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Qinghua Guo
University of Western Australia, qguo@uow.edu.au

Defeng (David) Huang
University of Western Australia, david.huang@uwa.edu.au

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Abstract

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GMP-Based Channel Estimation for Single-Carrier Transmissions over Doubly Selective Channels

Qinghua Guo, *Member, IEEE*, and Defeng (David) Huang, *Senior Member, IEEE*

***Abstract*—We present a graph-based channel estimation approach for SC-IFDE (single-carrier transmissions with iterative frequency domain equalization) without CP (cyclic prefix) over doubly selective channels using the recently developed Gaussian message passing (GMP) technique. A direct application of the GMP updating rules in the FFG (Forney-style factor graph) of the SC-IFDE system model incurs high complexity. Approximate updating rules are therefore developed to overcome this problem. The proposed GMP-based channel estimation approach has similar complexity as the low-complexity Kalman-filtering based frequency domain channel estimation approach in the literature, but significantly outperforms the latter due to its enhanced capability in capturing the time correlation information of doubly selective channels through bidirectional processing.**

***Index Terms*—Doubly selective channels, frequency domain equalization (FDE), Gaussian message passing (GMP).**

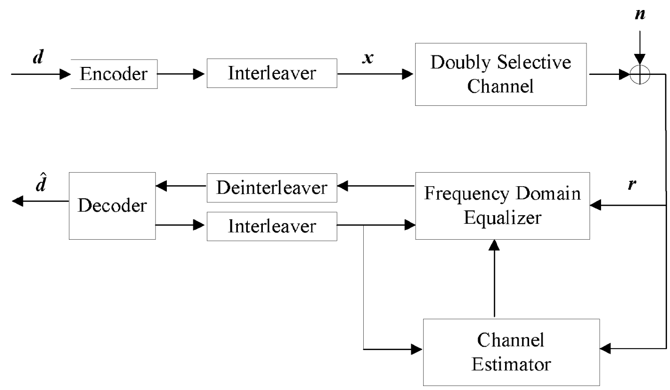

Fig. 1. System structure of coded SC-IFDE without CP.

I. Introduction

SINGLE-CARRIER block transmissions with iterative frequency domain equalization (SC-IFDE) are promising techniques to combat inter-symbol interference (ISI) [1], [2]. However, over doubly selective channels (i.e., time-varying ISI channels), the use of cyclic prefix (CP) in SC-IFDE incurs high power and spectral overheads due to the short channel coherence time and long channel memory. Recently, SC-IFDE without CP has been investigated in [4]–[6], [9], in which the feedbacks from the decoder are used to restore the effect due to the removal of CP. As shown in Fig. 1, the iterative receiver of SC-IFDE without CP consists of a channel estimator, a frequency domain equalizer, and a soft-in soft-out decoder, working iteratively [4]–[6], [9]. In this letter, we focus on designing the low-complexity channel estimator, which provides channel state information to the equalizer.

A low-complexity frequency domain channel estimator has been proposed in [6] for SC-IFDE without CP, where Kalman-filtering is performed on a per-tone basis by ignoring the correlation among different tones, followed by an across-tone refinement process, and the time correlation of the channel is exploited by a unidirectional recursion. In this letter, we extend the unidirectional frequency domain channel estimation approach to bidirectional processing with the recently developed Gaussian message passing (GMP) technique [10], thereby enhancing the capability of the channel estimator in capturing the channel correlation information, leading to significant performance improvement.

By representing the SC-IFDE system model with a Forney-style factor graph (FFG) [10], channel estimation can be performed through local message computations and exchanges between the basic building blocks in the FFG based on the GMP updating rules in [10]. However, a direct application of the GMP updating rules incurs high complexity. To overcome this problem, approximate message updating rules are developed to make the proposed channel estimation approach with similar complexity as that in [6], but significantly outperforms the latter as demonstrated by simulation results. Moreover, the proposed approach facilitates parallel processing due to the nature of the graph-based methodology, which is desirable from the system implementation perspective.

