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### A new approach to harmonic allocation for medium-voltage installations

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## A new approach to harmonic allocation for medium-voltage installations

### Abstract

Distributors need to allocate a maximum allowed level of harmonic current to medium-voltage customers to keep voltage distortion acceptable. The paper describes a new approach, based on the concept of voltage droop, requiring much less calculation and data than required by the present approach based on an IEC technical report. The discrepancy between the new method and the present is studied by comparing some carefully selected scenarios. It is shown that the proposed method gives results within 20% of the standards-based approach which makes it a very attractive alternative for harmonic allocation. © Institution of Engineers Australia, 2013.

### Keywords

harmonic, allocation, medium, voltage, installations, approach

### Disciplines

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# A New Approach to Harmonic Allocation for MV Installations

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**Abstract**— Distributors need to allocate a maximum allowed level of harmonic current to MV customers to keep voltage distortion acceptable. The paper describes a new approach, based on the concept of voltage droop, requiring much less calculation and data than required by the present approach based on an IEC technical report. The discrepancy between the new method and the present is studied by comparing some carefully selected scenarios. It is shown that the proposed method gives results within 20% of the standards-based approach which makes it a very attractive alternative for harmonic allocation.

**Keywords**— distribution systems, harmonics, IEC standards, harmonic allocation, voltage droop

## I. NOMENCLATURE

Symbol	Meaning
$E_{hi}$	Emission allocation of current at harmonic "h" for load "i"
h	Harmonic order
$k_h$	Harmonic allocation constant
$L_h$	LV harmonic voltage limit
SCR	Load short-circuit ratio; fault level divided by maximum demand
$S_i$	Max demand of load "i"
$V_d$	Voltage droop
$V_{hi}$	Harmonic voltage caused by load "i" at its point of connection
$x_{hi}$	Harmonic reactance seen by load "i"
$\alpha$	Summation law exponent

## II. INTRODUCTION

Distributors are required, under Australian harmonic standard AS/NZS 61000.3.6 [1], to keep harmonic voltage levels on their network below the acceptable limits. The main concern is harmonic levels in MV distribution systems which depend mainly on the harmonic current drawn by the MV installations. The standard gives some principles by which each MV installation's harmonic current allowance can be determined as will be detailed in Section III.

Reference [1] is largely based on an IEC document having the status of a technical report because the international community could not come to final agreement on how harmonic allocation should be done. The IEC counterpart should be viewed as a list of ideas and guidelines rather than a final normative statement of how the allocation process should be carried out. This has caused difficulties in Australia where [1] has been called up by the National Electricity Rules and has

legal authority. Consequently there has been much work in Australia to find satisfactory analytical techniques and this has led to the publication of [2] which details some aspects of MV harmonic allocation, i.e. radial distribution systems without spurs.

Reference [3] shows that implementing the IEC guidelines in a rigorous manner requires a detailed harmonic study, requiring data on the maximum demand and impedance at the point of connection for all locally connected MV loads. Assumptions need to be made regarding the effect of LV loads connected to the system and MV loads which may be connected in future. This involves guesswork and judgement and there is much scope for utility/customer conflict.

The authors have developed an alternative approach based on the concept of "voltage droop" which is the fundamental voltage drop between a load and a hypothetical upstream Thevenin voltage source [4]. We can define the voltage droop due to a particular load or the voltage droop at the end of a particular feeder due to all loads in the local power system. Our particular interest is the maximum voltage droop which can occur in the power system. This is likely to be at the end of a long LV feeder. Since each upstream transformer can regulate over a range of about 10%, and there are about three effective such levels, one would expect that the maximum voltage droop in a power system (to be given the symbol  $V_d$ ) is limited to be about 30%. The approach can be used for harmonic allocation in distribution systems of any topology providing the harmonic impedance at harmonic "h" is "h" times the fundamental reactance. This requires that transmission lines are sufficiently short for line capacitance to be negligible and all shunt-connected capacitors to be detuned. It should be noted that if these assumptions do not hold, there are major issues for all harmonic allocation schemes presently used.

