New results with near- Yang sequences

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Abstract
We construct new TW -sequences, weighing matrices and orthogonal designs using near-Yang sequences. In particular we construct new OD(60(2m + 1) + 4t; 13(2m+ 1), 13(2m+ 1), 13(2m+ 1), 13(2m+ 1) and new W(60(2m+ 1) + 4t; 13s(2m+ 1)) for all t ≥ 0, m ≤ 30, s = 1,2,3,4.

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New Results with Near-Yang Sequences
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Abstract. We construct new TW-sequences, weighing matrices and orthogonal designs using near-Yang sequences. In particular we construct new OD(60(2m + 1) + 4t; 13(2m + 1), 13(2m + 1), 13(2m + 1), 13(2m + 1)) and new W(60(2m + 1) + 4t; 13s(2m + 1)) for all t ≥ 0, m ≤ 30, s = 1, 2, 3, 4.

1. Introduction
For definitions we refer the reader to [9, Introduction] and [11, Section 2]. We give one new definition.

Definition 1. (near-Yang sequences) A triple (F, G, H) of sequences is said to be a set of near-Yang sequences for length n (abbreviated as NY(n)) if the following conditions are satisfied.

(i) F = (f_k) is a (0, 1, -1) sequence of length n.
(ii) G = (g_k) and H = (h_k) are sequences of length n with entries 0,1,-1, such that G ± H = (g_k ± h_k) and G - H = (g_k - h_k) are both (0,1,-1) sequences of length n.
(iii) \( g_s + g_{s-s-1} \equiv 0 \pmod{2} \) for s = 1, ..., \( \frac{n}{3} \)
(iv) \( N_F(s) + N_G(s) + N_H(s) = 0 \), s = 1, ..., n - 1.

where

\[ N_X(s) = \sum_{i=s}^{n-s} x_i x_{i+s} \]

2. Computational Results
In Koukouvinos, Kounias, Seberry, Yang and Yang [6] it is shown that if in (ii) of the definition G ± H are both (1, -1) sequences then conditions (i), (ii) and (iv) imply condition (iii) but this is not true for near-Yang sequences. These sequences are normal sequences NS(\ell).

We searched for normal sequences NS(\ell). NS(\ell) do exist for the following lengths \( \ell \in \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 20, 25, 26, 29, 32, \ldots \} \) and they do not exist for \( \ell \in \{6, 14, 17, 21, 22, 23, 30, 46, 56, 62, 78, \ldots \} \).

Table 1: Normal sequences via Simulated Annealing.

<table>
<thead>
<tr>
<th>Length $\ell$</th>
<th>Sequences</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$F = +++++-++----+++-+++-+$</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>$G = 0+-0000+000000+0000--0$</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>$H = -00++0-+++++0+-+00$</td>
<td>14</td>
</tr>
<tr>
<td>25</td>
<td>$F = ++++++-++-++++---+++$</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>$G = 0-0-0+0+0-0+0-0-0+0+0$</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>$H = -0-0-0+0+0-0+0-0+0-0+0$</td>
<td>13</td>
</tr>
</tbody>
</table>

94, . . . [3, 6, 16]. We note here that we found normal sequences of length 25 and 20 (Table 1) using simulated annealing; this is described in Gysin [3]. Sequences of length 25 can be obtained from Turyn sequences of lengths 13 and 12 and a complete search for these was carried out over 20 years ago. It is known that there are eight inequivalent sets of Turyn sequences of lengths 13 and 12 [2] and hence by the construction discussed in [6] probably at least sixteen inequivalent sets of normal sequences of length 25. It would be interesting to know if there are $NS(25)$ which cannot be made from Turyn sequences. There exist $NS(20)$ which cannot be made from Turyn sequences, an example is given in Table 1. In those small cases where $NS(\ell)$ do not exist we searched for $NY(n)$, which contain more zeros in appropriate positions. We obtained the following new results:

$NY(n)$ with weight $u = 12$ exist for the following lengths: $n \in \{7, 11, 13, 15\}$.

In Table 2 two conditions were imposed in counting the number of inequivalent triples of sequences: two triples of sequences were considered equivalent if one triple of sequences can be changed into the other triple of sequences by reversing and/or negating one or more sequences of the triple; if the three sequences $F$, $G$, and $H$ all started or ended with '0' they were considered to be of smaller length and not counted for this length $n$.

This allows us to find new 4-complementary sequences of lengths $15(2m + 1)$, $23(2m + 1)$, $27(2m + 1)$, $31(2m + 1)$ and weights $13(2m + 1)$, $m \leq 30$.

3. Construction

Definition 2. (suitable sequences) [5, 8, 11] $A, B, C, D$ are suitable sequences $SS(m + p, m; w)$ with elements 0, 1, −1 of lengths $m + p$, $m + p$, $m$, $m$ and total total weight $w$ if $A$ and $B$ are disjoint, $C$ and $D$ are disjoint and $A, B, C,$ and $D$ have zero non-periodic autocorrelation function.

We use a modified version of Yang's [5, 8, 16] theorem.
Table 2: New near-Yang sequences.