Bold uppercase (lowercase) letters are used to denote matrices (column vectors). The superscript "^{T}" and "^{H}" denote the transpose and conjugate transpose operations, respectively. Expectation and variance are represented by $E(\cdot)$ and $V(\cdot)$, respectively. The letter $\boldsymbol{I}$ denotes an identity matrix with proper size. $Diag(\cdot)$ denotes a diagonal matrix with the entries in the bracket on its diagonal.

II. System Model

The symbol-spaced channel taps are assumed to be independent complex stationary Gaussian stochastic processes, and the autocorrelation function for each tap is the zeroth order Bessel function of the first kind [7]. The power of the lth channel tap is represented by q^l.

In SC-IFDE without CP, a conventional block single-carrier transmitter is employed. To avoid the spectral overhead due to the pilot signal, we use the superimposed training approach (it

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The authors are with the School of Electrical, Electronic and Computer Engineering, The University of Western Australia, Perth WA, Australia (e-mail: qhguo@ee.uwa.edu.au; huangdf@ee.uwa.edu.au).

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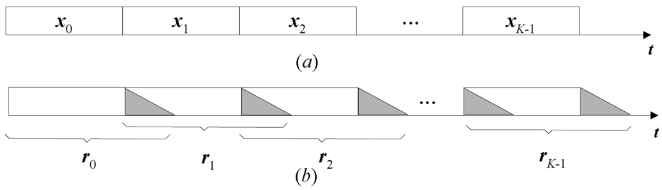

Fig. 2. (a) Transmitted signal $\boldsymbol{x}$ is partitioned into a number of short segments $\{\boldsymbol{x}_k\}$. Each segment is assumed to undergo a static ISI channel. (b) ISI causes inter-segment interference, as illustrated by the shadowing parts.

was called implicit training in [8]),[1] i.e., the length-J signal block $\boldsymbol{x}=\boldsymbol{p}+\boldsymbol{s}$, where $\boldsymbol{s}$ denotes data symbols, formed from a codeword after symbol mapping, and $\boldsymbol{p}$ denotes pilot symbols. To facilitate processing at the receiver, as shown in Fig. 2(a), the transmitted signal block $\boldsymbol{x}$ is partitioned into a number of short segments $\{\boldsymbol{x}_k, k=0,1,\ldots,K-1\}$, each with length M by assuming that $J=KM$ (note that such a partition does not affect the structure of the transmitted signal). Each segment $\boldsymbol{x}_k$ is assumed to undergo a static ISI channel $\boldsymbol{h}_k=[h_k^0,h_k^1,\ldots,h_k^L]^T$ ($\boldsymbol{h}_k$ is a complex Gaussian random vector according to the channel model), where L is the channel memory length.

We use the following segment-level first-order auto-regressive model to approximately characterize the time-varying channel [12]

$$\boldsymbol{h}_k=\alpha\boldsymbol{h}_{k-1}+\boldsymbol{w}_k \tag{1}$$

where $\{\boldsymbol{w}_k, k=0,1\ldots\}$ denotes a zero mean white Gaussian process with diagonal covariance matrix $V(\boldsymbol{w}_k)$ for each $\boldsymbol{w}_k$. The values of α and $V(\boldsymbol{w}_k)$ can be determined as in [12].

As shown in Fig. 2(b), due to the channel delay spread, the received signal $\boldsymbol{r}_k$, corresponding to the transmitted signal segment $\boldsymbol{x}_k$, has length $N=M+L$ (we assume that $L<M$). By using the soft feedbacks from the decoder, the inter-segment interference can be gradually mitigated with the iteration between the equalizer and the decoder. As in [6], the residual inter-segment interference is assumed to be ignored, and then we have the following model

$$\boldsymbol{r}'_k=\boldsymbol{x}_k*\boldsymbol{h}_k+\boldsymbol{n}_k \tag{2}$$

where "$*$" denotes the linear convolution operation, $\boldsymbol{n}_k$ denotes the additive Gaussian noise with variance $2\sigma^2\boldsymbol{I}$, $\boldsymbol{r}'_k$ is the counterpart of $\boldsymbol{r}_k$ after the inter-segment interference cancellation.

