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Abstract

Two principals engage in Hotelling competition for an agent's services under incomplete information as to her outside option (location). This renders the agent's participation decision probabilistic from the perspective of each principal. Regardless of the market structure at equilibrium the optimal contract features a trade-off between participation probability and incentives. Rent and effort are inversely related and non-monotonic in the agent's transport cost and so in market structures; they increase (decrease) with competition. Uncertainty as to the agent's location may increase or decrease the rent compared to full information. This correspondingly harms or benefits principals.

Disciplines

Business

Publication Details

Roger, G. (2016). Participation in moral hazard problems. *Games and Economic Behavior*, 95 10-24.

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June 10, 2015

Abstract

Two principals engage in Hotelling competition for an agent's services under incomplete information as to her outside option (location). This renders the agent's participation decision probabilistic from the perspective of each principal. Regardless of the market structure at equilibrium the optimal contract features a trade-off between participation probability and incentives. Rent and effort are inversely related and non-monotonic in the agent's transport cost and so in market structures; they increase (decrease) with competition. Uncertainty as to the agent's location may increase or decrease the rent compared to full information. This correspondingly harms or benefits principals.

Keywords: moral hazard, asymmetric information, contract, participation constraint, principal-agent. JEL Classification: D82,D86.

1 Introduction

The canonical model of moral hazard takes the agent's outside option as exogenous and known to the principal designing the incentive contract. Assuming so is natural to focus attention on the incentive problem, which is then the sole source of frictions.

This assumption does not match most situations. In the labor market for example an employer must overcome both the “compensating differential” (see for example Rosen, 1983) and the terms of any competing offer.¹ It is easy to argue that in most cases that information is private to

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¹ “[F]irms and workers exchange wage-job attributes bundles in an implicit market.” (Ho, 2012)

the prospective employee. We see this routinely in academic recruiting, where new hires have idiosyncratic preferences. Some people refuse to work for tobacco manufacturers or for defense contractors on ethical grounds. Overcoming this compensating differential can be costly: when he was hired away from Royal Bank of Scotland by Westpac (an Australian bank) CEO Brian Hartzler was reportedly paid a lump-sum of \$7M to be lured.

Casting aside the question of participation is not without loss because the participation decision interacts with the incentives through wealth effects. That is, information about the agent's outside option has a bearing on the optimal contract. A new *participation-incentive* trade-off emerges, with consequences for the power of incentives and therefore the optimal action.

To introduce competition and uncertainty I embed a principal-agent problem in a Hotelling model. Principals are located at the extremes of an interval containing a single agent whose location is her private information. The Hotelling structure allows for the interpretation of the total transport cost (distance $\times$ unit cost) as the compensating differential. Alternatively the distance between the agent and a principal can be interpreted as the degree of fitness of a match. A better match is more productive here because it allows for stronger incentives to be offered.

The participation-incentive trade-off arises because how important it is to secure participation depends on the principals' expected payoff – not on the agent's exogenous outside option. To transfer utility to the agent most efficiently principals improve the insurance properties of the contract. Inframarginal types respond by selecting a lower action than they otherwise would, which entails a social cost. This connection between insurance and effort underpins most results.

When principals compete even the marginal agent receives a rent, and increasing competition exacerbates rent-giving: the (endogenous) outside option of the agent increases, and the principals must offer more. This also induces weaker incentives through the insurance effect. Hence with more substitutable, or with more profitable, principals, the agent receives a higher rent and works less. This is reminiscent of the dot-com bubble of the late 1990's, when firms competed for workers for very little effort in return.² To counter this costly rent-giving employers, if they could, should

²Witness "tolinrome"'s (a pseudo) account: "Everyday we had catered lunches, I mean nice stuff [...] IT employees had their DSL bills paid every month and they paid for everyone's cell phone [...]. They even flew me twice to HQ in SF for a meeting and paid everything [...] I worked about 2 hours a day and spent the rest of the time cruising around SF. Another time we had an IT meeting there, flew me out again, stayed at a ranch in Napa valley area, horseback riding, spa, everything." Source: <http://techtalk.dice.com/t5/Off-Topic-Other-Archive/Crazy->

emphasize differentiation.

Incomplete information has ambiguous effects on rents and effort that depend on the agent's location. When she is very contestable, incomplete information tames Bertrand competition for the agent because the participation-incentive trade-off caps rent-giving. So if they could, firm should commit to remain ignorant of the worker's outside option some of the time. Conversely, a principal cannot take advantage of the agent's proximity and offer a "cheap" contract; the participation-incentive trade-off, not the actual location, pins the marginal participating type.

The optimal action is non-monotonic in the transport cost. For a high transport cost the principals operate as local monopolists; the outside option is high and renders effort costly. Participation mechanically increases as the transport cost drops, which modifies the participation-incentive trade-off; the cost of effort decreases. Incentives become stronger, until principals start competing. At that point the marginal type becomes increasingly contestable and is thus able to extract an increasing rent; then effort drops. To return to the introductory application, both compensating differential and competing offer(s) matter, depending on the equilibrium market structure.

This paper belongs to the broad literature on moral hazard (Mirrlees, 1975; Holmström, 1977, 1979; Rogerson, 1985b; Page, 1987; Jewitt, 1988; Conlon, 2008, 2009). More closely related is the work of Kadan and Swinkels (2010), who study a principal's incentives to alter the agent's action as payment constraints vary. Their principal employs more than one agent whose reservation utilities are observable. The participation problem is standard but with more than one agent, the principal may either ask less effort of everyone, or employ fewer people and ask more effort of them. The single-agent framework neutralizes these incentive problems and focuses on participation.

I purposefully do not adopt a model of common agency (Aubert, 2005; Célérier, 2012). Common agency drastically modifies the principals' incentives in their contract offers, and therefore the agent's action in response. In Bisin and Guaitoli's work (2004) competing principals may offer contracts that induce the low action in equilibrium; they feature full insurance and generate zero profit. Principals may induce the high action and secure positive profits by also offering latent contracts that are not active in equilibrium. These latent contracts deter the other principals from offering more attractive contracts. Attar et al (2006, 2007a, 2007b) show that restricting attention to take-it-or-leave-it offers in such a context entails a loss of generality. Exclusive contracting (here)

stories-from-the-dotcom-peak/td-p/128704

allows for take-it-or-leave-it offers and affords a clean characterization. Parlan and Rajan (2001) also study common agency however without asymmetric information; instead failure is strategic. The incentives of agents to engage in strategic failure weakens the principals' incentives to compete for agents; it breaks the Bertrand logic. Here the Bertrand logic is broken by the participation-incentive trade off.

There is a burgeoning literature on moral hazard in a market context (Besley and Ghatak, 2005; Dam and Perez-Castrillo, 2006; Macho-Stadler et al, 2014; Serfes, 2008; Terviö, 2008). All these papers feature heterogenous agents, an assignment problem and no private information. A good match is important because agent's characteristics affects their productivity; for each (publicly observable) type there exists an optimal contract. Through competitive matching an agent's outside option is given by the next best match – the competing offer in this paper. But here heterogeneity is orthogonal to productivity so types cannot be screened and heterogeneity does not enter the incentive problem. This paper also departs from Besley and Ghatak (2005) precisely because the agent's type affects her outside option but not her productivity. Serfes (2008) and Dam and Pérez-Castrillo (2006) show that identical principals obtain identical profits (zero when the market is short on agents). Here the principals always receive ex ante positive profits under uncertainty thanks to the participation-incentive trade-off, even though the market is always short on agents.

As in Jullien (2000) and Maggi and Rodriguez-Clare (1995) the outside option is type dependent. But it does not affect the agent's production technology, and so does not directly affect incentives. This paper also bears an obvious connection to the work of Rochet and Stole (2002), who study random outside option under adverse selection. Further discussion of the relation between their work and this one is postponed to Section 5.

