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New weighing matrices

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Abstract

New weighing matrices and skew weighing matrices are given for many orders $4t \leq 100$. These are constructed by finding new sequences with zero autocorrelation. These results enable us to determine for the first time that for $4t \leq 84$ a $W(4t, k)$ exists for all $k = 1, \dots, 4t - 1$ and also that there exists a skew-weighing matrix (also written as an $OD(4t; 1, k)$) for $4t \leq 80$, t odd, $k = a^2 + b^2 + c^2$, a, b, c integers except $k = 4t - 2$ must be the sum of two squares.

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NEW WEIGHING MATRICES*

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SUMMARY. New weighing matrices and skew weighing matrices are given for many orders $4t \leq 100$. These are constructed by finding new sequences with zero autocorrelation. These results enable us to determine for the first time that for $4t \leq 84$ a $W(4t, k)$ exists for all $k = 1, \dots, 4t - 1$ and also that there exists a skew-weighing matrix (also written as an $OD(4t; 1, k)$) for $4t \leq 80$, t odd, $k = a^2 + b^2 + c^2$, a, b, c integers except $k = 4t - 2$ must be the sum of two squares.

1. INTRODUCTION

An *orthogonal design* of order n and type (s_1, s_2, \dots, s_u) ($s_i > 0$), denoted $OD(n; s_1, s_2, \dots, s_u)$, on the commuting variables x_1, x_2, \dots, x_u is an $n \times n$ matrix A with entries from $\{0, \pm x_1, \pm x_2, \dots, \pm x_u\}$ such that

$$AA^T = \left(\sum_{i=1}^u s_i x_i^2 \right) I_n$$

Alternatively, the rows of A are formally orthogonal and each row has precisely s_i entries of the type $\pm x_i$. In [3], where this was first defined, it was mentioned that

$$A^T A = \left(\sum_{i=1}^u s_i x_i^2 \right) I_n$$

and so our alternative description of A applies equally well to the columns of A . It was also shown in [3] that $u \leq \rho(n)$, where $\rho(n)$ (Radon's function) is defined by $\rho(n) = 8c + 2^d$, when $n = 2^a b$, b odd, $a = 4c + d$, $0 \leq d < 4$.

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A *weighing matrix* $W = W(n, k)$ is a square matrix with entries $0, \pm 1$ having k non-zero entries per row and column and inner product of distinct rows zero. Hence W satisfies $WW^T = kI_n$, and W is equivalent to an orthogonal design $OD(n; k)$. The number k is called the *weight* of W .

Given the sequence $A = \{a_1, a_2, \dots, a_n\}$ of length n the *non-periodic autocorrelation function* $N_A(s)$ is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s}, \quad s = 0, 1, \dots, n-1, \quad \dots (1)$$

If $A(z) = a_1 + a_2 z + \dots + a_n z^{n-1}$ is the associated polynomial of the sequence A , then

$$A(z)A(z^{-1}) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}), \quad z \neq 0. \quad \dots (2)$$

Given A as above of length n the *periodic autocorrelation function* $P_A(s)$ is defined, reducing $i + s$ modulo n , as

$$P_A(s) = \sum_{i=1}^n a_i a_{i+s}, \quad s = 0, 1, \dots, n-1. \quad \dots (3)$$

Sequences with zero periodic autocorrelation function or zero non-periodic autocorrelation function, of length t , are used to form the first rows of four circulant matrices which are then used in the Goethals-Seidel array to form matrices of order $4t$ of the required type (if orthogonal designs) or weight (if weighing matrices). In the case of sequences with zero non-periodic autocorrelation function, the sequences are first padded with sufficient zeros added to the end to make their length t .

The results used are:

Theorem 1 [5, Theorem 4.49]. *If there exist four circulant matrices A_1, A_2, A_3, A_4 of order n satisfying $\sum_{i=1}^4 A_i A_i^T = fI$ where f is the quadratic form $\sum_{j=1}^u s_j x_j^2$, then there is an orthogonal design $OD(4n; s_1, s_2, \dots, s_u)$.*

Corollary 1. *If there are four $\{0, \pm 1\}$ -sequences of length n and weight w with zero periodic or non-periodic autocorrelation function then these sequences can be used as the first rows of circulant matrices which can be used in the Goethals-Seidel array to form $OD(4n; w)$ or a $W(4n, w)$. If one of the sequences is skew-type then they can be used similarly to make an $OD(4n; 1, w)$. We note that if there are sequences of length n with zero non-periodic autocorrelation function then there are sequences of length $n + m$ for all $m \geq 0$.*