Note that $\boldsymbol{x}_k=\boldsymbol{p}_k+\boldsymbol{s}_k$ consists of both the pilot component $\boldsymbol{p}_k$ and the data component $\boldsymbol{s}_k=[s_{k,0},\ldots,s_{k,M-1}]^T$, and the means and variances of $\{s_{k,m}\}$[2] (i.e., $\{E(s_{k,m})\}$ and $\{V(s_{k,m})\}$) are available from the output of the decoder [4]–[6], [9]. As a result, $\boldsymbol{x}_k$ can be represented as

$$\boldsymbol{x}_k=\boldsymbol{b}_k+\boldsymbol{e}_k \tag{3}$$

[1]Since the time-multiplexed pilots approach can be regarded as a special case of the superimposed training, it can also be handled by the proposed channel estimation approach.

[2]Note that $\{s_{k,m}\}$ are approximately statistically independent of each other due to the use of the interleaver.

where the deterministic vector $\boldsymbol{b}_k=\boldsymbol{p}_k+E(\boldsymbol{s}_k)$, and $\boldsymbol{e}_k$ is a zero mean random vector with covariance matrix $V(\boldsymbol{e}_k)=Diag\big(V(s_{k,0}),\ldots,V(s_{k,M-1})\big)$.

As discussed in [9], (2) can be reformulated as

$$\boldsymbol{r}'_k=\tilde{\boldsymbol{x}}_k\otimes\tilde{\boldsymbol{h}}_k+\boldsymbol{n}_k \tag{4}$$

where "$\otimes$" denotes the cyclic convolution operation, and $\tilde{\boldsymbol{x}}_k$ and $\tilde{\boldsymbol{h}}_k$ are length-N vectors, formed by appending a proper number of zeros to the tails of $\boldsymbol{x}_k$ and $\boldsymbol{h}_k$, respectively. Define $\boldsymbol{F}$ as the normalized discrete Fourier transform (DFT) matrix with size $N\times N$ (i.e., the (m,n)th entry of $\boldsymbol{F}$ is given by $N^{-1/2}e^{-i2\pi mn/N}$, where $i=\sqrt{-1}$). By applying DFT to both sides of (4), we have

$$\boldsymbol{y}_k=Diag(\boldsymbol{t}_k)\boldsymbol{c}_k+\boldsymbol{u}_k \tag{5}$$

where

$$\boldsymbol{c}_k=\sqrt{N}\boldsymbol{F}\tilde{\boldsymbol{h}}_k=\sqrt{N}\boldsymbol{F}\boldsymbol{S}\boldsymbol{h}_k \tag{6}$$

with $\boldsymbol{S}=[\boldsymbol{I}_{(L+1)\times(L+1)},\boldsymbol{0}_{(L+1)\times(N-L-1)}]^T$, $\boldsymbol{y}_k=\boldsymbol{F}\boldsymbol{r}'_k$, $\boldsymbol{t}_k=\boldsymbol{F}\tilde{\boldsymbol{b}}_k$, $\boldsymbol{u}_k=Diag(\boldsymbol{F}\tilde{\boldsymbol{e}}_k)\boldsymbol{c}_k+\boldsymbol{n}_k$, and $\tilde{\boldsymbol{b}}_k$ and $\tilde{\boldsymbol{e}}_k$ are constructed by appending a proper number of zeros to the tails of $\boldsymbol{b}_k$ and $\boldsymbol{e}_k$, respectively. The zero-mean random vector $\boldsymbol{u}_k$, which characterizes the distortion due to both noise and the uncertainty of the estimate of $\boldsymbol{x}_k$, is assumed to be Gaussian. Let $c_{k,m}$ denote the mth entry in $\boldsymbol{c}_k$. Assuming that $|c_{k,m}|^2\approx E(|c_{k,m}|^2)=\sum_l q^l$ and $V(\tilde{\boldsymbol{e}}_k)\approx v\boldsymbol{I}$ (v is the average of $\{V(s_{k,m})\}$), we have

$$V(\boldsymbol{u}_k)\approx\beta\boldsymbol{I} \tag{7}$$

where $\beta=(v\sum_l q^l+2\sigma^2)$.