<table>
<thead>
<tr>
<th>Length n</th>
<th>No of Seq.</th>
<th>Sequence Examples</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>(F = +++0-+-)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(G = 0++0+-0)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(H = +00000+)</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>(F = -0+0++0000+)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(G = 0000000+0-0)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(H = +00+00000-00)</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>24</td>
<td>(F = +0+00+0-0+00)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(G = 0+0000000+0)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(H = +00+00000-00)</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>26</td>
<td>(F = +000000+0000-)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(G = 0+0000+0-00000)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(H = +000+000000-00)</td>
<td>4</td>
</tr>
</tbody>
</table>

Theorem 1. Let \(A, B, C, D\) be \(SS(m+p, m; w)\) and \(F, G, H\) be \(NY(n)\) with total weight \(u\) and \(0', 0\) be sequences of zeros of length \(m+p\) and \(m\) respectively and \(X^*\) be the reverse sequence of \(X\) then

\[
Q = \{A_{f_2}, C_{g_1} - D_{h_1} ; 0', 0 ; A_{f_m-1}, C_{g_2} - D_{h_2} ; 0', 0 ; \ldots ; A_{f_1}, C_{g_m} - D_{h_m} ; 0', 0 ; B^*, 0\} \\
R = \{B_{f_2}, D_{g_1} + C_{h_1} ; 0', 0 ; B_{f_m-1}, D_{g_2} + C_{h_2} ; 0', 0 ; \ldots ; B_{f_1}, D_{g_1} + C_{h_1} ; 0', 0 ; -A^*; 0\}
\]

\[
S = \{0', 0 ; A_{g_2} + B_{h_1} ; -C_{f_1} ; 0', 0 ; A_{g_m-1} + B_{h_2} ; -C_{f_2} ; \ldots ; 0', 0 ; A_{g_1} + B_{h_m} ; -C_{f_m} ; 0', D^*\}
\]

\[
T = \{0', 0 ; -B_{g_1} + A_{h_1} ; D_{f_1} ; 0', 0 ; -B_{g_m-1} + A_{h_1} ; D_{f_2} ; \ldots ; 0', 0 ; -B_{g_2} + A_{h_1} ; D_{f_1}; 0', C^*\}
\]

are TW-sequences of length \((2m+p)(2n+1)\) and total weight \((u+1)w\).

This gives many new TW-sequences, weighing matrices and orthogonal designs. Many other corollaries are also possible.

Example 1. Let \(F = \{++0-+-\}, G = \{0000+0\}, H = \{+00+0+\}\) and \(A, B, C, D\) be suitable sequences of length \(m+p\) and \(m\) and total weight \(w\). Then
with 0' and 0 zero vectors of length $m + p$ and $m$ respectively we have

$$Q = \{-A, -D; 0', 0; A, C; 0', 0; -A, -D; 0', 0; 0', 0; A, D; 0', 0; A, -D; 0', 0; B, 0; 0, 0\}$$

$$R = \{-B, C; 0', 0; B, D; 0', 0; -B, C; 0', 0; 0', 0; 0, 0\}$$

$$S = \{0', 0; B, -C; 0', 0; A, -C; 0', 0; B, C; 0', 0; -A, 0\}$$

$$T = \{0', 0; A, D; 0', 0; -B, D; 0', 0; -A, D; 0', 0; 0', 0; -B, C; 0', 0; A, -C; 0', 0; B, C; 0', 0; 0', 0; -B, 0; 0, 0\}$$

are TW-sequences of of length $15(2m + p)$ and total weight $13w$.

**Corollary 1.** Suppose there are suitable sequences of length $m + p$, $m + p$, $m$, $m$ and total weight $w$, SS($m + p, m; w$) and near-Yang sequences of length $n$ and total weight $u$. Then there are TW-sequences of length $(2n + 1)(2m + p)$ and total weight $(u + 1)w$.

**Corollary 2.** Suppose there exist SS($m + p, m; w$). Then since there are near-Yang sequences of length 7 and total weight 12 there are TW-sequences of length 15($2m + p$) and total weight 13$w$.

From [11] we see SS($m + 1, m; 2m + 1$) exist for all $m \leq 30$ hence there exist TW-sequences of length 15($2m + 1$) and weight 13($2m + 1$) for all $m \leq 30$. Using theorems 3.6, 3.7, 3.8 of [11] we have $OD(60(2m + 1) + 4t; 13(2m + 1), 13(2m + 1), 13(2m + 1), 13(2m + 1))$ for all $t \geq 0$ and $m \leq 30$. Furthermore $Q, R, S, T$ can be used in the Goethals-Seidel array to form $OD(4t; 13w, 13w, 13w, 13w)$ for every $t > 15(2m + 1)$.

Recalling that variables in an OD can be set equal or set zero to give weighing matrices, we obtain $W(60(2m + 1) + 4t; 13s(2m + 1))$ and $W(4t; 13sw), s = 1, 2, 3, 4$. Since $NS(n)$ are $NY(n)$ with total weight $2n$.

**Corollary 3.** Suppose there exist SS($m + p, m; w$). Then since there are normal sequences of length 25 and total weight 50 there are TW-sequences of length 51($2m + p$) and total weight 51$w$. If $w = 2m + p$ then we have $T$-sequences.

Again using theorems 3.6, 3.7, 3.8 of [11] we have $OD(104(2m + 1) + 4t; 51(2m + 1), 51(2m + 1), 51(2m + 1), 51(2m + 1))$ for all $t \geq 0$ and $m \leq 30$. Furthermore $Q, R, S, T$ can be used in the Goethals-Seidel array to form $OD(4t; 51w, 51w, 51w, 51w)$ for every $t > (2n + 1)(2m + 1)$.

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References