Weighing matrices have long been studied because of their use in weighing experiments as first studied by Hotelling [7] and later by Raghavarao [12] and others. For more applications of weighing matrices see Banerjee [1] and Harwit and Sloane [6]. There are a number of conjectures concerning weighing matrices:

Conjecture 1. *There exists a weighing matrix $W(4t, k)$ for $k \in \{1, \dots, 4t\}$.*

Conjecture 2. *When $n \equiv 4 \pmod{8}$, there exists a skew-weighing matrix (also written as an $OD(n; 1, k)$) when $k \leq n - 1$, $k = a^2 + b^2 + c^2$, a, b, c integers except that $n - 2$ must be the sum of two squares.*

Conjecture 3. *When $n \equiv 0 \pmod{8}$, there exists a skew-weighing matrix (also written as an $OD(n; 1, k)$) for all $k \leq n - 1$.*

The reader is referred to Geramita and Seberry [5] for all other undefined terms.

In Geramita and Seberry [5] the status of the weighing matrix conjecture is given for $W(4t, k)$, $k \in \{1, \dots, 4t\}$ and $t \in \{1, \dots, 21\}$. We give new results including resolving the conjecture in the affirmative for all $4t \leq 84$.

2. SOME NEW SEQUENCES WITH ZERO AUTOCORRELATION

Tables 1 to 6 give new results which allow us to settle in the affirmative Conjecture 2 for orders 68 and 76. They also eliminate most unsolved cases for Conjecture 1 in order 92 and Conjecture 2 in order 84.

TABLE 1. SEQUENCES OF LENGTH 19 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=19	Sequences with zero non-periodic autocorrelation function
1,62	$\{0++++-+-0+a-0+-+---0\}$, $\{+---++-++0---0++++\}$, $\{+---++-++0++-+0----\}$, $\{0++++-+-0+0+0-+-++0\}$
1,66	$\{0++++-+-++a---+---0\}$, $\{0++++-+-++0+-+---++0\}$, $\{+++0+-----+---++0\}$, $\{+++0+-----++---+0\}$
1,68	$\{+++0+-----a+++0+---\}$, $\{+++---++0++-+---++\}$, $\{+++---++0---+---++\}$
1,72	$\{---++++-+a+-----++\}$, $\{+---++-++0+---++\}$, $\{+---++-++0+---++\}$, $\{---++++-+0+---++\}$

3. NUMERICAL CONSEQUENCES

We use the tables of Appendix H and extend Theorem 4.149 of Geramita and Seberry [5].

We note that sequences with periodic autocorrelation function zero exist for $W(4t, 4t - 1)$, $W(4t, 4t)$ for all $t \in \{1, \dots, 31\}$ [15]. We use the $(OD; 1, 1, 64)$ for $n \geq 17$ and the $(OD; 1, 1, 80)$ for $n \geq 21$ from [11]. Hence using the results of [8], [9], [10], [11] and those given in Tables 1, 2 and 3, we have:

TABLE 2. SEQUENCES OF LENGTH 19 WITH ZERO PERIODIC AUTOCORRELATION FUNCTION

Length=19	Sequences with zero periodic autocorrelation function
1,61	{0 0 - + + - + - - a + + - + - - + 0 0}, {+++++ + - - - + - + - + + + 0 0 0}, {+ + - + + + - - - + + - + - + 0 0 + 0 0}, {- + 0 - - - + + - + + + + - - - 0 0}
1,67	{+ - - - 0 + + - + a - + - - 0 + + + -}, {0 + + + + + - + + + 0 + - + + - - - -}, {- + + + + - + - + + - - - + - + 0 + 0}, {- 0 + + - + + + - + + + + - - - - 0}
1,69	{0 + - - + + + - + a - + - - - + - 0}, {- + + + + + - - - + - + - - + 0 0 + -}, {+ + - 0 + - + - - + - - - + + + + -}, {- 0 + + + + + + + - + - + - + + + -}
1,70	{+ + + + + - + - + a - + - - - - -}, {+ 0 - + - + + - - + - + + + - + + +}, {+ + - + + + - - 0 + + + - - + + - 0}, {- + + + 0 - + + - + + + - 0 - - - + +}
1,1,74	{+ + - + - + + + - a + - - - + - + - -}, {- + - - + - - - - b + + + + - + + - +}, {- + + - - - - - + + + - - - - + + -}, {- + + - + - + + + + + + - + - + + -}

