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Timoghy J. Green  
*University of Wollongong*

Wen Xu  
*University of Wollongong*

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## Population inversion in an optically pumped single quantum well

### Abstract

An optically pumped intersubband laser generator is proposed in which the continuum states above an  $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}-\text{GaAs}-\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$  single quantum well with a width of  $L=17$  nm serve as the highest level in a four-level laser system. The design allows much greater flexibility in the choice of pumping source and simplifies considerably the device fabrication. We have obtained the electronic subband structure of the proposed device and utilized a simple rate equation approach to examine the electron density in different states under optical pumping.

### Keywords

single, well, population, inversion, quantum, pumped, optically

### Disciplines

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# Population inversion in an optically pumped single quantum well

T. J. Green and W. Xu<sup>a)</sup>

*Institute for Superconducting and Electronic Materials, Department of Materials Engineering and Department of Engineering Physics, University of Wollongong, New South Wales 2522, Australia*

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An optically pumped intersubband laser generator is proposed in which the continuum states above an  $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}-\text{GaAs}-\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$  single quantum well with a width of  $L=17$  nm serve as the highest level in a four-level laser system. The design allows much greater flexibility in the choice of pumping source and simplifies considerably the device fabrication. We have obtained the electronic subband structure of the proposed device and utilized a simple rate equation approach to examine the electron density in different states under optical pumping. © 2000 American Institute of Physics. [S0021-8979(00)07317-5]

## I. INTRODUCTION

In a low-dimensional semiconductor system (LDSS), the conducting electrons are confined within nanometer distances. As a consequence, the electron kinetic energy ( $\hbar^2 k^2/2m^*$ ), the electronic subband separation ( $\varepsilon_{n'} - \varepsilon_n$ ), the Fermi energy ( $E_F$ ), the phonon energy ( $\hbar\omega_Q$ ), etc. are comparable to the energy of far-infrared (FIR) or terahertz (THz) photons ( $\hbar\omega \sim \text{meV}$ ). Therefore, in a LDSS, FIR emission and absorption can be achieved via energy transfer during electronic transition events. Furthermore, the rate of electronic scattering via coupling with impurities and phonons is also in the order of THz. This implies that FIR emission and absorption can significantly modify the processes of momentum and energy relaxation for excited electrons in a LDSS. Hence, LDSSs such as heterojunctions, quantum wells,  $\delta$ -doped layers, superlattices, etc. can be used as optical generators and/or detectors working at THz frequencies.

Recently, it has been realized that GaAs-based LDSSs can be designed to serve as FIR laser generators from which coherent electromagnetic (EM) radiation can be generated via intersubband electronic transitions. Although the present investigation into intersubband laser emission in LDSSs is dominated by the quantum cascade laser realized from semiconductor superlattices,<sup>1</sup> optically pumped intersubband lasers (OPISLs) based on polar-semiconductor quantum well structures have been proposed and studied since 1995 by several groups.<sup>2-5</sup> From a fundamental point of view, the study of OPISLs provides an opportunity to examine the excitation and relaxation of electrons in a LDSS via intersubband transition events caused by electron interactions with photons and phonons. Moreover, it has been realized that GaAs-based quantum cascade lasers cannot produce long-wavelength laser radiation because of problems with thermal management, where the laser emission is suppressed when the radiation frequency approaches that of the optical phonons. In contrast, OPISLs<sup>2-5</sup> are designed to take advantage of the electron-phonon interactions and, so, can achieve

longer wavelength laser emission. Therefore, OPISLs may provide a new source of FIR or THz semiconductor laser radiation.