The role of the present paper is to estimate the accuracy in the proposed approach and to investigate if it is acceptable for practical systems. Sections III and IV summarise the IEC guidelines and the new harmonic allocation approach respectively. The next section studies the difference or "margin" between a strict IEC allocation and the new method. Section VI gives the application of the new approach to both a homogenous system (all feeders and loads identical) and a more realistic system with a mix of strongly and weakly loaded feeders.

### III. IEC GUIDELINES

The major IEC guidelines for harmonic allocation have been adopted without change in [1].

- (a) Under time-varying conditions, harmonic quantities are to be characterised by their 95% values.
- (b) Diversity between independent harmonic sources can be represented by an exponential summation law.

$$V_{\text{tot}} = \sqrt[\alpha]{V_1^\alpha + V_2^\alpha} \quad (1)$$

where  $\alpha$  depends on the harmonic order.

- (c) All present and projected customers are assumed to be drawing their full harmonic allocation which should be such that, when the system is fully loaded, the maximum harmonic voltage reaches the limit.

Reference [1] suggests that the allocation in distribution systems should give each installation a harmonic VA proportional to the maximum demand, with an allowance for diversity and this approach has been followed in [2]. This is achieved for a load with maximum demand  $S_i$  by a harmonic current allocation of

$$E_{1hi} = \frac{k_h S_i^{1/\alpha}}{\sqrt{x_{hi}}} \quad (2)$$

where  $k_h$  is called the Allocation Constant and needs to be determined for each supply substation. Guideline (c) above requires a complete harmonic study of the substation load with every significant connected customer needing to be represented and  $k_h$  varied until the harmonic voltage limit is just reached. This requires knowledge extensive knowledge of all relevant present day loads, including upstream and LV loads, information which can be difficult to assemble. Guesses need to be made regarding the magnitude and point of connection of all future loads in the subsystem. It is unsatisfactory that a standard could lead to a result which is so poorly defined.

### IV. NEW APPROACH

#### A. Summary of theory

The proposed new approach, presented in [4] will be summarised here. It is best introduced by neglecting diversity, that is taking  $\alpha = 1$ . Suppose every customer is allocated the same percentage harmonic current. At the  $h^{\text{th}}$  harmonic, all MV power system impedances are "h" times larger than the corresponding fundamental reactances and the IX drop at the "h" harmonic is simply proportional to the fundamental voltage drop. The maximum harmonic voltage in the subsystem will occur at the end of the most heavily loaded LV feeder and can be shown to be equal to the voltage droop limit, scaled up by "h" (because of the increase in system reactance) and scaled down by the fraction of harmonic current to fundamental current.

When there is diversity, the link between harmonic voltage drop and voltage droop can be maintained very closely, as discussed in [4], by an allocation law which is also dependent on impedance at the point of connection

$$E_{1hi} = k_h \frac{S_i^{1/\alpha}}{x_{1i}^{1-\frac{1}{\alpha}}} \quad (3)$$

$E_{1hi}$  is the allocated current,  $S_i$  is the maximum demand of load "i" being assessed,  $x_{1i}$  the upstream fundamental impedance at its point of connection and  $k_h$  an allocation constant to be determined. In order to limit the maximum harmonic voltage at the far end of LV feeders,  $k_h$  should be chosen from

$$k_h = \frac{L_h}{hV_d^{1/\alpha}} \quad (4)$$

where  $L_h$  is the LV harmonic limit.