Theorem 2. *There exists an orthogonal design $OD(4n; 1, k)$ when*

- (i) *for $n \geq t$, $t = 3, 5, 7, 9$ with $k \in \{x : x \leq 4t - 1, x = a^2 + b^2 + c^2\}$;*
- (ii) *for $n \geq 11$, with $k \in \{x : x \leq 43, x = a^2 + b^2 + c^2, x \neq 42\}$;*
- (iii) *for $n \geq 13$, with $k \in \{x : x \leq 51, x = a^2 + b^2 + c^2\}$;*
- (iv) *for $n \geq 15$, with $k \in \{x : x \leq 59, x = a^2 + b^2 + c^2\}$;*
- (v) *for $n \geq 17$, with $k \in \{x : x \leq 67, x = a^2 + b^2 + c^2, x \neq 61, 66\}$;*
- (vi) *for $n \geq 19$, with $k \in \{x : x \leq 75, x = a^2 + b^2 + c^2, x \neq 61\}$.*
- (vii) *for $n \geq 21$, with $k \in \{x : x \leq 83, x = a^2 + b^2 + c^2, x \neq 61, 77, 78, 82\}$.*

TABLE 3. SEQUENCES OF LENGTH 21 WITH ZERO PERIODIC AUTOCORRELATION FUNCTION

Length=21	Sequences with zero periodic autocorrelation function
1,57	{---+--000-a+000+---+}, {++000+++---+---000-++}, {+000-+++0+0-+-+---+0-0}, {+-0-+0+0+---00-+00+-}
1,61	{+++--+0-+0+a-0-+0-+---}, {+-+--+---+0+++--+0+000}, {+++--+0++++---+0-++0000}, {+00-000-++0-+---+0++}
1,62	{---+0-0-0a0+0+0-+++}, {+0+-+---0+0+0+++--+}, {+++0-+---00+-0+-0+-0+}, {0+-+---+0-00-+-+0+++}
1,67	{0-+--+0+-a+-0+---+0}, {-0-+--+0+0+0+-+---+}, {+0+-+---00-+---+0-0}, {00-+---+---+---+---+0-0}
1,73	{+++++---+0a0+---+---}, {-+-0+++--+0+++0-+---}, {+-0+++--+0-+-+0-+---}, {-+---+---+---+---+0+0+}
1,74	{+---+---+---a+---+---}, {-+---+---+00+---+---+---}, {+000-+---+---+---+---+}, {+-+---+00+---+---+0+0-+}
1,75	{+-+---+---+0a0-+---+---}, {-0+0+---+---+---+---+}, {-+---+---+---+---+00+-}, {+-+---+---+---+---+00-}
1,76	{+---+---+---a+---+---+}, {-0-0+++++---+---+0+-}, {-+---+---+---+---+0+-}, {+0-0+---+---+---+0++}
1,77	{-+---+---+0-a+0-+---+}, {-0+++++---+---+---+}, {+0+++0+-+---+---+---}, {-+---+0-+---+---+---+}
79	{+-+---+---+---+---+0+---}, {0+++---+---+0+-+---+}, {-+---+---+---+---+0+-}, {+-+---+---+---+---+}
1,1,82	{-+---+---+---a+---+---+}, {++++---+---+---+---+}, {-+---+---+---+---+---+}, {-+---+---+---+---+---+}

(viii) for $n \geq 23$, with $k \in \{x : x \leq 91, x = a^2 + b^2 + c^2, x \neq 61, 77, 78, 82, 85, 86, 89, 90, 91\}$.

(ix) for $n \geq 25$, with $k \in \{x : x \leq 99, x = a^2 + b^2 + c^2, x \neq 61, 77, 78, 82, 85, 86, 89, 90, 91, 92, 93, 94, 97, 98, 99\}$.

All are constructed by using four circulant matrices in the Goethals-Seidel array.

Proof. We have the results for $n \geq 13$ for all k except 46 and 49 from [11]. The $OD(52; 1, 46)$ is given in [10, Table 2]. An $OD(52; 1, 49)$ is given in [5, Theorem 8.38]. The $OD(4n; 1, 46)$ and $OD(4n; 1, 49)$ for $n \geq 15$ are given in Table 1 of [8].

