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In the present study, I examine a modelling strategy as employed by a teacher in the context of an algebra lesson. The actions of this teacher suggest that a modelling approach will have a greater impact on enriching student learning if we do not lose sight of the need to manage associated cognitive loads that could either aid or hinder the integration of core concepts with processes that are at play. Results here also show that modelling a problem that is set within an authentic context helps learners develop a better appreciation of variables and relations that constitute the model. The teacher's scaffolding actions revealed the use of strategies that foster the development of connected, meaningful and more useable algebraic knowledge.

Recent reform agendas in mathematical teaching and learning have been underpinned by the need to foster robust and meaningful learning, one that aids in the long-term retention of core concepts and principles, and application in a wide variety of contexts. This latter shift has been instrumental in the development and characterisations of notions of numeracy as the utilisation of mathematics in understanding real-life contexts and solution of problems.

As a consequence, an important goal of mathematics teaching is now seen as developing students' structural understanding of concepts and the embedding of concepts in realistic contexts. Determining and utilising effective and appropriate strategies to promote enduring understandings is crucial to quality learning outcomes in mathematics. A principal element of such understandings would be the establishment of links among concepts, facts, conventions and procedures. This need to examine connections that students construct has been endorsed by major curriculum reform documents (Board of Studies, 2002; National Council of Teachers of Mathematics, 2000). How can we elucidate these relations that are central to connected learning? The mathematics community has taken this issue by adopting a model or modelling perspective in analysing effective mathematical practices.

Modelling and Cognitive Load

Modelling perspectives have been the subject of considerable interest and study in the past two decades and there is an emerging consensus regarding the efficacy of this approach in engaging learners in powerful ways. In a survey of the different interpretations of modelling particularly in relation to mathematics education, Kaiser and Sriraman (2006) identified the cognitive perspective as being relatively new and one that had the potential to provide insights into various processes that students and teachers activate during the course of constructions of models. Their analysis suggests that, from a cognitive framework, modelling 'starts with a descriptive position' outlining the path and associated knowledge that is activated in negotiating the path to developing an appropriate model or models.

While there have been attempts to examine modelling from a cognitive angle and students' responses to tasks (Stillman, 2004; Tan, 2007), little effort has been invested in elucidating the cognitive architecture underlying processes that support the development of models. Research findings regarding how learners utilise their limited mental resources and associated cognitive load provide us with new tools to examine modelling processes more closely.

Chinnappan and Chandler (2010) have provided an overview of the relationship between three types of cognitive load and their influence on mathematical learning and retention of learning. An important point in their arguments is that effective instructional approaches must aim to maximise students' ability at processing information in the Working Memory and storing the resulting information in their Long-term Memory for future accessibility. This could be achieved by engaging students with mathematical tasks that have appropriate *Intrinsic* and *Germane* loads. Whether one views modelling as a 'vehicle' or 'content' (Stillman, Brown, & Galbraith, 2008), the task needs to have elements that are quantitatively and qualitatively complex (relative to the learner) in order to induce optimum Intrinsic cognitive load. Likewise, a mathematical task that constrains students to develop models of a concept or problem by reflecting on and accessing other concepts, and build new relational understandings can be seen as one that supports Germane cognitive load. Both these loads have positive effects on the generation and subsequent evaluation of models. *Extraneous* load, on the other hand, could hinder students' learning by having students attend to the less relevant parts of the problem. This load is largely associated with the format of instruction adopted by the teacher during lessons.

Intrinsic, Extraneous and Germane Loads

Intrinsic load is related to the complexity of a task. Memorising the formula for working out the perimeter of a rectangle, $p = 2(l + w)$, is low in complexity and therefore would impose low intrinsic load. In order to process this information, students simply need to consider the two elements: length (l) and width (w). They would not be required to concurrently process any other information. Thus, recalling a simple formula could be taught with little interaction or no interaction with other elements of information. However, applying the formulae for perimeter and area of a rectangle in order to determine the dimension of a rectangle that has maximum area for a given perimeter is one that has high intrinsic load. Clearly, intrinsic load will also be dependent on the knowledge and experiences of the learner.