It is sometimes convenient to use expressions involving the load short-circuit ratio or SCR defined as the fault level at the point of load connection divided by the load maximum demand. Reference [4] shows

$$\frac{E_{1hi}}{I_{i1}} = k_h \text{SCR}_i^{1-\frac{1}{\alpha}} \quad (5)$$

#### B. Example

Consider a 1MVA load connected to a point where the fault level is 120MVA, giving a SCR of 120. Assume for the 5<sup>th</sup> harmonic, for which  $\alpha = 1.4$ , an LV limit of 5.5%. Step (i)

determine  $k_h$  from (4)  $k_h = \frac{0.055}{5 \times 0.30^{1/1.4}} = 0.026$ . Step (ii) determine  $\text{SCR} = 120\text{MVA}/1\text{MVA} = 120$ . Step (iii) determine

$I_5$  as a fraction of  $I_1$  from (5)  $\frac{E_{1hi}}{I_{i1}} = 0.026 \times 120^{\left(1-\frac{1}{1.4}\right)} = 0.10$ .

Hence the allocated 5<sup>th</sup> harmonic current is 10% of the fundamental current.

#### C. Discrepancy between strict IEC and new approaches

The method is exact when all loads are supplied from the one feeder as demonstrated by the example in Fig. 1 involving three installations. The reactance values are given so that the fundamental reactance to node "i" is  $x_i$ . First we estimate the fundamental voltage droop, with the assumption that the fundamental current is equal to the maximum demand in per unit. For harmonic order "h" we shall take the LV limit to be  $L_h$ .

$$V_d = V_{d1} + V_{d2} + V_{d3} = x_1 S_1 + x_2 S_2 + x_3 S_3 \quad (6)$$

From (4)

$$k_h = \frac{L_h}{hV_d^{1/\alpha}} = \frac{L_h}{h(x_1 S_1 + x_2 S_2 + x_3 S_3)^{1/\alpha}} \quad (7)$$

The allocated harmonic currents which can be injected by

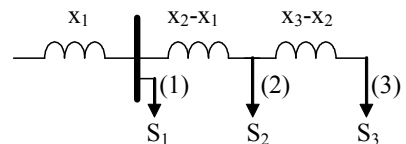


Fig. 1 – Single feeder distribution system

loads  $S_1$ - $S_3$  are then

$$E_{lh1} = \frac{k_h S_1^{1/\alpha}}{x_1^{1-1/\alpha}}, E_{lh2} = \frac{k_h S_2^{1/\alpha}}{x_2^{1-1/\alpha}}, E_{lh3} = \frac{k_h S_3^{1/\alpha}}{x_3^{1-1/\alpha}} \quad (8)$$

As a check we compare the total harmonic voltage produced against the limit. The harmonic voltages produced by each load at the point of connection and therefore at the end of the feeder are

$$V_{h1} = k_h h(x_1 S_1)^{1/\alpha}, V_{h2} = k_h h(x_2 S_2)^{1/\alpha}, V_{h3} = k_h h(x_3 S_3)^{1/\alpha} \quad (9)$$

Taking each term to the power of  $\alpha$  and adding

$$V_h^\alpha = k_h^\alpha h^\alpha (x_1 S_1 + x_2 S_2 + x_3 S_3) = L_h^\alpha \frac{(x_1 S_1 + x_2 S_2 + x_3 S_3)}{(x_1 S_1 + x_2 S_2 + x_3 S_3)} = L_h^\alpha \quad (10)$$

showing that the allocation method is exact in this case.

When there is more than one feeder, there is a need to distinguish between the combination of different sources acting at one point, and the addition voltages from a single current source acting through impedances in series. The first follows the summation law and is correctly accounted for while the second add arithmetically (KVL) and are not. The practical outcome is the incorrect representation of a source connected to one feeder and acting on another. This can be demonstrated by means of a two feeder system Fig. 2.

