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Mohan Chinnappan
University of Wollongong, mohan@uow.edu.au

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Introduction

Debates about children's understanding of mathematics has focused on how children make use of school mathematics in their every day lives. The notion of numeracy has been gaining increasing currency is recent curriculum reforms that support the need for mathematics and mathematics teaching to pay attention to involving children in the learning of mathematics in ways that will make sense to each child. Children's everyday life abounds with mathematics objects and concepts. An awareness of the mathematics in the environment is a function of students' ability to interpret the relevance of both the formal and informal mathematics they have been experiencing in the classroom. Regardless of the context in which children engage with mathematics, children's ability to reason and make decision with mathematics provide teachers insight into developments in children's understanding. These understandings are built on children's personal experiences, intuitions and formal knowledge taught in the classroom.

The development of numeracy skills have been received high priority in recent years. For example, the National Numeracy goal states children should attain a satisfactory level of numeracy (DETYA, 2000). While there are different ways to visualise numeracy, broadly, it refers to the ability and disposition of children to use mathematics effectively to meet the demands of life at home and work (McIntosh, 1990; Willis, 1998, 2000). This includes 'fundamental mathematical concepts needed to access personally new mathematical ideas, and the confidence and competence to make sense of mathematical and scientific arguments in decision-making situations' (Willis, 1990: 22). Thus, to be numerate is to be able apply mathematics beyond the classroom and across curriculum. More importantly, from the literacy perspective, numeracy demand that children construct and sustain arguments during the course of hypothesizing and generating solutions to both classroom-based and real-life problems.

Practice That Scaffolds Numeracy

The development of the foregoing numeracy outcomes among our children would depend on a range of factors not least the quality of meaningful learning experiences provided by their teachers during formal and informal classroom contexts. This line of argument brings into focus the critical role of teachers and their pedagogical decisions in the design of appropriate lessons and consequent rich learning experiences that would engage children in ways that would optimise engagement with mathematical concepts and their use to solve a range of problems. But the role of literacy skills in the process of engagement has received less attention.

As young children bring their own understandings to the learning situation, teachers need to be skilful in harnessing these unique understandings of individual learners in the development of lessons such that all children gain access to the range and depth of fundamental mathematics concepts that Willis refers to. Recent research that had focused on immersion and guided learning (Carpenter, Fennema, Franke, Levi and Empson, 2000) had yielded important insights into the role of scaffolding in effective teaching practices. Research has shown that good teaching plays a central role in scaffolding the numeracy development of young children but often the complexity of this task is underestimated. In particular, studies of scaffolds seem to downplay the semiotic systems that are used by children in making meaning.

Mathematics educators and teachers have invested considerable effort in exploring instructional strategies that would help children develop better grasp of mathematical concepts. One stream of inquiry about teaching approaches has focused on teaching practices that aid in the construction of powerful and meaningful understanding of mathematics and its utility. Developments in cognitive psychology and domain expertise have yielded further significant insights into what we mean by powerful understandings. Mathematics curriculum reforms call for teaching and learning experiences that optimise the development of substantive understanding of mathematics (NCTM, 2000). The question remains as to how we can characterize this type of understanding? Discussions about understandings and the manifestation of understandings within rich domains such as mathematics have focused on the structure of knowledge that is constructed by students and its impact on meaning making. The notion of structure implies the existence of links among strands of knowledge. Prawat

(1989) argued that a well-organized knowledge structure is indicative of sophisticated understanding of the subject matter knowledge. This type of knowledge is qualitatively superior to one that is less well organized and aids in the accessing and use of that knowledge during problem solving and other performances. The assimilation of new mathematical knowledge with prior knowledge raises questions about the contents and nature of links teachers expect their students to construct.