A practical three-level OPISL was proposed<sup>3</sup> in 1995 using an AlGaAs-GaAs double quantum well structure and a CO<sub>2</sub> laser as the pumping source. Since then, some alternative designs, such as employing AlGaAs-GaAs step quantum wells which behave as three-<sup>4</sup> or four-level<sup>5</sup> OPISLs under CO<sub>2</sub> laser pumping, have also been proposed. In these devices the electrophonon resonance (EPR)<sup>6</sup> effect, i.e., the strongest scattering between electrons and longitudinal optical (LO) phonons occurs whenever the energy difference  $\varepsilon_{n'} - \varepsilon_n$  between subbands  $n'$  and  $n$  equals the energy of the LO phonon  $\hbar\omega_{\text{LO}}$ , is used to achieve population inversion through nonradiative electronic transitions. However, from both fabrication and application points of view, these devices suffer two major drawbacks: (i) they are not very easy to fabricate due to their relatively complicated structures. One has to control very accurately the barrier layer and Al content, respectively, for double and step quantum wells during sample growth; (ii) there is little flexibility in the choice of pumping source, since the energy of the pumping radiation must correspond to the separation between subbands and the CO<sub>2</sub> laser ( $\hbar\omega = 124$  meV) seems to be the only pumping source to make the device work.

## II. PROPOSED DEVICE

We propose a OPISL device which consists of an  $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}-\text{GaAs}-\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$  single quantum well with a GaAs layer thickness of  $L=17$  nm. We consider a typical modulation-doped structure with a donor (Si) concentration  $N_d = 2 \times 10^{18} \text{ cm}^{-3}$  in the AlGaAs layers, a spacer thickness  $s = 5$  nm measured from the AlGaAs/GaAs interfaces, and a background acceptor concentration  $N_a = 2 \times 10^{16} \text{ cm}^{-3}$  in the GaAs layer. The electronic subband structure for this device can be obtained from a standard self-consistent calculation<sup>7</sup> and the resulting potential profile is shown in Fig. 1.

The device parameters have been chosen such that for the three subbands present,  $\varepsilon_1 - \varepsilon_0 = 37.1$  meV (close to  $\hbar\omega_{\text{LO}} \cong 36.6$  meV),  $\varepsilon_2 - \varepsilon_1 = 57.6$  meV (much larger than  $\hbar\omega_{\text{LO}}$ ), while the energy separation between the top of the

<sup>a)</sup>Electronic mail: wxu@wumpus.its.uow.edu.au

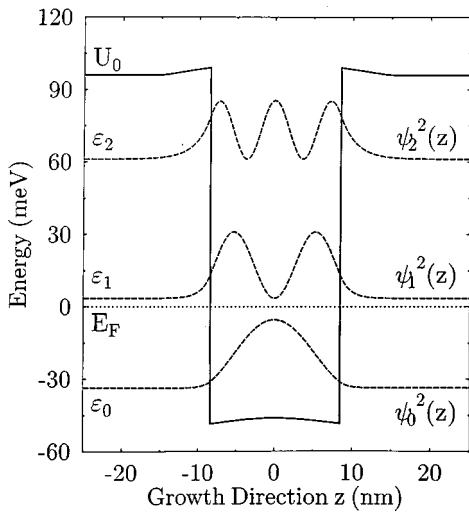


FIG. 1. Potential profile along the growth direction of an intersubband laser device based on an  $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$ - $\text{GaAs}$ - $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$  single quantum well. The results are obtained from solving self-consistently the Schrödinger equation for eigenfunction and eigenvalue and the Poisson equation for confining potential energy. The input parameters of the calculation are (1) the width of the well layer  $L=17$  nm, (2) the spacer thickness  $s=5$  nm measured from the  $\text{AlGaAs}/\text{GaAs}$  interfaces, (3) the modulation-doped donor (Si) concentration  $N_d=2 \times 10^{18} \text{ cm}^{-3}$ , (4) the background acceptor concentration  $N_a=2 \times 10^{16} \text{ cm}^{-3}$ , and (5) the average donor binding energy  $E_d=96$  meV measured from the bottom of the conduction band of  $\text{AlGaAs}$ . The output results of the calculation are (i) there are three subbands in the quantum well,  $\epsilon_0=-33.7$  meV,  $\epsilon_1=3.5$  meV, and  $\epsilon_2=61.1$  meV measured from the Fermi energy  $E_F$ ; (ii) for an  $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}/\text{GaAs}$  heterojunction, the conduction band discontinuity between  $\text{AlGaAs}$  and  $\text{GaAs}$  is  $U_0=147.5$  meV; (iii) the electron density of the system is  $n_e=9.4 \times 10^{11} \text{ cm}^{-2}$ ; and (iv) the depletion length is  $d=16.0$  nm.