The estimation of $\boldsymbol{h}_k$ (or $\boldsymbol{c}_k$) based on only its corresponding observation vector $\boldsymbol{y}_k$ by using an MMSE (minimum mean square error) estimator may not lead to satisfactory performance since the small channel coherence time of the doubly selective channels leads to a $\boldsymbol{y}_k$ with short length. In the following, we extend ththe unidirectional Kalman-filtering based approach in [6] to bidirectional processing and efficiently estimate each $\boldsymbol{h}_k$ based on all the observation vectors $\{\boldsymbol{y}_j, j=0,1,\ldots,K-1\}$ by using the GMP technique [10] operating in an FFG of the linear Gaussian system described by (1), (5) and (6).

III. GMP-Based Channel Estimation Approach

The GMP technique [10] provides a factor graph based signal processing approach for linear systems. An FFG of a linear system can be constructed using the basic building blocks: equality constraints, adders, and multipliers. Since the sum-product message updating rules for the basic building blocks preserve Gaussianity, the message passing in the FFG of a linear Gaussian system can be characterized by the updating of mean vectors and covariance matrices (or the inverses of covariance matrices and transformed mean vectors) for the basic building blocks [10]. Through local computations based on the GMP updating rules for the basic building blocks [10], the global information (means and variances) of the variables can be obtained.

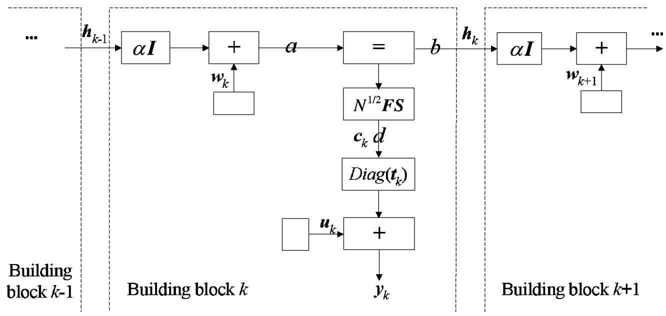

Fig. 3. An FFG of the system model described by (1), (5), and (6).

The FFG of (1), (5), and (6) for the kth segment is shown in the dash box in Fig. 3, where edges represent variables and boxes represent factors. By using the FFG of (1), (5), and (6) as the kth building block, a complete graph with K building blocks can be formed to include all the channel variables $\{\boldsymbol{h}_k, k = 0, \ldots, K-1\}$. Through a forward and a backward recursion using GMP in this graph, we can obtain the *a posteriori* means and variances of $\{\boldsymbol{h}_k, k = 0, \ldots, K-1\}$. However, due to matrix inversions and multiplications involved in the GMP updating rules, the complexity is $O((L+1)^3)$ per building block, which is a concern when L is large. It can be found that the most intensive computational load is on the local message computations between points a and b shown in Fig. 3. In the following, we develop approximate updating rules to reduce the complexity. Since the message passings between a and b are symmetric in forward and backward directions, the updating rules for both the directions are the same. As a result, we only describe the forward direction.

Given messages at a (i.e., the mean vector $\boldsymbol{m}_a$ and the covariance matrix $\boldsymbol{V}_a$, which are the *a priori* mean and variance of $\boldsymbol{h}_k$ provided by building block $k-1$), based on the GMP updating rules [10], the message at b (i.e., the mean vector $\boldsymbol{m}_b$ and the covariance matrix $\boldsymbol{V}_b$) can be computed as

$$\boldsymbol{m}_b = (\boldsymbol{V}_a^{-1} + N\boldsymbol{S}^T\boldsymbol{F}^H\boldsymbol{W}_d\boldsymbol{F}\boldsymbol{S})^{-1}(\boldsymbol{V}_a^{-1}\boldsymbol{m}_a + \sqrt{N}\boldsymbol{S}^T\boldsymbol{F}^H\boldsymbol{z}_d) \tag{8}$$

$$\boldsymbol{V}_b = \left(\boldsymbol{V}_a^{-1} + N\boldsymbol{S}^T\boldsymbol{F}^H\boldsymbol{W}_d\boldsymbol{F}\boldsymbol{S}\right)^{-1} \tag{9}$$