2 Model

Description. There are two principals, P_0 and P_1 , located at the extremes of an interval of length 1, and a lone agent who can contract with at most one principal. At cost $c(a)$, $c', c'', c''' \geq 0$, the agent takes an unobservable action $a \in \mathcal{A} \subset \mathbb{R}_+$, which yields a stochastic outcome $q \in [\underline{q}, \bar{q}] \equiv \mathcal{Q} \subset \mathbb{R}_+$ with conditional distribution $F(q|a)$ and corresponding density $f(q|a) > 0$. The likelihood ratio $f_a(q|a)/f(q|a)$ is increasing (MLRP), and concave in q . I also impose that $F_a(F^{-1}(q, a)|a)$

be convex in (q, a) , which is sufficient for Concave Local Informativeness (CLI). The agent receives a transfer t ; her net utility is given by $v(t, a) \equiv u(t) - c(a)$, where $u : \mathbb{R} \mapsto \mathbb{R}$ is a continuous, increasing and (strictly) concave function. The principal receives a net payoff $S(t; q) = q - t$ if contracting with the agent, and zero otherwise. A contract $\mathcal{C} = (t(q), a)$ is an effort recommendation a and a transfer function t . As in Holmström's original model, I take $t(q)$ to be equicontinuous.³ Throughout I assume that the conditions of the first-order approach (FOA) hold – see Jewitt (1988). In particular, u and u' do not diverge – see Moroni and Swinkels (2014).

An agent who does not contract receives 0. An agent has a type $d \in [0, 1]$, which is private information and follows the common knowledge, symmetric distribution $G(d)$ with log-concave density $g(\cdot) > 0$. Let $\gamma > 0$ denote the transportation cost in the Hotelling tradition; a lower γ thus means more substitution. If contracting with P_0 , the agent pays a cost $\gamma \cdot d$, which therefore represent her outside option. Let U^1 be the expected utility induced by P_1 's offer; the net utility of this contract is $U^1 - \gamma \cdot (1 - d)$. The agent takes P_0 's contract only if it dominates any alternative: $U^0 - \gamma \cdot d \geq \max \{0, U^1 - \gamma \cdot (1 - d)\}$. The outside option generated is private to the agent and treated as a random variable (by the principals). Furthermore, because $F(q|a)$ and $c(a)$ are independent of d , screening is not possible (see Proposition 6). The timing is standard: (i) the principals each offer a contract $\mathcal{C}_i$, $i = 0, 1$; (ii) the location d is known to the agent only; (iii) the agent accepts one contract or rejects them all. If accepting, she also chooses an action a ; (iv) action a generates an outcome $q \in \mathcal{Q}$.

For large enough γ , the principals do not compete for the agent at equilibrium; the market is not covered in equilibrium. The agent may work for either principal or not at all. For γ small enough, the principals do compete for the agent's services; the market is covered and the contracts are best response to each other; they form a Nash equilibrium.

This formulation allows for competition only at the participation stage; it nests three models including the Holmström (1979) model when d is known. Let the linear functionals

$$\begin{aligned}\pi(t, a) &\equiv \int_{\mathcal{Q}} [x - t(x)] dF(x|a) \\ U(t, a) &\equiv \int_{\mathcal{Q}} u(t(x)) dF(x|a) - c(a)\end{aligned}$$

and $\pi_a(t, a) \equiv \partial\pi(t, a)/\partial a$, $U_a \equiv \partial U(t, a)/\partial a$. Let $\delta U = u'f(q|a)$ denote the Fréchet derivative of

³See Holmström (1977) for details. Restricting attention to this family of functions $t(\cdot)$ comes at little cost.

U , and likewise for other functionals.

Preliminaries. In the standard model there is a single principal and the agent's outside option $\bar{u}$ is known to all parties. The principal maximizes $\pi(t, a)$ subject to participation ($U(t, a) \geq \bar{u}$) and incentive compatibility ($U_a(t, a) = 0$ under the FOA). This is a well known problem; Holmström 1977's seminal result is summarized as

$$\forall q, \quad \frac{1}{u'(t(q))} = \lambda + \mu \frac{f_a(q|a)}{f(q|a)}, \quad \lambda > 0 \quad (2.1)$$

$$\pi_a + \mu U_{aa} = 0. \quad (2.2)$$

Remark 1 *In the standard model the optimal action is decreasing in the outside option $\bar{u}$.*⁴

3 Analysis

Let $\tilde{d}$ denote the marginal participating agent. The agent's participation decision defines a random variable: the participation constraint is satisfied with probability

$$\Pr \left(U^0 \geq \max \left\{ \gamma \tilde{d}, U^1 - \gamma(1 - 2\tilde{d}) \right\} \right).$$

Re-arranging a pair of inequalities, P_0 's coverage is $G \left(\min \{ U^0/\gamma, (\gamma + U^0 - U^1)/2\gamma \} \right)$. Then

Problem 1

$$\max_{t,a} G \left(\min \left\{ \frac{U^0}{\gamma}, \frac{\gamma + U^0 - U^1}{2\gamma} \right\} \right) \pi(t, a)$$

s.t.

$$U_a(t, a) = 0 \quad (3.1)$$

The objective function includes the stochastic participation decision and is non-monotone concave, unlike the standard problem.⁵ This is depicted in Figure 1, where the horizontal axis represents an increase in t for any q . In the standard problem (left), increasing t is a strict cost for the principal; here (right) it increases the participation probability up to a point – hence non-monotonicity.

Principals are local monopolists whenever $\gamma > U^0 + U^1$; otherwise there is competition at equilibrium. Which of these inequalities holds is determined in equilibrium. For now I study each case in turn and postpone that other question.

⁴Proof upon request; in the same spirit as that of Proposition 1.

⁵Log-concavity of $g(\cdot)$ is sufficient for concavity of the objective $G(\cdot)\pi(t, a)$.

Figure 1: objective functions. Standard problem (left) and this problem (right)

3.1 Local monopolies

Since only one firm is considered in this section I drop the subscripts. The one complication is whether the marginal participating agent $\tilde{d}$ sits at location 1 or strictly below. The solution turns out to be continuous all the way to $\tilde{d} = 1$. In what follows I lay out P_0 's problem in either case.

At $\tilde{d} = 1, G(\tilde{d} = 1) = 1$, adopt the convention that $g(\tilde{d} = 1) = 0$; when $\tilde{d} < 1$, $G(\tilde{d}) < 1$ and $g(\tilde{d}) > 0$. If $\tilde{d} = 1$ the principal solves

Problem 2

$$\max_{t,a} \pi(t, a) \text{ s.t. } U_a(t, a) = 0 \text{ and } U(t, a) = \gamma$$

and if $\tilde{d} < 1$,

$$\max_{t,a} G\left(\frac{U^0}{\gamma}\right) \pi(t, a) \text{ s.t. } U_a(t, a) = 0$$

The second line comes directly from Problem 1. The first one features $G(U/\gamma) = G(\tilde{d} = 1) = 1$; at that point any agent participates so the rent should be capped at $U(t, a) = \gamma \cdot \tilde{d} = \gamma$. Attach Lagrange multiplier λ to that constraint; maximizing with respect to a always yields

$$G(U(t, a)/\gamma) \pi_a(t, a) + \mu U_{aa}(t, a) = 0, \quad (3.2)$$

almost as in the standard problem (and where $G(U(t, a)/\gamma)$ may be 1). Because $g(\cdot)$ is log-concave, so is $G(\cdot)$; with $\delta U(t, a) > 0$ and $\delta \pi(t) < 0$, the second-order condition is immediately verified.

