1998

Some necessary and sufficient conditions for two variable orthogonal designs in order 44

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Abstract

We give a new algorithm which allows us to construct new sets of sequences with entries from the commuting variables 0, ±a, ±b, ±c, ±d with zero autocorrelation function. We show that for twelve cases if the designs exist they cannot be constracted using four circulant matrices in the Goethals-Seidel array. Further we show that the necessary conditions for the existence of an OD(44;s_1,s_2) are sufficient except possibly for the following 7 cases. (7,32) (8,31) (9,30) (9,33) (11,30) (13,29) (15,26) which could not be found because of the large size of the search space for a complete search. These cases remain open. In all we find 398 cases, show 67 do not exist and establish 12 cases cannot be constructed using four circulant matrices. We give a new construction for OD(2n) and OD(n+1) from OD(n). The full OD(44;s_1,s_2,s_3,44-s_1-s_2-s_3) given in this paper yield at least 68 equivalence classes of Hadamard matrices.

Disciplines

Physical Sciences and Mathematics

Publication Details

Koukouvinos C, Mitrouli M and Seberry J, Some necessary and sufficient conditions for two variable orthogonal designs in order 44, Journal of Combinatorial Mathematics and Combinatorial Computing, 28, (1998), 267-287.

Necessary and sufficient conditions for some two variable orthogonal designs in order 44

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Dedicated to Professor Anne Penfold Street

Abstract

We give a new algorithm which allows us to construct new sets of sequences with entries from the commuting variables $0,\ \pm a,\ \pm b,\ \pm c,\ \pm d$ with zero autocorrelation function.

We show that for twelve cases if the designs exist they cannot be constracted using four circulant matrices in the Goethals-Seidel array.

Further we show that the necessary conditions for the existence of an $OD(44; s_1, s_2)$ are sufficient except possibly for the following 7 cases:

$$(7,32)\quad (8,31)\quad (9,30)\quad (9,33)\quad (11,30)\quad (13,29)\quad (15,26)$$

which could not be found because of the large size of the search space for a complete search. These cases remain open. In all we find 398 cases, show 67 do not exist and establish 12 cases cannot be constructed using four circulant matrices.

We give a new construction for $OD(2n)$ and $OD(n+1)$ from $OD(n)$.

The full $OD(44; s_1, s_2, s_3, 44 - s_1 - s_2 - s_3)$ given in this paper yield at least 68 equivalence classes of Hadamard matrices.

Key words and phrases: Autocorrelation, construction, sequence, orthogonal design.

AMS Subject Classification: Primary 05B15, 05B20, Secondary 62K05.

1 Introduction

Throughout this paper we will use the definition and notation of Koukouvinos, Mitrouli, Seberry and Karabelas [2].

We note from [3] that we have to test $\frac{1}{4}n^2 = 484$ cases. We find 398 cases, show 67 do not exist and establish 12 cases cannot be constructed using four circulant matrices. There are 7 open cases which could not be found because of the large size of the search space for a complete search.

2 New orthogonal designs

Theorem 1 *An $OD(44; s_1, s_2)$ cannot exist for the following 2−tuples (s_1, s_2):*

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(1, 7)	(3, 5)	(4, 23)	(6, 10)	(8, 14)	(10, 24)	(12, 20)	(15, 20)
(1, 15)	(3, 13)	(4, 28)	(6, 26)	(8, 30)	(11, 13)	(12, 21)	(15, 25)
(1, 23)	(3, 20)	(4, 31)	(7, 9)	(9, 15)	(11, 16)	(12, 29)	(16, 19)
(1, 28)	(3, 21)	(4, 39)	(7, 16)	(9, 23)	(11, 20)	(13, 19)	(16, 23)
(1, 31)	(3, 29)	(5, 11)	(7, 17)	(9, 28)	(11, 21)	(13, 27)	(16, 28)
(1, 39)	(3, 37)	(5, 12)	(7, 25)	(9, 31)	(11, 29)	(14, 18)	(17, 23)
(1, 42)	(3, 40)	(5, 19)	(7, 28)	(10, 17)	(12, 13)	(15, 16)	(19, 20)
(2, 14)	(4, 7)	(5, 27)	(7, 33)	(10, 22)	(12, 15)	(15, 17)	(19, 21)
(2, 30)	(4, 15)	(5, 35)	(7, 36)				

Proof. These cases are eliminated by the number theoretic necessary conditions given in [1] or [2, Lemma 3].

Example. To illustrate how we used the number theoretic conditions to establish the non-existence of an $OD(4n; 11, 20)$ we consider the product $11 \times 20 = 4^1 \times 55$ now this is a number of the form $4^a(8b + 7)$ which cannot be written as the sum of three squares and hence an $OD(4n; 11, 20)$ cannot exist.

Remark. A computer search, which we believe was exhaustive, was carried out which leads us to believe that

1. there are no 4-$NPAF(7, 19)$ sequences of length 7.
2. there are no 4-$NPAF(3, 31)$, 4-$NPAF(5, 30)$, 4-$NPAF(6, 29)$ and 4-$NPAF(8, 27)$ sequences of length 9. This means that there are also no 4-$NPAF(1, 5, 30)$, 4-$NPAF(1, 6, 29)$ and 4-$NPAF(1, 8, 27)$ of length 9.
3. there are no 4-$NPAF(2, 41)$ sequences of length 11. This means that there are also no 4-$NPAF(1, 2, 41)$ sequences of length 11.
4. there are no 4-$NPAF(6, 37)$ sequences of length 11.

Lemma 1 *$OD(44; 1, 1, 42)$ and an $OD(44; 1, 3, 40)$ do not exist (this is proved theoretically). The Geramita-Verner Theorem says that if an $OD(44; 3, 40)$ exists then an $OD(44; 1, 3, 40)$ will exist, and if an $OD(44; 1, 42)$ exists then an $OD(44; 1, 1, 42)$ will exist. Hence the $OD(44; 1, 42)$ and $OD(44; 3, 40)$ do not exist.*

Lemma 2 *The following $OD(44; 1, a, 43 - a)$ and $OD(44; a, 43 - a)$ cannot be constructed using four circulant matrices in the Goethals-Seidel array:*

(6, 37)	(10, 33)	(12, 31)	(13, 30)	(14, 29)	(16, 27)	(19, 24)	(20, 23)
(1, 6, 37)	(1, 10, 33)	(1, 12, 31)	(1, 13, 30)	(1, 14, 29)	(1, 16, 27)	(1, 19, 24)	(1, 20, 23)

Proof. By the Geramita-Verner theorem if an orthogonal design $OD(n; x_1, x_2, \cdots, x_{u-1}, x_u)$ with $\Sigma_{i=1}^{u}\ x_i = n - 1$ exists, $n \equiv 0(mod\ 4)$, then an $OD(n; 1, x_1, x_2, \cdots, x_{u-1}, x_u)$ exists.

Now for each of the cases in this lemma we have an $OD(44; a, 43 - a)$ and that is by the Geramita-Verner theorem an $OD(44; 1, a, 43 - a)$.

Using the sum-fill matrix method we write $1 = 1^2 + 0^2 + 0^2 + 0^2$, $a = a_1^2 + a_2^2 + a_3^2 + a_4^2$ and $43 - a = b_1^2 + b_2^2 + b_3^2 + b_4^2$. We require the sum-fill matrix to be a 3×4 orthogonal matrix with the first row containing 1, 0, 0, and 0, the second row containing $\pm a_1$, $\pm a_2$, $\pm a_3$, and $\pm a_4$, in some order and the third row containing $\pm b_1$, $\pm b_2$, $\pm b_3$, and $\pm b_4$, in some order.