The voltage drop at the end of feeder 1 is

$$V_{d1} = x_1 S_1 + x_0 S_2 \quad (11)$$

while for feeder 2

$$V_{d2} = x_0 S_1 + x_2 S_2 \quad (12)$$

We shall assume without loss of generality that  $V_{d1} \geq V_{d2}$ , so that

$$(x_1 - x_0) S_1 \geq (x_2 - x_0) S_2 \quad (13)$$

$$k_h = \frac{L_h}{h V_{d,\max}^{1/\alpha}} = \frac{L_h}{h (x_1 S_1 + x_0 S_2)^{1/\alpha}} \quad (14)$$

$$E_{lh1} = \frac{k_h S_1^{1/\alpha}}{x_1^{1-1/\alpha}}, E_{lh2} = \frac{k_h S_2^{1/\alpha}}{x_2^{1-1/\alpha}} \quad (15)$$

With more than one feeder, we do not determine the harmonic voltage at the point of load connection, but that imposed on the feeder with the largest harmonic voltage (or voltage drop), that is the harmonic voltage imposed at node (1) in this case.

$$V_{h1} = k_h h(x_1 S_1)^{1/\alpha}, V_{h2} = \frac{k_h h x_0 S_2^{1/\alpha}}{x_2^{1-1/\alpha}} \quad (16)$$

Taking each term to the power of  $\alpha$  and adding

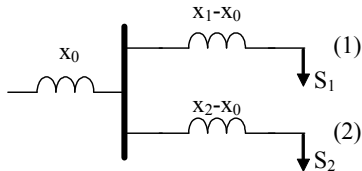


Fig. 2 – Two feeder system

$$V_h^\alpha = k_h^\alpha h^\alpha \left( x_1 S_1 + \frac{x_0^\alpha S_2}{x_2^{\alpha-1}} \right) = \frac{\left( x_1 S_1 + \frac{x_0^\alpha S_2}{x_2^{\alpha-1}} \right)}{(x_1 S_1 + x_0 S_2)} L_h^\alpha \quad (17)$$

Hence

$$V_h = \left( \frac{x_1 S_1 + \left( \frac{x_0}{x_2} \right)^{\alpha-1} x_0 S_2}{x_1 S_1 + x_0 S_2} \right)^{1/\alpha} L_h \quad (18)$$

which is always less than  $L_h$ , showing that the allocation method is not exact in this case. For example, if  $x_0 = 1$ ,  $x_1 = 5$ ,  $x_2 = 4$ ,  $S_1 = 1$ ,  $S_2 = 1$ , and  $\alpha = 1.4$ ,  $L_h = 1$  pu the inequality (13) is met and  $V_h = 0.95$  pu. However, if there is no diversity ( $\alpha = 1$ ), the above expression becomes 1 pu. for any combination of values of the other parameters.

Ideally, we would like a method which gives a harmonic voltage reaching the harmonic limit when all loads are connected and taking their full allocation. In the above example, the method is said to "underallocate", since the harmonic voltage only reaches 95% of the limit. The reserve harmonic capacity, 5% in this case, we shall call the "margin". It is desirable that the margin is not too large so that customers are given most of the harmonic capacity of the system. In practice, some margin is desirable to allow for contingencies such as the presence of embedded generation which can contribute to harmonic emission but is not considered in the voltage drop figure. We wish to demonstrate that the new method gives a margin which is usually no more than 20% of the harmonic voltage limit.

## V. THE MARGIN GIVEN BY THE NEW APPROACH

### A. Methodology

The equations for harmonic allocation are non-linear because of the summation law (1) and can only be solved exactly for some simple cases or for specific numerical cases. It would be impractical to find an exhaustive set of numerical studies that could be guaranteed to cover all situations of power system topology and reactance and load values which could arise in practice. We have approached this by carefully selecting a set of scenarios to give an estimate of the maximum margin from this new approach.

We begin with a system having two feeders with single loads at the extremity of each. This will demonstrate that the maximum margin can be estimated by studying the situation when the feeder impedances and the load maximum demands are identical. This case can be studied theoretically and it is then found that the maximum margin for a two feeder system is when the feeder impedance is about twice the supply impedance. The approach is then extended to a system having  $N$  identical feeders.

