flow rates of the air increases, the corresponding overlapping region defining the multi-stability tends to shrink. In the velocity phase portrait (Figure 5), as the flow evolves from point A to C, there is no operation zone for the corresponding mass flow rate of solids \( (m_s = 9 \text{ t/h}) \) beyond point C. On increasing the air velocity, there is a step-wise rise to the higher mass flow rate of solids at point E \( (m_s = 19 \text{ t/h}) \) from point C \( (m_s = 9 \text{ t/h}) \). Such sudden change is referred to as bifurcation instability. However, as we move from point F to D, the system will follow the path F to E to D, until the critical point D \( (m_s = 19 \text{ t/h}) \) is reached, the system will undergo a step drop, therefore, falling to point B at lower mass flow rate of solids \( (m_s = 9 \text{ t/h}) \). Therefore, it can be inferred that for higher solid flow rate (i.e., \( m_s = 19 \text{ t/h} \)), larger particle velocity \( (u_s = 4.7 \text{ m/s}) \) would be required to make the flow to start compared to lower solids flow rate (i.e., corresponding to \( m_s = 9 \text{ t/h} \)).

**Figure 6:** Change in pressure with increasing solids volumetric fraction

Figure 6 shows the trends of change in pressure with an increase in solids volume fraction. As the flow develops along the pipeline, the gas expands in the pipe at steady rate, thus continuously
decreasing the pressure along the pipeline length. It is very much obvious that the solids particle concentration decreases as the flow evolves due to the expansion of gas, thus a fluidized-dense phase flow gradually proceeds towards a dilute-phase regime. As a result, the pressure and volume fraction of solids drop as the gas-solids flow develop (Figure 6). On the other hand, volume fraction for gas increases along the direction of flow. As the flow evolves, the gas expands in the pipeline creating the drop in the corresponding pressure value. In the test rig schematic (Figure 1), there are closely coupled bends after 80 m of pipeline length, which are responsible for the steep rise in the trends of solids to gas velocity ratio, as the solids particles de-accelerates due to bends and change in direction of motion. This can be due to the drop in the gas pressure as it flows through the closely coupled bends and the accompanying effect of gas expansion. Bistable operational states have been observed for the region under the curve AB and CD in the air velocity versus pressure plot (Figure 7). The system exhibits multiple steady states for different combinations of the mass flow rates of solids and gas. In solids velocity versus pressure plot, the region under the curve EF shows the multiple operational states for the system (Figure 8). However, in Figure 8, the area under the curves AB and FG shows the bi-stable zones. The multi-stable states under the curve EF in Figure 8 are considerably important as the transition between the different mass flow rates of air and solids has been observed. Figure 8 shows permissible operation zone depicting upper and lower boundaries of the pneumatic conveying characteristics at the particular combination of the mass flow rate of gas and solids. Figure 8 considers the increase of gas velocity due to the gas compressibility for a given feed gas rate along the length of the pipeline.

**Figure 7:** Change in pressure with increasing actual air velocity.
Figure 8: Change in pressure with increasing particle velocity.

Figure 9 show the 3-dimensional representation of pressure drop versus actual air velocity and particle velocity. As can be comprehended from the Figure 9, corresponding to the different combinations of $u_s$ and $u_a$, pressure drop can be determined for each combination of the flow rates. Increase in the air and solids velocity results in the pressure drop in the system along the pipeline. Rate of pressure drop is significantly higher corresponding to the lower values of mass flow rate of air. Figure 9 gives the comprehensive trend of variation in pressure with change in velocities of gas and solids phase. In this plot, the variation of pressure can be predicted due to the change in both gas and solids phase velocities.

Figure 9: Change in pressure with increasing particle velocity and actual air velocity.