Extraneous load is induced solely by the instructional format that is used during teaching. Instructional format refers to the organization of texts and visuals that are used by teachers to help learners understand a given problem context. In certain formats of instructions, students are given texts only while others may involve a blend of texts and computer-based diagrams. Instructional formats that separate texts from diagrams increases extraneous load as students are constrained to integrate these sets of information before they could examine potential solution paths. Examples of this type of formats are commonly used in the teaching of geometry where teachers provide diagrams of angles, and provide the measure of the angles in statements that accompany the diagrams.

The third class of loads (Germane Load) has it roots in activities that directly relate and contribute to mathematical schema development and the automation of such schemas. Germane activities may include encouraging self-explanations, developing mental imagery, studying rich worked examples and reflecting on the relationships that exist among elements of a given problem. Germane load suggests that learning activities that are germane in nature bring about meaningful learning. For example, a learner's attempt to explain and justify a solution to a geometry proof problem contributes to germane load. From a mathematics pedagogical perspective, it is important that teachers introduce activities that foster germane load.

Modelling and Teaching for Connected Learning

One of the features of high-quality learning conditions is that students must be invited to interact meaningfully with mathematical content. Such engagements allow students to develop different perspectives about a concept. This latter aspect has been characterised as the development of multiple representations of concepts (Goldin, 2008). While a concept may assume multiple concrete manifestations depending on the situated nature of the concept, students with high degrees of understanding must be able to articulate the relations among the representations by activating appropriate reasoning skills. We contend that such connected learning can be seen in actions when students engage in modelling either a concept or a problem context.

The characterisation of models and processes involved in modelling provide us with insightful angles into the type of connections that are rich and powerful. Recent thinking on this issue of the need to examine relations and relational understandings that are embedded in models has focused on not only links but also strategies to support learners develop such links in authentic contexts (Blum, 2002; Seino, 2005; Stillman, Brown & Galbraith, 2008).

A number of studies have explored the type of understandings developed by students, and the quality of teaching provided by teachers that supported the development of models (Coad, 2006; Galbraith, Stillman & Brown, 2006). The increasing emphasis on model-based teaching approaches and its relations to student understanding have, however, provided limited insight into how teachers go about marshalling their content knowledge of mathematics during actual teaching episodes. Hill, Sleep, Lewis and Ball (2007) suggested future research needs to examine teacher knowledge *in situ* so that we could gain a better understanding of the nature of pedagogical content knowledge that is actually activated *during* the lesson as opposed to that accessed in non-teaching contexts.

In the present study, we explore this issue by examining how a teacher could exploit an authentic context through which students are scaffolded into modelling a problem using algebraic concepts.

Theoretical Framework

Schemas and the Construction of Models

Studies of mathematical problem solving processes (English & Halford, 1995; Lesh, 2006; Mayer, 1975) have generated a number of frameworks for the examination of links including the construct of *schemas*. Schemas have multiple attributes, but in general terms, refer to a cluster of knowledge that contains information about core concepts, the relations between these concepts and knowledge about how and when to use these concepts. As chunks of knowledge, schemas guide the assimilation of new information, the integration of the new information with existing knowledge, the retrieval and subsequent use of that knowledge in making sense of new contexts including model development. Thus, from a schema-based perspective, when teachers and students acquire mathematical concepts, principles, and procedures they connect these into prior schemas that in turn provide the knowledge base for further mathematical activity such as problem exploration. These connections and reorganisation of the knowledge base result in the construction of newer and more powerful schemas. Sweller (1989) argued that the degree of sophistication of schemas has a strong influence on how teachers and students categorise, model and solve problems. Clearly, one would expect teachers to have built up a wider schema base in comparison to their students.

As in other areas of mathematics, schemas provide an important theoretical tool with which to study the organisation of algebraic knowledge and the use of that knowledge during modelling. For example, within the broad area of high school algebra, learners develop a schema for the concept of variable. As the variable schema develops, that schema will be enlarged to include concepts about equation and function. This line of reasoning led Chinnappan and Thomas (2003) to develop a framework for visualising knowledge connections in problem and concept modelling (Figure 1).