### B. Two feeder system

The system and its equations have been given in Section IV.C. The equations have been set up in a spreadsheet and attempts made, using Excel's Solver Add-in, to find the values

TABLE I RESULTS OF EXPLORATION OF TWO FEEDER SYSTEM

Case	$x_1$	$x_2$	$S_1$	$S_2$	$V_h/L_h$	Comments
1	10	10	1	1	0.96	Starting point
2	6713	2.32	0.00035	1.76	0.91	All parameters changed at once
3	10737	10	0.00084	1	0.96	$x_1$ & $S_1$ changed from starting point
4	10	2.24	1	7.22	0.92	$x_2$ & $S_2$ changed from starting point
5	2.93	2.93	1	1	0.94	$x_1$ & $x_2$ changed while forced to be equal

of input parameters giving a minimum value of  $V_h/L_h$  corresponding to the maximum margin. Some simplification is possible.  $x_0$  can be chosen as 1 pu without loss of generality. Similarly,  $h$  and  $L_h$  can be taken as 1. Results are summarised in Table I.

When attempts are made to change any of the data individually, Solver gives the starting point as the minimum (0.96). When all parameters are changed at once (Case 2) a somewhat lower value of 0.91 is achieved. If just  $x_1$ ,  $S_1$  together or  $x_2$ ,  $S_2$  together are changed, intermediate values are obtained as shown for Cases 3, 4. Solver makes no changes when  $x_1$  and  $x_2$  are allowed to change together. However, if  $x_1$  and  $x_2$  are forced to be equal to a new variable, and that variable is changed, we obtain a minimum of 0.94 when  $x_1 = x_2 = 2.93$ . It is also observed that the minimum occurs in every case when both feeders have the same voltage droop. We conclude

- The minimisation problem for the 2-feeder system, including the voltage droop constraints, is both theoretically and numerically very difficult.
- Most minimum values (Cases 2 and 3 in particular) involve unrealistic combinations of parameters.
- An estimate of the margin to be expected for realistic cases can be found by examining the case where all feeders have the same loading and the same reactance (Case 5), corresponding to each feeder having the same voltage droop.

C. N feeder system

The system is illustrated in Fig. 3. All loads are taken equal with value 1 pu, since scaling the loads should have no impact on the margin. A per unit system has been chosen to give a supply side reactance of 1pu. It is assumed that each feeder's load can be lumped at a point where there is a reactance of  $x$  pu to the supply bus. Using these units,  $x$  can be interpreted as the ratio of the feeder to the supply side reactance.

The load voltage droops shall be estimated from the

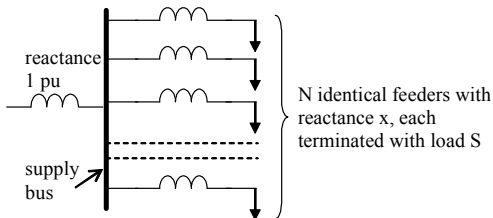


Fig. 3 – System with N homogenous feeders

product of the load maximum demand (equal to the fundamental current in per unit) times the fundamental reactance. The voltage droop caused by each load at the supply bus is 1 pu. The voltage caused by a load at the end of its feeder is  $1 + x$ . The voltage droop at the end of any one feeder is the sum of two quantities, that from the directly connected load and the  $N - 1$  contributions from loads connected to the remaining feeders.

$$V_d = 1 + x + (N - 1) = N + x \tag{19}$$

$$k_h = \frac{L_h}{hV_d^{1/\alpha}} = \frac{L_h}{h(N+x)^{1/\alpha}} \tag{20}$$

For each load the allocated current is

$$E_{th} = \frac{k_h 1^{1/\alpha}}{(1+x)^{1-\frac{1}{\alpha}}} = \frac{L_h}{h(1+x)^{1-\frac{1}{\alpha}}(N+x)^{1/\alpha}} \tag{21}$$

The harmonic voltage caused by each load at the supply bus is

$$V_{h.bus} = \frac{L_h}{(1+x)^{1-\frac{1}{\alpha}}(N+x)^{1/\alpha}} \tag{22}$$

The harmonic voltage caused by each load at its point of connection is

$$V_{h.pc} = h(1+x)I_h = \frac{(1+x)^{1/\alpha} L_h}{(N+x)^{1/\alpha}} \tag{23}$$