Figure 9 seems to indicate each of the plot reaches a minimum value (minimum value of pressure) for a certain combination of air and solids velocities. The plots also show that same pressure values can be achieved for different combination of air and solids velocities. Figure 10 shows the 2-dimensional representation of solids friction factor versus particle velocity and actual air velocity plots. Solid lines represent the variation of solids friction factor with respect to gas velocity ($u_a$), while the dash-dot lines represent the variation with respect to solids velocity ($u_s$).
Figure 10: Variation of solids velocity (dash-dot lines) and gas velocity (solid lines) with respect to solids friction factor

It can be observed in the above figure that there are different combinations of $u_s$ and $u_a$ which could lead to the same value of solid friction factor, e.g. $\lambda_s = 0.01424$ can be achieved corresponding to $u_s = 2.23 \text{ m/s}$ and $u_a = 6.8 \text{ m/s}$ as well as for $u_s = 5.0 \text{ m/s}$ and $u_a = 8.3 \text{ m/s}$. Similarly, $\lambda_s = 0.008$ can be achieved corresponding to $u_s$ and $u_a$ combinations of 3.74 m/s and 9.11 m/s as well as 6.94 m/s and 10.22 m/s. In Figure 10, it has been observed that the range of solids friction factor is larger for the lower mass flow rate of air as compared to the higher mass flow rates of air. The solids friction factor is more dominant in lower air flow rates. It is also important to mention that the solids friction factor decreases more steeply for lower mass flow rates of air with increasing air and solids velocities, whereas, for higher mass flow rates of air the system experiences gradual decrease in solids friction factor. Higher values of mass flow rate of solids experience faster rate of drop in solids friction factor than its lower counterpart.

In an attempt to provide validation of the findings of this study, it may be noted that various efforts to model solids friction factor [2] have found that solids friction factor models developed from the experimental data obtained from the tapping point locations P9-P10 and P11-P12 of the 69 mm I.D x 168 m long pipe (see Figure 1) have resulted into different set of models (using the same dimensionless parameters). In other words, the developed models for solids friction factor got changed depending on the location of tapping points (i.e. along the length of flow). For the
sake of completeness, two sets of such models using different dimensionless parameters are provided below.

Models developed for solids friction factor [2] using different dimensionless number based on P9-P10 tapping point data are given as:

$$\lambda_s = 20.807(m^*)^{-0.71}(Fr)^{-2.1131}$$  \hspace{1cm} (25)

$$\lambda_s = 14.5(m^*)^{-0.63}(Fr)^{-2.01}(\rho)^{0.207}$$  \hspace{1cm} (26)

Models developed for solids friction factor [2] using different dimensionless number using P11-P12 tapping point data are given as:

$$\lambda_s = 1.998(m^*)^{-0.36}(Fr)^{1.657}$$  \hspace{1cm} (27)

$$\lambda_s = 1.85(m^*)^{-0.353}(Fr)^{-1.66}(\rho)^{-0.071}$$  \hspace{1cm} (28)

Comparing between equations (25) and (27) and between equations (26) and (28) clearly demonstrate that the models representing the flow condition are different along the flow direction.

Mallick [2] and subsequently Setia et al. [9] hypothesized that such dependencies of the developed models on the location of tapping points could be due to the gradual deformation of the structure of the moving dunes along the flow direction. To cater for the same, Setia et al. [9] provided a two-layer modelling approach in which the flow was assumed to be compromising of a fluidized dune (dense-phase) flowing along the bottom of pipe and a suspension flow (dilute-
phase) occurring on top of the dense-phase. Setia et al. [2] considered that the dune structure was gradually diminishing along the flow with more and more particles getting suspended in the dilute-layer. This model of Setia et al. [9] with the consideration of a possible change in dune structure along the direction of flow provided reliable scale up predictions [9]. Figure 11 shows constant solids loading ratio and mean Froude number lines for the P9-P10 section superimposed on the straight pipe (P9-P10) PCC. Experimental data points are also superimposed on the PCC.