Two characteristics are important to understanding algebraic schemas: *organisation* and *spread*. Organisation refers to the establishment of connections between ideas, whereas spread refers to the extent of those connections. An algebraic schema is said to be sophisticated or complex when it has a high degree of organisation, spread and multiple points to connect with.

A research strategy to examine the quality of mathematical schemas, and how students and teachers attempt to exploit these schemas is to observe the interplay between components of schemas that are activated during a problem-modelling episode. In the present study, the focus is on the accessing and use of algebraic schemas by a teacher and his students as they attempt to co-construct a model for a problem at hand.

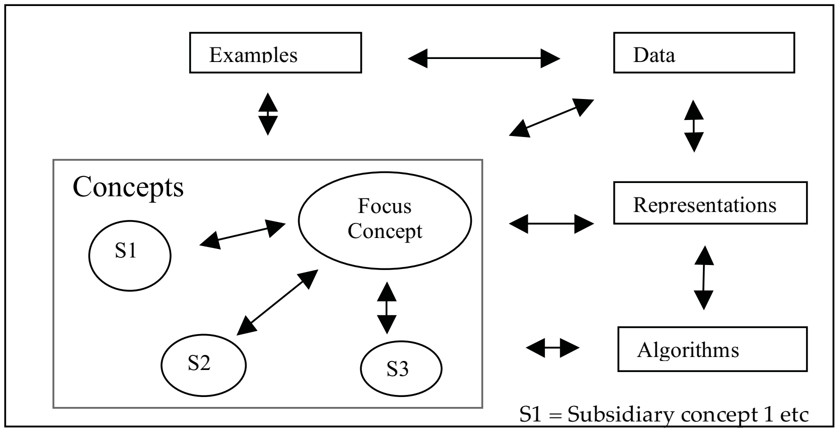

Figure 1. A structural framework for teachers' mathematical knowledge and modelling of a focus concept (Chinnappan & Thomas, 2003, p. 155).

Figure 1 shows a framework that was developed by Chinnappan and Thomas (2003) in their investigation of teachers' knowledge and the use of their knowledge in modelling a focus concept. A *Focus Concept* is buttressed by other concepts, *Subsidiary Concepts* (SC1, SC2 and so on). The *Examples*, *Data*, *Representations* and *Algorithms* were hypothesised as entities that can be utilised to situate the Focus Concept in a meaningful manner. This embedding of the *Focus Concept*, can, and does often, occur during the modelling of real-life problems, thus constituting a case of mathematical modelling. Stillman, Brown and Galbraith (2008, p. 142) acknowledged the above interpretation of mathematical modelling in commenting that the Chinnappan and Thomas (2003) framework saw "mathematical modelling primarily as a means of motivating, developing, and illustrating the relevance of particular mathematical content".

Thus, we feel confident in drawing on the above framework in examining the evolution of a model for a problem during the course of a lesson that was analysed in the present study. The framework acts as a window through which the researcher is able to observe the integration of schemas by both the teacher and his/her students in their attempts to generate the problem model. In this sense, Figure 1 helps us see the 'big picture' concerning teacher-student knowledge interactions that are aimed at purposeful application of prior knowledge (Prawat, 1989). One can also detect several levels of schemas within the framework. For example, the connections between the subsidiary concepts and the focus concept constitute a schema in its own right. At a second level, the establishment of links amongst *Examples*, *Data*, *Representations* and *Algorithms*, further forms a macro schema.

Context and Goals of the Research

The data for the present study were based on actions of one high school teacher of a Year 9 class during an algebra lesson. During the course of the lesson, the teacher invites and works with his students in modelling the *Chewing Gum Problem*. The observations of modelling episodes in a classroom situation are quite unlike laboratory-based activities. The events that unfold in the former context are seldom predictable and continuously changing. While the teacher may set the agenda for the lesson, once the lesson commences the participants may pursue different goals as they engage in the modelling process. Our aim was to capture the actions (and associated schemas) of the participants in these ever-changing classroom dynamics. Thus, it was decided that schema activation and actions aimed at model generation as units of analysis would be appropriate for the study. For purposes of anonymity, the teacher will be referred to as Simon, and students will be labelled as S1 for student 1, S2 for student 2 and so on.