In the past year or so, debate among teacher educators about best classroom practices have contributed to the emergence of a model of teaching called productive pedagogies. This model incorporates a series of important dimensions that has been shown to be important for students' active involvement in the learning process and attain the type of understanding envisaged by proponents of deep learning. While discussions continue about the efficacy of this model, it is acknowledged that productive pedagogies do provide a useful framework that aid teachers bring greater level of focus to the provision of learning opportunities which would assist students extend prior knowledge in useful ways. The use of this framework to guide decisions about teaching is expected to aid teachers to think deeply about the mathematics in question and consider the different ways students could come to make decisions on the basis of mathematics discussed in the classroom and beyond. Teacher could draw on productive pedagogies in order to examine the nature of students' engagement with mathematics and understandings that are constructed as a consequence of learning experiences provided in the classroom. These reflections could provide fruitful directions concerning future strategies to help move the students to higher planes of understandings. While the framework identifies numerous important themes, in this paper I will examine the role of communication in the construction of connections in mathematics and its relationship to numeracy.

Connectedness in Mathematics

The establishment of links among the apparently disparate mathematics entities can be analyzed within the framework of connectedness. Mayer (1975) examined the notion of connectedness as involving the accumulation of new information in long-term memory, adding new nodes to memory and connecting the new nodes with components of the existing network. He identified two types of connectedness: internal and external. *Internal connectedness* refers to the degree to which new nodes of information are connected with one another to form a single well-defined structure or schema. This sense of connectedness refers to both the presence of nodes related to a schema and the quality of the relationships established among those nodes. The broad notion of quality here can be related, in part, to what Anderson (2000) refers to as 'strength' of a memory trace. Seen in this way, the stronger the connections among the nodes in a particular schema, the better the quality of that well-defined structure.

Mayer (1975) visualized *external connectedness* as the degree to which newly established knowledge structures are connected with structures already existing in the learner's knowledge base. For example a student might be expected to relate a schema for proportion with schemas for ratio or fraction. These external connections between proportion and ratio or fraction will have a certain quality that would impact on children's ability to use them in order to solve problems or provide alternative representations. In such a case, the new schema for proportion will have both a certain quality in its internal structure (internal connectedness) and a certain quality in its connections to related schemas (external connectedness). This analysis of connectedness was used to support the argument that the linking of the different pieces of knowledge of geometry and algebra reflect deeper and richer understandings (Chinnappan, 1998; Chinnappan and Thomas, 2003). Mayer's analysis of connectedness, while useful for the exploration of organizational features of conceptual aspects of mathematical understanding, is somewhat limited in that it does not identify the mechanisms that aid children's construct the type of links that results in one of the two form of connectedness. It is here that pedagogies that are productive need to focus on the linguistic and situatedness aspects of numeracy. Let us now consider these two dimensions in the context of children learning about fractions that would lead them to be numerate.

Connectedness in Fraction Number Knowledge

Several attempts have been made to capture the complexity of fraction numbers and children's sense of these numbers. The most detailed analysis of fraction numbers was undertaken by Kieren (1988). His analysis showed that the fraction number knowledge consists of many interwoven strands. He identified eight levels in his description of fraction number knowledge. This is a hierarchical model in which the higher levels of knowledge are based on developments at lower levels. An important outcome of this model is the specification of concepts that act as building blocks for the maturing of understanding of fraction numbers among young children. These structures, which appear at levels three and four consist of *sub-constructs*: partitioning, unit forming, quotients, measures, ratio and operations. These sub-constructs among others play a key role in young children's interpretation of fractions and the use of this understanding to interpret real-life situations.

Kieren's framework is useful in that it one is able to identify the internal and external links or connectedness that is specific to this domain of primary mathematics. The links are important for the analysis of

the organizational quality of the knowledge that drives children's representation of fractions in a range of contexts. A further advancement of the model is that it identifies the array of prior knowledge that young children bring to their constructions of formal and informal representations of fractions. I argue that children's domain knowledge base for fractions is better analyzed with reference to this model as it details the structure as well as the relations among the sub-constructs and their components.
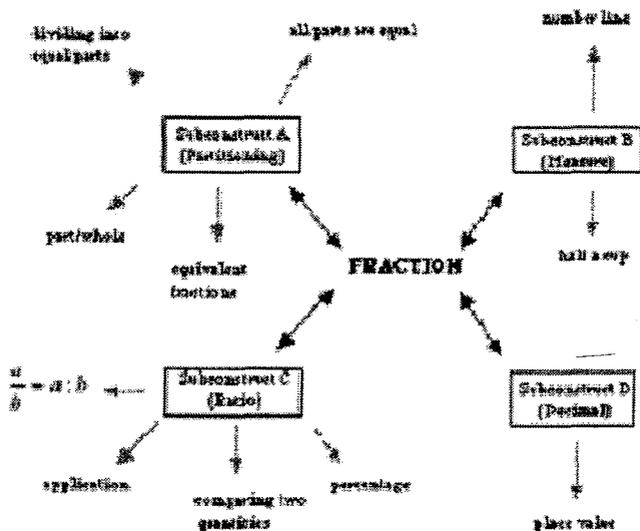