**Figure 11**: Straight pipe PCC based on P9-P10 data with constant solids loading ratio and constant Froude number lines superimposed

Figure 11 shows that it is possible for two sets of different air velocities (corresponding to mean Froude numbers of 6 and 10) and solids loading ratio (90 and 60), the pressure drop would be the same (660 Pa/m). This information may be useful in designing parallel streams of gas-solids flow systems, ranging from circulating fluidized beds to cyclones to pneumatic transport systems. Elyasi et al. [22] carried out an analytical and numerical study on the distribution of gas-solid pneumatic flow passing through a ‘‘Y branch’’ in a gas solids flow system. It was concluded that for the case of identical vertical parallel pipes, external effects cause maldistribution of flow. The results of both 2-D and 3-D multiphase computational fluid dynamic simulations and stability analysis confirmed the analytical conclusions. Zhang et al. [23] introduced the ratio of mass flow rate ratio of solids to gas to represent multi-phase interaction. They used direct Lyapunov method to determine the instability of uniformity. The Lyapunov function was expressed by the total energy dissipation to give a universal criterion for predicting mal-distribution. Based on the stability analysis of uniformity, the geometric design
principle of parallel cyclones was provided. In a recent study, Zhang et al. [24] carried out research to provide a universal framework to detect the instability of uniform gas solids fluidization based on Prigogine’s minimum of entropy production. They commented that the total solids pressure loading is a vital independent parameter as it assesses the interaction between chaotic entropy production across solids bed and regular entropy input from the distributor. They also provided a phase diagram showing the effects from operational parameter, geometrical characteristics of distributor and property of solids on the instability of uniformity.

In a recent work, Lu et al. [30] conveyed pulverized coal through a pneumatic conveying set up operating in dense-phase flow regime. The set-up considered a pipe having 15 mm internal diameter and 25 m length. Lu et al. [30] provided different values of experimental particle velocities for different line pressures. Using the parametric relations obtained through the phase-space analysis provided above for particle velocity, a relation for particle velocity has been developed [25]. The predicted values of particle velocities have been compared with the experimental particle velocity values given by Lu et al. [30] for different operating pressures. The results are provided in Figure 12. The average relative deviation has been calculated to be in the range of 30%. Considering the differences in particle properties of fly ash and pulverized coal and the complex flow situations of dense-phase pneumatic conveying of fluidized powders, such deviation may be expected.

**Figure 12:** Evaluation of predicted particle velocity values against the experimental data of pulverized coal
The stability analysis results in this paper would be helpful in understanding and choosing the suitable flow rate conditions in the pneumatic conveying systems. This study can be extended to the different samples and with much larger range of mass flow rates of solids and air. The developed flow model is based on fundamental flow equations (mass, momentum and energy balance). In this model, the effect of frictional forces due to gas and solids phases have been considered along with velocities, volume fraction, pressure, density and the area of pipeline cross-section. The dynamics of such a system is dependent on the above enlisted quantities. Change in any quantity would directly influence the dynamics of the system. This approach is much more fundamental and we can extract the fundamental information about the behaviour of the system, such as, its stability and then corresponding stability zone, general as well as system/sample specific operational conditions. Empirical power function based modelling approaches do not adequately address the fundamental relation between different flow parameters. The approach depicted in this work is an attempt towards fundamentally understanding and modelling the flow condition.

Conclusion

The flow dynamics of gas-solids flow and stability of four critical conveying parameters: pressure drop, particle and solids velocities, and solid volume fraction has been studied for fly ash samples. The possible change in the characteristics of dunes and transport mechanisms in the direction of flow has been validated by the change in slopes of straight-pipes pneumatic conveying characteristics along the length of pipe. Two- and three-dimensional phase space
portraits with different combinations of conveying parameters show that as the flow develops, solids and actual air velocities increase, solids volume fraction decreases (along the direction of flow) and pipeline pressure drops. The solids friction factor has decreased with the increasing actual gas and solids velocities. Multi-stable operating zones for the sample of fly ash has been predicted at certain mass flow rate of air and solids. The behavior of solids friction factor for the fly ash samples has been studied as the flow develops in the pneumatic conveying system. Lower mass flow rates of air experience the steeper fall in the solids friction factor which is more dominant in such systems.