The harmonic voltage at the supply bus caused by all loads is the combination of these terms using the summation law

$$V_h^\alpha = \frac{(N-1)L_h^\alpha}{(1+x)^{\alpha-1}(N+x)} + \frac{(1+x)L_h^\alpha}{(N+x)} = \frac{(1+x)}{(N+x)} \left\{ \frac{(N-1)}{(1+x)^\alpha} + 1 \right\} L_h^\alpha \tag{24}$$

giving 
$$\frac{V_h}{L_h} = \left[ \frac{(1+x)}{(N+x)} \left\{ \frac{(N-1)}{(1+x)^\alpha} + 1 \right\} \right]^{1/\alpha} \tag{25}$$

The margin can be found by examining the difference between the quantity  $V_h/L_h$  and one. To obtain a preliminary idea of the variation of this multivariable function, some graphs have been given for some typical values. Fig. 4 shows the variation of  $V_h/L_h$  with  $N$  for three values of  $x$  determined for  $\alpha = 1.4$ . The graph has been extended beyond the normal range of  $N$  (10-20) to show the asymptotic variation of the function. The variation for  $\alpha = 2$  is similar, except that it deviates from unity by about 50% more. For  $\alpha = 1$  (no diversity), (25) is identical to one and there is no margin.

We see that the variation gets worse with increasing number of feeders. The values asymptotically approach a value which can be found from (25) by finding the limit as  $N$

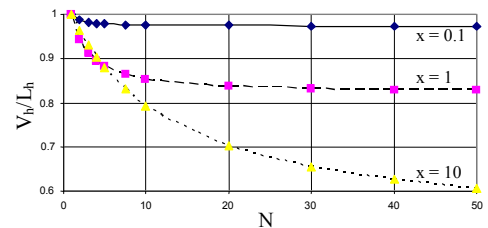


Fig. 4 – Variation of  $V_h/L_h$  with  $N$  for three values of  $x$

TABLE II ASYMPTOTIC VALUES OF  $V_h/L_h$

x	$\alpha = 1.4$	$\alpha = 2$
0.1	0.97	0.95
1	0.82	0.71
10	0.50	0.30

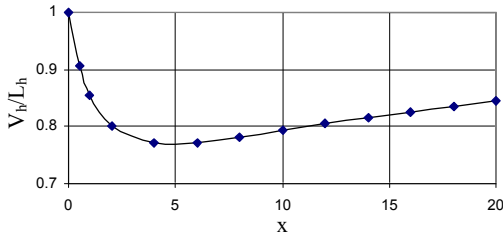


Fig.5 – Variation of  $V_h/L_h$  with x

approaches infinity to give

$$\lim_{N \rightarrow \infty} \frac{V_h}{L_h} = \frac{1}{(1+x)^{1-\frac{1}{\alpha}}} \quad (26)$$

This has been determined in the Table II. The margin also increases with  $\alpha$ , being zero for no diversity ( $\alpha = 1$ ). The margin appears to increase with x because of the restricted number of points shown. Consider the variation with x for a realistic value of  $N = 10$ . Fig. 5 shows that, for a particular value of N, there is a value of x giving the largest margin ( $x \sim 5$  gives 0.78).

The value of x giving the largest margin can be determined by the standard process of differentiating (25). Some simplification can be achieved by taking this equation to the power of  $\alpha$  and using L'Hospital's rule to find where the minimum occurs. Re-expressing the RHS of (25)

$$\frac{(1+x) \left\{ \frac{(N-1)}{(N+x)} + 1 \right\}}{\left( \frac{(N-1)(1+x)^{1-\alpha} + 1+x}{(N+x)} \right)} \quad (27)$$

Applying L'Hospital's rule gives

$$\frac{(N-1)(1+x)^{1-\alpha} + 1+x}{(N+x)} = \frac{(N-1)(1-\alpha)(1+x)^{-\alpha} + 1}{1} \quad (28)$$

Some manipulation gives

$$(1+x)^{1-\alpha} = (1-\alpha)(1+x)^{1-\alpha} + 1 + (N-1)(1-\alpha)(1+x)^{-\alpha} \quad (29)$$

It is not possible to solve analytically for x, but rearranging allows an expression for N

$$N = \frac{(1+x)^\alpha - 1 - \alpha x}{(\alpha - 1)} \quad (30)$$

For a given N, the value of x satisfying the RHS of (30) gives the feeder reactance, relative to the supply reactance, giving the largest margin.

