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Laterally loaded rigid piles with rotational constraints

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Laterally loaded rigid piles with rotational constraints

Abstract

Elastic-plastic, closed-form solutions were developed recently by the author, to capture the nonlinear response of laterally loaded rigid piles. Presented in compact form, the solutions are convenient to use, and sufficiently accurate despite using only two input parameters of the net limiting force per unit length p_u along the pile, and a subgrade modulus k . Nevertheless, piles may be subjected to limited cap-restraints or loading below ground surface, which alter the response remarkably. This paper provides explicit expressions for estimating loading capacity of anchored piles and develops new solutions for lateral piles with cap-rotation by stipulating a constant p_u or a linear increasing p_u (Gibson p_u) with depth. Lateral loading capacity H_o (at the tip-yield state and yield at rotation point state) and maximum bending moment M_m (at the tip-yield state) are presented against loading locations, and in form of the lateral capacity H_o - M_o (applied moment) locus. The capacity is consistent with available solutions for anchored piles, and caissons with either p_u profile, allowing a united approach from lateral piles to anchored piles. The new solutions are also presented in charts to highlight the impact of rotational stiffness of pile-cap on nonlinear response, offering a united approach for free-head piles through fixed-head piles. Several advantages of the solutions are identified against the prevalent p - y curve based approach. To estimate the key parameter p_u , values of the resistance factor N_p (=ratio of pile-soil limiting resistance over the undrained shear strength s_u) are deduced using the current expressions against available normalised pile capacity involving the impact of gapping (between pile and soil), pile movement mode, pile slenderness ratio, inclined loading angle (anchored piles) and batter angles (lateral piles). The N_p is characterised by: (i) An increase from 5.6-8.6 to 10.14-11.6, as gapping is eliminated around lateral piles and caissons, and from 1.0-6.1 to 2.8-9.8, as translation is converted into rotation mode of footings. (ii) Similar variations with slenderness ratio between anchors and caissons (without gapping), and among anchors, caissons and pipelines (with gapping). And (iii) A reduction with loading angles (anchors) resembling that with batter angles (piles). © 2013 Elsevier Ltd.

Keywords

rotational, constraints, laterally, loaded, piles, rigid

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ABSTRACT

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This paper provides explicit expressions for estimating loading capacity of anchored piles and develops new solutions for lateral piles with cap-rotation by stipulating a constant p_u or a linear increasing p_u (Gibson p_u) with depth. Lateral loading capacity H_o (at the tip-yield state and yield at rotation point state) and maximum bending moment M_m (at the tip-yield state) are presented against loading locations, and in form of the lateral capacity H_o - M_o (applied moment) locus. The capacity is consistent with available solutions for anchored piles, and caissons with either p_u profile, allowing a united approach from lateral piles to anchored piles. The new solutions are also presented in charts to highlight the impact of rotational stiffness of pile-cap on nonlinear response, offering a united approach for free-head piles through fixed-head piles.

Several advantages of the solutions are identified against the prevalent p-y curve based approach. To estimate the key parameter p_u , values of the resistance factor N_p (= ratio of pile-soil limiting resistance over the undrained shear strength s_u) are deduced using the current expressions against available normalised pile capacity involving the impact of gapping (between pile and soil), pile movement mode, pile slenderness ratio, inclined loading angle (anchored piles) and batter angles (lateral piles). The N_p is characterised by: (i) An increase from 5.6~8.6 to 10.14~11.6, as gapping is eliminated around lateral piles and caissons, and from 1.0~6.1 to 2.8~9.8, as translation is converted into rotation mode of footings. (ii) Similar variations with slenderness ratio between anchors and caissons (without gapping), and among anchors, caissons and pipelines (with gapping). And (iii) A reduction with *loading angles* (anchors) resembling that with batter angles (piles).

Keywords: soil/structure interaction, theoretical analysis, piles, plasticity, limit state design/analysis, bearing capacity, anchor piles

1. INTRODUCTION

Offshore exploration has propelled analytical, numerical and experimental investigation into bearing capacity of anchored piles [1-6]. This is seen in development of a complicated strength mobilization (SM) method [7], finite element method (FEM) [8] and plastic limit analysis (PLA) [9], among many others. The study provides the evolution law of the capacity with depth of loading attachment e^- for a constant p_u with depth (p_u = net force per unit length along the pile, $[FL^{-1}]$) and a linearly increase p_u (Gibson p_u). [Note the symbol e is taken as negative (e^-) for depth of attachment to distinguish it from the positive (e^+) loading above ground level]. The FEM and PLA analyses also reveal the variational law of the capacity with loading inclination angle (against horizon). To conduct practical design via these methods, one needs to determine the p_u that should incorporate the combined interaction among pile-movement (translation or rotation) mode, gapping and loading angle. This can be difficult and may be further complicated by ~ 4 times reduction in the p_u from free-head to fixed-head conditions [10-12]. A realistic p_u may be deduced by fitting available numerical and test results using closed-form solutions, as is evident in the deduced p_u profiles for 52 laterally loaded, flexible piles [13]. The corresponding closed-form solutions for lateral piles with rotating pile-cap, however, are not available. The impact of rotational constraints on the piles by the depth of attachment and/or cap- restraint remains to be determined.

The rigid piles refer to those with a pile-soil relative stiffness E_p/G_s being higher than $0.8322(l/d)^4$ [14] (Note: E_p is Young's modulus of an equivalent solid pile $[FL^{-2}]$, G_s is soil shear modulus $[FL^{-2}]$, d is an outside diameter of a cylindrical pile $[L]$, and l is the pile embedded length $[L]$). It should be cautioned that in the use of rigid-pile solutions to predict response of flexible piles [15], bending failure needs to be assessed against maximum bending moment in piles rather than against the applied bending moment.

The 52 p_u profiles deduced from test piles (e^+) in clay, sand or multi-layered soil [13] allow the inadequacy of some prevalent p_u profiles to be revealed. To obtain p_u profile for an anchored pile or its like, pertinent literature for piles, caissons and footings in cohesive soil are reviewed herein. Murff and Hamilton [16] gained an elegant solution for estimating the p_u profile along rigid piles, but for the inability to incorporate the reverse resistance observed above pile-tip level. Aubeny et al [17] conducted the FEM and PLA

analyses on laterally loaded caissons with a slenderness ratio l/d of 2~10. They demonstrated (i) a ~10% variation in the lateral capacity from anisotropic to isotropic strength profiles (smaller variation for $l/d > 6$, and no gapping between caisson and soil); (ii) a normalised rotation depth z_r/l of 0.74~0.78 (FEM), or 0.70~0.76 (PLA) upon loading at mudline (without gapping between caisson and soil); (iii) a normalised capacity $H_o/(s_u dl)$ (ie. N_p) of 4.2~4.8 (without gapping) or 2.3~3.5 (with gapping), respectively for $l/d = 2\sim 10$ and $e = 0$ (e is a real or fictitious eccentricity of the load above ground level; H_o is the lateral capacity; s_u is undrained shear strength), which are rather close to those gained for anchor plates in clay [18]; and (iv) an $H_o/(s_u dl)$ of 10.2~11.9 (without gapping) or 5.2~8.6 (with gapping), respectively for loading at mid-depth ($e/l = -0.5$). Yun and Bransby conducted FEM analysis on footing in cohesive soil [19], which resembles a short-rigid pile (with a full-length gap on one side). They also provided normalised capacity. These values of normalised capacity reviewed will be used to deduce the values of the factor N_p for caissons, footings and anchored piles in cohesive soil.