Next maximizing pointwise with respect to t one has⁶

$$\frac{1}{u'(t)} = \begin{cases} \lambda(\gamma) + \mu^M \frac{f_a}{f}, & \text{if } \tilde{d} = 1; \\ \frac{g(U(t,a)/\gamma)}{G(U(t,a)/\gamma)} \frac{\pi(t,a)}{\gamma} + \hat{\mu}^M \frac{f_a}{f}, & \text{if } \tilde{d} < 1. \end{cases} \quad (3.3)$$

⁶ $U(t, a) = \int u(t(z)) dF(z|a) - c(a)$ is a linear functional and the critical d satisfies $U(t, a) = \gamma \cdot d$.

where $\hat{\mu}^M = \mu/G(U(t, a)/\gamma) = -\pi_a/U_{aa}$. The first line is as the standard case. The second equality defines a fixed-point problem in a space $\mathcal{T}$ of functions defined on $\mathcal{Q}$. The solution, if it exists, is the transfer function $t(q)$. It turns out to be less problematic than at first glance. The new term $\frac{g(U(t, a)/\gamma)}{G(U(t, a)/\gamma)} \frac{\pi(t, a)}{\gamma}$ in (3.3) is a linear functional from $\mathcal{T}$ to $\mathbb{R}$. So for a given action a , the first-order condition (3.3) rewrites

$$\frac{1}{u'(t)} = \alpha + \hat{\mu}^M \frac{f_a}{f}, \quad \alpha \in \mathbb{R}_+$$

which now resembles the standard Condition (2.1).⁷ Recalling that $U(t, a)/\gamma = \tilde{d}$,

Theorem 1 *A solution to Problem 2 is a transfer function t^M , an action a^M and a pair of real numbers $\alpha, \hat{\mu}^M$ such that Conditions (3.2), (3.3), the constraint (3.1) and $U(t, a) = \gamma \cdot \tilde{d}$ for some $\tilde{d}$ simultaneously hold, with*

$$\alpha^M(\tilde{d}) = \begin{cases} \lambda(\gamma), & \text{if } \tilde{d} = 1; \\ \frac{g(U(t, a)/\gamma)}{G(U(t, a)/\gamma)} \frac{\pi(t, a)}{\gamma}, & \text{if } \tilde{d} < 1. \end{cases} \quad (3.4)$$

The solution is continuous at $\tilde{d} = 1$, i.e.

$$\lim_{\tilde{d} \rightarrow 1} \alpha(\tilde{d}) = \lambda(\gamma).$$

The principal faces a trade-off between incentives and participation. This trade-off is captured by the term $[g(\bar{u}/\gamma)/G(\bar{u}/\gamma)][\pi(\bar{u}/\gamma)/\gamma]$ and is depicted in the right-hand panel of Figure 1. When the solution is interior (i) participation is not guaranteed, even if it is profitable to the principal (who would benefit from more information to increase it) and (ii) the expected marginal benefits of the principal and the agent are equalized. That is,

$$\mathbb{E}_q \left[\frac{1}{u'(t^M(q))} \right] = \frac{g(\bar{u}/\gamma)}{G(\bar{u}/\gamma)} \frac{\pi(\bar{u}/\gamma)}{\gamma}, \quad (3.5)$$

where the right-hand side is the expected marginal benefit to the principal of increasing t , and $\mathbb{E}_q \left[\frac{1}{u'(t^M(q))} \right] = \mathbb{E}_q [(u')^{-1}(t^M(q))]$ the expected marginal cost. In the standard problem $\mathbb{E}_q [1/u'(t^S(q))] = \lambda^S(\bar{u})$: the marginal cost is equated to the agent's opportunity cost, regardless of the principal's benefit. Uncertainty about the agent's outside option restores some bargaining power.

⁷ $\alpha < 0 \Leftrightarrow \pi(t, a) < 0$ and the principal prefers to not contract.

Holmström's (1977) insight is affirmed by Condition (3.3): the transfer $t^M(q)$ tracks the likelihood ratio f_a/f . However surplus sharing varies according to the principal's marginal benefit of participation. This alters the incentive properties of the optimal contract. For any d , denote by $(t^S(d), a^S(d))$ the contract under full information (i.e. solving (2.1) and (2.2)).

Proposition 1 *Take $\tilde{d} \in (0, 1]$ such that $U(t^M, a^M) = \gamma \cdot \tilde{d}$. For all $d < \tilde{d}$, $a^M(d) < a^S(d)$.*

The principal induces a lower action from inframarginal types than if their outside option were known and they were offered the corresponding optimal contract. Their incentives are too weak. Increasing the probability $G(U(t, a)/\gamma \geq d)$ of acceptance requires increasing the induced expected utility $U(t, a)$. The principal transfers utility most cheaply to the risk-averse agent by improving the insurance properties of the contract – not just to by increasing the transfer. To improve insurance he increases t in low-income states (where $f_a < 0$) and decreases it in high-income states (where $f_a > 0$).⁸ This is bad for incentives. The expected utility $U(t^M, a^M)$ just satisfies type $\tilde{d}$ but entails excessive insurance for all others accepting the contract. The principal would benefit from information about the agent's type to provide stronger incentives.

3.2 Competitive setting

When γ is low enough the principals compete for the marginal agent who is identified by the standard condition $\hat{d} = (\gamma + U^0 - U^1)/2\gamma$: the outside option is the (endogenous) competitor's offer. Without loss I seek a symmetric Nash equilibrium of the contract game.⁹ P_0 solves

Problem 3

$$\max_{t_0, a_0} G\left(\frac{1}{2}\left[1 + \frac{U^0 - U^1}{\gamma}\right]\right) \int [x - t_0(x)] dF(x|a_0) \quad s.t. \quad (3.1)$$

where $U^i \equiv U(t_i, a_i)$. Because the agent chooses a , *after* agreeing to participate, subgame perfection in a immediately yields the envelope condition (3.2) – up to the exact definition of $\hat{d}$. Optimizing with respect to the transfer gives the best response

$$\frac{1}{u'(t_0)} = \frac{g(\hat{d})}{2\gamma G(\hat{d})} \pi(t_0) + \frac{\mu}{G(\hat{d})} \frac{f_a}{f}, \quad (3.6)$$

and similarly for P_1 . By extension of Theorem 1 (in the Appendix) a solution to (3.6) exists.

⁸ U is concave: take a variation with respect to t : $u'f > 0$ (with the envelope condition $U_a = 0$) and $u''f < 0$.

⁹There exists a unique equilibrium of this game; strict diagonal concavity of the objective function is satisfied (Rosen, 1965). Proof upon request.

Proposition 2 *In the unique, symmetric Nash equilibrium of the contract game, for each principal, the equilibrium contracts are characterized by (a^C, t^C) such that*

$$\frac{1}{2}\pi_a + \mu U_{aa} = 0 \quad (3.7)$$

$$\frac{1}{u'(t, a)} = \frac{g(\frac{1}{2})}{\gamma}\pi(t, a) + 2\mu\frac{f_a}{f} \quad (3.8)$$

The contract commits a fixed share $g(1/2)/\gamma$ of the principal's profits π to the agent.¹⁰ The competitor's offer depends on γ , which parametrizes the intensity of competition. Participation remains stochastic, so the participation-incentive trade-off remains. Because contracting is exclusive the incentive component $(\mu/G(\hat{d}))(f_a/f)$ of the best reply is substantively unchanged.