where the diagonal matrix

$$\boldsymbol{W}_d = \beta^{-1} Diag(\boldsymbol{t}_k) Diag(\boldsymbol{t}_k^H) \tag{10}$$

is the inverse of the covariance matrix of $\boldsymbol{c}_k$ at point d in Fig. 3 (see (7) for the definition of β), and

$$\boldsymbol{z}_d = \boldsymbol{W}_d\boldsymbol{m}_d = \beta^{-1} Diag(\boldsymbol{t}_k^H)\boldsymbol{y}_k \tag{11}$$

is the transformed mean vector of $\boldsymbol{c}_k$. The calculations of (8) and (9) are of high complexity due to the matrix inversions.

A straightforward way to reduce the complexity of calculating (8) and (9) is to approximate $\boldsymbol{W}_d$ as $\mu\boldsymbol{I}$, where μ is the average of the diagonal elements of $\boldsymbol{W}_d$. By using this approximation for all the building blocks, $\boldsymbol{V}_a$ is also diagonal. Hence the matrix inversions in (8) and (9) are trivial. However, this approximation results in poor performance (see the simulation results in Section IV). The reason can be explained as follows. Since $\boldsymbol{t}_k = \boldsymbol{F}\tilde{\boldsymbol{b}}_k$, the amplitude of the entries of $\boldsymbol{t}_k$ may fluctuate significantly, resulting in significant fluctuation of the diagonal entries of $\boldsymbol{W}_d$ [see (10)]. It can be seen from (8) that the averaging operation to $\boldsymbol{W}_d$ thereby destroys the information from the observation $\boldsymbol{y}_k$ [through $\boldsymbol{z}_d$, see (11)], which has a major contribution for the estimation of $\boldsymbol{h}_k$ [i.e., $\boldsymbol{m}_b$ in (8)]. In the following, we develop alternative approximations to avoid the averaging operation to $\boldsymbol{W}_d$ by averaging the diagonal elements of $\boldsymbol{V}_a$ instead.

Since $\boldsymbol{S} = [\boldsymbol{I}_{(L+1)\times(L+1)}, \boldsymbol{0}_{(L+1)\times(N-L-1)}]^T$, we can have the approximations $(\boldsymbol{V}_a^{-1} + N\boldsymbol{S}^T\boldsymbol{F}^H\boldsymbol{W}_d\boldsymbol{F}\boldsymbol{S})^{-1} \approx \boldsymbol{S}^T(\boldsymbol{S}\boldsymbol{V}_a^{-1}\mathbf{S}^T + N\boldsymbol{F}^H\boldsymbol{W}_d\boldsymbol{F})^{-1}\boldsymbol{S}$ and $\boldsymbol{F}\boldsymbol{S}\boldsymbol{S}^T\mathbf{F}^H \approx \boldsymbol{I}$. Applying them to (8) and (9), we have

$$\boldsymbol{m}_b \approx \boldsymbol{S}^T\boldsymbol{F}^H\left(\boldsymbol{C} + N\boldsymbol{W}_d\right)^{-1}\left(\boldsymbol{C}\boldsymbol{F}\boldsymbol{S}\boldsymbol{m}_a + \sqrt{N}\boldsymbol{z}_d\right) \tag{12}$$

$$\boldsymbol{V}_b \approx \boldsymbol{S}^T\boldsymbol{F}^H\left(\boldsymbol{C} + N\boldsymbol{W}_d\right)^{-1}\boldsymbol{F}\boldsymbol{S} \tag{13}$$

where $\boldsymbol{C} = \boldsymbol{F}\boldsymbol{S}\mathbf{V}_a^{-1}\boldsymbol{S}^T\boldsymbol{F}^H$. By approximating $\boldsymbol{C}$ as $(1/v_a)\boldsymbol{I}$, where v_a is the average of the diagonal entries of matrix $\boldsymbol{V}_a$, we have

