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Abstract

The design of a class of well-structured low-density parity-check (LDPC) codes, namely linear interleaver based concatenated zigzag (LICZ) codes, is investigated. With summary distances as the design metric, short LICZ codes with large minimum distances can be constructed. Moreover, an efficient cycle-based method is proposed to compute the minimum distances of LICZ codes. Simulation results show that LICZ codes outperform both CZ codes with random interleavers and LDPC codes by the progressive edge growth algorithm.

Keywords

codes, linear, zigzag, joint, concatenated, design, interleaver

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Joint linear interleaver design for concatenated zigzag codes

D.S. Lin, S. Tong and S.Q. Li

The design of a class of well-structured low-density parity-check (LDPC) codes, namely linear interleaver based concatenated zigzag (LICZ) codes, is investigated. With summary distances as the design metric, short LICZ codes with large minimum distances can be constructed. Moreover, an efficient cycle-based method is proposed to compute the minimum distances of LICZ codes. Simulation results show that LICZ codes outperform both CZ codes with random interleavers and LDPC codes by the progressive edge growth algorithm.

Introduction: As a special class of structured low-density parity-check (LDPC) codes, concatenated zigzag (CZ) codes have been reported to perform within 1 dB of the Shannon limit at long block lengths [1]. However, for short and medium lengths, CZ codes with random interleavers show relatively higher error floors, which suggests their poor minimum distances, d_{min} , and motivates the design of good joint interleavers.

It is well known that interleaver design plays an important role in concatenated coding schemes. Compared to random interleavers, algebraic interleavers are of practical interest owing to their regular structures, which facilitate analytical design and simple hardware implementation. As one class of the simplest algebraic interleavers, linear interleavers are simply defined by angular coefficients, and can be generated on-line. Moreover, it has been shown that multiple turbo codes (MTCs) with carefully designed linear interleavers perform very well [2]. This result motivates the application of joint linear interleavers in the design of CZ codes, resulting in a class of well-structured LDPC codes, namely linear interleaver based CZ codes (LICZ).

Our design focuses on the error floor performance, which is mainly due to d_{min} . Although there are already several practical methods for evaluating d_{min} , such as the ‘error impulse’ method [3] (however, this method cannot guarantee the true d_{min} is found) and the method due to Garelo *et al.* [4], all these methods are still time consuming and not suitable to be incorporated into a code design procedure. Following [2], we adopt summary distances as the design metric.

CZ codes and linear interleavers: An (I, J) zigzag code can be obtained by inputting IJ information bits into an accumulator with a generator of $1/(1+D)$ and then puncturing $(J-1)$ parity bits per J parity bits. An (I, J, K) CZ code is a parallel concatenation of K constitute (I, J) zigzag codes linked by interleavers [1]. In fact, CZ codes can also be interpreted as a special class of LDPC codes based on a semi-random parity check matrix [1]. However, compared to general LDPC codes, CZ codes not only admit linear time encoding, but also exhibit a faster convergence speed under a belief propagation decoder.

A length- N linear interleaver $\pi\{\pi_0, \pi_1, \dots, \pi_{N-1}\}$ is defined by $\pi_i = iq \bmod N, i = 0, 1, \dots, N-1$, where q is coprime to N and referred to as the angular coefficient. Hence, a K -dim CZ code is specified by a length- K angular coefficient vector $q = (q_0, q_1, \dots, q_{K-1})$ with $q_0 = 1$. Alternatively, a K -dim joint linear interleaver can also be represented as an index permutation matrix as follows:

$$\Pi = \begin{bmatrix} \pi_0^{(0)} & \pi_1^{(0)} & \dots & \pi_{N-1}^{(0)} \\ \pi_0^{(1)} & \pi_1^{(1)} & \dots & \pi_{N-1}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_0^{(K-1)} & \pi_1^{(K-1)} & \dots & \pi_{N-1}^{(K-1)} \end{bmatrix} \quad (1)$$

Each transposed column of Π , denoted as $\phi_i(\pi_i^{(0)}, \pi_i^{(1)}, \dots, \pi_i^{(K-1)})$, $i = 0, 1, \dots, N-1$, is referred to as a position vector [2].

Summary distances for CZ codes: Summary distance is only a property of the interleavers, irrelevant to component encoders [2]. Here, we modify this concept and relate summary distance to the component zigzag code. Typically, low weight codewords are generated by small even weight input sequences, and then only even length summary distances are defined.

Definition: For a $CZ(I, J, K)$ code, the length- $2L$ summary distance $D_{2L}(\phi_{2L})$ for a set ϕ_{2L} of $2L$ position vectors $\phi_{i_l}, l = 1, 2, \dots, 2L$, is

defined as

$$D_{2L}(\Phi_{2L}) = \sum_{j=0}^{K-1} \delta(\psi_{2L}^{(j)}) \quad (2)$$

where $\psi_{2L}^{(j)} = \{i_1^{(j)}, i_2^{(j)}, \dots, i_{2L}^{(j)}\}$ is a permuted version of $\{\pi_{i_1}^{(j)}, \pi_{i_2}^{(j)}, \dots, \pi_{i_{2L}}^{(j)}\}$ with $i_1^{(j)} < i_2^{(j)} < \dots < i_{2L}^{(j)}$, and $\delta(\psi_{2L}^{(j)}) = \lfloor \sum_{l=1}^{2L} (i_{2l}^{(j)}/J) - \lfloor i_{2L-1}^{(j)}/J \rfloor \rfloor \cdot \lfloor x \rfloor$ denotes the largest integer not greater than X and $\lfloor X \rfloor = \min(\lfloor X \rfloor, I - \lfloor X \rfloor)$. Note that if the operation $\lfloor \cdot \rfloor$ is removed, $D_{2L}(\phi_{2L})$ is just the true parity weight due to the weight- $2L$ input sequence with $2L$ 1's located in positions $i_l, l = 1, 2, \dots, 2L$, thus offering a lower bound on the parity weight. Moreover, a ‘quasi-shift-invariant’ property holds for D_{2L} , i.e. $D_{2L}(\phi_{i_t+j_t}, \dots, \phi_{i_{2L}+j_t}) = D_{2L}(\phi_{i_t}, \dots, \phi_{i_{2L}}), t \in \mathbb{R}_N$.