3.3 Equilibrium market structure

Whether the principals compete depends on γ . For any economy $(\mathcal{Q}, c(\cdot), F(q|a))$, there exists a critical value of γ such that

$$U(t^M, a^M) = \gamma \cdot \frac{1}{2} = U(t^C, a^C),$$

where $1/2$ is the location of the agent that is just attracted under either regime. Let this relation identify γ^* . For $\gamma \leq \gamma^*$ competition prevails; otherwise either principal is a monopolist. As shown in Proposition 2 the transition is smooth.

4 Information, market structure and outcomes

Effort may vary according to the information structure as well as market structure. I study each in turn, beginning with the impact of uncertainty in a (imperfectly) competitive market.

4.1 The impact of uncertainty

If d were known principals would engage in Bertrand competition for the agent. This process implies that $U^0 - \gamma \cdot d = U^1 - \gamma \cdot (1 - d)$; so the agent would always contract with the principal closest to her. Let $\bar{U}^1$ denote the level of utility such that $\pi^1(\bar{U}^1) = 0$; this is the most utility P_1 can bestow to the agent. Let $d^0(\gamma)$ the corresponding type; it is defined by the relation

$$\bar{U}^1 - \gamma \cdot (1 - d) = 0,$$

¹⁰Integrate (3.8) over $Q : \mathbb{E} \left[\frac{1}{u'(t, a)} \right] = \frac{g(\frac{1}{2})}{\gamma}\pi(t, a)$.

so $d^0(\gamma) \in [0, 1/2]$ (otherwise P_1 cannot even contest the agent), and is increasing in γ (similarly $d^1(\gamma)$ for P_1). There are two cases to consider. First, for any $d < d^0(\gamma)$, P_0 would be a monopolist in equilibrium if d were known because P_1 could not offer more than $\bar{U}^1$. Second, for $d^0(\gamma) \leq d \leq 1/2$, the agent would be contestable by both principals. Figure 2 helps fixing ideas (horizontal axis).

Proposition 3 *Whether incomplete information generates higher rents and induces a lower action than complete information depends on the agent's location.*

1. *If $d \leq d^0(\gamma)$, P_0 would be a monopolist under complete information. The rent is higher and the equilibrium action is lower than if the outside option were known; i.e. $a^C < a(d)$.*
2. *If $d > d^0(\gamma)$, principals would compete under complete information. There exists a $d_c^0(\gamma) \in (d^0(\gamma), 1/2]$ such that*
 - (a) *If $d \in (d^0(\gamma), d_c^0(\gamma))$, the equilibrium rent is higher and the action is lower than if the outside option were known; i.e. $a^C < a(d)$; and*
 - (b) *if $d \in (d_c^0(\gamma), 1/2]$, the equilibrium rent is lower and the action is higher than if the outside option were known; i.e. $a^C > a(d)$*

Incomplete information induces a *lower* rent (and higher action) when the agent is highly contestable. If they had an information acquisition technology, principals should commit themselves to not use it at least some of the time. Conversely, if agents could credibly disclose some information, they should claim indifference. Cremer (1995) also shows that less information about the agent's type is better in a two-period model of moral hazard and adverse selection. His principal can generate stronger effort incentives through willful ignorance about the agent's productivity. Then following an adverse outcome in period one there cannot be any renegotiation and the agent may exert a second-period effort in excess of the second-best. Here ignorance introduces the participation-incentive trade-off, which curbs rent giving. If the principals knew the agent's location they would *always* compete away their rents (item 2 of the Proposition). So the lack of information acts like a commitment device; in Cremer (1995) it prevents renegotiation, here it limits competition.

On Figure 2 we see that when $\tilde{d} > 1/2$, P'_0 's profit under complete information is naught, the expected profit (π^N) under incomplete information is positive. The complete-information profit

Figure 2: profit functions and thresholds across regimes

increases when the agent is closer to P_0 ($< 1/2$); at the point $d_c^0(\gamma)$ the two are equal. So on the range $[d_c^0(\gamma), 1/2]$, P_0 is better off not knowing $\tilde{d}$. As $\tilde{d}$ decreases further this is no longer true; the lack of information leads P_0 to offer too much to the agent. The slope of π_0 changes at the point $d^0(\gamma)$ precisely because the agent becomes contestable: moving slightly to the right of $d^0(\gamma)$ not only increases $\gamma \cdot d$, it also invites offer from P_1 .

Under Item 1 the agent is too far away from P_1 to even be contestable under complete information. But when $\tilde{d}$ is unknown the principals engage in wasteful competition for the agent: P_0 must offer a contract that is attractive to a type (fictitiously) located at least at $1/2$. Then the logic of Proposition 1 applies.

Item 2 asserts that when the agent is highly contestable ($> d_c^0(\gamma)$), incomplete information tames principal competition. Suppose the agent is located at $d = 1/2$. Under complete information competition dissipates all the principals' profit; the agent's rent is the highest and she works the least. Under asymmetric information, the participation-incentive trade-off guarantees that not too much utility is transferred to the agent – see the FOC (3.6). Each principal ends up with a participation probability of $1/2$ and positive expected profits (Condition 3.8). As the agent moves towards P_0 , that principal's full-information profit increases and reaches the level of the incomplete-information profit at $d_c^0(\gamma)$. Past that point the logic of Item 1 prevails again.

4.2 Market structure, rent and action

Here I attempt to understand how participation levels and effort behave as γ varies over wide enough a range for either monopoly or duopoly to arise in equilibrium.

Proposition 4 *The participation probability $G(\tilde{d})$ increases as γ decreases and $G(\tilde{d}) = 1 \quad \forall \gamma \leq \gamma^*$. The rent is U-shaped and the effort hump-shaped in γ ; they, respectively,*

- *decreases (increases) as γ increases towards γ^* (under competition); and*
- *increases (decreases) beyond γ^* (under monopoly).*

Under monopoly a lower γ has two effects. First there is a direct effect: fix the contract (fix U^0), decreasing γ mechanically increases the participation probability $G(U^0/\gamma)$. This is the market share effect (extensive margin). As participation increases exogenously through the market share effect, the principal offers a steeper contract that induces a higher action. This is the second effect – the margin effect (intensive margin). More can be demanded from the agent because the cost of any action decreases.

As γ keeps decreasing below γ^* , one switches to the competitive regime. In equilibrium, the exogenous market share effect disappears altogether (see Condition (3.8)). In response to competition rents must increase in the form of a lower-power contract to secure participation (from the marginal agent). A natural implication is that when agents are more contestable principals should differentiate more to preserve the incentive power of monetary transfers.

4.3 Welfare implications

The impact of uncertainty on welfare is not entirely obvious *prima facie*. The principal is effort-constrained: $\mu^M, \mu^C > 0$ so that he would like a higher action. However the optimal contract presents the risk-averse agent with better insurance, at the cost of a lower action. Let $\tilde{d}_0, \tilde{d}_1$ denote the marginal participating types with P_0, P_1 ($\tilde{d}_0 = \tilde{d}_1 = 1/2$ under competition). Let social welfare

$$W(a) \equiv G(\tilde{d}_0) [\pi^0(t, a) + U^0(t, a)] + [1 - G(\tilde{d}_1)] [\pi^1(t, a) + U^1(t, a)] - [1 - G(\tilde{d}_1) + G(\tilde{d}_0)] \bar{u}.$$

The first two terms are the expected joint payoff from the relationship with either principal, if it is entered into; the last one is the value of the exogenous outside option if no contract is accepted. Although a higher action shifts the distribution $F(x|a)$ of the output in a first order sense, it may not be chosen in equilibrium because too expensive. Recall the definition of $d_c^0(\gamma)$, then

Proposition 5 *Suppose $d \leq 1/2$; P_0 contracts with the agent and the optimal contract induces*

- lower welfare if $d < d_c^0(\gamma)$ (whether under monopoly or duopoly), and
- higher welfare if $d > d_c^0(\gamma)$ (always under competition)

than when the outside option is known. Similarly when $d \geq 1/2$; then P_1 contracts with the agent, the critical cut-off is $d_c^1(\gamma)$ and the inequalities are reversed.