The two primary goals of the study were:

1. To identify critical engagements and events during the lesson that involved modelling; and
2. To provide a schema-based analysis of the understandings displayed by the teacher and students.

Results and Discussion

The events of the lesson were classified into five phases. The parsing of the lesson into phases was guided by our framework for modelling (Figure 1). While these phases are related, there is a discernible difference among these in their foci. The value of a phase-based analysis of mathematical modelling activities was highlighted by a related study conducted by Galbraith, Stillman and Brown (2006).

Phase 1: Advance Organiser

In this introductory phase, the teacher establishes continuity with the previous lesson by revisiting processes that are involved in the solution of a linear equation which is the focus concept in this modelling exercise. This move acts as an advanced organiser and sets up the context for the lesson. Student, S1, ponders about by why he needs to study algebra (Line 2). Simon, the teacher, repeats the question to the class at large and solicits their responses. He makes an attempt to provide valuable linkages between algebra and potential career options of students (Line 3).

Teacher: We're just going to start off the lesson recapping quickly the equations we got up to yesterday, which were pretty substantial equations. Four-step equations that looked possibly like this: $4m + 2 = 2m + 18$. And what we might do is a few, just to make sure people understood the process of solving these equations because, as I said, the mathematics is quite involved. (line 1)

S1: Why do we need to learn algebra? (line 2)

Teacher: OK, John wants to know why we need to learn algebra. Do you know what sort of job you're going to do when you leave school? (*Student shrugs.*) Have no idea? (*Student nods head 'no'.*) Who does? (*Students raise hands.*) (line 3)

The equation in Step 1 provided the students with an opportunity to revisit their previous work involving algebraic work. While the equation is abstract, it has the potential to lead students to ones that are more complex and grounded in a real-life problem (Phase 2).

Phase 2: Setting the Context

In this phase, Simon develops a real-life problem context with the view to showing the practical use of algebra (Line 4). In so doing, Simon is creating relevance and brings a level of authenticity to the problem. This is a co-learning process in which students are asked to consider ways to recycle chewing gums that are found stuck on the under side of their desks. This gums activity draws on all the dimensions that are featured in Figure 1. As it will become clear, the teacher takes the lead in helping the students negotiate these dimensions during various phases of the lesson. The gums activity commenced with students counting the number of gums stuck on their desks. However, as the problem develops, students are required to access and make new connections with multiple knowledge components through the process of modelling. Thus, the gums activity is far from trivial and exerts a high intrinsic cognitive load which in turn demands deeper processing on the part of the students.

While the teacher did not pose the problem at the outset, he takes the students along a journey in which the problem emerges and students are scaffolded in the modelling process. Students are asked to turn their desks over and locate, and count the number of gums found underneath their desks. In this phase, the actions of the students are physical and there are interactions.

Line 5 indicates one student counting aloud the number of gums under her desk. The teacher reminding students to write the number of gums down (Line 6) is evidence of helping them to record and monitor data generation (Figure 1).

Teacher: All right. So there's a problem here. I need to stop – because we're doing algebra here – pretty heavy duty algebra here. I've got a lot of people who don't think it will be useful for them. Well, let me see if I can convince you that learning algebra will do two things: (a) It may expand your mental capabilities to problem solve or to learn other things in your life better. (b) Maybe you don't know you'll be using algebra, when you are. Let me show you something. S6, can you push those books onto S7's desk, then turn your desk upside down for me, please? Can you count the pieces of gum? (*S6 counts*) (line 4)

S6: 1,2,3, …, 36 (line 5)