**List of symbols and abbreviations**

\( A \quad \text{Cross sectional area of control volume [m}^2\text{]}\)

\( A_{11}, A_{12}, A_{21}, A_{22}, A_{33}, A_{44} \quad \text{Coefficients of linearized evolution equations [1/m]}\)

\( A_{13}, A_{23} \quad \text{Coefficients of linearized evolution equations [1/s]}\)

\( A_{14}, A_{24} \quad \text{Coefficients of linearized evolution equations [ms/kg]}\)

\( A_{31}, A_{32} \quad \text{Coefficients of linearized evolution equations [s/m}^2\text{]}\)

\( A_{34} \quad \text{Coefficients of linearized evolution equations [s}^2\text{/kg]}\)

\( A_{41}, A_{42} \quad \text{Coefficients of linearized evolution equations [kg/(m}^3\text{s)}]}\)

\( A_{43} \quad \text{Coefficients of linearized evolution equations [Pa/m]}\)

\( c_1 \quad \text{Coefficient of eigenvalue equation [1/m]}\)

\( c_2 \quad \text{Coefficient of eigenvalue equation [1/m}^2\text{]}\)

\( c_3 \quad \text{Coefficient of eigenvalue equation [1/m}^3\text{]}\)
$c_4$ Coefficient of eigenvalue equation [1/m^4]

$d_p$ Particle diameter [µm]

$d_{50}$ Median particle diameter [µm]

$Fr = V/(gD)^{0.5}$ Froude number of flow

$Fr_m = V_m/(gD)^{0.5}$ Mean Froude number of flow for a straight section

$F_a$ Frictional force due to air phase [N]

$F_s$ Frictional force due to solids phase [N]

$F_{p-f}$ Volumetric force between particles and fluids [N/m^3]

$f_a$ Frictional force per unit volume due to air phase [N/m^3]

$f_s$ Frictional force per unit volume due to solids phase [N/m^3]

$f_{c,ij}$ Contact force between $i^{th}$ and $j^{th}$ particles [N]

$f_{d,ij}$ Damping force between $i^{th}$ and $j^{th}$ particles [N]

$f_{pgf,i}$ Pressure gradient force on $i^{th}$ particle [N]

$g$ Acceleration due to gravity [m/s^2]

$l_i$ Particle moment of inertia of $i^{th}$ particle [kg m^2]

$L$ Length of horizontal pipe or test section [m]

$L_v$ Length of vertical pipe or test section [m]

$m_r, m_a$ Mass flow rate of air [kg/s]

$m_s$ Mass flow rate of solids [kg/s]

$m_i$ Mass of the $i^{th}$ particle [kg]

$m^* = m_s/m_a$ Solids loading ratio

$P$ Pressure [Pa]
\(T_{t,ij}\)  
Tangential component of the torque between \(i^{th}\) and \(j^{th}\) particles [N m]

\(T_{r,ij}\)  
Rolling friction torque between \(i^{th}\) and \(j^{th}\) particles [N m]

\(u_a\)  
Actual gas velocity [m/s]

\(u_s\)  
Particle or dune velocity [m/s]

\(u, V\)  
Gas velocity [m/s]

\(v_i\)  
Translational velocity of \(i^{th}\) particle [m/s]

\(V_m\)  
Mean gas velocity for a straight section

**Greek symbols**

\(\rho, \rho_a, \rho_f\)  
Density of air [kg/m\(^3\)]

\(\rho_s\)  
Particle density [kg/m\(^3\)]

\(\rho_b\)  
Loose-poured bulk density [kg/m\(^3\)]

\(\rho_{fl}\)  
Fluidized bulk density [kg/m\(^3\)]

\(\lambda\)  
Eigenvalue [1/m]

\(\lambda_f\)  
Air/gas only friction factor

\(\lambda_s\)  
Solids friction factor through straight pipe

\(\varepsilon_f, \varepsilon_a = V_a/(V_a + V_s)\)  
Volume fraction of air

\(\varepsilon_s = V_s/(V_a + V_s)\)  
Volume fraction of solids

\(\tau\)  
Fluid viscous stress tensor [kg/(m s\(^2\)])

\(\omega_i\)  
Angular velocity of \(i^{th}\) particle [s\(^{-1}\)]