Figure 1: Fraction schema

Kieran's sub-constructs provide important conceptual points at which to anchor learning activities leading to the development of numeracy involving fraction numbers. Let us explore the characteristics of such actions and activities that teachers could draw upon that would encourage children to talk and reflect on. At the bottom left-hand corner of Figure 1, subconstruct C (Ratio) includes a component, a/b = a:b.

Here is an opportunity for the teacher and the children to question why these two modes of indicating ratios are the same. The questions could lead to discussions and debate about the role of conventions denoting not only in fractions but in other contexts. If a/b is used to compare the *relative* size of two quantities, what are some of the quantities that can be compared? For example, in my class the ratio of girls to boys is 3:2. What does this mean? Children could be asked to pose questions and seek answers. Posing questions and providing potential solutions could be done in written or spoken mode. 3:2 is a part-to-part ratio. How can this ratio be transformed to express the part-to-whole relationship in this context? Children could explore the connection between relative size and the total number of children. Further discussions along these lines could be used to shift children's thinking to equality of ratios and proportions. For example, children could be asked to decide if ratios a:b and ar:br represent the same relative amounts? If so why? This could present the context to make external connections with equivalency of fractions (Sub-construct A).

Connectedness and Mathematics Classroom Culture

A schema-based analysis of connectedness vis-à-vis fractions needs extension with a view to disentangling the complexity of personal experiences of individual students as they grapple with mathematics and develop multiple understandings. Students' engagement with mathematics is not confined to classroom actions but would influence their cognition outside the classroom. For example, a child's understanding of slope would influence and influenced by the constructions made from the classroom instruction about fractions. There is a widely accepted view that students do attempt to make sense of school mathematics by examining real, practical or hypothetical problems beyond the classroom. A solid understanding of core concepts could better facilitate this integration of classroom mathematics with real-life problems – an important requirement for children to become numerate (Willis, 2000).

Perspectives about how children come to know and make sense of mathematics have contributed to the emergence of the socio-constructivist theoretical framework. According to this framework mathematics learning is seen as an individual as well a shared activity during which students should be encouraged to

investigate, argue, justify and test conjectures (Cobb, 1995). Such activities need a classroom-learning environment that supports children to question what teachers and peers say about a particular mathematics concept. Teaching approach needs to move away from the transmission mode towards one that fosters free and open inquiry and debate. The social context in which learning takes place is as important as the concepts themselves in the growth of the type of understanding that are constructed by the learners (Lee, 1998).

Pedagogies that bring into focus the active and shared meaning-making aspect of mathematics learning would also practice different norms in the classroom. In these mathematics classrooms the rules of student behaviour and mathematical discourse are negotiable. Students are offered multiple opportunities to engage constructively and critically with mathematical ideas and teachers need to display an understanding of the learning environment for each student in the class including their background and beliefs about what it means 'to do mathematics.' Thus, the scaffolding of learning has emerged to be a core activity for teachers and students working in group-based activities.

But how does this type of learning environment contribute to connectedness and deep learning? Clearly, the higher level of input especially from students during their critical evaluation of mathematical concepts would help individual children to reflect on their own understandings and reconstruct new links and nodes in their existing schemas. The new perspectives that debate and problem solving brings to the examination of a focus concept can be expected to aid in children building the elements that are necessary for the growth of external connectedness. For example, young children working on the fractions will be motivated to explore meaningful contexts where part-whole relations that are embedded in the fraction number are given concrete and richer representations. Thus, an open-ended and an interpretive view of mathematics help catalyze the construction of powerful connections by the learner. However, this view of learning as shared meaning making, while it is consistent with the framework of productive pedagogies, raises further questions about how we can make judgments about what constitutes 'good' links.