Teacher: OK, stop there. You've got over 35 pieces of gum. Now, some of you people think maybe I picked that desk on purpose. Thanks, S6, have a seat. Some of you might think that I put those pieces of gum there, because we're not allowed to have gum in school. We're not allowed to have gum in school, are we? Everybody, look under your desk. Count the pieces of gum. Why do we plant gum under every person's desk? (*Students start to move desks around.*) Don't move anything, just count the pieces of gum. Remember the number. As a matter of fact, write the number down. When you've done it, write the number down. (line 6)

Teacher: And put your desks back up. Now, let's do some maths. Some fun maths. And maybe, maybe I can convince you that something important we can connect to mathematics and connect to algebra, and connect to maybe innovation, lateral thinking, problem solving, and maybe. I don't know if I can convince all the people who said 'I want to be an actor', like S22. Let's find out. S5, how many? (line 7)

S5: 21. (line 8)

Phase 3: Developing the Context

Having sought a consensus on the average number of gums below each desk (Line 9), students are invited to work out the number of gums in their class. This is the phase where students begin to engage in the modelling process with teacher taking the lead by having students respond to a series of general questions. In a sense, the teacher scaffolds the process by the linking of the questions with prompts. During this process the teacher encourages students to use estimation as a legitimate mathematical tool. There is also evidence of the concept of average being utilised in the course of this phase. Students were asked to count the number of gums under their own tables. We have 36 (Line 1), 21 (Line 8) and then students were required to estimate an average figure (Line 9). In order to estimate, students would have to have some notion of the 'centre'. The estimate of 25 (Line 10) thus constitutes evidence of understanding of 'average'.

There is an opportunity to get students to compute multiplication without the aid of calculators but the teacher misses this. Alternatively, it is possible that the teacher is attempting to reduce Extraneous Load so that students could be cognitively engaged with the more demanding aspects of model generation.

Similarly, Line 11 shows that the teacher was aware of the importance of engaging the students in developing important relations among the various components of the model rather than deviating their attention by having them work out the exact number of gums in the classroom. "We'll run with that" is an indication of this tactic. This can be seen as a further strategy to reduce Extraneous cognitive load that Chinnappan and Chandler (2010) argued could increase load on the Working Memory thereby hindering the allocation of valuable mental resources to the more important facets of modelling. Seeking support from other students (Line 14) regarding the accuracy of their computations is also a move to keep all students involved. In Line 16, the teacher, again, invokes a reflection move that is indicative of introduction of Germane cognitive load.

Teacher: OK, how about we estimate an average amount of gum on the bottom of the desks. Just estimate. We don't need to do the mathematics here. Let's do some estimation. So, if you had to estimate how many pieces of gum would you say, on average, is under a desk in this room? Just have a guess. S13? (line 9)

S23: 25 (line 10)

Teacher: 25? (*Writes 25 on the board*) Who would agree with that? Shall we run with that? (*Class nods in agreement.*) OK, watch this. Grab your calculator. Everybody have a calculator on their desk? (*Students nod.*) According to our group here, we have approximately 25 pieces of gum under every desk. Some have a few more, some have a few less. We'll run with that. (line 11)

Teacher: How many desks do we have in our room – approximately. S17? (*Student S17 passes.*) How many desks do we have approximately in this room? S18? 30? Let's go with 30. Do this for me on your calculator: 25 times 30. (*Teacher adds ' × 30 ' onto the board.)* Alright, I need someone to give me an answer here. Somebody different, maybe. S19? (line 12)

S19: 750. (line 13)

Teacher: Is that right? Can someone verify that for me? (*students nod agreement*). Let's write that (*teacher writes '750 p. of gum' on the board*). OK. 750 pieces of gum. 750 pieces of gum in the room. How many classrooms do we have in this school? Approximately – just at a guess. About 50? Who would disagree with that? (*class nods agreement*) About 50? OK. Let's do another calculation. Let's do 750 times – which is what we have in each room, times 50 rooms (*teacher writes on board '750 × 50'*) – won't take long because you all have calculators, don't you? S17? (line 14)

S17: 37,500 (line 15)