The study to date has revealed that under a lateral load H_t and moment loading $M_o (= H_t e)$, response of lateral piles [see Fig. 1a] is readily captured using a load transfer approach [14], underpinned by a series of springs (with a subgrade modulus k) distributed along the pile shaft (with limited impact of interaction among the springs). Each spring has a limiting force per unit length p_u [varying with depth z , see Figs. 1b, 1c₁ and 1c₂]. With the ideal elastic-plastic p - $y(u)$ curves, Guo [20, 10] developed nonlinear closed-form solutions for free-head, rigid piles. The solutions reflect the consequence of mobilizing the *on-pile force* per unit length (p) along the *limiting* p_u in the depth zones of $0 \sim z_o$ and $z_1 \sim l$, as indicated by the solid lines in Fig. 1c₁ for a Gibson p_u profile, and in Fig. 1c₂ for a constant p_u profile, respectively. The two p_u profiles generally bracket real p_u distributions with depth along piles [21, 22]. The associated solutions well capture nonlinear pile response against those based on real nonlinear p - y curves, despite use of only two input-parameters k and p_u [13]. Two critical states are defined: (i) *Tip-yield state* at which the on-pile force p at the pile-tip just attains the limiting p_u with $p = p_u$ at $z_1 = l$ [Fig. 1c₁ or 1c₂]; and (ii) the unachievable *Yield at rotation point (YRP) state* with the slip depths z_o, z_1 merging with the depth of rotation z_r [i.e. $z_o = z_r = z_1$]. The values of lateral loads H_t at the two states are taken as loading capacities H_o .

This paper provides new expressions for estimating bearing capacity of anchored-piles, and develops new solutions for rigid piles with rotating cap for constant or Gibson p_u profile. The expressions are compared with centrifuge tests, numerical (FEM, PLA) results, and complicated SM (rotation-dependent) solutions, and are subsequently used to examine the impact of p_u profiles (with constant k), loading eccentricity, and yielding (at tip and rotation point) states on the capacity, displacement and rotation. The new solutions are used to examine the impact of rotational stiffness of pile-cap on nonlinear response, and to deduce the expressions for the capacity. Finally, the critical resistance factor N_p for cohesive soil is deduced by matching the new expressions with the available normalised capacity involving the impact of gapping, translation or rotation, and *loading angles*. The N_p is then synthesised into explicit expressions to facilitate estimating the p_u .

2. ELASTIC-PLASTIC SOLUTIONS

2.1 Model for Laterally Loaded Rigid Piles

In the framework of load transfer approach for lateral piles or anchored piles, the pile-soil interaction is characterised by the p_u and the k profiles.

In the elastic zone between depths z_0 and z_1 [Fig. 1c₁ or 1c₂], the *on-pile force (per unit length)*, p [FL⁻¹] ($< p_u$) at any depth is proportional to the local displacement, u and the modulus of subgrade reaction, kd [FL⁻²] [see Fig. 1b]:

$$p = kdu \quad (\text{Elastic state}) \quad (1)$$

where k [FL⁻³] is the gradient of the p - u curve with $k = k_0 z^n$, which is obtained using an average shear modulus $\tilde{G}_s$ over the pile embedment, and the expressions provided in Table 1. Later, a Gibson k ($n = 1$) profile may be characterised by the gradient k_0 [FL⁻³⁻ⁿ]. The slip (plastic) zone of $0 \sim z_0$ (prior to the tip-yield state) or both zones of $0 \sim z_0$ and $z_1 \sim l$ (after the tip-yield state) are developed once the p attains the p_u :

$$p_u = A_r z^n d \quad (2)$$

where A_r [FL⁻²⁻ⁿ] is the gradient of the limiting p_u profile, with $A_r = N_p s_u$ and $n = 0$ for constant p_u , and $n = 1$ for Gibson p_u ; s_u is an average undrained shear strength over the pile embedment [FL⁻²]. The A_r for Gibson p_u may be estimated, for instance, by employing Hansen's plasticity solutions [23] with frictional angle and effective unit

weight of subsoil [20, 13]. The N_p is equal to 2.6~2.7 for footings with $l/d \leq 1.0$ [10] and 2~4 for slope stabilising rigid piles [24] with gapping between pile and soil. Further investigation into the N_p is conducted later.

At the transition (slip) depth z_0 from the plastic to the elastic state, the limiting force per unit length p of Eq. (1) is equal to the p_u of Eq. (2), at which the displacement threshold u^* (see Fig. 1) is deduced as

$$u^* = N_p s_u / k \text{ (Constant } p_u \text{ and } k) \quad (3a)$$

$$u^* = A_r / k_0 \text{ (Gibson } p_u \text{ and } k), \text{ or } u^* = A_r z / k \text{ (Gibson } p_u \text{ and constant } k) \quad (3b)$$

The pile-tip threshold displacement $-u^*$ is shown in Figs. 1d₁ and 1d₂. Mathematically speaking, the *on-pile force profile* is a consequence of the p_u mobilised by a linear, rigid pile (with deflection $u = \omega z + u_g$) [20, 25] rotating about a depth $z_r (= -u_g/\omega)$ to an angle ω [note ω is in radian, and u_g is a ground-level deflection, see Figs. 1d₁ and 1d₂]. More specifically speaking, the Gibson profile is characterised by $p_u = A_r z_0 d$ and $u^* (= \omega z_0 + u_g)$, or Eq. (3b)) at the slip depth z_0 , and by $p_u = -A_r z_1 d$ and $-u^* (= \omega z_1 + u_g)$ at the slip depth z_1 , respectively. The constant profile is described by $p_u = N_p s_u d$ and $u^* = N_p s_u / k$ at the depth z_0 , and by $p_u = -N_p s_u d$ and $-u^* = N_p s_u / k$ at the depth z_1 . Both p_u and u^* profiles are schematically illustrated as the dashed lines in Figs. 1d₁ and 1d₂, respectively.

In the context of load transfer model, elastic-plastic solutions were developed to capture response of a rigid pile, concerning three typical pairs of the p_u and k profiles: (1) a constant p_u and a constant k [10]; (2) a Gibson p_u and a constant k ; and (3) a Gibson p_u and a Gibson k [20]. The response is presented in non-dimensional form such as normalised capacity $\bar{H}_o = H_o / (A_r d l^{1+n})$, normalised maximum bending moments $\bar{M}_m = M_m / (A_r d l^{2+n})$, and applied bending moment (at ground level) $\bar{M}_o = M_o / (A_r d l^{2+n})$, etc. The free-head solutions are simplified next to determine the capacity H_o of anchored-piles at both the tip-yield state and the YRP state. They are also used to examine the response of load, displacement, rotation and maximum bending moment of anchored piles. Afterwards, in the same framework, elastic-plastic solutions are developed to incorporate the impact of rotational stiffness of pile-cap on the pile response.