However, as we illustrate for $OD(1,20,23)$, this is not possible for the cases mentioned in the enunciation. Using the sum-fill matrix method for $OD(1,20,23)$, $1 = 1^2+0^2+0^2+0^2$, $20 = 4^2+2^2+0^2+0^2$ and $23 = (-1)^2+2^2+3^2+3^2$. There is no way to form an orthogonal matrix unless both 20 and 23 can be written as the sum of ≤ 3 squares. □

Theorem 2 *There are $OD(44; s_1, s_2, s_3, 44-s_1-s_2-s_3)$ constructed using four sequences to obtain four circulant matrices for use in the Goethals-Seidel array for the following $2-$tuples:*

1, 43	2, 2, 20, 20	7, 37	15, 29
1, 2, 41	2, 6, 12, 16	1, 9, 34	1, 17, 26
2, 2, 4, 36	2, 8, 16, 16	1, 11, 32	1, 18, 25
2, 2, 8, 32	5, 39	13, 31	21, 23

Corollary 1 *By suitably choosing the variables of the known $OD(44; s_1, s_2, s_3, 44-s_1-s_2-s_3)$ to be replaced by ± 1 these lead to at least 36 algebraically inequivalent Hadamard matrices of order 44. By suitably choosing the variables of the known $OD(44; s_1, 44-s_1)$ to be replaced by ± 1 these lead to at least 12 more algebraically inequivalent Hadamard matrices of order 44.*

Corollary 2 *By suitably choosing the variables of the known $OD(44; 1, s_1, 35-s_1)$ to be replaced by ± 1 we obtain at least 20 algebraically inequivalent skew-Hadamard matrices of order 44. The number depends on whether each skew-Hadamard matrix is equivalent to its transpose or not.*

3 New Algorithm

The algorithm previously used to find OD via four sequences of length $t \leq 10$ was prohibitively slow for length 11. Hence we tried a new algorithm, which depended on the previous algorithm, to find first a $W(4t,k)$ made with four sequences of length t with $PAF = 0$ or $NPAF = 0$. In the new algorithm $MATLAB$ was used to set up a series of equations to be solved for each individual k and then all solutions to these equations were found.

Example. We illustrate the algorithm by trying to construct the $OD(44; 11, 27)$. We first notice that 11 has a unique decomposition into squares $11 = 3^2+1^2+1^2+0^2$, while 27 has three decompositions into four squares. All three can be used in this construction as they must be able to be used in an integer matrix (the sum-fill matrix) which is orthogonal. Hence we use $27 = 3^2+3^2+3^2+0^2 = 4^2+1^2+1^2+3^2 = 5^2+1^2+1^2+0^2$. So we have the matrices

$$\begin{bmatrix} 3 & 1 & 1 & 0 \\ 0 & 1 & -1 & 5 \end{bmatrix}, \quad \begin{bmatrix} 3 & 1 & 1 & 0 \\ -1 & 4 & -1 & 3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 3 & 1 & 1 & 0 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

We now fill each of the positions which are represented by 0 by one of 17 variables $x_1,\ x_2, \cdots,\ x_{17}$. We now use $MATLAB$ to expand the first rows to make four circulant 11×11 matrices with row inner product zero: this corresponds to forming four sequences with $PAF = 0$. The equations will be those that involve some x_j, $1 \leq j \leq 17$ with a, and those which have no terms in a.

This gives at most 6 independent equations. A search is now made through the 17 variables, allowing them to assume the values 0, ± 1, where six of them must always be zero, and using the extra constraints that

$$\sum_{i=1}^{3} x_i = -1, \quad \sum_{i=4}^{5} x_i = -1, \quad \sum_{i=6}^{11} x_i = 3, \quad \sum_{i=12}^{17} x_i = 0.$$

We start with the following four sequences of length 11 and PAF = 0.

```
1 1 1 - 1 1 - 1 0 0 0
1 1 1 - - - 1 - - 0 0
0 0 0 0 1 1 - 1 - 0 0
1 1 1 - 0 0 0 0 1 0 0
```

We replace the ± 1 by a variable such as a and we replace the 17 zeros by the variables. Thus we have the sequences

$$\begin{array}{ccccccccccc} a & a & a & -a & a & a & -a & a & x_1 & x_2 & x_3 \\ a & a & a & -a & -a & -a & a & -a & -a & x_4 & x_5 \\ x_6 & x_7 & x_8 & x_9 & a & a & -a & a & -a & x_{10} & x_{11} \\ a & a & a & -a & x_{12} & x_{13} & x_{14} & x_{15} & a & x_{16} & x_{17} \end{array}$$

We then use MATLAB to set up a series of equations, that when solved, yield, among others, the following solution:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}
0	$-b$	0	0	$-b$	b	b	$-b$	b	0	b	b	$-b$	$-b$	0	0	b

We now replace the variables in the original four sequences by these solutions to obtain the $OD(44; 11, 27)$. □

Remark. Using this algorithm we tested all unknown two variable cases and found 7 cases which we were unable to resolve due to the extremely large search space. We estimate that a complete search for the $OD(44; 7, 32)$ using this algorithm requires 2^{37} operations. □

4 New Results

Theorem 3 *Write* $X(a,b) = \{e_1x_1, e_2x_2, \cdots, e_{n-1}x_{n-1}, e_nx_n\}$, $Y(a,b) = \{f_1y_1, f_2y_2, \cdots, f_{n-1}y_{n-1}, f_ny_n\}$ *for the sequences of length n, NPAF = 0, where* e_i *and* f_i *are chosen from* a, b *where a, b are commuting variables and* x_i, y_i *have elements* $0, \ \pm 1$ *and the sequences* $X(\pm 1, \pm 1)$ *and* $Y(\pm 1, \pm 1)$ *have NPAF = 0. Suppose a occurs a total of* s_1 *times and b a total of* s_2 *times then we say the two sequences we have are* $2\text{-}NPAF(n; s_1, s_2)$.

Write $\alpha_i = b$ *if* $e_i = a$ *and* $\alpha_i = a$ *if* $e_i = b$ *for i = 1,...,n, and similarly,* $\beta_i = b$ *if* $f_i = a$ *and* $\beta_i = a$ *if* $f_i = b$ *for i = 1,...,n.*

Then (i)

$$X(a,b), Y^*(b,a) \text{ and } Y(a,b), X^*(-b,-a)$$

where Z^* *denotes the reverse of the sequence Z or*

$$\{e_1x_1, e_2x_2, \cdots, e_{n-1}x_{n-1}, e_nx_n, \beta_ny_n, \beta_{n-1}y_{n-1}, \cdots, \beta_2y_2, \beta_1y_1\}$$

and

$$\{f_1y_1, f_2y_2, \cdots, f_{n-1}y_{n-1}, f_ny_n, -\alpha_nx_n, -\alpha_{n-1}x_{n-1}, \cdots, -\alpha_2x_2, -\alpha_1x_1\}$$

are two sequences with elements $\{0, \ \pm a, \ \pm b\}$ *with NPAF = 0. These sequences are* $2\text{-}NPAF(2n; 2s_1, 2s_2)$.

(ii) If x_{n-1} and y_{n-1} are both zero then the sequences

$$\{e_1x_1, e_2x_2, \cdots, \beta_n y_n, e_n x_n, \beta_{n-2}y_{n-2}, \cdots, \beta_2 y_2, \beta_1 y_1\}$$

and

$$\{f_1y_1, f_2y_2, \cdots, -\alpha_n x_n, f_n y_n, -\alpha_{n-2}x_{n-2}, \cdots, -\alpha_2 x_2, -\alpha_1 x_1\}$$

are two sequences with elements $\{0, \ \pm a, \ \pm b\}$ with NPAF = 0. These sequences are 2-$NPAF(2n-2; 2s_1, 2s_2)$.

(iii) Similarly with 4-$NPAF(n; s_1, s_2)$, $X(a,b)$, $Y(a,b)$, $Z(a,b)$ and $W(a,b)$ we have

$$X(a,b), Y^*(b,a), \quad Y(a,b), X^*(-b,-a), \quad Z(a,b), W^*(b,a) \ \text{ and } \ W(a,b), Z^*(-b,-a)$$

where Z^* denotes the reverse of the sequence Z are 4-$NPAF(2n; 2s_1, 2s_2)$.

(iv) Similarly with 4-$NPAF(n; s_1, s_2)$, if the second last element of each of the four sequences is zero then proceeding as in (ii) we obtain 4-$NPAF(2n-2; 2s_1, 2s_2)$.

(v) Similarly if there are 4-$NPAF(n; s_1, s_2)$, and the second last element of two of the sequences is zero and the last element of two of the sequences is zero then combining the methods of (ii) and (iii) we can get 4-$NPAF(2n-2; 2s_1, 2s_2)$.

Proof. The proof follows by writing out the sequences and checking the NPAF.