Teacher: 37,500 pieces of gum (*teacher adds that figure on board calculation*) in our school. Would you say that? Now, stop and think about that for a minute. Some rooms might be newer than others, would you agree? Less gum. Shall we knock off 7,500 just in case? (*class nods agreement*) And let's round it off to 30,000 pieces of gum in our school (*teacher writes figure '30,000' on board and circles it*). (line 16)

Phase 4: Model Development

In this phase, the teacher continues with the modelling process by developing important relations among the relevant variables. In Line 17, he works out the number of plastic soldiers that can be made from 30000 gums based on the assumption that 10 gums will be needed to make one soldier. The dialogue with the students continues where the teacher poses a problem that requires students determine the profit that can be made if they sold all the gums at a particular cost per gum. In Line 18, students are asked to work out the profit if they could sell each soldier for one dollar, assuming the cost to convert the gums to soldiers is $2000. The relationship is captured into a linear equation (Line 19).

Teacher: So there's 30,000 pieces of gum, yes? If we divide it by 10, it will tell us how many little soldiers we can make. Do you agree with that? (*Students nod in agreement.*) (Line 17)

Teacher: Then whatever the cost is, if you multiply by the cost, it tells us how much money you will make. Do you agree with that? For example, we put a dollar there (*Teacher points to the formula on the board)*, so each one is multiplied by one dollar. (Line 18)

Teacher: Now, what if we say we subtract the $2000 amount we need to spend? (*Teacher writes formula* ' $P = C(\frac{30000}{10}) - 2000$ ' *on board.*) So this is what this mathematical expression [sic] tells us? How many dollars we can make. This mathematical expression [sic] tells us how much money we're going to make from selling them. (Line 19)

Teacher: This mathematical expression tells us how much money we make, take away the money we have to spend, which leaves us with the profit. Do you agree? Let's hope that's right. (Line 20)

While Simon attempted to capture the relations between profit, number of gums, and selling price, there was an opportunity at Line 19 and 20 to extend the above relations into a model or general equation, $p = c\left(\frac{n}{q}\right) - e$, where p = profit, c = selling price of a soldier, n = number of gums in the school, q = number of gums required to make one soldier and e = cost involved in sterilising the gums. Alas, this was not exploited by Simon. It must be acknowledged that the students did not generate the above equation as a model but the generation of the equation would have been a useful extension by the teacher.

Phase 5: Model Exploration and Evaluation

In this final phase of the lesson, students are invited to examine all the variables embedded in the model with the view to maximising values for p, profit that can be made with the sale of recycling the gums in the form of plastic soldiers (Line 27). By inviting students to substitute different values for variables c and e on the right-hand side of the equation, the teacher brings life to the model. Here we see evidence of the plugging of Data (Figure 1) that is geared towards the focus concept (Linear equation). Through this approach students are supported in better understanding the relations among the variables that they can relate to.

The component of the model that is indicated by $\frac{n}{q}$ can be subjected to discussion, and in this lesson it is assumed to remain constant in the earlier part of Phase 5. While this is appropriate, the model exploration continues by examining the effect on profit (p) if n and q assumed different values (Line 31). This can be seen as a test of the elasticity of the model.

S28: 1000 (Line 21)