2.2 Ultimate Capacity

Assuming a constant p_u ($n = 0$) and a constant k with depth, the normalised capacity $\bar{H}_o$ at the tip-yield state or the YRP state [10] is given respectively by:

$$\bar{H}_o = \frac{H_o}{N_p s_u dl} = \frac{-3\bar{e} + (9\bar{e}^2 + 6\bar{e} + 3)^{0.5}}{3(1 + \bar{e}) + (9\bar{e}^2 + 6\bar{e} + 3)^{0.5}} \quad (\text{Tip-yield state}) \quad (4)$$

$$\bar{H}_o = \frac{H_o}{N_p s_u dl} = 2[-\bar{e} + (\bar{e}^2 + \bar{e} + 0.5)^{0.5}] - 1 \quad (\text{YRP state}) \quad (5)$$

where $\bar{e} = e/l$, and $e = M_o/H_o$, or loading eccentricity. The normalised moment $\bar{M}_m$ at the tip-yield through the YRP state is given by

$$\bar{M}_m = \bar{M}_o + 0.5\bar{H}_o^2 \quad (6a)$$

where at tip-yield state, $\bar{M}_o = \bar{H}_o \bar{e}$ with $\bar{H}_o$ determined from Eq. (4); and at YRP state, the $\bar{M}_o$ is correlated with the $\bar{H}_o$ (depending on loading direction), e.g. for the fourth quadrant, by

$$\bar{M}_o = -\left[\frac{1}{4}(\bar{H}_o + 1)^2 - \frac{1}{2}\right] \quad (6b)$$

Stipulating a Gibson p_u ($n = 1$) and a constant k , the capacity $\bar{H}_o$ at the tip-yield state or the YRP state [20] is given respectively by

$$\bar{H}_o = \frac{H_o}{A_r dl^2} = \frac{0.5[-(3\bar{e} + 1) + (9\bar{e}^2 + 12\bar{e} + 5)^{0.5}]}{3(1 + \bar{e}) + (9\bar{e}^2 + 12\bar{e} + 5)^{0.5}} \quad (\text{Tip-yield state}) \quad (7)$$

$$\bar{H}_o = \frac{H_o}{A_r dl^2} = 0.25(\sqrt[3]{A_0} + \sqrt[3]{A_1} - \bar{e})^2 - 0.5 \quad (\text{YRP state}) \quad (8)$$

where

$$A_j = (-\bar{e}^3 + 2 + 3\bar{e}) + (-1)^j [(2 + 3\bar{e})(-2\bar{e}^3 + 2 + 3\bar{e})]^{0.5} \quad (j = 0, 1) \quad (9)$$

The normalised moment $\bar{M}_m$ at the tip-yield through the YRP state is given by

$$\bar{M}_m = \bar{M}_o + \frac{1}{3}(2\bar{H}_o)^{1.5} \quad (10a)$$

where at tip-yield state $\bar{M}_o = \bar{H}_o \bar{e}$ with $\bar{H}_o$ gained from Eq. (7), and at YRP state, the $\bar{M}_o$ is correlated with the $\bar{H}_o$, e.g. for the fourth quadrant, by

$$\bar{M}_o = -\left[\frac{2}{3}(\bar{H}_o + 0.5)^{1.5} - \frac{1}{3}\right] \quad (10b)$$

Eq. (4) or (7), and Eq. (5) or (8) allow the normalised capacity of lateral piles (with $\bar{e} \geq 0$) to be calculated for the tip-yield and the YRP states, respectively. The capacities for piles with an l/d of 4-20 were obtained using the Broms' solutions [21] underpinned by an 'around-pile gap' to a depth of $1.5d$, and were normalised by taking $N_g = 9$. All the normalised capacities are plotted in Fig. 2, which indicate that Broms' solutions are well bracketed by the current solutions of constant p_u ($n = 0$) and Gibson p_u ($n = 1$). Note both solutions neglect the longitudinal resistance along the shaft and transverse shear resistance on the pile-tip.

2.3 Capacity of Anchored Piles and Loading Depth

Guo [10] demonstrates that upon approaching YRP state, lateral piles (e^+) may rotate about a depth of $(0.5 \sim 0.707)l$, and render a normalised capacity of $\bar{H}_o = \sim 0.414$ for constant p_u ; or rotate about a depth of $(0.707 \sim 0.794)l$ with an $\bar{H}_o = \sim 0.113$ for Gibson p_u . A higher loading capacity is attained by shifting the depth of loading attachment (rotation) downwards (with $e < 0$ in anchored piles), as is demonstrated in Fig. 3 for three pairs of the p_u and k profiles, at the tip-yield and the YRP states:

- (i) At the tip-yield state, the normalised capacity $\bar{H}_o$ for either p_u profile (constant k), was calculated using Eqs. (4) and (7), respectively. It is plotted in Fig. 3 as the curves $a_c b_c$ and $b_c c_c$ (constant p_u), and as the dash line ab (Gibson p_u). The $\bar{H}_o$ for a Gibson p_u (Gibson k) was obtained using the expressions in [20] and is depicted as the dash line $a_g b_g$.
- (ii) At the YRP state, the capacities for constant p_u and Gibson p_u (regardless of the k profiles) were calculated using Eqs. (5) and (8) respectively, and are plotted as solid lines in Fig. 3. They slightly exceed the $\bar{H}_o$ at the tip-yield state at $\bar{e} = 0 \sim -0.4$ (constant p_u) or $0 \sim -0.667$ (Gibson p_u), but become indistinguishable from the latter at $\bar{e} = -0.5$ (constant p_u) or -0.667 (Gibson p_u).

Irrespective of the tip-yield or the YRP state, a maximum capacity occurs at translation mode (discussed later), with $\bar{H}_o = 1.0$ at $-\bar{e} = 0.5$ for constant p_u , and $\bar{H}_o = 0.5$ at $-\bar{e} = 0.667$ (constant k) ~ 0.725 (Gibson k) for Gibson p_u . The capacity increases by ~ 2.73 ($= 1/0.414$) times (constant p_u) and by 3.84 ($= 0.5/0.13$) times (Gibson p_u) as the attachment depth is shifted to $0.5l$ and $(0.667 \sim 0.725)l$ below the ground level, respectively. This is physically associated with the shift of the on-pile resistance along two-sides of the pile (divided by rotation depth) to one-side (translation) for a specified p_u of $N_{psu}d$ or A_rzd .

The current capacity $\bar{H}_o$ (at the tip-yield state) is compared with published results in Figs. 4a and 4b for constant p_u , and Gibson p_u , respectively. The $\bar{H}_o$ shows

- A similar increase with the normalised depth to $-\bar{e} = 0.5$ and decrease afterwards [see Fig. 4a] against the PLA analysis (with/without gapping between caisson and surrounding soil), and $\sim 15\%$ less than the SM prediction on caissons [7].
- A fair comparison, see Fig. 4a, with the measured capacities, at three typical e/l ratios, of the anchored piles (tested in centrifuge) with flanges [26].
- An excellent agreement between the current solution (Gibson p_u and Gibson k) and the SM prediction [7]; and between the current solution (Gibson p_u and constant k) and the PLA prediction [9].

