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Abstract

A low-complexity low density parity check (LDPC) coded bit-interleaved coded modulation with iterative decoding (BICM-ID) scheme with orthogonal modulations is proposed. With a novel mapping strategy of coded bits to symbols, the proposed scheme is equivalent to a generalised LDPC code with Hadamard constraints and thus orthogonal demodulation can be merged into the iterative LDPC decoding process, resulting in a simpler implementation and a lower computational complexity.

Keywords

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Low-complexity LDPC coded BICM-ID with orthogonal modulations

S. Che and S. Tong

A low-complexity low density parity check (LDPC) coded bit-interleaved coded modulation with iterative decoding (BICM-ID) scheme with orthogonal modulations is proposed. With a novel mapping strategy of coded bits to symbols, the proposed scheme is equivalent to a generalised LDPC code with Hadamard constraints and thus orthogonal demodulation can be merged into the iterative LDPC decoding process, resulting in a simpler implementation and a lower computational complexity.

Introduction: Recently, coded modulation using low density parity check (LDPC) codes [1] has been extensively investigated. A popular and pragmatic approach to LDPC coded modulation is bit-interleaved coded modulation (BICM) [2]. To improve the system performance further, iterative processing between demodulation and channel decoding is employed, termed as BICM with iterative decoding (BICM-ID) [3]. However, the performance gain of BICM-ID is achieved at the cost of increased computational complexity.

In this Letter, a low complexity LDPC coded BICM-ID scheme with orthogonal modulations (OMs) is proposed, its feature being its novel mapping of coded bits to symbols. The mapping strategy is based on two properties: one is the equivalence of OMs and Hadamard codes and the other is an inherent single parity check (SPC) relation of Hadamard codes. It can be shown that, with the proposed mapping strategy, our scheme is equivalent to generalised LDPC (GLDPC) codes with Hadamard constraints [4]. Thus, the demodulation process can be merged into iterative GLDPC decoding, which greatly simplifies the receiver structure. Moreover, the computational complexity of orthogonal demodulation can be significantly reduced by using fast Hadamard transform (FHT) and *a posteriori* probability FHT (APP-FHT) [5]. The proposed scheme can be used in some military communication systems and underwater acoustic communications using orthogonal modulations such as frequency shift keying (FSK) or pulse position modulation (PPM).

Orthogonal modulations and Hadamard codes: The i th symbol of an M -ary ($M = 2^m$) orthogonal modulation can be represented as the i th column e_i of an $M \times M$ identity matrix ($i = 0, 1, 2, \dots, M-1$), while an $M \times M$ Hadamard matrix H_M can be constructed recursively as

$$H_M = \begin{bmatrix} +H_{M/2} & +H_{M/2} \\ +H_{M/2} & -H_{M/2} \end{bmatrix} \text{ with } H_1 = [+1] \quad (1)$$

A length M Hadamard code is constructed from an $M \times M$ Hadamard matrix H_M defined over $\{\pm 1\}$ by mapping $+1 \rightarrow 0$ and $-1 \rightarrow 1$, which results in a matrix C_M , and taking the columns c_i of C_M as codewords [4]. In fact, column h_i of H_M can also be viewed as the BPSK modulated version of Hadamard codeword c_i with BPSK modulation defined by $0 \rightarrow +1$ and $1 \rightarrow -1$. Both OMs such as FSK and PPM and Hadamard matrices are popular examples of orthogonal sets, which is the inherent reason for their equivalence as stated in the following lemma.

Lemma 1 (equivalence of an M -ary OM and a length M Hadamard code): For an M -ary OM and a length M Hadamard code, symbol e_i and BPSK modulated Hadamard codeword h_i are related by $h_i = H_M e_i$ and $e_i = H_M h_i / M$.

In systematic encoding of a length M Hadamard code, the information bit positions are $1, 2, \dots, 2^{m-1}$. There is an SPC relation between all the information bits and the last parity check bit, as summarised in the following lemma.

Lemma 2 (an inherent SPC relation of Hadamard codes): For any codeword c_i of a length $M (=2^m, m > 1)$ Hadamard code, there is an SPC relation given by

$$\sum_{j=0}^{m-1} \oplus c_i(2^j) \oplus c_i(2^m - 1) = 0 \quad (2)$$

where $c_i(j)$ denotes the j th bit of the codeword c_i and $\oplus$ denotes mod-2 addition.