Teacher:	1000. Only 1000 dollars. Wow. Profit $1000 (*Teacher writes on board*). Interesting, isn't it? Who can verify that's correct? (*Students nod 'yes'.*) OK. Are you happy with that? (Line 22)
S5:	No. Maybe. (line 23)
S7:	Maybe we should only sterilise a few and see how we go. (Line 24)
Teacher:	So you're saying not sterilise them? (Line 25)
S7:	No, sterilise, like some, and see how we go without telling them – a test run. (Line 26)
Teacher:	So, do a small order – a small section. Not spend too much money and see if it's successful. A good head on your shoulders, S22. That's good. What about if we did something slightly different? Give them names. Turn them into something a bit more saleable and we can charge more. How can we maximise our profit? Somewhere in here (*Teacher pointing to formula on board*). Now, S22 said we could cut our expenses. Good. How else can we maximise our profit? Have a look at what we've got here. (*S20 raises hand.*) (Line 27)
Teacher:	S20? (Line 28)
S20:	Put the price up by $1. (Line 29)
Teacher:	Excellent. How else? So, we can raise it $1 upwards. We can lower the expenses. How else can we raise the possibility of profit? There is one under there, staring us in the face… Less gum. Do we really need 10 pieces of gum to make that small figurine? No. What do you think, S21? (Line 30)
Teacher:	Well, let's decide what happens if we change one aspect of this formula. Let's decide that there's 8 pieces of gum under there and all we need is 5 pieces. All we need to do is replace the n with 5 and see what happens. Can you do that calculation on your calculator, please, and see what happens? (*Students do calculation*) (Line 31)
Teacher:	When you get an answer, just put your hand up. Anybody? (Line 32)
S30:	Is it 4000? (Line 33)
Teacher:	Is it? OK. Have a look at this. 4000 dollars. We've actually quadrupled our profits by halving the amount of gum we use. And what if we decide to bump up the price and see what will happen then? Let's change that $1 to $2 and see what happens in our formula now. Don't forget we need to put '2' in here (*Teacher points to formula*). The cost is $2. And our new answer? Somebody else maybe, S19? (Line 34)
S19:	$8000. (Line 35)

Reflections

The teacher's actions and his interactions with the students can be interpreted from the perspective of cognitive load. A cognitive load-based analysis, in turn, provides one with windows into the accessing of mathematical schemas by the students and the co-construction of the model for the focus problem. The *Chewing Gum Problem*, in its totality, can be seen as having high intrinsic load for students who have to process and connect multiple sources of information during the different phases of the lesson. In his attempts to induct the students to *develop* the abstract model for the chewing gum problem, the teacher takes the students through a series of moves that aim at the identification of a real-life problem and transformation of that problem into its abstract model. We accept that the problem may not induce high intrinsic load for the high-achievers in the class but for most of the students, it appears to engage them by imposing a reasonable level of intrinsic load that is necessary for meaning and connected learning.

Throughout the lesson, Simon keeps the students on task by not splitting their attention with information that could distract them. At the beginning of the lesson, he revisits the structure of linear equations from a previous lesson as well as engages students with questions about the value of algebra before moving onto the context for the chewing gum problem. There were instances of his repeating or rephrasing what students had said or inferred only as a strategy to initiate and maintain the flow of information all of which were geared towards the emergence of the abstract model. Thus, overall, there was evidence of reduction in the imposition of extraneous cognitive load.

Simon uses questions effectively as a means of identifying potential points of entry for student input. These actions prompt students to access and make active use of prior knowledge (e.g., the concept of average), while helping students to accommodate and assimilate new information to existing knowledge. The knowledge integration contributes to the growth of connected bodies of knowledge (schemas) that are instrumental for the retrieval and use of knowledge in future activities (Prawat, 1989; Rumelhart & Ortony, 1977).

The teacher poses questions and prompts students to look for reasonable answers and test for reasonableness of their answers. In so doing, students are called to be actively involved in justifying their actions. Students are constrained to reflect on their responses as evidenced during Phase 5 where the impact of substituting values for a number of variables on the profit that can be made is highlighted. He guides students and takes on a spectator's role in the construction of the model until it is necessary to intervene, with little evidence of 'giving away' answers. There are frequent attempts to remind students to reflect on and monitor their actions in the model generation process. This strategy results in sustainable understandings not just rote learning of formulae and methods. From a cognitive load perspective, the foregoing actions induce Germane load, thus, facilitating schema automation (Sweller, 1989).

What does this study tell us about modelling? Kaiser and Sriraman (2006) posited that, from a cognitive perspective, modelling involves the reconstruction of routes and identification of barriers for learners. The events of the lesson reported in this study, I argue, provides support for the above position but from a theoretical framework that was underpinned by issues of knowledge organisation and integration, that is, schema. The framework that is adopted here (Figure 1) identified important elements that need to be orchestrated for the model to emerge. In this study, I have shown how the teacher helped students collect and assemble the *Data* component of the modelling process. During the process, students were aided in the accessing of concepts such as average and algorithmic knowledge in order to perform computations. Thus, one can see evidence for the activation of *Algorithms* and *Concepts* that anchor modelling, as supported by the framework.