The ultimate lateral-moment loading capacity of anchored piles and lateral piles is governed by the same p_u profile. Eqs. (5) and (6b) provide a concave portion of the $\bar{M}_o \sim \bar{H}_o$ curve in the fourth quadrant of Fig. 5a for a constant p_u , which overlaps the loading capacity locus for lateral piles [10]. Eqs. (8) and (10b) offer the same portion of locus in Fig. 5b for a Gibson p_u . The higher capacity $\bar{H}_o$ (e.g. $0.414 \sim 1.0$ for constant p_u) of an anchored pile is owing to rotation about a depth z_r either within its body at $-\bar{e} = 0 \sim 0.33$ [e.g. $\bar{z}_r = z_r/l = 0.791 \sim 0.833$ for $\bar{e} = -0.25$, and $0.865 \sim 1.055$ for $\bar{e} = -0.35$], or outside its body as $-\bar{e}$ approaches 0.5 [e.g. $\bar{z}_r = 5.3 \sim 83.7$ ($\bar{e} = -0.499$), and ∞ ($-\bar{e} = 0.5$), respectively]. These diverse rotation depths resemble the mechanisms revealed in the SM method [7] that are characterised by four sets of complex expressions. The normalised capacity or moment as shown in Fig. 3 are independent of the slenderness ratio l/d ,

gapping development, rotation-translation mode, and loading angle, but the normalier N_p (or A_r) is, as discussed later.

The normalised moment $\bar{M}_m$ was estimated, using Eqs. (6a) and (10a), respectively, for the tip-yield state [10], and is shown in Fig. 6. The figure indicates a limited effect of the p_u profiles on the $\bar{M}_m$ in anchored piles at $-\bar{e} = 0.25 \sim 0.49$ (and $\bar{M}_m = 0$ at $-\bar{e} = 0.5$ (constant p_u) or 0.667 (Gibson p_u)), compared to a remarkable effect on the $\bar{M}_m$ in lateral piles with $e+$.

Normalised responses of $\bar{H}_t$, $\bar{u}_g$, $\bar{\omega}$, and $\bar{M}_m$ were obtained using the free-head solutions [10] for a few typical e/l ratios in the range of $3 \sim -0.5$ ($n = 0$) or $3 \sim -0.667$ ($n = 1$), and are shown in Fig. 7. With anchored piles having a constant p_u , Fig. 7 shows that a loading at middle depth of embedment ($\bar{e} = -0.5$) causes pure translation of the pile ($\omega = 0$), zero maximum bending moment ($M_m = 0$, see Fig. 6 as well), and a limited elastic displacement $\bar{u}_g = 1$ at $\bar{H}_t = 1$ (not shown completely in the figure). A slip from the anchored pile-base may commence shortly after or even simultaneously with the incipient of the top-slip z_0 . This causes a limited impact of base-slip on the capacity, as is detected from the discrepancy between the tip-yield and the YRP states (see Fig. 3), and the small gap between the current and the previous solutions (see Fig. 4b). As for a Gibson p_u , (1) similar features to constant p_u are noted, albeit that the translation occurs at $\bar{e} = -0.667$ (constant k) ~ -0.725 (Gibson k); (2) the $\bar{M}_m$ is doubled by using $p_u = 2A_r dz$ (e.g. at tip-yield state and $\bar{e} = 0.01$, $\bar{M}_m = 0.07728$ in Fig. 7, compared to $\bar{M}_m = 0.03864$ in Fig. 6), and it occurs prior to reaching the tip-yield state. These conclusions for anchored piles, while expected, should be verified against displacement performance once available.

2.4 New Solutions for Lateral Piles with Rotating Cap

Piles are often installed in a group and cast into a pile-cap to restrain pile-head rotation. In reality, the pile-cap with a rotational stiffness k_r will rotate to an angle ω [27]. With a head restraining bending moment $M_0 = k_r \omega$, new closed-form solutions are developed herein in the context of load transfer approach. The development employs the same technology as that for free-head piles [20], and it is thus not elaborated herein. For

instance, given a *constant* k [= $p/(dz)$, from Eq. (1)], and a constant p_u [= $N_p s_u z$, Eq. (2)], Eq. (3a) and the expression of $u = \omega z + u_g$ allow the unknown rotation ω and groundline displacement u_g of a rigid pile to be determined, which in turn allow the pile response to be related to the rotational stiffness, k_r [FL/radian], in addition to the eccentricity, e [L], of the head load, and the slip depth, z_o [see Fig. 1a], as outlined below

$$\bar{H}_t = \frac{0.5[(1 + 2\bar{z}_o)(\bar{z}_o - 1)^2 + 12\bar{k}_r]}{(\bar{z}_o - 1)^2(2 + \bar{z}_o + 3\bar{e}) + 6\bar{k}_r} \quad (11)$$

$$\bar{u}_g = \frac{2 + 3(1 + \bar{z}_o^2)\bar{e} + \bar{z}_o^3 + 6\bar{k}_r}{(2 + \bar{z}_o + 3\bar{e})(1 - \bar{z}_o)^2 + 6\bar{k}_r} \quad (12)$$

$$\bar{\omega} = \frac{-3(1 + 2\bar{e})}{(2 + \bar{z}_o + 3\bar{e})(1 - \bar{z}_o)^2 + 6\bar{k}_r} \quad (13)$$

$$\bar{z}_r = \frac{-u_g}{\omega l} \quad (14)$$

$$\bar{z}_m = \bar{H}_t, \quad \bar{M}_m = \bar{H}_t(\bar{e} + 0.5\bar{z}_m) + \bar{k}_r\bar{\omega} \quad (15)$$

$$\bar{z}_*^3 + 3\bar{e}\bar{z}_*^2 - 1.5(1 + 2\bar{e})\bar{z}_* + 0.5(1 + 12\bar{k}_r) = 0 \quad (16)$$

where $\bar{H}_t = H_t/(N_g s_u dl)$, normalised pile-head load; $\bar{u}_g = u_g k/(N_g s_u)$, normalised groundline displacement; $\bar{\omega} = \omega kl/(N_g s_u)$, normalised rotation angle; $\bar{M}_m [= M_m/(N_g s_u dl^2)]$ and $\bar{M}_o [= M_o/(N_g s_u dl^2)]$, normalised maximum bending moment, and moment at ground level; $\bar{k}_r = k_r/(kl^3)$, normalised rotational stiffness of the pile-cap; $\bar{z}_o = z_o/l$, normalised slip depth; $\bar{z}_r$ and $\bar{e}$ were defined earlier. The solutions for a Gibson p_u ($= A_r dz$) with a constant k (i.e. $p = kdu$, thus $u^* = A_r z_o/k_o$), or with a Gibson k (i.e. $p = k_o dzu$, thus $u^* = A_r/k_o$) were also developed and are provided in Table 1.