Table III shows, for each value of N, the value of x giving the largest margin and the computed value of  $V_h/L_h$ . Fig. 6 shows the variation of  $V_h/L_h$  vs. N. A typical value of N is 10 for which the maximum margin occurs when the equivalent feeder impedance is  $x = 4.8$  times the supply impedance, a typical value in practice. Here the computed  $V_h/L_h$  is 0.77 giving a margin of 23%.

TABLE III VALUES CORRESPONDING TO LARGEST MARGIN FOR EACH VALUE OF N

N	1	2	5	10	20	50	100
x	1.32	1.93	3.22	4.80	7.23	12.60	19.40
$V_h(\text{min})$	1.00	0.94	0.84	0.77	0.70	0.60	0.54

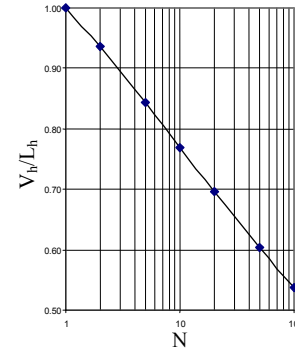


Fig. 6 – Variation of  $V_h/L_h$  with N for x giving maximum margin.

Thus about 23% of the capacity of the local power system to absorb harmonics is unused. This is not a major issue for several reasons

- This applies only when each feeder has the same equivalent impedance and is loaded identically. This is seldom the case and the margin is generally less than given by Fig. 6.
- Some reserve margin is useful for contingencies e.g. (a) additional harmonic contributions from embedded generation such as rooftop PV units, (b) higher emissions than allowed by IEC guidelines for some loads connected in the past under previous harmonic allocation procedures, (c) some amplification due to nearby capacitors which are not fully detuned.

## VI. TYPICAL DISTRIBUTION SYSTEM STUDIES

### A. System values

It would be impractical to find a power system (zone substation and its loads) that everyone would agree was typical. We shall take one which has been treated for harmonic allocation studies in the past [5] and which has originally appeared in [1] Appendix I – see Fig. 7.

The fault level at the 20kV bus is 234MVA while the reactance of the 20kV feeder is  $0.35\Omega/\text{km}$ . We convert to per

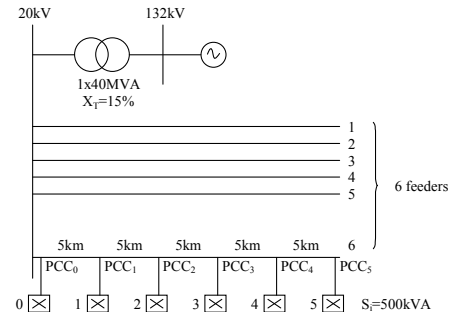


Fig. 7 - Homogenous study system from [1,5]

unit using a base of 50MVA as used in [5]. At 20kV,  $Z_B = 20^2/50 = 8\Omega$ . The upstream supply reactance seen at the 20kV bus is  $50/234 = 0.21\text{pu}$ . The reactance of each 5km section =  $5 \times 0.35 = 1.75\Omega = 0.22\text{pu}$ . Each load is 0.01pu.