Example. We use $\bar{a}$ to mean $-a$ and $\bar{c}$ to mean $-c$. To illustrate part (v) of the theorem we note that

$$\begin{array}{ccccccc} c & a & \bar{c} & c & 0 & \bar{c} & 0 \\ c & a & \bar{c} & \bar{c} & 0 & c & 0 \\ c & 0 & c & 0 & \bar{c} & 0 & \bar{c} \\ c & 0 & c & 0 & c & 0 & c \end{array} \quad \text{and} \quad \begin{array}{ccccccc} c & a & \bar{c} & c & a & \bar{c} & 0 \\ c & a & \bar{c} & \bar{c} & \bar{a} & c & 0 \\ c & 0 & c & 0 & \bar{c} & 0 & \bar{c} \\ c & 0 & c & 0 & c & 0 & c \end{array}$$

are 4-$NPAF(7; 2, 16)$ and 4-$NPAF(7; 4, 16)$, respectively.

In fact we note

$$\begin{array}{ccccccc} c & a & \bar{c} & c & 0 & \bar{c} & 0 \\ c & a & \bar{c} & \bar{c} & 0 & c & 0 \\ c & 0 & c & b & \bar{c} & 0 & \bar{c} \\ c & 0 & c & 0 & c & 0 & c \end{array} \quad \text{and} \quad \begin{array}{ccccccc} c & a & \bar{c} & c & a & \bar{c} & 0 \\ c & a & \bar{c} & \bar{c} & \bar{a} & c & 0 \\ c & 0 & c & b & \bar{c} & 0 & \bar{c} \\ c & 0 & c & 0 & c & 0 & c \end{array}$$

are 4-$NPAF(7; 1, 2, 16)$ and 4-$NPAF(7; 1, 4, 16)$, respectively.

We also note that

$$\begin{array}{ccccccccccc} c & a & \bar{c} & c & c & \bar{c} & c & 0 & c & 0 & c \\ c & a & \bar{c} & c & \bar{c} & \bar{c} & \bar{c} & 0 & \bar{c} & 0 & \bar{c} \\ c & a & \bar{c} & \bar{c} & c & c & c & b & \bar{c} & 0 & \bar{c} \\ c & a & \bar{c} & \bar{c} & \bar{c} & c & \bar{c} & \bar{b} & c & 0 & c \end{array} \quad \text{and} \quad \begin{array}{cccccccccccc} c & a & \bar{c} & c & a & \bar{c} & c & a & \bar{c} & \bar{c} & \bar{a} & c \\ c & a & \bar{c} & c & a & \bar{c} & \bar{c} & \bar{a} & c & c & a & \bar{c} \\ c & 0 & c & 0 & \bar{c} & c & \bar{c} & c & b & \bar{c} & 0 & \bar{c} \\ c & 0 & c & 0 & \bar{c} & \bar{c} & \bar{c} & \bar{c} & \bar{b} & c & 0 & c \end{array}$$

are 4-$NPAF(11; 2, 4, 32)$ and 4-$NPAF(12; 2, 8, 32)$, respectively.

$$\begin{array}{ccccccccccc} c & a & \bar{c} & c & c & \bar{c} & c & 0 & c & 0 & c \\ c & a & \bar{c} & c & \bar{c} & \bar{c} & \bar{c} & 0 & \bar{c} & 0 & \bar{c} \\ c & a & \bar{c} & \bar{c} & c & c & c & 0 & \bar{c} & 0 & \bar{c} \\ c & a & \bar{c} & \bar{c} & \bar{c} & c & \bar{c} & 0 & c & 0 & c \end{array} \quad \text{and} \quad \begin{array}{cccccccccccc} c & a & \bar{c} & c & a & \bar{c} & c & a & \bar{c} & \bar{c} & \bar{a} & c \\ c & a & \bar{c} & c & a & \bar{c} & \bar{c} & \bar{a} & c & c & a & \bar{c} \\ c & c & c & c & \bar{c} & c & \bar{c} & c & & & & \\ c & \bar{c} & c & \bar{c} & \bar{c} & \bar{c} & \bar{c} & \bar{c} & & & & \end{array}$$

are 4-$NPAF(11;4,32)$ and 4-$NPAF(12;8,32)$, respectively.

Lemma 3 *If there exist* 2-$NPAF(n;s_1,s_2)$ *then there exist* 4-$NPAF(n+1;2,2,2s_1,2s_2)$.

Corollary 3 *Since there exist* 2-$NPAF(n;s_1,s_2)$ *for the values listed in the table we get the corresponding larger* 4-$NPAF(n+1;2,2,2s_1,2s_2)$.

2-$NPAF(n;s_1,s_2)$	$\Rightarrow$	4-$NPAF(n+1;2,2,2s_1,2s_2)$
(9;13)		(10;2,2,26)
(11;13)		(12;2,2,26)
(14;17)		(15;2,2,34)
(18;25)		(19;2,2,50)
(4;4,4)		(5;2,2,8,8)
(6;2,8)		(7;2,2,4,16)
(6;5,5)		(7;2,2,10,10)
(8;8,8)		(9;2,2,16,16)
(10;10,10)		(11;2,2,20,20)
(14;13,13)		(15;2,2,26,26)

Corollary 4 *Using the previous theorem we see that*

4-$NPAF(n;s_1,s_2)$	$\Rightarrow$	4-$NPAF(2n;2s_1,2s_2)$
NPAF(5;1,18)		NPAF(10;2,36)
NPAF(5;1,19)		NPAF(10;2,38)
NPAF(5;2,17)		NPAF(10;4,34)
NPAF(5;2,18)		NPAF(10;4,36)
NPAF(5;3,17)		NPAF(10;6,34)
NPAF(7;3,18)		NPAF(14;6,36)
NPAF(5;4,16)		NPAF(10;8,32)
NPAF(7;4,17)		NPAF(14;8,34)
NPAF(7;4,18)		NPAF(14;8,36)
NPAF(5;5,14)		NPAF(10;10,28)
NPAF(5;5,15)		NPAF(10;10,30)
NPAF(7;5,16)		NPAF(14;10,32)
NPAF(7;5,17)		NPAF(14;10,34)
NPAF(7;5,18)		NPAF(14;10,36)
NPAF(5;6,14)		NPAF(10;12,28)
NPAF(7;6,16)		NPAF(14;12,32)
NPAF(7;7,14)		NPAF(14;14,28)
NPAF(7;7,15)		NPAF(14;14,30)
NPAF(5;8,11)		NPAF(10;16,22)
NPAF(5;8,12)		NPAF(10;16,24)
NPAF(5;9,10)		NPAF(10;18,20)
NPAF(5;9,11)		NPAF(10;18,22)
NPAF(7;9,12)		NPAF(14;18,24)

Theorem 4 *The sequences given in the Appendices can be used to construct the appropriate designs to establish that the necessary conditions for the existence of an* $OD(44;s_1,s_2)$ *are sufficient, except possibly for the following* 12 *cases which cannot be constructed from four circulant matrices:*

$$
\begin{array}{cccccc}
(5,38) & (6,37) & (8,35) & (10,33) & (12,31) & (13,30) \\
(14,29) & (15,28) & (16,27) & (19,24) & (20,23) & (21,22).
\end{array}
$$

and the following 7 cases which are undecided:

$$
(7,32) \quad (8,31) \quad (9,30) \quad (9,33) \quad (11,30) \quad (13,29) \quad (15,26)
$$

Remark. There are 484 possible 2−tuples. Table 1 lists the 398 which correspond to designs which exist in order 44: 67 2-tuples correspond to designs eliminated by number theory (NE). For 12 cases, if the designs exist, they cannot be constructed using circulant matrices (Y). 7 cases remain undecided.