Typical responses of $\bar{H}_t \sim \bar{u}_g$, and $\bar{H}_t \sim \bar{\omega}$ to the stiffness $\bar{k}_r$ ($= 0, 0.05, 0.1, 0.2, 0.4, 0.8, 1.5, 3.0$, and 10) were obtained for $n = 0$ and $n = 1$. They are plotted in Figs. 8a and 8b, respectively, which exhibit the following salient features:

- A maximum lateral load for fixed-head ($\bar{k}_r > 10$) rigid piles is mobilised upon reaching a unit normalised displacement ($\bar{u}_g = 1$, see Fig. 8a), as anticipated.
- Given a constant p_u and a constant k , the rotation can be reduced by increasing the stiffness k_r (see Eq. (13)), and at an infinitely large k_r , the fully fixed-head ($\omega = 0$) occurs. The normalised load $\bar{H}_t$ reaches a maximum of 1.0 (the normalised capacity) as the slip depth extends from ground level to the pile tip ($\bar{z}_o \rightarrow 1$).

• With a Gibson p_u and a constant k , the rotation may also be reduced by increasing stiffness k_r . A fully fixed-head would theoretically occur at a normalised slip depth of 1.732~2.0 [using $\bar{z}_o = -1.5\bar{e} + 0.5(9\bar{e}^2 + 24\bar{e} + 12)^{0.5}$]. In other words, the fixed-head is practically unattainable given $\bar{z}_o \leq 1$, as is noted for Gibson p_u and Gibson k (see Table 2). The normalised load (or capacity) $\bar{H}_t$ reaches a maximum of 0.5 (the fixed-head capacity) as the slip depth extends to the entire pile length ($\bar{z}_o \rightarrow 1$).

The z^*/l for the tip-yield state may be estimated using Eq. (16) and the expressions in Table 1. For instance, given $k_r/(kl^3) = 0.02$ for a constant p_u and a constant k , the tip-yield occurs at z^*/l of 0.49 ($\bar{e} = 0$), and 0.462 ($\bar{e} = 0.03$), respectively. At a large $\bar{k}_r$ value, the yield between pile and soil will occur almost simultaneously over the entire pile length. Theoretically speaking, fully fixed-head conditions will never occur in laterally loaded rigid piles with a constant p_u through a Gibson p_u , rotation and/or cracking may be induced instead.

More importantly, the new solutions cover the free-head ($k_r = 0$) to fixed-head cases. Taking $k_r = 0$, for instance, Eqs. (11) and (16) reduce to

$$\bar{H}_t = \frac{0.5(1 + 2\bar{z}_o)}{2 + \bar{z}_o + 3\bar{e}} \quad (11a)$$

$$\bar{z}_* = -(1.5\bar{e} + 0.5) + 0.5(3 + 6\bar{e} + 9\bar{e}^2)^{0.5} \quad (16a)$$

Substituting Eq. (16a) into Eq. (11a) allows Eq. (4) to be deduced. Likewise, with a Gibson p_u and a constant k , the expressions in Table 1 offer

$$\bar{H}_t = \frac{0.5\bar{z}_o}{2 + \bar{z}_o + 3\bar{e}} \quad (11b)$$

$$\bar{z}_* = -(1.5\bar{e} + 0.5) + 0.5(3 + 12\bar{e} + 9\bar{e}^2)^{0.5} \quad (16b)$$

Substituting Eq. (16b) into Eq. (11b) enables Eq. (7) to be gained. Furthermore, the solutions for a Gibson p_u and a constant k can be deduced using the expressions in Table 1 as well.

Conversely, the existing solutions for free-head piles with $M_o = H_e$ (Guo 2012) are used to predict the response profiles of the current semi-fixed head piles (with $\bar{k}_r > 0$) by replacing the M_o with $M_o = H_e + k_r\omega$. For instance, the $M(z)$ for a constant p_u (constant k) at the pre-tip yield state is extended to

$$\bar{M}_1(\bar{z}) = \bar{H}_t(\bar{e} + \bar{z}) + \bar{k}_r\bar{\omega} - 0.5\bar{z}^2 \quad (17a)$$

$$\bar{H}_1(\bar{z}) = \bar{H}_t - \bar{z} \quad (0 < z \leq z_o) \quad (17b)$$

$$\begin{aligned} \bar{M}_2(\bar{z}) = & \bar{H}_t(\bar{e} + \bar{z}) + \bar{k}_r\bar{\omega} - (\bar{z}_o\bar{z} - 0.5\bar{z}_o^2) \\ & - \frac{\bar{\omega}}{6}(\bar{z}^3 - 3\bar{z}\bar{z}_o^2 + 2\bar{z}_o^3) - 0.5\bar{u}_g(\bar{z} - \bar{z}_o)^2 \end{aligned} \quad (18a)$$

$$\bar{H}_2(\bar{z}) = \bar{H}_t - \bar{z}_o - 0.5\bar{\omega}(\bar{z}^2 - \bar{z}_o^2) - \bar{u}_g(\bar{z} - \bar{z}_o) \quad (z_o < z \leq l) \quad (18b)$$

Note the tip-yield state is excluded herein, as yielding may occur simultaneously over the pile length under working load. In light of Eqs. (17) and (18), typical shear force and bending moment profiles were obtained for $\bar{z}_o = 0.4$ (corresponding to a factor of safety of 2.5 before reaching ultimate $\bar{z}_o = 1$) for nine typical values of normalised rotation stiffness. They are plotted in Figs. 9a₁ and 9b₁. The same profiles for Gibson p_u (constant k) were obtained using the expressions in Table 1, which are plotted in Figs. 9a₂ and 9b₂.

3. FACTORS N_p AND $N_{p\alpha}$ FOR COHESIVE SOIL

We have established non-dimensional solutions for the capacity of anchored piles, and for lateral piles with rotating cap. The remaining question is how to estimate the normaliser N_p (e.g. in Eq. (4)) or A_r (see Table 1), as it varies from lateral piles to anchored piles, and with loading angle. A rotation (largely observed on lateral piles) may shift to translation (observed in anchored piles) under the combined lateral-moment loading, depending on pile-head and base restraints, depths of stiff layers, and loading positions. It is legitimate to integrate together the values of N_p or A_r across footing, lateral piles (caissons) and anchored piles, or even pipelines [10]. This is addressed next for cohesive soil. Note we defer the search for A_r expression, as there is scarcity of measured p_u profiles for anchored piles in drained or cohesionless soil, despite the extensive p_u profiles deduced for lateral piles [20]. To match a measured load-displacement curve of a semi-fixed head pile using free-head and fixed-head solutions, the associated N_p^{FreH} for free-head ($\bar{k}_r = 0$) solutions may be reduced by 75% (to $0.25N_p^{\text{FreH}}$) when fixed-head (say, $\bar{k}_r > 10$) solutions [12] are adopted. The real value of N_p for a typical $\bar{k}_r$ may be interpolated from the two extreme values, which is referred herein to as capping effect. The study on the N_p is thus narrowed down to free-head cases.