TABLE 4. SEQUENCES OF LENGTH 21 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=21	Sequences with zero non-periodic autocorrelation function
1,68	{++ 0+-+0+- a +-0-+-0-}, {-+-+--+0--0+0---+-+}, {-+-+--+0--0--0+--+}, {++ 0+-+0+-0-+0+--+0+}
1,72	{0+-+--+--+ a ---+--+0}, {+0+-+--+0---+--+0+}, {+0+-+--+0+--+---0-}, {0+-+--+--+0+--+--+0}
74	{-+0+--+0+0-+--+0---+}, {-+0+--+0+0+--+0+--+}, {-+-+--+0+--+--+}, {-+-+--+0+--+--+}
1,76	{+++0---+ a -+--+0---}, {-+++--+--+0+--+--+}, {-+++--+--+0+--+--+}, {+++0---+--+0+--+--+}

The existence of $OD(4n; 1, 57)$ $n \geq 15$ and $OD(4n; 1, 67)$ for $n \geq 17$ is established in [9].

For $n \geq 17$ [11] gives the result except for $k = 46, 49, 61, 62$. The result for 62 is given in Table 1. An $OD(68; 1, 66)$ does not exist as its existence would imply the existence of an $OD(68; 1, 1, 66)$ by the Geramita-Verner Theorem [5, Theorem 2.20] and the existence of that design requires that 66 should be the sum of two squares which it is not. Hence the $OD(68; 1, 66)$ does not exist. The value 60 was erroneously included in [11] Theorem 8: it should not have been included as 60 is a number of the form $4^a(8b + 7)$ and so it cannot be written as three squares as required by the conditions of Conjecture 2.

$OD(4n; 1, k)$ for $k = 70$ exist for $n = 19$ from Table 2, $k = 21$ from [9], and for $n \geq 23$ from Table 5.

TABLE 5. SEQUENCES OF LENGTH 23 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=23	Sequences with zero non-periodic autocorrelation function
1,70	{0-+0++++-+ a -++---- 0-+0}, {-++0+0+0+-+ 0-++ 0-++ 0+--}, {-++0+0+0+-+ 0+-- 0+-- 0-++}, {0-+0++++-+ 0+--++++ 0+-0}
1,74	{-0-00++-++- a +---+-00+0+}, {++++0++++-+ 0-+-+++-+}, {++++0++++-+ 0+-+---++-}, {-0-00++-++- 0-+-+00-0-}
1,84	{+-+--++-- a +++--+-+}, {+-+0-++++ 0-+-+ 0++++}, {+-+0-++++ 0+-+ 0----}, {+-+--++-- 0----++-+-+}
1,88	{-+-+--++++ a ----+-+--}, {+-+--++--++++ 0-+-+----}, {+-+--++--++++ 0+-+----}, {-+-+--++++++ 0++++-+-+}

$OD(4n; 1, k)$ for $k = 68, 72$ and $n \geq 19$ are given in Table 1, the result for $k = 66$ and 68 with $n \geq 19$ is given in [5, Table H.2]. $OD(4n; 1, 76)$ for $n \geq 21$ is given in Table 4 and $OD(4n; 1, k)$ for $k = 68, 72$ and 83 are given in [9]. The $OD(4n; 1, k)$ for $k = 84, 88$ and $n \geq 23$ are given in Table 4. The $OD(4n; 1, 75)$ for $n = 21$ is from Table 3 and for $n \geq 23$ from Table 5.

The $OD(4n; 1, k)$ for $k = 69, 74$ and 75 exist for $n = 19$ from Table 2 and for $n \geq 21$ from [9]. The $OD(76; 1, 73)$ is given in [8] and the $OD(4n; 1, 73)$, $n \geq 21$, in [9].

The $OD(4n; 1, 96)$, $n \geq 25$ is given in Table 6.

TABLE 6. SEQUENCES OF LENGTH 25 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=25	Sequences with zero non-periodic autocorrelation function
1,96	{+-+--++++- a +++--+-+}, {++++-+--+++++0+---+--+}, {++++-+--+++++0-+++-+--}, {+-+--++++- 0----++-+-+}

Theorem 3. *There exists a $W(4n; k)$ when*

- (i) for $n \geq t$, $t = 3, 5, 7, 9, 11, 13, 15, 17, 19$ with $k \in \{x : x \leq 4t\}$;
- (ii) for $n \geq 21$, with $k = 1, \dots, 78, 80, 81, 82$;
- (iii) for $n \geq 23$, with $k = 1, \dots, 78, 80, 81, 82, 85, 86, 88, 89, 90, 92$.

All are constructed by using four circulant matrices in the Goethals-Seidel array.

Proof. We use Theorem 2, the $W(4t, 4t)$ and $W(4t, 4t - 1)$ from [15] and Theorem 3 from [11].

