New results with near- Yang sequences

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Abstract

We construct new TW -sequences, weighing matrices and orthogonal designs using near-Yang sequences. In particular we construct new OD(60(2m + 1) + 4t; 13(2m+ 1), 13(2m+ 1), 13(2m+ 1), 13(2m+ 1) and new W(60(2m+ 1) + 4t; 13s(2m+ 1))for all t ≥ O, m ≤ 30, s = 1,2,3,4.

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New Results with Near-Yang Sequences

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Abstract. We construct new TW-sequences, weighing matrices and orthogonal designs using near-Yang sequences. In particular we construct new $OD(60(2m+1)+4t; 13(2m+1), 13(2m+1), 13(2m+1), 13(2m+1))$ and new $W(60(2m+1)+4t; 13s(2m+1))$ for all $t \geq 0$, $m \leq 30$, $s = 1,2,3,4$.

1. Introduction

For definitions we refer the reader to [9, Introduction] and [11, Section 2]. We give one new definition.

Definition 1. (near-Yang sequences) *A triple (F, G, H) of sequences is said to be a set of near-Yang sequences for length n (abbreviated as $NY(n)$) if the following conditions are satisfied.*

(i) $F = (f_k)$ *is a* $(0,1,-1)$ *sequence of length* n.

(ii) $G = (g_k)$ *and* $H = (h_k)$ *are sequences of length* n *with entries* $0, 1, -1$, *such that* $G+H = (g_k + h_k)$ *and* $G - H = (g_k - h_k)$ *are both* $(0,1,-1)$ *sequences of length* n.

(iii)

$$\begin{aligned} g_s + g_{n-s+1} &\equiv 0 \pmod 2 \\ h_s + h_{n-s+1} &\equiv 0 \pmod 2 \end{aligned} \qquad s = 1, \dots, [\tfrac{n}{2}]$$

(iv) $N_F(s) + N_G(s) + N_H(s) = 0, \quad s = 1, \dots, n-1.$

where

$$N_X(s) = \sum_{i=1}^{n-s} x_i x_{i+s}.$$

2. Computational Results

In Koukouvinos, Kounias, Seberry, Yang and Yang [6] it is shown that if in (ii) of the definition $G \pm H$ are both $(1,-1)$ sequences then conditions (i), (ii) and (iv) imply condition (iii) but this is not true for near-Yang sequences. These sequences are normal sequences $NS(\ell)$.

We searched for normal sequences $NS(\ell)$. $NS(\ell)$ do exist for the following lengths $\ell \in \{1,2,3,4,5,7,8,9,10,11,12,13,15,16,18,19,20,25,26,29, 32,\dots\}$ and they do not exist for $\ell \in \{6,14,17,21,22,23,30,46,56,62,78,$

Table 1: Normal sequences via Simulated Annealing.

Length ℓ	Sequences	Weight
20	F = −−++++++−++−+−+++−+−	20
	G = 0+−000+000000+000−+0	6
	H = −00+++0−++−−+0−−−00+	14
25	F−−+−+−+++−−−++−−+−−+	25
	G = 0−0−0+0+0+0−0+0−0−0−0+0+0	12
	H = −0−0−0+0+0−0+0−0+0+0−0−0+	13

$94, \ldots\}$ [3, 6, 16]. We note here that we found normal sequences of length 25 and 20 (Table 1) using simulated annealing; this is described in Gysin [3]. Sequences of length 25 can be obtained from Turyn sequences of lengths 13 and 12 and a complete search for these was carried out over 20 years ago. It is known that there are eight inequivalent sets of Turyn sequences of lengths 13 and 12 [2] and hence by the construction discussed in [6] probably at least sixteen inequivalent sets of normal sequences of length 25. It would be interesting to know if there are $NS(25)$ which cannot be made from Turyn sequences. There exist $NS(20)$ which cannot be made from Turyn sequences, an example is given in Table 1. In those small cases where $NS(\ell)$ do not exist we searched for $NY(n)$, which contain more zeros in appropiate positions. We obtained the following new results:

$NY(n)$ with weight $u = 12$ exist for the following lengths: $n \in \{7, 11, 13, 15\}$.

In Table 2 two conditions were imposed in counting the number of inequivalent triples of sequences: two triples of sequences were considered equivalent if one triple of sequences can be changed into the other triple of sequences by reversing and/or negating one or more sequences of the triple; if the three sequences F, G and H all started or ended with '0' they were considered to be of smaller length and not counted for this length n.

This allows us to find new 4-complementary sequences of lengths $15(2m+1)$, $23(2m+1), 27(2m+1), 31(2m+1)$ and weights $13(2m+1), m \leq 30$.

3. Construction

Definition 2. (suitable sequences) [5, 8, 11] *A, B, C, D are suitable sequences $SS(m+p, m; w)$ with elements 0, 1, -1 of lengths $m+p$, $m+p$, m, m and total total weight w if A and B are disjoint, C and D are disjoint and A, B, C, and D have zero non-periodic autocorrelation function.*

We use a modified version of Yang's [5, 8, 16] theorem

Table 2: New near-Yang sequences.