With the normalised capacities for free-head cases (see Table 2), the N_p values for a uniform shear strength profile were deduced against solutions for a constant p_u (although the actual p_u may be somewhere between uniform p_u and Gibson p_u [13]), involving the impact of pile-movement mode, gapping, and slenderness ratios, as is elaborated below:

- With regard to $l/d = 2 \sim 10$ and $e = 0$, the N_p was deduced as 10.14~11.6 (no gapping) and 5.92~8.33 (with gapping), respectively. (i) the no-gapping N_p was deduced using $H_o/(s_u dl) = 0.414N_p = 4.2 \sim 4.8$ (PLA on caissons), as is gained using Eq. (5) at YRP state. And (ii) As gapping (still translation movement) evolves, the N_p observes $0.414N_p = 2.45 \sim 3.5$ (PLA on caissons). The no-gapping N_p of 10.14~11.6 (also see Table 2) is well bracketed by 9.14~11.94 of the upper bound solution [28] for translating piles. The N_p is thus independent of the mobilisation of limiting force on either two-sides of a rotating pile (see Fig. 1c₂, the current solutions) or one-side of a translating pile (the upper bound approach). The gapping N_p of 5.92~8.33 agrees well with 5.6~8.6 [17] mentioned earlier, and is 40~80% the no-gapping N_p .
- As for short piles with $l/d = 0.08 \sim 1$ or footings [19], the N_p for $\bar{e} = -0.5$ (translational movement) was deduced as 1.0~6.1 using $N_p = H_o/(s_u dl)$ at the YRP state; and the N_p for $e = \infty$ (rotational movement) was gained as 2.8~9.8 on the base of $0.25N_p = 0.7 \sim 2.45 [= M_o/(s_u dl^2)]$ at YRP state, Guo 2012].
- The N_p varies with pile-movement modes, and may be estimated using Eq. (19a) and Eqs. (19b and c) for pure rotation and rotation-translation (R-T), respectively.

$$N_p = 2 + 8.9 \arctan(0.9l/d) \quad (\text{Pure rotation}) \quad (19a)$$

$$N_p = 1 + 7.5 \arctan(0.8l/d) \quad (\text{R-T mode without gapping}) \quad (19b)$$

$$N_p = 1 + 5 \arctan(\varepsilon l/d), \varepsilon = 0.7 \sim 1.0 \quad (\text{R-T mode with gapping}) \quad (19c)$$

Eq. (19a) fits well with those induced from rotation of footing under moment loading and with $N_p = 15$ from cone penetration (CPT) tests. A higher N_p than 15 [29] is likely seen for CPT tests that force rotational soil movement around the cone tip, as is evident from the analytical solutions for a cone angle of $30^\circ \sim 180^\circ$ [30]. Eq. (19b) matches closely with those deduced from footing and caissons (lateral loading) without gapping. Eq. (19c) is based on those for footing, caissons (lateral loading with gapping) and anchors. Eqs. (19a, b and c) with $\varepsilon = 0.7$ are plotted in Fig. 10.

As a comparison, the upper bound solutions [16] were obtained for a rough-surface pile in uniform and non-uniform shear strength profiles; and for a smooth surface pile in the same strength profiles. They are plotted in Fig. 10, and confirm the current variation law of the N_p with l/d ratio, despite the scatter for $l/d < 1$. The latter may be caused by replacing the embedment depth of anchors or pipelines with the pile-length l in

determining the N_p values using empirical expressions deduced numerically for vertical anchors without gapping [31] and with gapping [32] deduced from centrifuge tests for pipelines with gapping [33].

Lateral loading is pragmatically applied at an angle α (>0) (between the horizon and loading direction) on anchored piles or exerted horizontally on a group of rake piles (with a batter angle of α , see the insert of Fig. 11). Measured capacities from centrifuge tests on anchored piles [34] were normalised using that of $\alpha = 0$ case, as were the measured lateral capacities from laboratory 1-g model tests [35] on rake piles. The normalised capacities were taken as the ratios of $N_{p\alpha}/N_p$ [$N_{p\alpha} = p_u/(s_u d)$, a new version of N_p involving an inclined loading or batter angles], and are plotted in Fig. 11. The figure demonstrates a starkly similar variation in the $N_{p\alpha}/N_p$ (thus p_u) with the angle α between the anchored and battered piles, regardless of e^+ or e^- . The impact of batter angle α on lateral piles is well mimicked by factoring the p_u for ‘vertical piles’, as is revealed in analysing the centrifuge tests on pile groups [36], and in finite element modelling on single piles [37]. The new factor $N_{p\alpha}$ thus should be equally sufficiently accurate to anchored piles, as they are governed by the same limiting p_u . The $N_{p\alpha}$ is well fitted (see Fig. 11) by

$$N_{p\alpha} = N_p \exp(-0.014\alpha) \quad (20a)$$

Assuming an identical variation law to inclined anchor in clay [32], the $N_{p\alpha}$ may follow

$$N_{p\alpha} = N_{p\alpha=90^\circ} + (N_{p\alpha=0^\circ} - N_{p\alpha=90^\circ}) \left(\frac{90^\circ - \alpha}{90^\circ} \right)^2 \quad (20b)$$

For convenience, we drop the subscript ‘ α ’, and rewrite Eq. (19a), (19b) and (19c) as

$$N_p = [2 + 8.9 \arctan(0.9l/d)] \exp(-0.014\alpha) \quad (\text{Pure rotation}) \quad (21a)$$

$$N_p = [1 + 7.5 \arctan(0.8l/d)] \exp(-0.014\alpha) \quad (\text{R-T mode without gapping}) \quad (21b)$$

$$N_p = [1 + 5 \arctan(0.7l/d)] \exp(-0.014\alpha), \quad \varepsilon = 0.7 \sim 1.0 \quad (\text{R-T mode with gapping}) \quad (21c)$$

The N_p of Eq. (21a) ~ (21c) may be used in Eqs. (4) ~ (6) to calculate response of anchored piles under inclined loading, as the impact of the loading position on the $N_{p\alpha}$ is limited [8]. It may be reduced further to cater for the shadowing and capping effect to model capped piles.

Plasticity limit analysis (PLA) and finite element analyses were conducted [9] to gain lateral capacity of caissons (with length over diameter ratios of 2, 6 and 10) under various loading angle. Given $-\bar{e} = 0, 0.25, 0.5, 0.75$ and 1.0 , the normalised capacities ($= N_{p\alpha}/N_p$ for a uniform shear strength profile) are plotted in Fig. 11. The ratio for each $\bar{e}$ hardly reduces at $\alpha = 0 \sim 15^\circ$, but drops quickly at $\alpha > 75^\circ$, which is not seen in the cited test data (further experiment is required). The faster reduction in $N_{p\alpha}$ with the α may be catered for by replacing the factor -0.014α in Eq. (20) with -0.018α .

It is customary to estimate the p_u utilising N_p and undrained shear strength. As for Gibson p_u , the p_u may be obtained using plasticity solutions [23, 13] with cohesion and frictional angle of soil [38] especially for piles in layered soil. The p_u may be fitted to gain the gradient A_r , which allows the corresponding solutions to be adopted, as is noted for flexible piles in stiff clay [20]. Further study on the factor A_r for anchors, anchored piles in Gibson p_u is warranted.