P indicates that 4-PAF sequences with length 11 exist; n indicates 4-$NPAF$ sequences with length n exist.

s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n
1	1	1	2	10	3	3	21	NE	4	34	10	6	15	7	7	34	P
1	2	1	2	11	5	3	22	7	4	35	11	6	16	7	7	35	P
1	3	1	2	12	5	3	23	7	4	36	10, 11	6	17	7	7	36	NE
1	4	2	2	13	5	3	24	7	4	37	P	6	18	7	7	37	11
1	5	2	2	14	NE	3	25	7	4	38	11	6	19	7	8	8	5
1	6	3	2	15	5	3	26	9	4	39	NE	6	20	7	8	9	5
1	7	NE	2	16	5	3	27	9	4	40	11	6	21	7	8	10	5
1	8	3	2	17	5	3	28	9	5	5	3	6	22	7	8	11	5
1	9	3	2	18	5	3	29	NE	5	6	3	6	23	9	8	12	5
1	10	3	2	19	7	3	30	9	5	7	3	6	24	8	8	13	7
1	11	3	2	20	6	3	31	10	5	8	5	6	25	9	8	14	NE
1	12	4	2	21	7	3	32	9	5	9	5	6	26	NE	8	15	7
1	13	5	2	22	7	3	33	9	5	10	5	6	27	9	8	16	7
1	14	5	2	23	7	3	34	10	5	11	NE	6	28	9	8	17	7
1	15	NE	2	24	7	3	35	11	5	12	NE	6	29	P	8	18	7
1	16	5	2	25	9	3	36	11	5	13	5	6	30	9	8	19	9
1	17	5	2	26	7	3	37	NE	5	14	5	6	31	10	8	20	7
1	18	5	2	27	9	3	38	11	5	15	5	6	32	10	8	21	9
1	19	5	2	28	8	3	39	11	5	16	7	6	33	$P, 20$	8	22	8
1	20	7	2	29	9	3	40	NE	5	17	7	6	34	10	8	23	9
1	21	7	2	30	NE	3	41	11	5	18	7	6	35	P	8	24	9
1	22	7	2	31	9	4	4	2	5	19	NE	6	36	11	8	25	9
1	23	NE	2	32	9	4	5	3	5	20	7	6	37	Y	8	26	9
1	24	7	2	33	9	4	6	3	5	21	7	6	38	11	8	27	P
1	25	7	2	34	9	4	7	NE	5	22	9	7	7	4	8	28	9
1	26	9	2	35	10	4	8	3	5	23	7	7	8	6	8	29	P
1	27	7	2	36	10, 11	4	9	5	5	24	9	7	9	NE	8	30	NE
1	28	NE	2	37	11	4	10	5	5	25	9	7	10	5	8	31	
1	29	9	2	38	10, 11	4	11	5	5	26	9	7	11	7	8	32	10
1	30	11	2	39	11	4	12	5	5	27	NE	7	12	7	8	33	P
1	31	NE	2	40	11	4	13	5	5	28	9	7	13	5	8	34	11
1	32	9	2	41	P	4	14	5	5	29	9	7	14	7	8	35	Y
1	33	9	2	42	11	4	15	NE	5	30	10	7	15	7	8	36	11
1	34	11	3	3	2	4	16	5	5	31	9	7	16	NE	9	9	5
1	35	11	3	4	3	4	17	7	5	32	10	7	17	NE	9	10	5
1	36	11	3	5	NE	4	18	7	5	33	10	7	18	7	9	11	5
1	37	11	3	6	3	4	19	7	5	34	P	7	19	8	9	12	7
1	38	11	3	7	3	4	20	7	5	35	NE	7	20	9	9	13	6
1	39	NE	3	8	3	4	21	7	5	36	11	7	21	7	9	14	7
1	40	11	3	9	3	4	22	7	5	37	P	7	22	9	9	15	NE
1	41	11	3	10	5	4	23	NE	5	38	Y	7	23	9	9	16	7
1	42	NE	3	11	5	4	24	7	5	39	11	7	24	9	9	17	7
1	43	11	3	12	5	4	25	9	6	6	3	7	25	NE	9	18	7
2	2	1	3	13	NE	4	26	8	6	7	5	7	26	9	9	19	7
2	3	2	3	14	5	4	27	9	6	8	5	7	27	9	9	20	9
2	4	2	3	15	5	4	28	NE	6	9	5	7	28	NE	9	21	9
2	5	3	3	16	7	4	29	9	6	10	NE	7	29	9	9	22	9
2	6	2	3	17	5	4	30	9	6	11	5	7	30	P	9	23	NE
2	7	3	3	18	7	4	31	NE	6	12	5	7	31	10	9	24	9
2	8	3	3	19	7	4	32	9	6	13	7	7	32		9	25	9
2	9	5	3	20	NE	4	33	10	6	14	5	7	33	NE	9	26	9

Table 1: The existence of $OD(44; s_1, s_2)$.

s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n
9	27	9	10	31	P	12	15	NE	13	25	P	15	21	9	17	25	P
9	28	NE	10	32	11	12	16	7	13	26	P	15	22	P	17	26	P
9	29	P	10	33	Y	12	17	9	13	27	NE	15	23	P	17	27	P
9	30	20	10	34	11	12	18	8	13	28	P	15	24	P	18	18	9
9	31	NE	11	11	6	12	19	9	13	29		15	25	NE	18	19	P
9	32	$P, 15$	11	12	7	12	20	NE	13	30	Y	15	26		18	20	10
9	33	20	11	13	NE	12	21	NE	13	31	P	15	27	$P, 20$	18	21	P
9	34	P	11	14	7	12	22	9	14	14	7	15	28	Y	18	22	10
9	35	P	11	15	7	12	23	NE	14	15	P	15	29	P	18	23	P
10	10	5	11	16	NE	12	24	9	14	16	8	16	16	8	18	24	11
10	11	7	11	17	7	12	25	P	14	17	P	16	17	9	18	25	P
10	12	7	11	18	9	12	26	P	14	18	NE	16	18	9	18	26	P
10	13	7	11	19	9	12	27	20	14	19	9	16	19	NE	19	19	P
10	14	7	11	20	NE	12	28	10	14	20	9	16	20	9	19	20	NE
10	15	7	11	21	NE	12	29	NE	14	21	9	16	21	11	19	21	NE
10	16	7	11	22	9	12	30	$P, 13$	14	22	9	16	22	10	19	22	P
10	17	NE	11	23	9	12	31	Y	14	23	P	16	23	NE	19	23	P
10	18	7	11	24	9	12	32	11	14	24	P	16	24	10	19	24	Y
10	19	P	11	25	9	13	13	7	14	25	P	16	25	P	19	25	P
10	20	8	11	26	P	13	14	9	14	26	10	16	26	11	20	20	10
10	21	9	11	27	P	13	15	7	14	27	P	16	27	Y	20	21	P
10	22	NE	11	28	P	13	16	10	14	28	$P, 12$	16	28	NE	20	22	11
10	23	9	11	29	NE	13	17	9	14	29	Y	17	17	9	20	23	Y
10	24	NE	11	30		13	18	9	14	30	P	17	18	9	20	24	11
10	25	9	11	31	P	13	19	NE	15	15	9	17	19	9	21	21	11
10	26	9	11	32	P	13	20	9	15	16	NE	17	20	11	21	22	Y
10	27	P	11	33	P	13	21	9	15	17	NE	17	21	P	21	23	P
10	28	10	12	12	7	13	22	P	15	18	9	17	22	P	22	22	11
10	29	P	12	13	NE	13	23	9	15	19	9	17	23	NE			
10	30	10	12	14	7	13	24	P	15	20	NE	17	24	P			

Table 1(Cont): The existence of $OD(44; s_1, s_2)$.

References

[1] A.V.Geramita, and J.Seberry, *Orthogonal designs: Quadratic forms and Hadamard matrices*, Marcel Dekker, New York-Basel, 1979.

[2] C.Koukouvinos, M.Mitrouli, J.Seberry, and P.Karabelas, On sufficient conditions for some orthogonal designs and sequences with zero autocorrelation function, *Australas. J. Combin.*, 13, (1996), 197-216.

[3] C.Koukouvinos and Jennifer Seberry, New orthogonal designs and sequences with two and three variables in order 28, *Ars Combinatoria*, (to appear).

Appendix A: Order 40 (Sequences with zero non-periodic autocorrelation function)