For $d < d_c^0(\gamma)$, the loss of welfare due to a lower action dominates the benefit of enhanced insurance, and conversely for $d > d_c^0(\gamma)$. This result relies on the fact that the Pareto frontier tracks the agent's action. Better insurance for the agent also implies rent giving, which decreases the value of effort for the principal. When $d > d_c^0(\gamma)$, asymmetric information caps the agent's rent thanks to the participation-incentive trade-off (recall Proposition 4).

5 Discussion

Screening. Menus of contract cannot make the principal better off in this model. Consider a single principal who uses a direct revelation mechanism to elicit the agent's private information of the form $(t(q; \bar{u}), a(\bar{u}))$. Denote by $\tilde{u}$ be the agent's private information (type) and by $\hat{u}$ her message. Truthful revelation requires $U(t(\cdot; \tilde{u}), a(\tilde{u}); \tilde{u}) \geq U(t(\cdot; \hat{u}), a(\hat{u}); \tilde{u}), \forall \tilde{u}, \hat{u}$. Equivalently the agent solves $\hat{u} \in \arg \max_{\bar{u} \in \mathcal{U}_0} U(t(\cdot; \hat{u}), a(\hat{u}); \tilde{u})$; taking a variation $\Delta t_{\bar{u}}$ of t w.r.t. $\bar{u}$ at $\hat{u} = \tilde{u}$,

$$u'(t(q))\Delta t_{\bar{u}}|_{\bar{u}} f(q|a) + U_a \frac{\partial a}{\partial \bar{u}} \Big|_{\bar{u}} = 0 \quad (5.1)$$

By (3.1), $U_a = 0$ and $u'(t(q))f(q|a) > 0$; so truthful revelation requires $\Delta t_{\bar{u}}|_{\bar{u}} = 0$. There can be no discrimination on the basis of the outside option.

Proposition 6 *A menu of contracts contingent on the agent's outside option cannot do better than the non-linear contract given by (3.2) and (3.3).*

There is no *direct* connection between the agent's type and her technology $(F(q|a), c(a))$, so the output q is not informative of the type. The single-crossing property does not hold – this is the first term of (5.1).¹¹ A stochastic mechanism also does not help, for the same reason. Truthful revelation requires some ex post observability of $\bar{u}$, which is beyond the scope of this paper.

¹¹The single-crossing condition is lost in a broad sense: the game is not even supermodular.

Connection to Rochet and Stole (2002). These authors (now RS) study a problem of non-linear pricing where two principals compete on a Hotelling line; the location of the agent is private information and orthogonal to the quality of the good. RS find three main results: (i) less distortion for all types (with no distortion at the bottom), (ii) limited participation under monopoly and (iii) for sufficiently competitive principals, no distortion at all. Below I contrast their findings to mine, in the same order.

First, RS establish there is less distortion than the standard case for all types. The reason is a trade-off between participation and incentives: inducing more participation is achieved by increasing rents; under adverse selection this means reducing distortions in the allocation. The same trade-off exists here but under moral hazard rents are delivered through more distortion, not less. Second, RS show that a monopolist nonetheless never wants to induce full coverage: he should raise his price instead. This too is motivated by the trade-off between participation and incentives: the monopolist is better-off shutting down a measure of agents so as to not give away too much rent. Here too the monopolist shuts down some agents (when $\tilde{d} < 1$), for the same reason: inducing more participation becomes too costly. However in some departure, full participation nonetheless may be optimal here – when the outside option parameter γ is small enough. This difference is grounded in the nature of the problem. In RS, the cost of shutting down the agent increases with the type; shutting down very low types is almost costless, but the benefit (rent-saving) is always positive. Here the cost of shutting down an agent is constant; it is independent of the agent's type and never vanishes.

Third, RS find there is no distortion at all when agents are sufficiently contestable; this generates the first-best allocation. Principals compete for agent(s) by handing out rents; under adverse selection this means reducing the incentive distortions, which improve welfare. Here fierce principal competition is bad for welfare. It also requires handing out rents to agent, which means improving insurance, now worsening welfare. The outcomes are different because the distortions induced by the principals under adverse selection are privately optimal and socially inefficient but the rents are neutral. Under moral hazard the distortions are also socially inefficient but the rents are *not* neutral on effort; they amplify the distortions.

Last, RS note that the rent and the allocation rule (the control) are jointly determined. Here too the rent and the transfer function (the control) are jointly determined.

The linear-CARA-normal model. The CARA-normal-linear framework of Holmström and Milgrom (1987) has become a workhorse of applied research. However it offers insights about the effects of stochastic participation that do not extend to a more general setting.

To make the point consider the monopoly problem. Let $t = \alpha + \beta q$ be the tariff offered, $c(a) = (c/2)a^2$ and $u = -e^{-r(t-c(a))}$, where r is the coefficient of risk aversion. Let also $q \sim \mathcal{N}(0, \sigma^2)$. Upon accepting the contract the agent's problem (i.e. (3.1)) is unchanged. The principal solves

$$\max_{\alpha, \beta} G \left(-e^{-r[\alpha + \beta^2/2c - (r/2)\beta^2\sigma^2]} \right) \left[\frac{\beta}{c} - \left(\alpha + \frac{\beta^2}{c} \right) \right],$$

whence $\beta = 1/(1 + rc\sigma^2)$; the stochastic nature of the outside option has no consequences on incentives. This outcome owes precisely to that specification, which neutralizes wealth effects. The agent's optimal action defined as $a = \beta/c$ is independent of level of utility; therefore the equilibrium slope parameter β is independent of $\bar{u}$. This is clearly not true according to the standard condition $1/u' = \lambda^S + \mu^{SB} f_a/f$ (where $\lambda \equiv \lambda(\bar{u})$), and even less in the FOCs (3.3) and (3.8). The CARA specification understates the importance of the participation problem.

6 Conclusion

This paper presents a model of contracting under moral hazard when the agent's outside option is unknown to principals competing *à la* Hotelling. The model captures both imperfect principal competition and a stochastic outside option. While in the standard model the agent always or never participates, here she does with a non-degenerate probability. With a risk-averse agent increasing the participation probability is best achieved by offering her better insurance; that is, incentives are necessarily weakened.

Asymmetric information turns out to help competing principals when the agent is highly contestable, that is, when principals must transfer a large fraction of surplus. Because the optimal contract entails a trade-off between incentives and participation, it guarantees that principals do not transfer all the surplus to the agent. The agent's rent is capped and the incentives are not completely diluted. The equilibrium action is also responsive to the market structure. It is non-monotonic in the Hotelling "transportation cost". Common agency, where the agent may enter into multiple contracts, is left for future research.

APPENDIX

A Additional material

A.1 Elements of the construction of Theorem 1

The basic idea is to view the first-order conditions as Kuhn-Tucker conditions rather than a fixed-point problem. The proofs of Lemmata 2-4 are available upon request.