well  $U_0$  and subband 2 is  $U_0 - \epsilon_2 = 36.0$  meV, again close to  $\hbar\omega_{\text{LO}}$ . In the absence of pumping only the lowest subband is occupied by electrons. With the application of a pumping field of frequency  $\hbar\omega > U_0 - \epsilon_0 = 129.7$  meV (or wavelength  $\lambda < 9.2 \mu\text{m}$ ), the device can behave as a four-level laser generator with the continuum states acting as the highest level. The device operates in the following way: (1) Electrons in the lowest subband are pumped into continuum states above the well. (2) Relaxation of electrons from continuum states above the well into subband 2 and from subband 1 into subband 0 via electron-LO-phonon emission scattering will be relatively rapid due to the electrophonon resonance effect. (3) Because  $\epsilon_2 - \epsilon_1$  and  $\epsilon_2 - \epsilon_0$  are much larger than  $\hbar\omega_{\text{LO}}$ , electrons in subband 2 cannot be scattered quickly into the lower subbands via nonradiative electronic transition channels. Thus, the electron populations in subbands 2 and 1 can be inverted and intersubband laser emission can be generated via radiative electronic transitions between these two subbands.

The principal advantages of the present laser device are as follows: (i) The device looks simpler, is much easier to fabricate and, as a result, should be produced more cheaply. (ii) The design allows much greater flexibility in the choice of pumping source. It is well known that in sharp contrast to the strict selection rules for optical pumping of electrons between bound levels in a two-dimensional electron gas (2DEG) system, bound-to-continuum pumping can be more

easily and efficiently achieved.<sup>8</sup> The theoretical results [see Eq. (5) below] indicate that the optical pumping efficiency, i.e., the rate to pump electrons from a bound level with energy  $\epsilon_n$  to continuum states, is proportional to  $I_0/\sqrt{\hbar\omega + \epsilon_n - U_0}$ , where  $I_0$  and  $\omega$  are, respectively, the intensity and frequency of the (EM) pumping field. Therefore, we can use any FIR source (not necessarily a FIR laser) as the pumping field to operate the device as long as  $\hbar\omega > U_0 - \epsilon_0 \cong 130$  meV is satisfied. (iii) Due to the simple sample structure, we can easily produce a multiple quantum well system based on this single quantum well laser to enhance optical gain, laser efficiency, laser output power, etc. Thus, the main disadvantages of the OPISL devices proposed previously can be largely overcome.

### III. THEORETICAL RESULTS

To examine theoretically the conditions under which the electron populations in subbands 2 and 1 can be inverted in the present OPISL device, we derive the rate equation from the steady-state Boltzmann equation. We consider a 2DEG system in which bound-to-bound ( $2\text{D} \leftrightarrow 2\text{D}$ ) and bound-to-continuum ( $2\text{D} \leftrightarrow 3\text{D}$ ) transitions are present. At the first moment, the mass-balance equation (i.e., the rate-balance equation or rate equation) can be derived by multiplying  $\Sigma_k$  on both sides of the Boltzmann equation. After (1) including interactions between electrons and LO phonons and between electrons and pumping field, (2) using a Maxwellian distribution function for the statistical distribution of the electrons, where the normalization factors for electrons in different levels are determined via the condition of electron number conservation, and (3) considering that the optical pumping field is linearly polarized along the growth direction of the quantum well and that only bound-to-continuum pumping is present, the rate equations for a stationary population density in the present device are