Length n	No of Seq.	Sequence Examples	Weight
7	2	$F = +++0-+-$	6
		$G = 0++0-+0$	4
		$H = +00000+$	2
		$F = +++0-+-$	6
		$G = 0+000+0$	2
		$H = +0+0-0+$	4
11	12	$F = -0+0+00000+$	4
		$G = 0+0+000+0-0$	4
		H = +0 + 00000-0+	4
13	24	$F = +0+0+0-0+000-$	6
		$G = 0+000000000+0$	2
		$H = +00+00000-00+$	4
15	26	$F = +0+00000+00000-$	4
		$G = 0+0000+0-0000+0$	4
		$H = +00+0000000-00+$	4

Theorem 1. *Let A, B, C, D be $SS(m+p, m; w)$ and F, G, H be $NY(n)$ with total weight u and $0', 0$ be sequences of zeros of length $m+p$ and m respectively and X^* be the reverse sequence of X then*

$$Q = \{Af_n, Cg_1 - Dh_1; 0', 0; Af_{n-1}, C_{g_2} - D_{h_2}; 0', 0; \ldots; A_{f_1}, C_{g_n} - D_{h_n}; 0', 0; B^*, 0\}$$
$$R = \{Bf_n, Dg_n + Ch_n; 0', 0; Bf_{n-1}, Dg_{n-1} + Ch_{n-1}; 0', 0; \ldots; B_{f_1}, D_{g_1} + C_{h_1}; 0', 0; -A^*; 0\}$$

$$S = \{0', 0; Ag_n + Bh_1, -Cf_1; 0', 0; Ag_{n-1} + Bh_2, -Cf_2; \ldots; 0', 0; A_{g_1} + B_{h_n}, -C_{f_n}; 0', D^*\}$$
$$T = \{0', 0; -Bg_1 + Ah_n, Df_1; 0', 0; -Bg_2 + Ah_{n-1}, Df_2; \ldots; 0', 0; -Bg_n + Ah_1, Df_n; 0', C^*\}$$

are TW-sequences of length $(2m+p)(2n+1)$ and total weight $(u+1)w$.

This gives many new TW-sequences, weighing matrices and orthogonal designs. Many other corollaries are also possible.

Example 1. Let $F = \{+++0-+-\}$, $G = \{0+000+0\}$, $H = \{+0+0-0+\}$ and A, B, C, D be suitable sequences of length $m+p$ and m and total weight w. Then with $0'$ and 0 zero vectors of length $m+p$ and m respectively we have

$$Q = \{-A, -D; 0', 0; A, C; 0', 0; -A, -D; 0', 0; 0', 0; 0', 0; A, D; 0', 0; A, C; 0', 0; A, -D; 0', 0; B^*, 0\}$$

$$R = \{-B, C; 0', 0; B, D; 0', 0; -B, -C; 0', 0; 0', 0; 0', 0; B, C; 0', 0; B, D; 0', 0; B, C; 0', 0; -A^*, 0\}$$

$$S = \{0', 0; B, -C; 0', 0; A, -C; 0', 0; B, -C; 0', 0; 0', 0; 0', 0; -B, C; 0', 0; A, -C; 0', 0; B, C; 0', D^*\}$$

$$T = \{0', 0; A, D; 0', 0; -B, D; 0', 0; -A, D; 0', 0; 0', 0; 0', 0; A, -D; 0', 0; -B, D; 0', 0; A, -D; 0', C^*\}$$

are TW-sequences of of length $15(2m+p)$ and total weight $13w$.

Corollary 1. *Suppose there are suitable sequences of length $m+p, m+p, m, m$ and total weight w, $SS(m+p, m; w)$ and near-Yang sequences of length n and total weight u. Then there are TW-sequences of length $(2n+1)(2m+p)$ and total weight $(u+1)w$.*

Corollary 2. *Suppose there exist $SS(m+p, m; w)$. Then since there are near-Yang sequences of length 7 and total weight 12 there are TW-sequences of length $15(2m+p)$ and total weight $13w$.*

From [11] we see $SS(m+1, m; 2m+1)$ exist for all $m \leq 30$ hence there exist TW-sequences of length $15(2m+1)$ and weight $13(2m+1)$ for all $m \leq 30$. Using theorems 3.6, 3.7, 3.8 of [11] we have $OD(60(2m+1)+4t; 13(2m+1), 13(2m+1), 13(2m+1), 13(2m+1))$ for all $t \geq 0$ and $m \leq 30$. Furthermore Q, R, S, T can be used in the Goethals-Seidel array to form $OD(4t; 13w, 13w, 13w, 13w)$ for every $t > 15(2m+1)$.

Recalling that variables in an OD can be set equal or set zero to give weighing matrices, we obtain $W(60(2m+1)+4t; 13s(2m+1))$ and $W(4t; 13sw)$, $s = 1, 2, 3, 4$. Since $NS(n)$ are $NY(n)$ with total weight $2n$.

Corollary 3. *Suppose there exist $SS(m+p, m; w)$. Then since there are normal sequences of length 25 and total weight 50 there are TW-sequences of length $51(2m+p)$ and total weight $51w$. If $w = 2m+p$ then we have T-sequences.*

Again using theorems 3.6, 3.7, 3.8 of [11] we have $OD(104(2m+1)+4t; 51(2m+1), 51(2m+1), 51(2m+1), 51(2m+1))$ for all $t \geq 0$ and $m \leq 30$. Furthermore Q, R, S, T can be used in the Goethals-Seidel array to form $OD(4t; 51w, 51w, 51w, 51w)$ for every $t > (2n+1)(2m+1)$.

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