Lemma 1 *Suppose the outside option $\bar{u}$ is known to the principal. A transfer function $t^S(\cdot)$ is optimal if and only if it takes the form defined in (2.1) for multipliers $\lambda^S, \mu^S \geq 0$, with $U_a = 0$, $U(t, a) = \bar{u}$ and $\lambda^S [U(t, a) - \bar{u}] = 0$. These multipliers exist; furthermore, $\lambda^S, \mu^S > 0$.*

See Jewitt, Kadan and Swinkels (2008) for a proof. Fix the action a , the first-order condition (2.1) defines a function $t[\lambda^S, \mu^S; a](q)$. With this, the constraints $U_a = 0$, $U(t, a) = \bar{u}$ define functions $\lambda^S[\bar{u}, a]$ and $\mu^S[\bar{u}, a]$ as solutions to the system

$$\mathbb{E}_q \left[u \circ (u')^{-1} \left(\frac{1}{\lambda^S + \mu^S \frac{f}{f_a}} \right) \right] = \bar{u} + c(a) \quad (\text{A.1})$$

$$\mathbb{E}_q \left[u \circ (u')^{-1} \left(\frac{1}{\lambda^S + \mu^S \frac{f_a}{f}} \right) \frac{f_a}{f} \right] = c'(a) \quad (\text{A.2})$$

Lemma 2 *Fix a . $\lambda[\bar{u}; a]$ is increasing in $\bar{u}$.*

Proof: Because $u'(\cdot)$ is decreasing, so is $(u')^{-1}$. The term $(\lambda^S + \mu^S f/f_a)^{-1}$ clearly decreases in λ^S , hence $(u')^{-1} \left(\frac{1}{\lambda^S + \mu^S \frac{f}{f_a}} \right)$ is an increasing function of λ^S , therefore so is the LHS of the first equation. It follows that $\lambda^S[\bar{u}; a]$ is increasing. ■

At a solution the multiplier μ^S is strictly positive. Substitute $t[\lambda^S, \mu^S; a](q) = t[\bar{u}, a](q)$ in Condition (2.2), $\mu[\bar{u}, a] = -\frac{\pi_a}{U_{aa}}$ defines now $a(\bar{u})$. Thus,

Lemma 3 *Given $\bar{u}$, a solution is a tuple $(t[\bar{u}](q), \lambda^S(\bar{u}), \mu^S(\bar{u}), a(\bar{u}))$ such that Conditions (2.1), (2.2), $U_a = 0$, $U(t, a) \geq \bar{u}$ all bind.*

For a given $\bar{u}$ a solution exists (Lemma 1) and can be computed (Lemma 3). The optimal transfer is then given by Condition (??). Condition (B.1) pins the value of d – the marginal agent.

Lemma 4 *The multiplier μ^M of Condition (3.3) is strictly positive.*

A.2 A useful Lemma

The following will be useful to prove Propositions 4 and 5.

Lemma 5 *In the competitive setting under incomplete information effort decreases in the intensity of competition γ , i.e. $da_i/d\gamma > 0$, $i = 0, 1$.*

Proof: Let $a^C = a_i^C, t^C = a_i^C, i = 0, 1$ denote the symmetric optimal contract solving Problem 3. Let $\mathcal{L}(\gamma, a^C) \equiv \max_a \mathcal{L}(\gamma, a)$ be the maximum of the Lagrangean of Problem 3 under this contract; that is, $\mathcal{L}_a(\gamma, a^C) = 0$. Everything is clearly continuous and differentiable. Thus

$$\frac{d}{d\gamma} \mathcal{L}_a(\gamma, a^C) = \frac{\partial^2 \mathcal{L}(\gamma, a^C)}{\partial a \partial \gamma} + \frac{\partial^2 \mathcal{L}(\gamma, a^C)}{\partial a^2} \frac{da}{d\gamma} = 0$$

and I need to show $\frac{da}{d\gamma} > 0$, for which $\frac{\partial^2 \mathcal{L}(\gamma, a^C)}{\partial a \partial \gamma} > 0$ is sufficient ($\frac{\partial^2 \mathcal{L}(\gamma, a^C)}{\partial a^2} < 0$). It is easier to work with the conditions of the problem of the agent, namely $U_a = 0$.¹² The identity $\frac{d}{d\gamma} U_a \equiv 0$ rewrites

$$\int u' \Delta t dF_a + U_{aa} \frac{da}{d\gamma} = 0$$

where Δt denotes a variation in t with respect to γ : $\Delta t = \lim_{\gamma_2 \rightarrow \gamma_1} \frac{t[\gamma_2](q) - t[\gamma_1](q)}{\gamma_2 - \gamma_1}$ and $U_{aa} < 0$. So the sign of $da/d\gamma$ follows that of the first term. In that first term the action a remains constant. For some $\tilde{q}$,

$$f_a \begin{cases} < 0, & q < \tilde{q}; \\ = 0, & q = \tilde{q}; \\ > 0, & q > \tilde{q}. \end{cases}$$

Take any $\gamma_2 \downarrow \gamma_1$, the corresponding transfers $t[\gamma_2], t[\gamma_1]$ must take the form $1/u' = \beta + \tilde{\mu} f_a/f$. Because the action is fixed, if $t[\gamma_1]$ passes through the point $\tilde{q}$, $t[\gamma_2]$ passes arbitrarily close to it (by continuity). Then, if the contract $t[\gamma_2](q)$ is steeper,

$$t[\gamma_2](q) \begin{cases} < t[\gamma_1](q), & q < \tilde{q} \text{ and} \\ > t[\gamma_1](q), & q > \tilde{q}. \end{cases}$$

so that

$$\Delta t \begin{cases} < 0, & q < \tilde{q} \text{ and} \\ > 0, & q > \tilde{q}. \end{cases}$$

¹²When the principal would like the agent to increase her effort, so would the agent – with the appropriate incentive. In Problem 3 the moral hazard constraint (3.1) implies that $a' = \arg \max U(t, a)$ implements $a^C = \arg \max \mathcal{L}(\gamma, a)$ and the multiplier μ is positive.

and $\int u' \Delta t dF_a > 0$ necessarily. The converse holds when $t[\gamma_2](q)$ is shallower. To complete the argument, recall that in equilibrium the marginal agent is always located at $d = 1/2$, and that she contracts with P_0 only if $U^0 \geq U^1 - \gamma/2$: the outside option of the marginal agent is decreasing in γ . Because U^0 and U^1 are strategic complements they both decrease as γ increases. One can construct an alternative transfer $t(q)$ such that $\int u(t[\gamma_1](x)) dF(x|a[\gamma_1]) < \int u(t[\gamma_2](x)) dF(x|a[\gamma_2])$ that also satisfies the moral hazard constraint. Take $t[\gamma_2](q)$ steeper than $t[\gamma_1](q)$ (so $t[\gamma_2](q)$ single-crosses $t[\gamma_1](q)$ from below at $\tilde{q}$). Then

$$\begin{aligned}
& \int_{\underline{q}}^{\tilde{q}} u(t[\gamma_2](x)) dF_a(x|a) + \int_{\tilde{q}}^{\bar{q}} u(t[\gamma_2](x)) dF_a(x|a) \\
&= \int_{\underline{q}}^{\tilde{q}} u(t[\gamma_1](x)) dF_a(x|a) - \int_{\underline{q}}^{\tilde{q}} [u(t[\gamma_1](x)) - u(t[\gamma_2](x))] dF_a(x|a) \\
&+ \int_{\tilde{q}}^{\bar{q}} u(t[\gamma_1](x)) dF_a(x|a) - \int_{\tilde{q}}^{\bar{q}} [u(t[\gamma_1](x)) - u(t[\gamma_2](x))] dF_a(x|a) \\
&= \int_{\underline{q}}^{\tilde{q}} u(t[\gamma_1](x)) dF_a(x|a) - \int_{\underline{q}}^{\tilde{q}} [u(t[\gamma_1](x)) - u(t[\gamma_2](x))] \frac{f_a}{f} dF(x|a) \\
&+ \int_{\tilde{q}}^{\bar{q}} u(t[\gamma_1](x)) dF_a(x|a) - \int_{\tilde{q}}^{\bar{q}} [u(t[\gamma_1](x)) - u(t[\gamma_2](x))] \frac{f_a}{f} dF(x|a) \\
&= c'(a) - \left(\int_{\underline{q}}^{\tilde{q}} [u(t[\gamma_1](x)) - u(t[\gamma_2](x))] \frac{f_a}{f} dF(x|a) + \int_{\tilde{q}}^{\bar{q}} [u(t[\gamma_1](x)) - u(t[\gamma_2](x))] \frac{f_a}{f} dF(x|a) \right) \\
&> c'(a)
\end{aligned}$$

where the penultimate line comes from the moral hazard constraint under γ_1 . The last line uses the fact that $u(t[\gamma_1](x)) - u(t[\gamma_2](x)) > 0$ and $\frac{f_a}{f} < 0$ to the left of $\tilde{q}$, and conversely to its right. So under $t[\gamma_2](q)$ the agent would rather pick a higher action, and it is cheaper to the principals. Hence $da/d\gamma > 0$. ■