$$\sum_{j=0}^3 (n_j \lambda_{ji} - n_i \lambda_{ij}) = 0. \quad (1)$$

Together with the condition of electron number conservation,

$$\sum_{i=0}^3 n_i = n_e, \quad (2)$$

we can determine the electron density in different levels. Here, the growth direction of the quantum well has been taken along the  $z$  axis, where  $i < 3$  and  $i = 3$  represent, respectively, the bound and the continuum levels,  $n_i$  is the areal electron density in the  $i$ th level, and  $n_e$  is the total electron density. It should be noted that  $n_3 \cong LN_{3\text{D}}$ , where  $L$  is the width of the quantum well structure and  $N_{3\text{D}}$  is the volume density for electrons in the continuum, since for a quantum well structure the conduction electrons are mainly confined within the well layer. Furthermore,  $\lambda_{ij}$  is the electron number scattering rate from level  $i$  to level  $j$ . The scattering rates due to electron-LO-phonon interactions (LO) via intersubband ( $nn' < 3$ ), continuum-to-bound ( $3n$ ) and bound-to-continuum ( $n3$ ) transitions are given, respectively, by

$$\lambda_{nn'}^{\text{LO}\pm} = \alpha L_0 \omega_{\text{LO}} (2\pi m^* \beta)^{1/2} \left( \frac{N_0}{N_0 + 1} \right) \times \int_0^\infty \frac{dq_z}{q_z} G_{nn'}(q_z) F(x_1^\pm, y_1^\pm), \quad (3a)$$

$$\lambda_{3n}^{\text{LO}\pm} = \alpha \hbar \omega_{\text{LO}}^2 \beta \frac{L_0}{L} \left( \frac{N_0}{N_0 + 1} \right) \times \int_{-\infty}^\infty dk_z e^{-\beta E_{k_z}} \int_{-\infty}^\infty \frac{dq_z}{|q_z|} S_n(k_z, q_z) F(x_2^\pm, y_2^\pm), \quad (3b)$$

and

$$\lambda_{n3}^{\text{LO}\pm} = \frac{\omega}{2} \left( \frac{\hbar \omega_{\text{LO}}^3 \beta}{\pi} \right)^{1/2} \left( \frac{N_0}{N_0 + 1} \right) \times \int_{-\infty}^\infty dk_z \int_{-\infty}^\infty \frac{dq_z}{|q_z|} R_n(k_z, q_z) F(x_3^\pm, y_3^\pm), \quad (3c)$$

where

$$x_1^\pm = \beta(E_{q_z} + \varepsilon_n - \varepsilon_{n'} \pm \hbar \omega_{\text{LO}})^2 / (4E_{q_z}), \quad (4a)$$

$$y_1^\pm = \beta(E_{q_z} + |\varepsilon_n - \varepsilon_{n'} \pm \hbar \omega_{\text{LO}}|)^2 / (4E_{q_z}), \quad (4b)$$

$$x_2^\pm = \beta(E_{q_z} + E_{k_z} + U_0 - \varepsilon_n \pm \hbar \omega_{\text{LO}})^2 / (4E_{q_z}), \quad (4c)$$

$$y_2^\pm = \beta(E_{q_z} + |E_{k_z} + U_0 - \varepsilon_n \pm \hbar \omega_{\text{LO}}|)^2 / (4E_{q_z}), \quad (4d)$$

$$x_3^\pm = \beta(E_{q_z} + E_{k_z} + \varepsilon_n - U_0 \pm \hbar \omega_{\text{LO}})^2 / (4E_{q_z}), \quad (4e)$$

and

$$y_3^\pm = \beta(E_{q_z} + |E_{k_z} + \varepsilon_n - U_0 \pm \hbar \omega_{\text{LO}}|)^2 / (4E_{q_z}). \quad (4f)$$