Design	A_1, A_2										A_3, A_4									
(1, 4, 10, 10)	*notinyet*																			
(1, 4, 32)	b	$-b$	$-b$	$-b$	a	a	b	b	b	$-b$	b	b	$-b$	b	c	$-b$	b	$-b$	$-b$	0
	b	$-b$	$-b$	$-b$	a	$-a$	$-b$	$-b$	$-b$	b	b	b	$-b$	b	0	b	$-b$	b	b	0
(2, 2, 18, 18)	d	c	a	$-c$	$-d$	c	$-c$	c	c	c	c	$-d$	b	d	$-c$	d	$-d$	d	d	d
	d	c	a	$-c$	$-d$	$-c$	c	$-c$	$-c$	$-c$	c	$-d$	b	d	$-c$	$-d$	d	$-d$	$-d$	$-d$
(2, 2, 34)	c	c	c	$-c$	$-c$	c	$-c$	$-c$	a	c	c	c	c	0	c	$-c$	c	$-c$	b	c
	c	c	c	$-c$	$-c$	c	$-c$	c	$-a$	$-c$	c	c	c	0	c	$-c$	c	c	$-b$	$-c$
(2, 4, 32)	b	b	$-b$	b	a	a	$-b$	$-b$	b	$-b$	b	$-b$	b	b	0	c	b	$-b$	b	b
	b	b	b	$-b$	a	$-a$	$-b$	$-b$	$-b$	b	b	b	b	$-b$	0	$-c$	b	b	b	$-b$
(2, 4, 16, 18)	d	c	a	$-c$	$-d$	b	c	$-d$	c	d	d	c	d	$-c$	d	b	$-c$	d	$-c$	$-d$
	d	c	a	$-c$	$-d$	$-b$	$-c$	d	$-c$	$-d$	d	c	d	$-c$	d	$-b$	c	$-d$	c	d
(2, 10, 10, 18)	d	$-c$	a	c	$-d$	b	d	$-c$	$-d$	$-c$	b	$-d$	b	d	c	d	b	d	$-b$	d
	d	$-c$	a	c	$-d$	$-b$	$-d$	c	d	c	b	$-d$	b	d	c	$-d$	$-b$	$-d$	b	$-d$
(2, 12, 22)	b	c	c	0	c	$-c$	b	a	$-b$	c	b	c	$-c$	0	$-c$	$-c$	b	c	b	$-c$
	b	c	c	0	c	c	$-b$	$-a$	b	$-c$	b	c	$-c$	0	$-c$	c	$-b$	$-c$	$-b$	c
(2, 35)	b	b	a	$-b$	$-b$	b	$-b$	b	b	b	b	b	$-b$	b	0	b	$-b$	b	b	0
	b	b	b	$-b$	b	b	b	$-a$	$-b$	$-b$	b	b	$-b$	b	b	$-b$	b	$-b$	$-b$	0
(3, 31)	b	0	$-b$	$-b$	$-b$	0	$-a$	$-b$	b	b	b	b	$-b$	0	a	$-b$	b	$-b$	$-b$	$-b$
	b	b	$-b$	b	$-b$	$-b$	a	b	b	$-b$	b	0	b	0	b	0	b	$-b$	b	b
(3, 34)	b	b	a	$-b$	$-b$	b	$-b$	b	b	b	b	b	$-b$	b	a	$-b$	b	$-b$	$-b$	0
	b	b	b	$-b$	b	b	b	$-a$	$-b$	$-b$	b	b	$-b$	b	0	b	$-b$	b	b	0

Appendix A(cont): Order 40 (Sequences with zero non-periodic autocorrelation function)

Design	A_1, A_2									
$(4,8,8,16)$	*notinyet*									
$(4,4,16,16)$	a	d	c	d	$-c$	a	$-d$	$-c$	$-d$	c
	a	d	c	d	$-c$	$-a$	d	c	d	$-c$
$(4,6,12,18)$	b	a	$-d$	c	d	d	a	$-b$	$-c$	$-d$
	b	a	$-d$	c	d	$-d$	$-a$	b	c	d
$(4,8,8,16)$	d	c	a	$-c$	d	d	b	$-a$	$-b$	d
	d	c	a	$-c$	d	$-d$	$-b$	a	b	$-d$
$(4,10,10,16)$	b	c	a	$-c$	b	b	d	$-a$	$-d$	b
	b	c	a	$-c$	b	$-b$	$-d$	a	d	$-b$
$(5,30)$	b	$-b$	$-b$	0	a	a	b	$-b$	b	$-b$
	b	0	b	$-b$	b	b	a	$-b$	$-b$	b
$(5,33)$	b	b	b	a	a	$-b$	$-b$	b	$-b$	b
	b	b	b	$-b$	$-a$	b	b	$-b$	b	0
$(6,31)$	b	$-b$	b	$-a$	$-a$	b	$-b$	$-b$	b	b
	b	b	b	b	0	$-b$	b	a	b	0
$(7,31)$	b	b	b	0	$-a$	$-b$	$-b$	b	$-a$	$-b$
	b	b	b	b	$-a$	b	a	$-b$	a	b
$(8,8,10,10)$	a	c	0	$-c$	a	a	c	d	c	$-a$
	a	c	0	$-c$	a	$-a$	$-c$	$-d$	$-c$	a
$(10,10,10,10)$	a	b	b	d	$-d$	$-b$	a	a	c	$-c$
	a	b	b	d	$-d$	b	$-a$	$-a$	$-c$	c

A_3, A_4									
b	c	$-d$	c	d	b	$-c$	d	$-c$	$-d$
b	c	$-d$	c	d	$-b$	c	$-d$	c	d
d	c	$-d$	c	$-b$	d	$-c$	d	c	d
d	c	$-d$	c	$-b$	$-d$	c	$-d$	$-c$	$-d$
d	b	0	b	$-d$	d	c	0	c	$-d$
d	b	0	b	$-d$	$-d$	$-c$	0	$-c$	d
b	c	d	c	$-b$	b	d	$-c$	d	$-b$
b	c	d	c	$-b$	$-b$	$-d$	c	$-d$	b
b	b	b	b	0	a	$-a$	b	b	$-b$
b	$-b$	$-b$	0	$-b$	$-b$	b	b	0	b
b	b	b	$-b$	$-b$	b	b	$-b$	b	0
b	b	b	a	$-a$	b	$-b$	b	$-b$	$-b$
b	b	$-b$	b	a	b	$-b$	$-b$	$-b$	$-b$
b	b	$-b$	$-b$	b	$-b$	a	$-a$	b	0
b	$-b$	$-b$	0	a	b	$-b$	b	$-b$	$-b$
b	b	$-b$	b	$-b$	b	$-a$	$-b$	$-b$	b
b	d	0	$-d$	b	b	d	$-c$	d	$-b$
b	d	0	$-d$	b	$-b$	$-d$	c	$-d$	b
d	c	c	$-a$	a	$-c$	d	d	$-b$	b
d	c	c	$-a$	a	c	$-d$	$-d$	b	$-b$

Appendix B: Order 44 (Sequences with zero non-periodic autocorrelation function)

Design						A_1,	A_2										A_3,	A_4				
(1, 4, 16, 16)	c	d	0	d	−c	a	c	−d	0	−d	−c	c	−d	b	d	c	0	c	d	−b	−d	c
	c	d	0	d	−c	0	−c	d	0	d	c	c	−d	b	d	c	0	−c	−d	b	d	−c
(1, 30)	b	b	0	a	0	−b	−b	0	0	0	0	b	−b	−b	0	−b	b	−b	b	b	−b	−b
	b	0	0	b	b	b	0	b	0	0	0	b	b	−b	−b	b	−b	b	−b	−b	−b	b
(1, 34)	b	−b	b	b	b	a	−b	−b	−b	b	−b	b	b	0	b	0	0	b	b	−b	0	−b
	b	b	b	0	0	0	−b	b	0	0	b	b	−b	b	b	b	−b	−b	b	b	−b	b
(1, 35)	b	−b	b	b	b	a	−b	−b	−b	b	−b	b	b	b	b	−b	b	0	−b	b	0	−b
	b	0	b	0	0	0	0	0	b	b	b	b	−b	b	−b	−b	−b	b	b	−b	−b	b
(1, 37)	b	−b	b	b	b	a	−b	−b	−b	b	−b	b	b	0	−b	b	b	−b	b	−b	−b	0
	b	b	0	−b	b	b	0	b	b	b	0	b	−b	b	b	0	−b	−b	−b	b	−b	b
(1, 38)	b	−b	b	b	b	a	−b	−b	−b	b	−b	b	−b	b	0	−b	−b	−b	0	b	−b	b
	b	b	b	0	b	0	b	0	−b	b	b	b	b	−b	−b	b	b	−b	b	−b	−b	−b
(1, 40)	b	−b	b	b	b	a	−b	−b	−b	b	−b	b	b	0	b	b	−b	−b	b	b	−b	−b
	b	b	b	b	b	0	−b	b	−b	b	b	b	−b	0	−b	−b	b	b	−b	b	−b	b
(1, 41)	b	−b	b	b	b	a	−b	−b	−b	b	−b	b	−b	b	b	−b	−b	b	b	0	−b	b
	b	b	0	−b	b	−b	b	b	b	b	b	b	b	b	b	−b	−b	b	−b	b	−b	−b
(2, 2, 4, 36)	b	a	a	a	−a	a	a	−a	d	a	−a	b	−a	−a	−a	a	−a	−a	−a	−c	a	a
	b	a	a	a	−a	a	−a	a	−d	−a	a	b	−a	−a	−a	a	−a	a	a	c	−a	−a
(2, 2, 8, 32)	a	d	c	−d	d	−d	d	−d	−d	c	d	b	d	c	−d	d	d	d	d	d	−c	−d
	a	−d	−c	d	−d	d	−d	d	d	−c	−d	b	−d	−c	d	−d	−d	−d	−d	−d	c	d
(2, 2, 20, 20)	−a	a	a	a	b	a	b	−b	−b	b	c	b	−b	−b	−b	a	−b	a	−a	−a	a	d
	−a	a	a	a	b	a	b	−b	−b	b	−c	b	−b	−b	−b	a	−b	a	−a	−a	a	−d
(2, 6, 12, 16)	d	0	b	0	d	−c	d	c	a	−c	−d	d	0	−d	0	b	−c	b	c	−d	c	d
	d	0	b	0	d	−c	−d	−c	−a	c	d	d	0	−d	0	b	−c	−b	−c	d	−c	−d