With regard to teaching of algebra, this study highlights two aspects that Kieran (2004) suggested to be key ingredients of effective learning. Firstly, that algebra is about identifying and understanding relationships among variables. Secondly, that these relational understandings need to be fore-grounded in meaningful activities. The actions of the lesson reported here demonstrate the marshalling of variables and the situation of the variable in a realistic context.

Mason, Stephens, and Watson (2009, p. 29) posit that in order to assist students decode the structure that underlies a problem or concept, teachers themselves need to be aware of structures and "have at hand strategies and tactics for bringing structural relationships to the fore". The actions of the teacher in the present study provide evidence of both the facets of discernment of structure and support for learners to appreciate the structure by the use of appropriate tactics. One could view the gradual unravelling of the structure as akin to emergence of the model. In this sense, the model or modelling process could be seen as a vehicle that mediates the consolidation of content as suggested by Stillman, Brown, and Galbraith (2008).

While we did not gather data on the teacher's knowledge base that drove his actions prior to the lesson, there is evidence here that he has built up and accessed a wide algebraic knowledge base that was rich in content and woven into practice. The linkages built during the lesson helped the students develop authentic and meaningful connections such as estimating the average number of gums in the class and computing total number of gums in the school. The modeling of the profit and variables that impacted on the profit had abstract and real-life attributes. This interlinking of concepts and playing them out during the course of the lesson by the use of interesting scenarios constitutes an important facet of teacher knowledge. In this regard, the data here reflect the pattern of results reported by Hill, Rowan and Ball (2005).

In a theoretical analysis of models and their role in supporting understandings that underpin formal mathematics, Gravemeijer (1999) identifies *emergent models*. These are models generated by students themselves and are argued to foster the constitution of formal mathematics. The emergent models are based on informal mathematical activity that gradually develops into a more general *model of* and *model for* contexts, the latter supporting advanced mathematical reasoning. The events reported in the lesson seem to parallel Gravemeijer's transitions in the way the model emerged with students working informally with concepts of average and advancing onto a more formal model that better captured the key relationships. As pointed out earlier, this formal model could have been translated into an equation that indicated the general relationships in an abstract manner. Such an equation, it would seem, constitutes a macro model that is sufficiently plastic for the teacher and students to manipulate values of the independent variables and examine their impact on the dependent variable, profit (p).

In the present study, the students are provided with numerous scaffoldings during the course of model construction by way of questions and prompts. The gradual removal of these scaffoldings and the impact that could have on the model construction and its quality by students of different ability levels constitutes an interesting area for future studies.

While student inputs into the lesson were somewhat scattered and minimal, their responses, when they did respond, indicate a level of engagement with, and understanding of, the modelling process. Data from Phase 1 indicated a high level of motivation on the part of the students in wanting to be part of the modelling activity. They were also actively involved in generating, fitting and evaluating data into the model. Here one evinces students' understanding of the four processes (Examples, Data, Representation, and Algorithms) that are involved in modelling a relatively complex problem. Data from Phase 5 provide evidence that students were examining the model more critically by exploring its behaviour with hypothetical data (Line 26).

Overall, there was evidence of co-learning amongst the teacher and students, and among students, all happening within an environment of open inquiry. Schoenfeld (1985) argues that this type of learning supports ownership of learning by the learner and assists in more meaningful learning that has a high degree of correlation with deeper understanding of concepts and retention of the concepts for future use.

The discourse reported here also shows that there was a high volume of teacher talk in comparison to student talk. The impression that one gets from the lesson is that this teacher adopts a 'traditional' approach to classroom teaching. While the approach may appear to be teacher-driven and conventional, the teacher embraces a constructive, fun-filled and action-based lesson that delves into the deeper facets of algebraic schema activation and modelling.

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