Here, the upper (lower) case refers to absorption (emission) of a LO phonon,  $\beta = 1/k_B T$ ,  $N_0 = (e^{\beta \hbar \omega_{\text{LO}}} - 1)^{-1}$  is the LO-phonon occupation number,  $\alpha$  is the electron-LO-phonon coupling constant ( $\alpha = 0.688$  for GaAs),  $L_0 = (\hbar/2m^* \omega_{\text{LO}})^{1/2}$ ,  $m^*$  is the effective electron mass ( $m^* = 0.068m_e$  for GaAs, with  $m_e$  being the rest electron mass),  $E_x = \hbar^2 x^2 / 2m^*$ , and  $F(x, y) = e^{-x} \text{erfc}(\sqrt{y})$  with  $\text{erfc}(y)$  being the probability integral. In Eq. (3)  $G_{nn'}(q_z) = |\langle n' | e^{iq_z z} | n \rangle|^2$ ,  $S_n(k_z, q_z) = |\langle n | e^{iq_z z} | k_z \rangle|^2$ , and  $R_n(k_z, q_z) = |\langle k_z | e^{iq_z z} | n \rangle|^2$  are, respectively, the form factors for electron-LO-phonon coupling via bound-to-bound, continuum-to-bound and bound-to-continuum transitions. Moreover, the rate to pump electrons from a bound level  $n$  to the continuum level 3 is

$$\lambda_{n3}^{\text{op}} = \frac{4}{\hbar m^*} \left( \frac{eF_0}{\omega} \right)^2 \frac{\Theta(x)}{\sqrt{x}} H_n(\sqrt{x}), \quad (5)$$

where  $x = 2m^*(\varepsilon_n - U_0 + \hbar \omega) / \hbar^2$ ,  $F_0$  is the electric field strength of the pumping EM field,  $\Theta(x)$  is the unit step function, and  $H_n(k_z) = |\langle k_z | \partial / \partial z | n \rangle|^2$  is the form factor for electron interaction with the EM field for bound-to-continuum transitions.

In order to save CPU time, for the numerical calculations we take  $|k_z\rangle = e^{ik_z z}$  while  $|n\rangle$  and  $\varepsilon_n$  are obtained by solving the Schrödinger equation for a rectangular single quantum

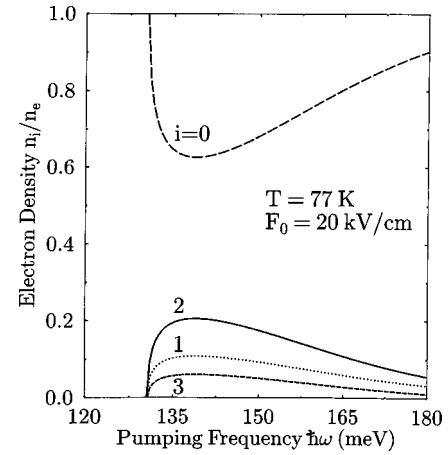


FIG. 2. Electron density in different levels  $n_i$  vs frequency of the pumping field at a fixed pumping intensity  $F_0$  and a fixed temperature.  $n_3$  is the averaged areal electron density in the continuum states above the quantum well and  $n_e$  is the total electron density.

well with  $L = 17$  nm and an Al content of 20%. The electron density  $n_i$  in different levels is shown in Fig. 2 as a function of pumping frequency at a fixed pumping intensity  $F_0$  and a fixed temperature. We see that when the condition  $\hbar \omega + \varepsilon_0 - U_0 > 0$  is satisfied, electrons in the bound levels can be pumped into the continuum states and the electron populations in subbands 2 and 1 can be inverted via electron-LO-phonon scattering. Due to the fact that the electron-photon scattering rate through bound-to-continuum transitions exhibits a much broader maximum than is observed for inter-subband optical pumping,<sup>8</sup> bound-to-continuum pumping can be effective over a wide range of frequencies. We find that the optimum pumping frequency is  $\hbar \omega \cong 139$  meV corresponding to a wavelength  $\lambda \cong 8.9 \mu\text{m}$ . The electron density  $n_i$  is plotted in Fig. 3 as a function of pumping intensity  $F_0$  at a fixed pumping frequency  $\hbar \omega = 139$  meV and a fixed temperature. When  $\hbar \omega + \varepsilon_0 - U_0 > 0$ , bound-to-continuum pumping can also be effective over a wide range of intensity and the magnitude of the population inversion between subbands 2 and 1 becomes larger with increasing  $F_0$ . The de-