4. CONCLUSIONS

This paper provides explicit expressions for estimating loading capacity of anchored piles and develops new solutions for lateral piles with rotating cap by stipulating a constant p_u or a linear increasing p_u (Gibson p_u) with depth. Lateral loading capacity H_o (at the tip-yield state and yield at rotation point state) and maximum bending moment M_m (at the tip-yield state) are presented against loading locations, and in form of the lateral capacity H_o - M_o (applied moment) locus. The capacity is consistent with available solutions for anchored piles, and caissons with either p_u profile, allowing a united approach from lateral piles to anchored piles. The new solutions are also presented in charts to highlight the impact of rotational stiffness of pile-cap on nonlinear response, offering a united approach for free-head piles through fixed-head piles.

The current solutions are underpinned by the mechanistic load-transfer model, which demonstrate the following advantages over the conventional p-y curve based approach:

- A displacement- based capacity is firmly defined as the load at the tip-yield state and/or at the yield at rotation point;
- The response of lateral piles and anchored piles is rigorously coupled by simply employing positive and negative loading eccentricity.

- The two input parameters k and p_u are well calculated using existing elastic and plastic solutions;
- In compact expressions, the solutions may be fitted to measured response of piles, allowing the parameters k and p_u to be deduced for piles in layered soil, and justified against available solutions; and finally
- A cap-stiffness is integrated into the compact expressions, allowing nonlinear response of free-head to fixed-head piles to be readily captured.

To facilitate estimating the key parameter p_u , values of the resistance factor N_p (= ratio of pile-soil limiting resistance over the undrained shear strength s_u) are deduced using the current expressions against available normalised pile capacity involving the impact of gapping (between pile and soil), pile movement mode, pile slenderness ratio, inclined loading angle (anchored piles) and batter angles (lateral piles). The N_p is characterised by:

(i) An increase from 5.6~8.6 to 10.14~11.6, as gapping is eliminated around lateral piles and caissons, and from 1.0~6.1 to 2.8~9.8, as translation is converted into rotation mode of footings. (ii) Similar variations with slenderness ratio between anchors and caissons (without gapping), and among anchors, caissons and pipelines (with gapping). And (iii) A reduction with *loading angles* (anchors) resembling that with batter angles (piles).

The use of rigid-pile solutions and a negative loading eccentricity to capture response of anchored piles is 'rigorous' and convenient, underpinned by the same parameters for lateral piles. The predicted capacity is on conservative side for $l/d < 3$ (owing to ignoring base resistance). The predicted displacement and determination of the value of A_r (for Gibson p_u) are yet to be corroborated once test data become available. It may be extended to model piles in multilayered soil, as was done previously for flexible piles.

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579 NOMENCLATURE

- 580 A_r = gradient of the limiting $[FL^{-2-n}]$;
581 $d(r_o)$ = outside diameter (radius) of a cylindrical pile (caisson) [L];
582 E_p = Young's modulus of an equivalent solid cylinder pile $[FL^{-2}]$;
583 e = eccentricity (free- length) [L];
584 e^- = depth of attachment for anchored piles;
585 e^+ = eccentricity (free- length), i.e. the height from the loading location to the
586 mudline; or $e = M_o/H_t$ for lateral piles [L];
587 $\bar{e}$ = e/l , normalised eccentricity;
588 $G_s, \tilde{G}_s$ = shear modulus of the soil, and average of the G_s $[FL^{-2}]$;
589 H_o = lateral capacity, which is the load H_t at tip-yield or YRP state [F];
590 $\bar{H}_o$ = $H_o/(A_r d l^{1+n})$, normalised capacity;
591 H_t = lateral load applied at an eccentricity of 'e' above mudline [F];
592 k = modulus of subgrade reaction $[FL^{-3}]$;
593 k_o = gradient of the modulus of subgrade reaction $[FL^{-3-n}]$, $k_o = k$ at $n = 0$;
594 k_r = rotational stiffness of a pile-cap $[FL/rad]$;
595 l = embedded pile (caisson) length [L];
596 LFP = net limiting force profile per unit length $[FL^{-1}]$;
597 M_m = M_{max} , maximum bending moment within a pile [FL];
598 $\bar{M}_m$ = $M_m/(A_r d l^{2+n})$, normalised maximum bending moment;
599 M_o = bending moment at the mudline level for free-head piles; or restraining bending
600 moment for semi- fixed-head piles [FL];
601 $\bar{M}_o$ = $M_o/(A_r d l^{2+n})$, normalised applied bending moment (at ground level);
602 n = power to the depth z , $n = 0$ for constant k or p_u , $n = 1$ for Gibson k or p_u ;
603 N_p = a resistance factor for limiting force per unit length;
604 $N_{p\alpha}$ = a resistance factor for limiting force per unit length under an inclined loading at
605 an angle of α (anchored piles), or a batter angle α (lateral piles);
606 p, p_u = force per unit length, and limiting value of the p $[FL^{-1}]$;
607 s_u = undrained shear strength $[FL^{-2}]$;
608 u = lateral displacement of a rigid pile [L];
609 u^* = local threshold u^* above which pile-soil relative slip is initiated [L];
610 u_g, u_t = lateral displacement at mudline level, and pile-head level, respectively [L];
611 YRP = yield at rotation point;
612 z = depth measured from the mudline [L];
613 z_m = depth of maximum bending moment [L];
614 $z_o(z_1)$ = slip depth initiated from mudline (pile-base) [L];
615 z_r = depth of rotation point [L];
616 $\bar{z}_r$ = z_r/l , normalised depth of rotation point;
617 z^* = slip depth at tip-yield state [L];
618 ω = rotation angle (in radian) of the pile;
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620 Superscripts '+' and '-' indicate a positive and negative value, respectively;

- 621 Superscript 'FreH' indicates free-head piles;
622 Bar '-' denotes normalised values (see Table 1)
623

Figure Captions

Fig. 1 Schematic analysis for a rigid pile [19]: (a) Pile - soil system, (b) Load transfer model, (c_i) p_u (LFP) profiles, (d_i) pile displacement features ($i = 1$ and 2 for Gibson p_u and constant p_u respectively)

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Fig. 6 Normalised maximum bending moment $\bar{M}_m$ versus normalised eccentricity (e/l) or depth of loading attachment

Fig. 7 Normalised response for typical ratios of e/l . (a) Pile-head load $\bar{H}_t$ and mudline displacement $\bar{u}_o$. (b) $\bar{H}_t$ and rotation $\bar{\omega}$. (c) $\bar{H}_t$ and maximum bending moment $\bar{M}_{max}$

Fig. 8 Response of (a) pile-head load $\bar{H}_t$ and groundline displacement $\bar{u}_g$, (b) $\bar{H}_t$ and rotation $\bar{\omega}$, ($n = 1$ for Gibson p_u & constant k , and $n = 0$ for constant p_u and constant k)

Fig. 9 Normalised profiles of $\bar{H}(z)$ and $\bar{M}(z)$ for typical normalised k_r at $z_o/l = 0.4$ (constant k): (a₁)-(b₁) constant p_u ; (a₂)-(b₂) Gibson p_u