**Abbreviations**

CFD  
Computational Fluid Dynamics
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>DEM</td>
<td>Discrete Element Model</td>
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<td>PCC</td>
<td>Pneumatic conveying characteristics</td>
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<td>I.D.</td>
<td>International diameter</td>
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References


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<th>Fly ash no.</th>
<th>$d_{10}$ (µm)</th>
<th>$d_{50}$ (µm)</th>
<th>$d_{90}$ (µm)</th>
<th>$\rho_s$ (kg/m$^3$)</th>
<th>$\rho_b$ (kg/m$^3$)</th>
<th>D (mm)</th>
<th>L (m)</th>
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<td>2300</td>
<td>700</td>
<td>69</td>
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**Table 1**: Physical properties of the fly ash conveyed.
Highlights:

- Flow governing equations have been solved numerically using RKF45 method
- Stability conditions for gas-solids flow system has been predicted
- Multiple feasible steady states and metastable dune flow condition were found
- Phase plots for air and solids velocity, pressure and solids friction is provided
- Comparison is given for different operating condition of air and solids flow rate
Graphics Abstract

The graph illustrates the relationship between \( \lambda_s \) (dimensionless) and \( u_s', u_a \) (m/s) for different mass flow rates. The points A, B, C, D, E, F, G, and H are marked on the graph, each corresponding to specific values of \( (m_s, m_a) \).

- **A**: \( (2.23, 0.01424) \)
- **B**: \( (3.74, 0.008) \)
- **C**: \( (4.70, 0.016) \)
- **D**: \( (6.94, 0.008) \)
- **E**: \( (2.2297, 0.0058) \)
- **F**: \( (5.72, 0.0037) \)
- **G**: \( (4.70, 0.0069) \)
- **H**: \( (7.45, 0.0039) \)

The lines are color-coded to represent different mass flow rates:

- **Red**: \( m_s = 19 \text{ t/h}, m_a = 0.08 \text{ kg/s} \)
- **Black**: \( m_s = 19 \text{ t/h}, m_a = 0.04 \text{ kg/s} \)
- **Green**: \( m_s = 09 \text{ t/h}, m_a = 0.08 \text{ kg/s} \)
- **Blue**: \( m_s = 09 \text{ t/h}, m_a = 0.04 \text{ kg/s} \)

The legend at the top right corner provides a key to the colors and mass flow rates.
Figure 7

- Red line: $m_s = 19$ t/h, $m_a = 0.08$ kg/s
- Black line: $m_s = 19$ t/h, $m_a = 0.04$ kg/s
- Green line: $m_s = 0.09$ t/h, $m_a = 0.08$ kg/s
- Blue line: $m_s = 0.09$ t/h, $m_a = 0.04$ kg/s

Points:
- A (8.14, 262025)
- B (9.11, 199526)
- C (11.57, 260000)
- C' (11.57, 192617)
- D (14.71, 166730)
- D' (14.71, 136210)

Axes:
- P (Pa) on the y-axis
- $u_a$ (m/s) on the x-axis
Figure 8
Figure 9

- $m_s = 19$ t/h, $m_a = 0.08$ kg/s
- $m_s = 19$ t/h, $m_a = 0.04$ kg/s
- $m_s = 09$ t/h, $m_a = 0.08$ kg/s
- $m_s = 09$ t/h, $m_a = 0.04$ kg/s

P (Pa)

$u_a$ (m/s)

$u_s$ (m/s)
Figure 10
Figure 11

The graph shows the relationship between pressure drop per unit length (Pa/m) and mass flow rate of conveying air (kg/s) for a 69mm-168mm pipe. The graph includes experimental data points and lines representing different conditions such as constant mean Froude number (Fr_m) and constant solids loading ratio. The graph also indicates flow rates corresponding to different pressure drop levels, such as 19 t/h, 14 t/h, and 9 t/h.
Figure 12

The graph shows the relationship between pressure drop (kPa) and particle velocity (m/s). The orange bars represent the experimental particle velocity, while the blue bars represent the predicted particle velocity. The pressure drop values range from 298 kPa to 981 kPa, and the particle velocity values range from 0 m/s to 8 m/s.