Minimum distance computation for CZ codes: Our investigations show that d_{min} 's of some good LICZ codes are due to large input weights (e.g. 16). Hence, Garelo *et al.*'s method is not suitable for LICZ codes, as it works efficiently only when d_{min} is due to input sequences with weight not greater than 8 [5]. Here, a new cycle-based method is proposed for computing d_{min} of CZ codes. Owing to space limitation, we give the following simple theorem without proof, which is the basis of the proposed cycle-based method.

Theorem 1: Let C be a binary code with a parity check matrix H , all the columns of which are of weight 2. Then, a vector c is a valid codeword of C if the nonzero elements among all the columns of H corresponding to the nonzero elements in c can form one or multiple disjoint cycles.

Now, consider the parity check matrix, H^2 , of a 2-dim CZ code. Obviously, except the two weight 1 columns corresponding to the last parity bits of the two component zigzag codes, all columns are of weight 2. In fact, the last parity bit in each zigzag code is the modulo-2 sum of all information bits and then the two last parity bits coincide. Using this fact, we can replace the two weight 1 columns with a combined weight 2 column, which is the sum of the two weight 1 columns, the resulting parity check matrix denoted as $\hat{H}^2$. Thus, all columns of $\hat{H}^2$ are of weight 2.

Now, the cycle-based approach to d_{min} of a K -dim CZ code is outlined as follows: 1. separate the K -dim CZ code into a 2-dim one and a $(K-2)$ -dim one; 2. for the 2-dim CZ code, find small weight ($\leq D$) codewords by searching all short cycles and their combinations in $\hat{H}^2$; 3. use the input sequences corresponding to the small weight codewords of the 2-dim CZ code found in step 2 as the input to the K -dim CZ code and find the minimum weight one from the generated codeword, which is d_{min} of the K -dim CZ code if its weight is not greater than D . Owing to the sparsity of $\hat{H}^2$, the computational complexity of searching short cycles is acceptable for short block lengths n (e.g. 500). In fact, the complexity of searching length $2d$ cycles, i.e. d linearly dependent columns, is of $O(n(J+1)^d)$, since all rows in $\hat{H}^2$ are of weight $(J+1)$ or $(J+2)$. By using the ‘quasi-shift-invariant’ property of linear interleavers, this complexity can be further greatly reduced.

Table 1: Minimum distances, d_{min} , of three rate-1/2, 4-dim LICZ codes and time required for computing d_{min}

(I, J, K)	$d_{min} / A_d (w/A_{w,d})$	Time (min)
(32,4,4)	14/46 (4/46)	1.18
(63,4,4)	18/168 (4/50,12/57,14/61)	540
(126,4,4)	20/124 (16/124)	6660

A_d denotes multiplicity, w the input weight, and $A_{w,d}$ the number of minimum distances due to input weight w

Numerical results: With summary distances as the design metric, a code design procedure, similar to that in [2], for LICZ codes is developed. For simplicity, only length-2 and length-4 summary distances are considered. Three rate-1/2 four-dimensional short CZ codes with lengths of 256, 504, and 1008 are presented. Their corresponding angular coefficient vectors are (1, 27, 41, 11), (1, 205, 233, 223), and (1, 79, 181, 463), respectively. Table 1 depicts their code parameters, including the minimum distances and their multiplicities A_d 's, along with the time required for computation of their minimum distances (a Pentium IV PC with a main frequency of 2 GHz is used). From Table 1, it is seen that for the (32, 4, 4) CZ code, the minimum distance is generated by some weight-4 input sequences; for the (63, 4, 4) one, by some

weight-4, weight-12, and weight-14 input sequences; and for the (126, 4, 4) one, by some weight-16 input sequences.

Fig. 1 shows the BER performances of the three LICZ codes, respectively. The maximum iteration number is 100. For comparison, also plotted in Fig. 1 are the BER performances of CZ codes with random interleavers, and LDPC codes with a degree distribution of $\lambda(X) = X/3 + 2X^3/3$, $\rho(X) = X^5$ same as that of the CZ codes, constructed using the PEG algorithm [6]. From Fig. 1, we can observe that the error floor performances of LICZ codes are remarkably better than those with random interleavers, and they are also slightly better than the LDPC codes constructed by the PEG algorithm. Moreover, LICZ codes are much simpler for practical implementation.

Fig. 1 BER performance comparison of CZ (1,4,4) ($l = 32, 63$, and 126) codes with random/linear interleavers and LDPC codes constructed by PEG algorithm

Conclusion: We investigated the application of joint linear interleavers to CZ codes. Using summary distances as the design metric, we constructed good LICZ codes. Moreover, a cycle-based method is proposed to compute the minimum distances of the CZ codes. Compared to CZ

codes with random interleavers the constructed LICZ codes have larger minimum distances and thus exhibit better error floor performance.

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