Appendix B(cont): Order 44 (Sequences with zero non-periodic autocorrelation function)

Design	A_1, A_2											A_3, A_4										
(2, 8, 16, 16)	b	d	c	−c	−d	b	d	−c	0	−c	d	b	−d	−c	−c	−d	−b	−d	c	a	−c	d
	b	d	c	−c	−d	b	−d	c	0	c	−d	b	−d	−c	−c	−d	−b	d	−c	−a	c	−d
(2, 37)	b	b	b	0	a	0	−b	−b	b	−b	b	b	b	b	0	b	0	−b	b	b	b	−b
	b	−b	b	−b	b	b	−a	−b	−b	b	b	b	b	b	b	−b	0	b	−b	−b	b	−b
(2, 39)	b	b	−b	−b	0	−b	a	b	−b	b	b	b	b	b	b	b	0	b	−b	b	−b	b
	b	b	b	−b	0	b	−a	−b	−b	b	−b	b	b	−b	−b	−b	b	b	−b	b	−b	−b
(3, 35)	b	0	b	b	0	a	−b	b	b	−b	0	b	b	b	b	b	−a	−b	−b	b	−b	b
	b	0	b	−b	0	a	b	−b	−b	b	0	b	b	b	b	−b	−b	b	b	−b	b	−b
(3, 36)	b	b	−b	b	0	−a	b	b	b	−b	0	b	−b	−b	−b	−b	0	b	−b	−b	b	b
	b	b	b	b	0	a	−b	b	−b	b	0	b	−b	−b	b	b	a	b	−b	b	−b	−b
(3, 38)	b	b	−b	−b	b	a	0	b	b	b	−b	b	−b	−b	−b	−b	0	a	−b	−b	b	−b
	b	−b	−b	−b	−a	b	−b	b	−b	−b	b	b	b	b	−b	b	−b	0	−b	−b	−b	b
(3, 39)	b	−b	b	b	b	a	−b	−b	−b	b	b	b	−b	−b	0	−b	−b	−b	a	b	−b	b
	b	b	−b	a	b	b	−b	0	−b	b	−b	b	−b	b	b	−b	b	b	b	b	b	−b
(3, 41)	b	b	b	b	b	a	−b	−b	b	−b	b	b	a	−b	b	−b	b	−b	−b	b	b	−b
	b	−b	−b	b	−b	−b	−b	−b	−b	a	b	b	b	b	−b	−b	b	b	b	−b	b	−b
(4, 35)	b	0	a	−b	a	b	−b	b	b	−b	−b	b	−b	b	b	b	0	b	0	b	−b	−b
	b	0	a	0	−a	−b	b	b	b	b	b	b	−b	−b	−b	b	b	−b	b	−b	b	b
(5, 36)	b	−b	−b	b	a	0	a	b	b	b	−b	b	−b	b	b	b	a	−a	b	b	b	−b
	b	−b	b	−b	a	−a	−a	b	b	b	b	b	b	b	b	−b	−b	b	b	−b	b	−b
(5, 39)	b	b	b	a	a	−b	−b	−b	b	−b	b	b	−b	b	b	b	a	−a	b	b	b	−b
	b	−b	b	b	−b	b	b	−a	−b	−b	b	b	b	b	b	−b	−b	b	b	−b	b	−b
(7, 37)	b	b	b	b	−a	−b	−a	−b	−b	b	−b	b	−b	−b	−b	a	b	−b	−b	−b	b	−b
	b	−b	b	b	−a	a	a	b	b	−b	b	b	−b	b	b	b	−b	a	−b	−b	b	b

Appendix C: Order 44 (Sequences with zero periodic autocorrelation function)

Design						A_1	A_2												A_3	A_4			
(1, 9, 34)	b	$-b$	$-b$	b	b	c	$-b$	$-b$	b	b	$-b$	b	b	b	b	b	b	b	$-b$	b	$-b$	$-b$
	a	$-a$	$-a$	$-a$	b	b	$-b$	b	$-b$	$-a$	$-b$	a	$-a$	b	$-b$	$-a$	$-b$	$-b$	$-b$	b	a	$-b$
(1, 11, 32)	b	$-b$	b	b	b	c	$-b$	$-b$	$-b$	b	$-b$	a	b	b	$-b$	$-b$	b	$-b$	b	b	b	b
	a	a	$-b$	a	b	$-b$	b	b	$-b$	$-b$	b	a	a	$-a$	$-a$	b	b	$-a$	a	$-a$	b	b
(1, 17, 26)	a	a	a	$-a$	a	c	$-a$	a	$-a$	$-a$	$-a$	b	b	b	b	b	$-b$	b	b	$-b$	a	$-b$
	a	a	b	a	b	$-b$	$-b$	a	$-b$	b	$-b$	b	b	b	a	b	$-b$	$-b$	b	b	$-b$	$-a$
(1, 18, 25)	a	a	a	$-a$	a	c	$-a$	a	$-a$	$-a$	$-a$	a	a	b	$-b$	b	a	$-b$	$-b$	$-b$	b	b
	a	a	$-b$	a	$-b$	$-b$	b	b	$-b$	b	b	a	$-a$	$-b$	$-b$	$-b$	b	$-b$	b	$-b$	$-b$	$-b$
(2, 12, 27)	b	b	$-b$	0	b	$-b$	$-b$	$-a$	0	0	$-a$	b	b	$-b$	b	b	b	b	a	c	0	$-a$
	b	b	$-b$	b	$-b$	b	$-b$	$-a$	$-a$	$-a$	a	b	b	$-b$	b	b	b	b	a	$-c$	0	$-a$
(2, 41)	a	$-b$	$-b$	b	$-b$	b	b	b	b	b	b	b	b	b	$-b$	b	b	b	$-b$	b	$-b$	$-b$
	a	b	b	$-b$	$-b$	$-b$	$-b$	b	$-b$	$-b$	$-b$	b	$-b$	$-b$	b	$-b$	b	$-b$	b	b	$-b$	0
(4, 37)	a	a	$-b$	$-b$	b	$-b$	b	$-b$	b	b	0	b	b	b	$-b$	b	$-b$	$-b$	b	$-b$	$-b$	$-b$
	a	$-a$	$-b$	$-b$	$-b$	$-b$	$-b$	$-b$	$-b$	b	0	b	b	$-b$	b	$-b$	$-b$	b	b	$-b$	$-b$	0
(5, 34)	a	a	$-a$	$-b$	$-b$	b	b	b	b	b	$-b$	$-b$	b	b	0	b	$-b$	b	b	b	b	0
	a	$-b$	a	b	$-b$	$-b$	$-b$	b	$-b$	b	b	$-b$	0	b	$-b$	b	$-b$	b	b	0	b	0
(5, 37)	a	$-b$	$-b$	$-b$	b	b	$-b$	b	b	a	0	b	$-b$	b	b	b	b	b	$-b$	b	b	0
	b	b	b	b	$-b$	$-b$	$-b$	$-a$	a	a	$-b$	b	b	$-b$	b	$-b$	b	$-b$	b	b	$-b$	$-b$
(6, 29)	b	0	0	b	b	0	b	$-a$	0	0	0	b	$-b$	b	b	b	b	$-b$	$-b$	b	a	$-a$
	b	$-b$	b	$-b$	$-b$	$-b$	b	b	0	0	a	b	0	b	b	b	$-b$	$-b$	b	$-b$	a	a
(6, 33)	b	b	$-b$	b	b	b	$-a$	$-b$	$-b$	0	0	b	$-b$	b	b	b	b	b	$-b$	b	$-a$	a
	b	b	a	$-b$	$-b$	b	$-b$	b	b	0	0	b	$-b$	b	b	0	$-b$	$-b$	b	$-b$	$-a$	$-a$
(6, 35)	a	a	b	b	$-b$	b	$-b$	$-b$	$-b$	$-b$	$-b$	a	$-b$	$-b$	b	b	$-b$	b	b	$-b$	b	0
	a	$-b$	b	b	b	$-b$	b	b	b	b	0	b	$-b$	b	$-b$	$-a$	a	b	b	$-b$	$-b$	0
(7, 30)	b	b	b	b	0	0	$-b$	b	b	$-a$	0	b	b	$-b$	b	a	0	$-b$	b	$-b$	a	$-a$
	b	b	b	$-a$	$-b$	$-b$	b	$-b$	$-b$	0	0	b	b	$-b$	0	$-b$	b	b	$-b$	b	a	a