B Proofs

Proof of Theorem 1: For completeness, the second line of Condition (3.3) is obtained from the FOC

$$g\left(\frac{U}{\gamma}\right) u' f \frac{\pi(t, a)}{\gamma} - G\left(\frac{U}{\gamma}\right) f + \mu u' f_a = 0$$

since $U(t, a)$ is a linear functional – and so pointwise differentiable. Re-arrange, divide by f, u' and G and set $\hat{\mu} = \frac{\mu}{G}$. Next,

Lemma 6 Let $\tilde{d} \in [0, 1)$. There exists a function $\alpha^M(d)$ such that

$$\alpha^M(\tilde{d}) = \frac{g(\tilde{d})}{G(\tilde{d})} \frac{\pi(\tilde{d})}{\gamma} \quad (\text{B.1})$$

Proof: The right-hand side of Condition (B.1) is decreasing in d . Indeed the first term is clearly decreasing in d since $g(\cdot)$ is assumed to be log-concave (see for example Bagnoli and Bergstrom, 2005) and the action a as fixed. Since a is fixed, the second term is also decreasing in d : increasing the agent's expected payoff $U(t, a)$ can only be achieved by decreasing the principal's expected payoff. A sufficient condition for the equality to hold is that the function $\alpha(d)$ be increasing in d . By Lemma 1 an $\alpha(d)$ exists for each d and by Lemma 2, $\alpha(d)$ is an increasing function of d . So indeed $\alpha = \frac{g(U(t, a)/\gamma)}{G(U(t, a)/\gamma)} \frac{\pi(t, a)}{\gamma}$. ■

When $\tilde{d} = 1$ the solution is standard. The last step is to show continuity of the solution at that point. Integrating over $\mathcal{Q}$,

$$\mathbb{E} \left[\frac{1}{u'(t)} \right] = \begin{cases} \lambda(\gamma), & \text{in the first instance; and} \\ \alpha(\tilde{d}), & \text{when } \tilde{d} < 1. \end{cases}$$

The function u' is continuous and monotone, therefore so is $1/u'$, and $\int_{\mathcal{Q}} dx$ is a bounded (so, continuous) operator. Therefore, letting t^α denote the transfer when $\tilde{d} < 1$,

$$\alpha(\tilde{d}) \rightarrow \lambda(\gamma) \Leftrightarrow \mathbb{E} \left[\frac{1}{u'(t^\alpha)} \right] \rightarrow \mathbb{E} \left[\frac{1}{u'(t^\lambda)} \right],$$

which is immediate. ■

Proof of Proposition 1: Here I show that for any $d < \tilde{d}$ the total cost of inducing an action is lower. For the standard problem, Jewitt has shown in two unpublished papers (1997, 2007) that the cost of an action defined as

$$\min \tilde{C}(a) = \int_{\mathcal{Q}} t(x) dF(a|x) \text{ s.t. } U \geq \bar{u} \text{ and } U_a = 0$$

with Lagrangian function $\mathcal{L}$ and multipliers λ, μ has a dual representation

$$\begin{aligned} C(a) &= \max_{\lambda, \mu} \min_u \mathcal{L} \\ &= \max_{\lambda, \mu} \left[\lambda(\bar{u} + c(a)) + \mu c'(a) - \int \rho \left(\lambda + \mu \frac{f_a}{f} \right) dF(a|x) \right] \end{aligned}$$

In the present problem, write

$$C(a) = \max_{\alpha, \hat{\mu}} \left[\alpha(\gamma \cdot d + c(a)) + \hat{\mu} c'(a) - \int \rho \left(\alpha + \hat{\mu} \frac{f_a}{f} \right) dF(a|x) \right]$$

where $\alpha, \hat{\mu}$ are defined in equation (3.3). So $\alpha, \hat{\mu}$ are increasing in d (by Lemma 2). To show $C(a)$ is increasing in d , rewrite it in three parts:

$$\begin{aligned} & \alpha \left[\gamma \cdot d + c(a) - \int \max_v v dF(x|a) \right] \\ & \hat{\mu} \left[c'(a) - \int \max_v v \frac{f_a}{f} dF(x|a) \right] = \hat{\mu} \left[c'(a) - \int \max_v v dF_a(x|a) \right] \\ & \int \max_v h(v) dF(x|a) \end{aligned}$$

Take a partial derivative with respect to d ; the last term is 0, and

$$\begin{aligned} \frac{\partial \alpha}{\partial d} \left[\bar{u} + c(a) - \int \max_v v dF(x|a) \right] + \alpha \cdot \gamma &> 0 \\ \frac{\partial \hat{\mu}}{\partial d} \left[c'(a) - \int \max_v v dF_a(x|a) \right] &= 0 \end{aligned}$$

because $\frac{\partial \alpha}{\partial d} [\bar{u} + c(a) - \int \max_v v dF(x|a)] = 0$ when the PC binds and similarly for the second line when the moral hazard constraint binds. Therefore $C'(a)$ is indeed positive. Last, CLI and $c''' \geq$ guarantee that $C(a)$ is also convex. Therefore in solving the problem

$$\max_a \int x dF(x|a) - C(a)$$

the principal selects a higher action for any $d < \tilde{d}$. ■

Proof of Proposition 2: First extend Theorem 1 to the best responses characterized by (3.6). For a given U^1 , (3.6) rewrites $\frac{1}{u'} = \beta + \hat{\mu} \frac{f_a}{f}$ where $\beta \equiv \beta[\hat{d}]$ is an increasing and continuous function of $\hat{d}$. Then apply Lemma 6, and the best response (3.6) is well defined. The first-order condition of the Lagrangean function of Problem 3 reads

$$G \left(\frac{1}{2} \left[1 + \frac{U^0 - U^1}{\gamma} \right] \right) \int [x - t(x)] dF_a(x|a) + \mu \left[\int u(t(x)) dF_{aa}(x|a) - c''(a) \right] = 0$$

and similarly for Principal P_1 . At a symmetric equilibrium, $U^0 = U^1$, so that $\hat{d} = 1/2$, which yields Conditions (3.7) and (3.8).

Similarly to the monopoly case hitting the boundary $\tilde{d} = 1$, there is a regime switch at $\hat{d} = 1/2$. It is isomorphic to the monopoly problem. More precisely, restate Problem 2 with $\tilde{d} = 1/2$, so that the constraint is $U(t, a) = \gamma \cdot \tilde{d} = \gamma/2$; the first-order condition is

$$\frac{1}{u'} = 2\lambda(\gamma) + \mu^C \frac{f_a}{f}.$$

Integrating over $\mathcal{Q}$, depending on γ , we may have

$$\mathbb{E} \left[\frac{1}{u'(t)} \right] = \begin{cases} 2\lambda(\gamma), & \text{if } \gamma \text{ is small enough; or} \\ \alpha(\tilde{d}), & \text{if } \gamma \text{ is large enough.} \end{cases}$$

So at $1/2, \alpha(1/2) = 2g(1/2)\pi/\gamma$ and in equilibrium $\lambda(\gamma) = 2g(1/2)\pi/\gamma = \beta[\hat{d} = 1/2]$. Hence by continuity of u' and of $\int dx$

$$\lim_{\tilde{d} \rightarrow \frac{1}{2}} \alpha(\tilde{d}) = \beta[1/2]$$

and the solution is also continuous at $1/2$. ■

Proof of Proposition 3: Recall the definition of $d^0(\gamma)$ and that it increases in γ .