This lemma can be easily proved by induction in a similar way to Theorem 4.2 in [4]. In the following, we propose a novel mapping strategy based on this inherent SPC relation and the equivalence between OMs and Hadamard codes.

Novel mapping strategy for orthogonal modulations: A conventional LDPC coded BICM-ID scheme operates as follows. An information block with k bits is first encoded to a length n LDPC codeword. Then, for an M -ary ($M = 2^m$) OM, the n coded bits are partitioned into n/m segments (assume that n is a multiple of m), each containing m coded bits and mapped to a symbol. Note that, owing to the inherent edge interleaver in the Tanner graph representation of a general LDPC code, the conventional interleaver between the channel encoder and modulator used in BICM can be omitted in LDPC coded BICM schemes. Since LDPC codes have been well studied in the last decade, we focus on the mapping strategy in the following.

Our scheme is different from the conventional one in the mapping strategy, which can be described as follows. Consider a degree m check node connected to m coded bits denoted as $c = (c(0), c(1), \dots, c(m-1))$. As $\sum_{i=0}^{m-1} \oplus c(i) = 0$, the freedom of the m bits is of $m-1$ and thus we only use the first $m-1$ coded bits for mapping. Denote the binary number formed by the first $m-1$ coded bits as $s = (c(m-2) \dots c(1)c(0))_2$. Then, c is mapped to the symbol e_s .

Example: Fig. 1 shows the mapping for a degree 4 check node in the factor graph of an LDPC code. The binary number formed by the first three coded bits is $(c(2)c(1)c(0))_2 = (110)_2 = 6$ and then the check node is mapped to e_6 of an 8×8 identity matrix. Assume that e_6 is transmitted through an AWGN channel. Then, the received signal is given by $y = e_6 + n$, where $n \sim N(0, \sigma^2 I)$. By lemma 1, a simple linear transformation yields $H_8 y = H_8 e_6 + H_8 n = h_6 + H_8 n$. Thus, the received symbol is equivalent to a noised BPSK modulated Hadamard codeword (see Fig. 1). Note that the original SPC relation between $\{c(i), i = 0, 1, 2, 3\}$ is preserved by the information bits $\{h_6(1), h_6(2), h_6(4)\}$ and the last parity bit $h_6(7)$ (see lemma 2 and Fig. 1) of the BPSK modulated Hadamard codeword h_6 . The example in Fig. 1 shows that with a simple linear transformation each check node is equivalent to a Hadamard codeword and then the LDPC coded orthogonal modulation scheme is equivalent to a GLDPC code with Hadamard constraints [4]. Thus, orthogonal demodulation can be merged into the iterative decoding of GLDPC with Hadamard constraints, which can be efficiently done by using FHT and APP-FHT [4, 5].

Fig. 1 Mapping of degree 4 check node to modulation symbol

Simulation results: We use a special class of LDPC codes, namely concatenated zigzag (CZ) code [6], as the channel code for its simple encoding and decoding structures. The transmitter consists of a randomly generated (1228, 5, 4) CZ code, which has a data length of 6140 and a code length of 11052, and a 64-ary orthogonal modulation. Fig. 2 shows the BER performance of the system used over a Rayleigh fading channel with noncoherent demodulation. From Fig. 2, it is seen that there is a gap of about 1.9 dB between our scheme and the channel capacity. No effort is made to optimise the employed CZ code. A similar result is reported in Table I of [7], where a rate 1/4 turbo coded BICM-ID system with a data length of 6138 and 64-ary noncoherent frequency shift keying operates about 1.67 dB from the capacity.

Fig. 2 BER performance in Rayleigh fading channel of (11052, 6140) CZ code with 64-ary noncoherent orthogonal modulation

Conclusion: A low complexity LDPC coded BICM-ID scheme with orthogonal modulations is proposed. Its novelty lies in the mapping of coded bits to symbols, which is based on the equivalence of orthogonal modulation and Hadamard codes and an inherent SPC relation of Hadamard codes. With the proposed mapping strategy, our scheme is equivalent to GLDPC codes with Hadamard constraints and thus the demodulation process can be combined into iterative GLDPC decoding, resulting in a simplified receiver structure. Primary simulation results validate the proposed scheme. Our future work could focus on the optimisation of LDPC codes for this scheme.

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