Appendix C(cont): Order 44 (Sequences with zero periodic autocorrelation function)

Design						A_1	A_2												A_3	A_4			
$(7,34)$	a	a	b	b	$-b$	$-b$	b	$-b$	$-b$	$-b$	0	a	$-b$	b	$-b$	b	$-b$	$-b$	b	b	$-b$	$-b$
	b	a	$-a$	b	b	$-b$	b	a	b	b	0	a	$-b$	$-b$	$-b$	b	b	b	b	$-b$	b	0
$(7,35)$	a	a	b	b	$-b$	b	$-b$	$-b$	$-b$	$-b$	$-b$	a	$-b$	$-b$	b	b	b	b	$-b$	$-b$	b	0
	a	b	b	$-b$	b	b	$-b$	b	b	b	0	a	$-a$	$-b$	b	b	$-b$	a	$-b$	b	$-b$	b
$(8,27)$	b	b	$-b$	0	b	$-b$	$-b$	0	0	0	0	b	b	$-b$	$-b$	$-b$	$-b$	b	$-a$	0	0	$-a$
	b	b	$-b$	b	$-b$	b	$-b$	$-a$	$-a$	$-a$	a	b	b	$-b$	b	b	b	b	0	a	$-a$	0
$(8,29)$	b	$-b$	b	b	0	$-b$	$-b$	b	$-b$	$-a$	$-a$	b	$-b$	b	b	b	b	b	$-b$	b	$-a$	a
	b	b	0	$-a$	$-b$	$-a$	b	$-b$	$-b$	0	0	b	b	0	$-a$	$-b$	a	$-b$	b	b	0	0
$(8,33)$	a	a	$-b$	$-b$	b	b	$-b$	b	b	b	0	b	$-a$	b	a	$-a$	b	$-b$	b	b	a	0
	b	$-b$	$-b$	a	b	a	b	$-b$	$-b$	$-b$	0	b	b	b	b	b	$-b$	b	$-b$	b	$-b$	$-b$
$(9,32)$	a	a	$-b$	b	b	$-b$	b	b	b	b	0	a	b	$-b$	$-a$	a	b	$-a$	$-b$	$-a$	b	$-b$
	a	$-b$	b	$-b$	$-b$	$-b$	$-b$	$-b$	a	b	0	b	b	b	$-b$	$-b$	$-b$	b	$-b$	b	$-b$	0
$(10,31)$	a	a	b	$-b$	b	b	$-b$	$-b$	$-b$	$-b$	0	b	$-b$	b	a	b	b	b	b	b	$-b$	0
	a	b	$-b$	b	$-b$	a	$-b$	b	b	$-b$	$-b$	a	a	b	$-a$	a	$-a$	b	b	$-b$	$-b$	0
$(11,27)$	b	b	b	$-b$	b	b	$-b$	b	0	$-a$	0	a	a	$-a$	a	b	b	$-b$	b	$-b$	0	a
	b	b	b	$-b$	$-b$	$-b$	b	$-b$	$-b$	0	$-a$	b	b	b	$-b$	a	$-a$	$-a$	0	b	0	a
$(11,28)$	b	b	b	0	0	$-b$	b	b	$-a$	$-a$	a	b	$-b$	b	$-b$	a	b	$-b$	$-b$	$-b$	$-a$	a
	b	b	b	$-a$	a	$-b$	b	$-b$	a	a	a	b	$-b$	$-b$	b	0	$-b$	b	b	b	0	0
$(11,31)$	b	$-b$	b	$-a$	0	$-b$	$-b$	b	b	a	a	b	$-b$	b	$-b$	a	b	$-b$	$-b$	$-b$	a	a
	b	$-b$	$-b$	a	$-a$	$-b$	b	b	b	$-a$	a	b	$-b$	$-b$	$-b$	$-b$	$-b$	b	$-b$	$-b$	0	$-a$
$(12,25)$	b	b	$-b$	b	b	$-b$	$-b$	$-a$	0	0	a	b	b	b	$-b$	b	$-b$	$-a$	a	0	$-a$	$-a$
	a	0	a	a	b	b	b	b	$-b$	b	$-a$	b	b	$-b$	0	$-b$	b	b	$-a$	0	0	$-a$
$(12,26)$	b	b	b	$-b$	b	b	$-b$	$-a$	$-a$	$-a$	a	b	$-b$	b	b	b	0	b	a	0	0	a
	b	b	b	$-b$	$-b$	$-b$	b	$-a$	$-a$	a	$-a$	b	$-b$	b	b	$-b$	0	$-b$	a	0	0	$-a$
$(12,30)$	b	b	b	b	$-b$	$-b$	b	0	$-b$	$-a$	$-a$	b	b	$-b$	a	$-a$	b	$-b$	b	$-b$	$-a$	$-a$
	b	$-b$	b	a	a	b	$-b$	b	b	$-a$	a	b	b	b	b	b	$-b$	$-b$	0	b	$-a$	a

Appendix C(cont): Order 44 (Sequences with zero periodic autocorrelation function)

Design						A_1	A_2												A_3	A_4			
(13, 22)	a	$-b$	$-b$	b	0	$-b$	$-b$	$-a$	$-b$	0	a		a	0	$-b$	a	$-b$	b	$-a$	$-b$	a	b	0
	a	0	b	a	b	b	0	b	$-b$	$-b$	0		a	$-a$	$-b$	0	$-b$	b	$-a$	b	$-a$	$-b$	0
(13, 24)	a	0	0	b	$-b$	b	b	0	$-a$	$-a$	$-a$		b	b	b	$-b$	b	b	$-b$	b	a	a	0
	b	b	b	$-b$	a	$-a$	$-a$	a	$-a$	0	$-a$		b	b	b	$-b$	$-b$	$-b$	b	$-b$	$-a$	0	0
(13, 25)	b	0	$-a$	$-a$	b	$-a$	$-b$	$-b$	$-a$	0	a		b	a	b	b	$-a$	b	0	b	$-a$	0	a
	b	b	$-b$	$-b$	$-b$	0	b	$-b$	b	a	$-a$		b	b	$-b$	b	$-b$	$-b$	$-a$	b	$-b$	0	$-a$
(13, 26)	b	b	b	$-b$	b	b	$-b$	$-a$	0	$-a$	0		b	$-b$	b	b	b	0	b	a	a	a	$-a$
	b	b	b	$-b$	$-b$	$-b$	b	$-a$	a	$-a$	$-a$		b	$-b$	b	b	$-b$	0	$-b$	0	$-a$	$-a$	a
(13, 28)	a	a	$-b$	$-b$	b	b	b	$-b$	b	b	0		b	$-a$	$-b$	$-a$	b	b	b	$-b$	b	$-b$	0
	a	$-b$	$-b$	$-b$	b	$-b$	a	b	$-b$	b	0		a	a	$-a$	a	$-a$	$-a$	$-b$	$-b$	$-b$	$-a$	$-b$
(13, 31)	a	a	a	b	$-b$	b	b	$-b$	$-b$	$-b$	$-b$		b	b	b	$-b$	b	b	b	$-b$	b	$-b$	$-b$
	a	b	b	b	$-b$	a	$-b$	b	b	$-b$	b		a	a	$-a$	a	$-a$	$-a$	$-b$	a	$-a$	$-b$	$-b$
(14, 15)	a	a	a	b	$-b$	0	0	0	0	$-b$	0		a	$-b$	0	0	b	0	0	0	$-b$	0	0
	a	$-b$	b	b	b	a	0	0	b	$-b$	0		a	a	$-a$	a	$-a$	$-a$	$-b$	a	$-a$	$-b$	$-b$
(14, 17)	a	a	a	$-b$	$-b$	0	0	0	0	0	0		b	a	$-b$	$-b$	b	$-b$	0	$-b$	b	b	0
	a	0	0	b	b	a	b	$-b$	0	b	0		b	$-a$	$-a$	a	$-a$	a	a	b	$-a$	a	0
(14, 23)	a	a	0	a	b	b	$-b$	$-b$	b	$-b$	$-b$		a	a	$-a$	a	0	$-b$	b	0	b	b	b
	a	$-a$	0	$-a$	b	b	0	b	$-b$	b	0		a	a	$-a$	$-a$	b	b	$-b$	0	$-b$	b	b
(14, 24)	b	0	b	$-a$	$-b$	$-a$	b	b	$-b$	a	$-a$		b	$-a$	b	a	b	a	$-b$	$-b$	b	0	$-a$
	b	0	b	a	$-b$	0	$-b$	$-b$	b	a	a		b	$-a$	b	a	b	0	b	b	$-b$	0	a
(14, 25)	b	$-a$	b	b	0	a	b	$-b$	$-b$	$-a$	a		b	b	$-b$	$-b$	$-b$	b	b	$-b$	b	$-a$	$-a$
	a	b	0	0	$-b$	b	$-b$	b	b	a	a		b	b	a	b	$-a$	b	0	0	$-a$	a	$-a$
(14, 28)	b	$-b$	b	$-b$	$-b$	$-b$	b	a	0	a	a		b	$-b$	$-b$	b	b	b	b	$-a$	a	a	a
	b	$-b$	$-b$	b	$-b$	$-b$	$-b$	a	0	a	$-a$		b	$-b$	$-b$	$-b$	$-b$	b	$-b$	$-a$	a	a	$-a$