The result for 75 in (i) comes from Theorem 1, Tables 3 and 5 and for 77, in (ii) from Theorem 1 and from Table 4. The result for 78, 80, 81, 82 and 84 in (ii) comes from [11, Table 8] as does the result for 86, 88, 90 and 92 in (iii). Table 5 gives 71, 85 and 89 of (iii). \square

Lemma 1. *The necessary conditions are sufficient for the existence of $OD(4n; 1, k)$ for $n = 3, 5, \dots, 19$ and $k \leq 4n - 1$. All are constructed from four circulant matrices in the Goethals-Seidel array.*

Proof. Use Theorem 2 for all the results for $n \leq 11$.

The $OD(52; 1, 46)$ and $OD(60; 1, 57)$ required are given in Tables 2 and 4 of [10].

The $OD(68; 1, k)$ for $k = 57, 61$, and 67 are given in [11, Table 6] and [10, Table 6]. An $OD(68; 1, 66)$ does not exist as was shown in the proof of Theorem 2 above. The value 60 was erroneously included in [11]: an $OD(68; 1, 60)$ does not exist.

The $OD(76; 1, k)$ for $k = 57, 61, 66, 67, 69, 70, 73, 74$ and 75 are given in Tables 2 and 3 of [8], and Table 2 of this paper. This gives us the result of the enunciation. \square

Lemma 2. *There exists an $OD(84; 1, k)$ for $k \in \{x : x \leq 83, x = a^2 + b^2 + c^2, x \neq 70, 78\}$ which are undecided. All may be constructed using four circulant matrices in the Goethals-Seidel array.*

Proof. The results for $k = 57, 61, 67, 73, 75, 77, 82$ and 83 which may be constructed using four circulants in the Goethals-Seidel array are given in Table 3 of this paper. \square

Lemma 3. *There exists an $OD(92; 1, k)$ for $n \geq 23$, with $k \in \{x : x \leq 91, x = a^2 + b^2 + c^2, x \neq 61, 77, 78, 82, 85, 86, 89, 90, 91\}$ which are undecided. \dagger*

Proof. Use Theorem 2. \square

Lemma 4. *There exists a $W(4n, k)$ for $k \in \{x : 0 \leq x \leq 4n\}$ with $n = 1, 3, \dots, 21$. All are constructed from four circulant matrices in the Goethals-Seidel array.*

Proof. Follows from [11], Lemma 1 and Tables 1 and 2. $k = 71$ is given in [9]. The sequences for $W(84, 79)$ and $W(84, 77)$ are given in Tables 3 and 4. \square

Lemma 5. *There exists a $W(92, k)$ for all k except possibly 79, 87, which are undecided. All are constructed by using four circulant matrices in the Goethals-Seidel array.*

Proof. The paper [11] gives all k except possibly 71, 73, 75, 77, 79, 83, 85, 87 and 89. Koukouvinos [9] gives $k = 83$. The sequences for 71, 73, 75, 77, 85, and 89 are given in Tables 2, 4 and 5. \square

Lemma 6. *There exists a $W(100, k)$ for all k except possibly 87, 91, 93, 95 which are undecided. All are constructed by using four circulant matrices*

in the Goethals-Seidel array.

Proof. The paper [11] gives all k except possibly 71, 77, 83, 87, 89, 91, 93, 95, 97. Koukouvinos [9] gives $k = 83$. The sequences for 71, 77 and 89 are given in Tables 4 and 5. The $OD(100; 1, 96)$ in Table 6 gives $k = 97$.

TABLE 7. SUMMARY OF THE CONJECTURES
TRUE SIGNIFIES THE CONJECTURE IS VERIFIED

Order	Applicable Conjecture	Unresolved Cases	Applicable Conjecture	Unresolved Cases
4	1	true	2	true
8	1	true	3	true
12	1	true	2	true
16	1	true	3	true
20	1	true	2	true
24	1	true	3	true
28	1	true	2	true
32	1	true	3	true
36	1	true	2	true
40	1	true	3	true
44	1	true	2	true*
48	1	true	3	true
52	1	true	2	true
56	1	true	3	true
60	1	true	2	true
64	1	true	3	true
68	1	true	2	true*
72	1	true	3	true
76	1	true	2	true
80	1	true	3	true
84	1	true	2	70,78
88	1	true	3	true
92	1	79,87	2	61,77,78,82 85,86,89,90,91
96	1	true	3	true
100	1	87,91,93,95	2	61,77,78,82,85, 86,89,90,91,92, 93,94,97,98*
104	1	95	3	94,95
112	1	true	3	true
120	1	true	3	true

* $OD(n; 1, n-2)$ is not possible as $n-2$ is not the sum of two squares

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