$$\boldsymbol{m}_b \approx \boldsymbol{S}^T\boldsymbol{F}^H\left(\frac{1}{v_a}\boldsymbol{I} + N\boldsymbol{W}_d\right)^{-1}\left(\frac{1}{v_a}\boldsymbol{F}\boldsymbol{S}\boldsymbol{m}_a + \sqrt{N}\boldsymbol{z}_d\right) \tag{14}$$

and $\boldsymbol{V}_b \approx \boldsymbol{S}^T\boldsymbol{F}^H\left(v_a^{-1}\boldsymbol{I} + N\boldsymbol{W}_d\right)^{-1}\boldsymbol{F}\boldsymbol{S}$, which can be further approximated as

$$\boldsymbol{V}_b \approx \gamma\boldsymbol{I} \tag{15}$$

where γ is the average of the diagonal entries of $\left(v_a^{-1}\boldsymbol{I} + N\boldsymbol{W}_d\right)^{-1}$. We can see that the averaging operation to $\boldsymbol{W}_d$ is avoided in (14), thereby protecting the information from $\boldsymbol{y}_k$, and the matrix inversion in (14) is trivial since $\boldsymbol{W}_d$ is diagonal [see (10)]. We call (14) and (15) as *Approximate Rule 1*.

We can approximate (14) further as

$$\boldsymbol{m}_b \approx \frac{\gamma}{v_a}\boldsymbol{m}_a + \sqrt{N}\boldsymbol{S}^T\boldsymbol{F}^H\left(\frac{1}{v_a}\boldsymbol{I} + N\boldsymbol{W}_d\right)^{-1}\boldsymbol{z}_d \tag{16}$$

to reduce the two DFT operations to one at the cost of slight performance loss (see the simulation results in Section IV). Equations (16) and (15) are called *Approximate Rule 2*.

After the forward and backward recursions, the final estimate of $\boldsymbol{h}_k$ is obtained by combining its messages in both directions [10]. It can be verified that, by using the *Approximate Rules 1* and *2*, all the matrix inversions involved in the calculations of the messages are trivial since all the corresponding matrices are diagonal. As a result, the complexity is dominated by the product of $\boldsymbol{F}$ (or $\boldsymbol{F}^H$) and a vector, which can be efficiently implemented using the fast Fourier transform (FFT). This leads to the complexity of the proposed approach $O(N\log_2 N)$ per building block (i.e., $O(\log_2 N)$ per symbol), which is similar to that in [6].

IV. Simulation Results

The system settings are as follows. The frequency domain equalizer with virtual zero-padding in [9] is employed. We set carrier frequency $f_0 = 2$ GHz, symbol duration $T_s = 0.25\ \mu$s, and mobile speed $v_m = 500$ km/h. Hence the normalized Doppler spread $f_D T_s = 0.00023$. The number of channel taps is 57 (i.e., $L = 56$) and $q^l = e^{-0.1l}$. Although we assume that the channel is static within one segment for the

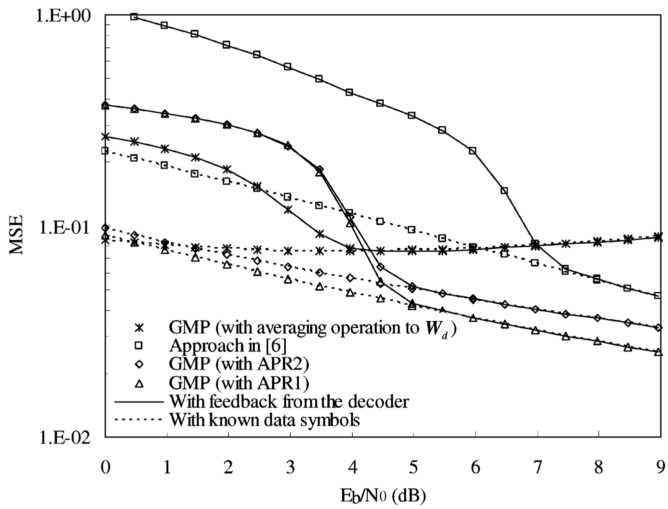

Fig. 4. MSE performance of various channel estimators. "APR1" and "APR2" denote *Approximate Rules 1* and *2*, respectively.