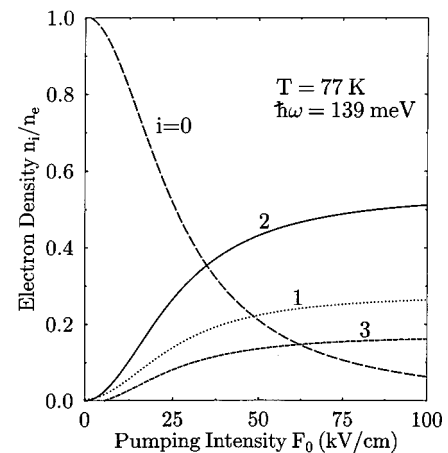


FIG. 3. Electron density in different levels vs intensity of the pumping field at a fixed pumping frequency and a fixed temperature.



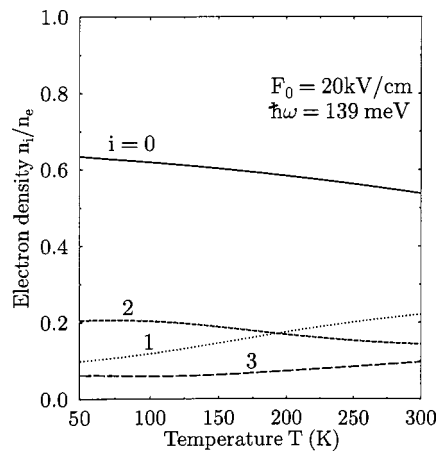


FIG. 4. Dependence of the electron density in different levels on temperature at a fixed pumping field with frequency  $\omega$  and electric field strength  $F_0$ .

pendence of the electron density  $n_i$  on temperature is shown in Fig. 4 at a fixed pumping field. Here we see that the population inversion between subbands 2 and 1 can only be achieved when  $T < 195$  K at  $\hbar\omega = 139$  meV and  $F_0 = 20$  kV/cm. At higher temperatures population inversion cannot be maintained due to the increased probability of LO-phonon absorption induced transitions from subbands 0 to 1 and from subband 3 to the continuum. With increasing intensity of the pumping field, the regime within which the population inversion can be observed is extended to higher temperatures. Our calculations show that a pumping field of  $F_0 \geq 35$  kV/cm is required for room temperature operation.

#### IV. CONCLUSION

In this article, we have proposed an OPISL device based simply on a single quantum well structure which has been designed to allow greater flexibility in the choice of pumping

sources. The results obtained from calculations show that a FIR source with  $\hbar\omega > 130$  meV (or wavelength  $\lambda < 9.2$   $\mu\text{m}$ ) and  $F_0 \cong 10$  kV/cm (or optical power  $I \cong 10$  kW/cm<sup>2</sup>) would be sufficient to operate the device as an intersubband laser with a frequency  $\hbar\omega \cong 57.6$  meV (or  $\lambda \cong 21.5$   $\mu\text{m}$ ). This wavelength is much longer than those generated by GaAs-based QCLs which are typically<sup>9</sup> limited to  $\lambda < 5$   $\mu\text{m}$ . For a field strength of  $F_0 = 20$  kV/cm, we find that population inversion can be maintained for temperatures up to  $T = 195$  K. In order to operate the device at room temperature, however, a pumping intensity of at least  $F_0 = 35$  kV/cm is required. It is our hope that the simple OPISL device proposed in this article can be realized and tested experimentally.

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