Case 1. For large enough transport cost γ , $\tilde{d} < d^0(\gamma)$, so the agent is not contestable under complete information. The characterization is standard; the agent does not receive a rent. However under incomplete information, the principal's offer attracts all agents up to $d = 1/2$, so $U^0 \geq \gamma/2 \geq \gamma \cdot \tilde{d} \geq 0$ (strictly for $\tilde{d} < 1/2$, or as soon as γ is small enough). The agent receives a rent, which we know weakens incentives. (Proposition 1).

Case 2. When γ is small enough ($d^0(\gamma) \leq \tilde{d} \leq 1/2$), P_0 competes *à la* Bertrand with P_1 under complete information (and always contracts with the agent). Let π^C denote the principals' payoff in a Nash equilibrium and $\pi^0(U^0)$ denote principal P_0 's profit under the optimal contract when he must give away utility U^0 . Under complete information, U^0 must satisfy $U^0 - \gamma \cdot \tilde{d} \geq \bar{U}^1 - \gamma \cdot (1 - \tilde{d})$. Letting $\bar{u} \equiv \bar{U}^1 - \gamma \cdot (1 - 2\tilde{d})$, P_0 's problem is again the standard problem, however the level of utility U^0 that must be offered under complete information depends on the location of the agent. It is helpful to understand the behaviour of $\pi^0(U^0)$ as $\tilde{d}$ changes.

Lemma 7 *The function $\pi^0(U^0)$ is everywhere non-increasing in $\tilde{d}$. It reaches a maximum at $\tilde{d} = 0$ and a minimum of zero at $\tilde{d} \in [1/2, 1]$. There is a kink at $\tilde{d} = d^0(\gamma)$, and $\frac{\partial}{\partial \tilde{d}} \pi^0(U^0)_{\tilde{d} > d^0(\gamma)} < \frac{\partial}{\partial \tilde{d}} \pi^0(U^0)_{\tilde{d} < d^0(\gamma)}$.*

Proof: When $\tilde{d} = 1/2$, $U^0 = \bar{u} = \bar{U}^1$, that is, $\pi^0 = 0$; for $\tilde{d} > 1/2$ the agent contracts with P_1 , so $\pi^0 = 0$. Next, for $\tilde{d} < 1/2$,

$$U^0 = \begin{cases} \gamma \cdot \tilde{d}, & \text{for } \tilde{d} < d^0(\gamma); \\ \bar{U}^1 - \gamma \cdot (1 - 2\tilde{d}), & \tilde{d} > d^0(\gamma). \end{cases}$$

U^0 is increasing in $\tilde{d}$ and steeper to the right of $d^0(\gamma)$ than to its left; so π^0 decreases in $\tilde{d}$, and more steeply so to the right of $d^0(\gamma)$. U^0 is minimized (i.e. π^0 maximized) for $\tilde{d} = 0$. Furthermore, because $U^0 = \gamma \cdot d^0(\gamma) = U^1 - \gamma[1 - d^0(\gamma)]$ (this is the definition of $d^0(\gamma)$) U^0 is continuous at that point. Therefore so is $\pi^0(U^0)$. ■

Under asymmetric information $\pi^C > 0$ always (by simple observation of the equilibrium condition (3.8)) and it is constant in the actual location of the agent. I claim that $\pi^0(U^0 = \gamma d^0(\gamma)) > \pi^C$. To see that, consider the limit case where γ is such that principals just compete; that is, $\tilde{d} = 1/2$. Then π^C is bounded by the monopoly profit *under asymmetric information* (they are equal at that point): $\pi^C \leq \pi^0(U^0(\tilde{d} = 1/2))$. By Proposition 1 we know that for any $d < \tilde{d} = 1/2$, the *full information* profit exceeds $\pi^0(U^0(\tilde{d} = 1/2))$, so this is true for $\tilde{d} = d^0(\gamma)$. Because $\pi^0(U^0)$ increases as d decreases, there exists some $d_c^0(\gamma) \in (d^0(\gamma), 1/2)$ such that

$$\pi^0(U^0) \begin{cases} > \pi^C, & \text{if } d \in (d^0(\gamma), d_c^0(\gamma)); \text{ and} \\ < \pi^C, & \text{if } d \in (d_c^0(\gamma), 1/2). \end{cases}$$

See Figure 2. To extend to effort levels, first recall that contracts are unique, so at the point $d_c^0(\gamma)$ the equilibrium action is the same under symmetric and asymmetric information. Because the solution under symmetric information must take the form $\lambda + \mu(f_a/f)$ we can apply Jewitt's dual representation as in the proof of Proposition 1, with effort monotonically increasing as $\tilde{d}$ decreases.

■

Proof of Proposition 4: For the first half apply the proof of Proposition 1 noting that $\bar{u} = \gamma \cdot d$; then the principal selects a lower action as γ increases. Calling on Lemma 5 proves the second half of the Proposition. ■

Proof of Proposition 5: This amounts to proving that welfare increases with the agent's action. Equilibrium welfare $W(a)$ rewrites

$$\begin{aligned} W(a) &\equiv G(\tilde{d}_0) \left[\int x dF(x|a) - T^0(a) + U^0(t, a) \right] + [1 - G(\tilde{d}_1)] \left[\int x dF(x|a) - T^1(a) + U^1(t, a) \right] \\ &\quad - \left[1 - G(\tilde{d}_1) + G(\tilde{d}_0) \right] \bar{u}, \end{aligned}$$

where $T(a) \equiv \int t(x) dF(x|a)$ is known to be an increasing, concave function (Conlon, 2008) and a is meant as the equilibrium action across either monopoly or duopoly equilibrium. Under monopoly,

$\tilde{d} = U^0/\gamma$, so for P_0

$$\begin{aligned}\frac{dW}{da^0} &= g(U^0/\gamma) \frac{U_a^0}{\gamma} \left[\int x dF(x|a) - T(a) + U^0(t, a) - \bar{u} \right] + G(U^0/\gamma) \left[\int x dF_a(x|a) - \frac{dT^0(a)}{da} + U_a^0 \right] \\ &= G(U^0/\gamma) \left[\int x dF_a(x|a) - \frac{dT^0(a)}{da} \right]\end{aligned}$$

since $U_a^0 = 0$. Under duopoly, $\tilde{d} = 1/2$, so $G(\tilde{d}) = 1 - G(\tilde{d}) = 1/2$ and equilibrium contracts are symmetric; then

$$\begin{aligned}\frac{dW}{da^i} &= \frac{1}{2} \left[\int x dF_a(x|a^i) - \frac{dT^i(a^i)}{da^i} - U_a^i \right] \\ &= \frac{1}{2} \left[\int x dF_a(x|a^i) - \frac{dT^i(a^i)}{da^i} \right]\end{aligned}$$

In either case, $\frac{dW}{da} > (<)0 \Leftrightarrow \int x F_a(x|a) - \frac{dT^i(a)}{da} > (<)0$. Because $\mu^M, \mu^C > 0$, both first-order conditions (3.2) and (3.7) immediately tell us that $\int x F_a(x|a) - \frac{dT^i(a)}{da} > 0$. ■

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