Appendix C(cont): Order 44 (Sequences with zero periodic autocorrelation function)

Design						A_1	A_2												A_3	A_4			
(14, 30)	a	a	a	b	b	$-b$	$-b$	b	$-b$	$-b$	$-b$		a	$-b$	b	b	b	b	b	$-b$	b	$-b$	b
	a	$-b$	b	$-b$	b	a	$-b$	$-b$	b	b	b		a	a	$-a$	a	$-a$	$-a$	$-b$	a	$-a$	$-b$	$-b$
(15, 22)	b	b	$-b$	$-a$	$-b$	0	$-a$	0	0	a	$-a$		b	$-b$	$-b$	b	b	b	b	$-a$	a	a	0
	b	$-b$	b	a	b	$-a$	$-a$	$-a$	$-a$	a	$-a$		b	b	b	$-b$	b	b	$-b$	0	a	0	0
(15, 23)	a	$-a$	a	a	0	$-b$	b	0	b	b	b		a	0	a	$-a$	$-b$	$-b$	b	b	$-b$	b	b
	a	a	$-a$	$-a$	b	b	$-b$	a	$-b$	b	b		a	a	0	a	$-b$	$-b$	0	$-b$	b	$-b$	0
(15, 24)	b	b	b	$-b$	b	b	$-b$	b	a	0	a		a	a	a	$-a$	$-b$	$-b$	b	$-b$	0	a	0
	b	b	b	$-b$	$-b$	$-b$	b	$-b$	a	$-a$	$-a$		b	b	b	$-b$	0	$-a$	0	$-a$	a	a	$-a$
(15, 27)	b	b	b	$-b$	$-a$	a	$-a$	$-a$	b	0	$-a$		b	b	b	$-b$	b	b	$-b$	b	$-b$	a	a
	a	$-a$	$-a$	0	b	b	$-b$	b	b	a	a		b	b	b	$-b$	$-b$	$-b$	b	$-b$	a	$-a$	a
(15, 29)	a	a	a	$-b$	$-b$	$-b$	b	$-b$	b	b	b		a	$-a$	b	$-b$	$-b$	$-b$	b	$-b$	$-a$	$-b$	$-b$
	a	a	b	$-b$	$-b$	b	$-b$	$-b$	b	$-b$	$-b$		a	$-a$	a	$-b$	$-a$	a	b	$-a$	$-a$	$-b$	$-b$
(17, 21)	b	$-a$	$-b$	b	b	$-a$	$-b$	$-a$	0	$-a$	0		b	b	b	$-b$	b	$-b$	a	0	a	$-a$	$-a$
	a	$-b$	b	$-b$	$-b$	$-b$	$-b$	$-a$	0	$-a$	0		b	$-a$	$-b$	$-a$	$-b$	a	b	$-a$	a	a	0
(17, 22)	a	$-a$	b	$-a$	b	b	0	$-b$	b	b	$-a$		a	0	b	$-a$	b	$-b$	$-a$	b	0	$-b$	a
	a	a	b	0	b	b	a	b	$-b$	$-b$	0		a	a	b	a	b	$-b$	$-a$	$-b$	a	b	$-a$
(17, 24)	a	0	a	$-a$	$-b$	$-b$	b	$-b$	a	a	$-a$		b	b	b	$-b$	b	b	$-b$	b	a	0	$-a$
	b	b	b	$-b$	a	$-a$	$-a$	a	0	a	a		b	b	b	$-b$	$-b$	$-b$	b	$-b$	$-a$	$-a$	$-a$
(17, 25)	b	b	$-b$	b	b	$-b$	$-b$	$-a$	$-a$	$-a$	a		b	b	b	$-b$	b	$-b$	$-a$	$-a$	a	$-a$	$-a$
	a	b	$-b$	b	b	b	b	0	$-a$	a	a		b	b	$-b$	a	$-b$	b	b	$-a$	a	$-a$	0
(18, 19)	b	$-b$	$-b$	a	a	a	a	$-a$	0	0	0		b	b	b	$-b$	$-b$	b	$-b$	$-a$	$-a$	a	a
	b	$-b$	$-b$	$-a$	a	$-a$	$-a$	0	0	0	$-a$		b	b	b	$-a$	b	$-b$	b	a	$-a$	a	0
(18, 21)	b	b	$-a$	$-b$	$-b$	$-a$	$-a$	a	0	a	$-a$		b	b	b	b	$-b$	b	a	$-a$	0	$-a$	0
	b	$-b$	b	b	$-b$	$-a$	$-a$	a	0	$-a$	0		a	a	a	a	$-a$	b	$-b$	b	b	b	$-b$

Appendix C(cont): Order 44 (Sequences with zero periodic autocorrelation function)

Design						A_1	A_2											A_3	A_4			
$(18,23)$	a	a	a	a	b	b	$-b$	b	$-b$	$-b$	$-b$	a	a	$-a$	$-a$	$-b$	$-b$	b	$-b$	b	$-a$	$-b$
	a	b	$-b$	$-b$	b	b	b	$-b$	b	b	b	a	$-a$	$-a$	a	$-a$	a	$-a$	a	a	$-b$	$-b$
$(19,22)$	b	$-a$	b	$-b$	$-b$	b	b	$-a$	$-a$	$-a$	a	b	$-b$	$-b$	$-b$	$-a$	$-b$	$-a$	$-a$	0	$-a$	a
	b	0	b	$-b$	$-b$	b	$-b$	$-a$	$-a$	a	a	b	$-b$	$-b$	$-b$	a	$-b$	a	$-a$	a	0	$-a$
$(19,23)$	b	$-b$	$-b$	$-a$	b	$-b$	$-b$	$-a$	a	$-a$	$-a$	b	b	b	$-a$	$-a$	$-b$	b	a	0	a	$-a$
	b	b	b	b	$-b$	b	$-b$	$-a$	$-a$	a	a	b	b	$-b$	a	$-a$	$-b$	b	$-a$	0	$-a$	$-a$
$(20,21)$	b	$-a$	$-b$	b	b	$-a$	$-b$	$-a$	$-a$	a	$-a$	b	b	b	$-b$	b	$-b$	a	a	a	0	$-a$
	a	$-b$	b	$-b$	$-b$	$-b$	$-b$	$-a$	0	$-a$	a	b	$-a$	$-b$	0	$-b$	$-a$	b	a	$-a$	a	a
$(21,23)$	a	a	a	$-b$	$-b$	$-b$	b	b	$-b$	b	b	a	$-a$	a	$-a$	a	a	$-a$	$-a$	a	a	b
	a	b	$-b$	a	$-b$	$-b$	$-b$	b	$-b$	b	$-b$	a	b	$-b$	$-b$	$-b$	$-b$	$-a$	$-a$	b	$-b$	$-b$