Connectedness and Language in Mathematics

Communication is an important prerequisite for students to develop competencies in problem solving, conjecturing and reasoning in mathematics. An important tool for communication in the mathematics classroom is language. Wertsch (1998) suggested that language is a cultural artifact or tool that mediates individual's cognition in mathematics. The mathematical language that is used by the mathematical community uses special words, symbols and representations with meanings that are different from their meaning in every-day language. The vocabulary of mathematics of mathematics that is used by mathematicians, mathematics textbooks and, to a large extent, mathematics teachers, tends to be at variance and conflict with that used by children. While mathematicians may understand each other, children often experience difficulties with the language of mathematics and every-day language (Sfard and Kieran, 2001). One of the difficulties here is that children attempt to extract the meaning that is embedded in what teachers and textbooks say in their own mathematical language. For example the notion of *function* and *roots* as used by the mathematical community has a special meaning in comparison to the every-day meaning of function. Through a process of acculturation students come to discriminate the duality of meanings that is associated with these two terms.

The failure of pedagogies to recognize learning mathematics as problems concerning language and communication exacerbates the situation for certain groups of students. Students might form the view that they have not grasped what is being taught when in fact the problem could be their difficulty to communicate their understandings and intuitions into mathematically precise statements. Many students' problems with mathematical understandings that are related to the solution of word problems can be attributed to problems of language. I see this tension between mathematical language and child's own language as impeding if not preventing the construction of important connections that are necessary for the growth of deeper levels of understanding. This gap in communication between teachers and students is further complicated by the multiple and idiosyncratic interpretations that children construct about a particular mathematical concept and their own linguistic backgrounds (Clarkson, 1992). The notion of ratio again provides us with a graphic example of this.

Situated Learning and Connectedness

While the cognitivists focus on the way children process information, the need to examine how and why individual students' construct different mathematical understandings had led educators to examine the situated nature of mathematical learning. There is now an emerging consensus that learning and the quality learning is a function of the context in and the activity through which learning takes place. The failure to see this important link between school mathematics and real-life mathematics has contributed to many of the difficulties experienced by children (Ginsburg and Allardice, 1984).

The situated view of learning is seen as participation in social activity. From this perspective, mathematical knowledge is regarded as something that the learner constructs within social contexts and grows

within that context as opposed to bits of information that lies inside one's head. The acquisition of knowledge of mathematical concepts, procedures and conventions, and the transformation of this knowledge is argued to occur in a complex socio-cultural environment. The social practice in which the participant engages in moulds the child's perceptions, understandings and mathematical realities.

Understanding mathematical learning as it emerges in activity is central to the situated view of learning. A key assumption here is that as teachers and students engage in learning activities, the tools that are employed during these activities structures the nature of student participation and meanings constructed by the participants (Lave, 1988; Wertsch, 1998). Any knowledge that an individual develops is seen as a product of and anchored by the tools that drive an activity. The relations between the members of the community are mediated by a variety of socio-semiotic resources which influence the meanings that individual develop through the course of interactions. For example, diagrams and the conventions that are used the construction of diagram constitute such resources (Saenz-Ludlow and Walgamuth, 2001). The framework of activity is consistent with that of productive pedagogies as it permits the teacher understand the dynamics and patterns of social interactions, which are, involved in the construction of meanings and identities that students and teachers develop with mathematics and its evolution.

Conclusion

From the perspective of connectedness, the building of deep understandings of mathematics for numeracy can be analyzed in terms of the quality and robustness of links that form the network of knowledge. Understanding is a developing phenomenon that can be characterized by the network of connections that building of which are facilitated by a range of literacy skills including visualizing, questioning and situating understandings in real-life contexts. Connections that are weak and fragile indicate a surface level understanding.

In order for children to build strong and well organized mathematical links, learning experiences need to focus not only on the development of representations of mathematics but also the construction and articulation of links among these representations through multiple literacies. With linguistic scaffoldings and appropriate patterns of communication children can achieve deeper understandings of mathematical problems and ideas than would be the case otherwise. But teachers need to embrace the framework of productive pedagogies in a critical manner with the view to analyzing the core dimensions including the construction of connections that reflect deep learning. Such practices indicate ' intelligent and adaptive action' (Shulman and Shulman, 2004: 263)

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