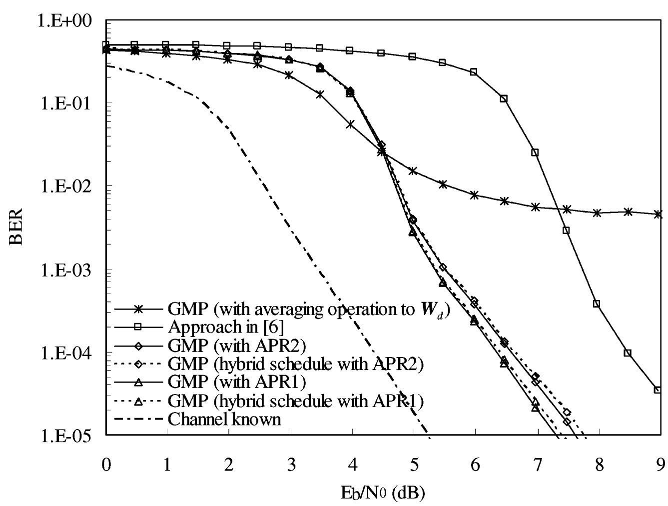

Fig. 5. BER performance of the system with various channel estimators. "APR1" and "APR2" denote *Approximate Rules 1* and *2*, respectively.

design of the channel estimator, the channels generated in the simulations vary symbol by symbol. To make the channel approximately static within each segment, we set the segment length $M = 200$ (i.e., $Mf_DT_s = 4.6\%$). For each channel realization, the average channel energy is normalized to 1, i.e., $J^{-1}\sum_{j=0}^{J-1}\sum_{l=0}^{L}|h_{j,l}^m|^2 = 1$ where $h_{j,l}^m$ is the lth channel tap at time j in the mth Monte Carlo simulation. A rate-1/2 convolutional code with generator $(23, 35)_8$, and QPSK modulation with Gray mapping are employed. The block length $J = 8000$, i.e., one block includes 40 segments. The cyclically extended Chu-sequence [1], [3] is used as pilot signal. The power ratio of the pilot signal to the data signal is 1/4, resulting in about 1 dB power loss. For comparison, the approach in [6] is used with superimposed training. The number of iterations is 10. The following mean square error (MSE)

$$\mathrm{MSE} = \frac{1}{Q}\sum_{m=1}^{Q}\left(\frac{1}{J}\sum_{j=0}^{J-1}\sum_{l=0}^{L}|\hat{h}_{j,l}^m - h_{j,l}^m|^2\right) \tag{17}$$

is used to evaluate the performance, where Q is the number of Monte Carlo simulations and $\hat{h}_{j,l}^m$ is the estimate of $h_{j,l}^m$.

The MSE and bit error rate (BER) performance are shown in Figs. 4 and 5, respectively, where the about 1 dB power overhead due to the pilot signal has been included in $\mathrm{E_b/N_0}$ (i.e., $\mathrm{E_b}$ is composed of both the data energy and the overhead due to the pilot signal). In Fig. 4, the MSE performance of the channel estimators with exact data symbols is also shown for reference. It can be seen that, with the $\mathrm{E_b/N_0}$ increasing, all the estimators approach the performance with exact data symbols, and the GMP-based channel estimators with the proposed approximate rules significantly outperform the one in [6]. However, the GMP-based approach with the averaging operation to $\boldsymbol{W}_d$ delivers poor MSE performance (this leads to poor BER performance of the system as shown in Fig. 5). In Fig. 5, the BER performance of the system using the GMP-based approach with hybrid schedule[3] (in which each subgraph consists of five building blocks) is also shown to demonstrate the feasibility of parallel processing. It can be seen that about 2 dB BER performance gain can be achieved by using any version of the proposed GMP-based approach compared with that using the approach in [6].

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[3]In the hybrid schedule, the whole graph is divided into a number of subgraphs, each consisting of a number of building blocks. The serial schedule [11] is used for message passing in each subgraph (which is handled by one processor) and the flooding schedule [11] is used for message passing between the subgraphs.