Fig. 10 Variation of N_p with gapping and movement (rotation, translation) modes

Fig. 11 Reduction of N_p with loading angle

647 Table 1 Solutions for a rigid pile at pre-tip and tip-yield states

Gibson p_u ($n=1$) and constant k ($n=0$)		Gibson p_u and Gibson k ($n=1$)
$u = \omega z + u_g$, $z_r/l = -u_g/(\omega l)$, $kd = \frac{3\pi\tilde{G}}{2} \{2\gamma_b \frac{K_1(\gamma_b)}{K_0(\gamma_b)} - \gamma_b^2 [\left(\frac{K_1(\gamma_b)}{K_0(\gamma_b)}\right)^2 - 1]\}$		
where $K_i(\gamma_b)$ ($i=0, 1$) is modified Bessel functions of the second kind, and of order i . $\gamma_b = k_1(r_o/l)$; $k_1 = 2.14 + \bar{e}/(0.2 + 0.6\bar{e})$, increases from 2.14 at $e=0$ to 3.8 at $e=\infty$.		
$p = kdu$, $p_u = A_r dz$, $u^* = A_r z_o/k$		$p = k_o dz u$, $p_u = A_r dz$, $u^* = A_r/k$
$\bar{H}_t = \frac{0.5\bar{z}_o[(\bar{z}_o-1)^2 + 6\bar{k}_r(2-\bar{z}_o)]}{(\bar{z}_o-1)^2(2+\bar{z}_o+3\bar{e}) + 6\bar{k}_r}$		$\bar{H}_t = \frac{1}{6} \frac{(1+2\bar{z}_o+3\bar{z}_o^2)(\bar{z}_o-1)^2 + 36\bar{k}_r}{[(2+\bar{z}_o)(2\bar{e}+\bar{z}_o)+3](\bar{z}_o-1)^2 + 12\bar{k}_r}$
$\bar{u}_g = \frac{[2+3\bar{e}+6\bar{k}_r]\bar{z}_o}{(2+\bar{z}_o+3\bar{e})(1-\bar{z}_o)^2 + 6\bar{k}_r}$		$\bar{u}_g = \frac{3+2[2+\bar{z}_o^3]\bar{e} + \bar{z}_o^4 + 12\bar{k}_r}{[(2+\bar{z}_o)(2\bar{e}+\bar{z}_o)+3](1-\bar{z}_o)^2 + 12\bar{k}_r}$
$\bar{\omega} = \frac{\bar{z}_o[\bar{z}_o^2 - 3 - 3(2+\bar{z}_o)\bar{e}]}{(2+\bar{z}_o+3\bar{e})(1-\bar{z}_o)^2 + 6\bar{k}_r}$		$\bar{\omega} = \frac{-2(2+3\bar{e})}{[(2+\bar{z}_o)(2\bar{e}+\bar{z}_o)+3](1-\bar{z}_o)^2 + 12\bar{k}_r}$
$\bar{z}_m = \sqrt{2\bar{H}_t}$		$\bar{z}_m = \sqrt{2\bar{H}_t}$ ($z_m \leq z_o$)
$\bar{M}_m = \bar{H}_t(\bar{e} + 2\bar{z}_m/3) + \bar{k}_r\bar{\omega}$ ($z_m \leq z_o$)		$\bar{M}_m = \bar{H}_t(\bar{e} + 2\bar{z}_m/3) + \bar{k}_r\bar{\omega}$ ($z_m \leq z_o$)
At tip-yield state		
$\bar{z}_*^3 + 3\bar{e}\bar{z}_*^2 + [3\bar{k}_r - (2+4.5\bar{e})]\bar{z}_* + (1+1.5\bar{e}+3\bar{k}_r) = 0$		$(\bar{z}_*-1)[\bar{z}_*^3 + (2\bar{e}+1)(\bar{z}_*^2 + \bar{z}_*) - (1+\bar{e})] + 12\bar{k}_r = 0$
$\bar{H}_t = H_t/(A_r dl^2)$, $\bar{u}_g = u_g k/A_r l$, $\bar{\omega} = \omega k/A_r d$, $\bar{M}_m = M_m/(A_r dl^3)$, and $\bar{k}_r = k_r/(kl^3)$		$\bar{H}_t = H_t/(A_r dl^2)$, $\bar{u}_g = u_g k_o/A_r$, $\bar{\omega} = \omega k_o l/A_r$, $\bar{M}_m = M_m/(A_r dl^3)$, $\bar{M}_o = M_o/(A_r dl^3)$, and $\bar{k}_r = k_r/(kl^4)$
Note: $\bar{z}_o = z_o/l$, $\bar{z}_r = z_r/l = -\bar{u}_g/\bar{\omega}$, $\bar{z}_m = z_m/l$, and $\bar{e} = e/l$. The solutions for free-head piles (Guo 2012) are used to predict response profiles by replacing $M_o = H_e$ with the counterparts of $M_o = H_e + k_r \omega$. e.g, the $M(z)$ for Gibson p_u and constant k is as follows:		
$\bar{M}_1(\bar{z}) = \bar{H}_t(\bar{e} + \bar{z}) + \bar{k}_r\bar{\omega} - \frac{1}{6}\bar{z}^3$ ($0 < z \leq z_o$)		
$\bar{M}_2(\bar{z}) = \bar{H}_t(\bar{e} + \bar{z}) + \bar{k}_r\bar{\omega} - (0.5\bar{z}\bar{z}_o^2 - \frac{\bar{z}_o^3}{3}) - \frac{\bar{\omega}}{6}(\bar{z}^3 - 3\bar{z}\bar{z}_o^2 + 2\bar{z}_o^3) - 0.5\bar{u}_g(\bar{z} - \bar{z}_o)^2$		
($z_o < z \leq l$, $z_1 \approx l$)		

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650 Table 2 Normalised capacities and N_p values deduced using Constant p_u & constant k

l/d	0.08	0.2	0.5	1.0	2.0	4.0	6.0	8.0	10
$H_o/(N_p s_u dl)$	1.0	2.0	3.6	6.1	4.2	4.5	4.75	4.8	4.8
N_p	1.0	2.0	3.6	6.1	10.14	10.87	11.47	11.59	11.59
References	Yun and Bransby [19], translation				Aubeny et al [17], rotation no gapping				
l/d	0.08	0.2	0.5	1.0	2.0	4.0	6.0	8.0	10
$M_o/(N_p s_u dl^2)^a$	0.7 ^a	0.925 ^a	1.3 ^a	2.45 ^a	2.45 ^b	2.8 ^b	3.1 ^b	3.3 ^b	3.45 ^b
$H_o/(N_p s_u dl)^b$									
N_p	2.8	3.7	5.2	9.8	5.92	6.76	7.49	7.97	8.33
References	Yun and Bransby [19], rotation				Aubeny et al [17], rotation with gapping				

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N.B. H_t = lateral load; e = free-length; l = embedded length; u_t = pile-head displacement; ω = angle of rotation (in radian); z = depth from mudline; z_o , and z_r = depths of slip, and rotation point.

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(a)

(b)

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