### B. Homogenous power system

The system given has been set up for illustrative purposes and the voltage droop need not correspond with 30% which we feel is typical in Australia. It is significant that no LV loads are shown. The voltage droop from source to the end of one of the 20kV feeders is the sum of the voltage droop due to the other 5 feeders ( $V_{d1}$ ) plus that due to the feeder under study ( $V_{d2}$ ).

$$V_{d1} = 0.21 \times 5 \times 6 \times 0.01 = 0.063.$$

$$V_{d2} = 0.01 \times (0.21 + 0.43 + 0.65 + 0.87 + 1.09 + 1.31) = 0.046.$$

Hence  $V_d = V_{d1} + V_{d2} = 0.011.$

For this example, we need to calculate our allocation constant as  $k_h = \frac{V_{h,\text{lim}}}{hV_d^{1/\alpha}}$ . For the 5<sup>th</sup> harmonic, with an

assumed limit of 5%,  $k_5 = \frac{0.05}{5 \times 0.011^{1/1.4}} = 0.048$ . This is about

twice the value to be expected with a droop of 30%. Table IV shows the calculation of the harmonic current allocations, compared with that given by a more complex calculation in [5].

In reviewing these results, we first we note that we should not expect the proposed method to agree at every load point with previous methods. The latter are based on constant harmonic VA allocation giving a current variation with the square root of fault level for equal load VA as here. In the proposed approach, the allocated currents in this situation will vary less sensitively with fault level, roughly as the fault level to the power of 0.3 (see (5)). The total 5<sup>th</sup> harmonic current allocated to a feeder can be estimated from summation of the individual load currents giving 124% (relative to 500kVA) in the proposed approach and 133% by the former "exact" approach, illustrating the margin. The maximum harmonic voltage which will occur with the proposed method has been computed as 4.3% for a limit of 5%, a margin of 14%

### C. Non-homogenous power system

The system of Fig. 7 has been modified to make it non-homogenous to explore how the margin might change. Feeder number one has been replaced by a stronger one of negligible length with a single load of 1.5 MVA (0.03 pu). Feeder number two has been replaced by a longer one with a fault level of 29MVA at the far end where a load of 1MVA is concentrated. These figures have been chosen to give exactly the same voltage droop as in the homogenous example. The maximum

TABLE IV COMPARISON OF ALLOCATED CURRENTS BY TWO METHODS

Node	0	1	2	3	4	5	
S	0.01	0.01	0.01	0.01	0.01	0.01	
FL	4.76	2.33	1.54	1.15	0.92	0.76	
SCR	476	233	154	115	92	76	Sums
$I_5(\%)$ - proposed	28%	23%	20%	19%	17%	17%	124%
$I_5(\%)$ - Ref [5]	38%	26%	21%	18%	16%	14%	133%

voltage droop now occurs only at the end of the weak feeder 2. Calculation gives a harmonic voltage of 4.4% at the end of feeder 2, giving a margin of 12%.

We see from the above that the system gives acceptable results for so-called typical systems, with less margin as the system becomes less homogenous.

## VII. CONCLUSIONS

The harmonic allocation principles in the IEC technical report are difficult to apply using methods published to date, mainly because of the data load and the need to estimate future scenarios that are convincing to all parties. A new approach has been described based on the voltage droop concept. It has been shown to be exact only when there is no diversity or only one feeder connected to the supply bus. Otherwise the method is pessimistic but there appears to be no clear analytical method for establishing its margin and its suitability for everyday calculations.

A full numerical study of a representative set of cases seems impracticable as there are far too many possibilities. A numerical study has been made of several scenarios to obtain an estimate of the accuracy of the proposed method for a two feeder system. The maximum margin, for realistic parameter values, occurs when the feeders are identical. It is assumed that this result applies to any number of feeders.

Numerical studies have been made for an N-feeder homogenous system which can be also studied analytically. It is shown that the margin increases monotonically with the number of feeders and with the value of  $\alpha$ . It is also shown that the margin is small for low and very large reactance feeders, relative to the supply impedance, with a maximum for intermediate values. An expression has been found allowing the value of feeder reactance and the corresponding value of margin to be determined for any value of N. With typical values of feeder number and reactance, the margin is shown to be at most 20%.

The new method has been demonstrated to have sufficient accuracy for engineering use and is convenient to apply with relatively small requirements for data.

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