1-1-2011

Consolidation by vertical drain beneath a circular embankment using analytical and numerical modelling

Cholachat Rujikiatkamjorn
University of Wollongong, cholacha@uow.edu.au

Buddhima Indraratna
University of Wollongong, indra@uow.edu.au

A Aljorany
ala@uow.edu.au

Consolidation by vertical drain beneath a circular embankment using analytical and numerical modelling

C. Rujikiatkamjorn & B. Indraratna
Centre for Geomechanics and Railway Engineering, Faculty of Engineering, University of Wollongong, Wollongong City, Australia

A. Aljorany
Dept. of Civil Eng, University of Baghdad, Iraq

Abstract
When prefabricated vertical drains (PVDs) are installed beneath circular embankment (i.e. circular oil tanks or silos), the system of vertical drains can be replaced by axisymmetric concentric rings with equivalent drain walls. A value for the equivalent coefficient of soil permeability must be obtained to provide the same degree of consolidation. A rigorous solution for PVDs installed under circular embankment is proposed and verified by comparing its results with conventional unit cell model. The model is then validated via the consolidation process by vertical drains at the Skå-Edeby circular test embankment (Area II). The calculated values of settlement, lateral displacement and excess pore water pressure provide good agreement with the field measurements.

1 INTRODUCTION

Installation of vertical drains is one of the most popular techniques for enhancing the geotechnical engineering characteristics of soft clay deposits. The main purpose of vertical drain is to speed up soil consolidation process by reducing the drainage path via radial drainage, thereby improving the shear strength of the soil while minimising its post-construction settlement (Holtz et al., 1991). The dissipation of excess pore water pressure occurs faster in the radial direction due to the greater coefficient of soil permeability in the horizontal direction and the reduced drainage path. The consolidation time can be reduced to achieve a required degree of consolidation by selecting a suitable drain spacing and an appropriate installation pattern (Jamiolkowski et al., 1983).

The theoretical solution of radial drainage consolidation was first introduced by Barron (1948) based on an axisymmetric unit cell. Further studies on the unit cell consolidation were carried out by Yoshikumi and Nakanodo (1979) and Hansbo (1981) which initiate the following well-known equation:

$$U_{\mathrm{h}} = 1 - \exp\left(-\frac{8T_{\mathrm{h}}}{\mu}\right), \tag{1}$$

In the preceding equation,

$$T_{\mathrm{h}} = \frac{c_{\mathrm{h}} \cdot t}{d_e^2} \tag{1a}$$

$$\mu = \ln\left(\frac{n}{s}\right) + \left(\frac{k_{\mathrm{h}}}{k_{\mathrm{s}}}\right)\ln(s) - 0.75 + \pi(2lz - z^2)\left(\frac{k_{\mathrm{h}}}{q_{\mathrm{w}}}\right) \tag{1b}$$

$$n = (d_{\mathrm{e}}/d_{\mathrm{w}}) \tag{1c}$$

$$s = (d_{\mathrm{s}}/d_{\mathrm{w}}) \tag{1d}$$

where, U_{h} = average degree of lateral consolidation, T_{h} = time factor, c_{h} = coefficient of consolidation in radial direction, t = time, d_{e} = diameter of soil cylinder, μ = factor accounting for smear and well resistance. d_{w} = equivalent diameter of the drain, d_{s} = diameter of smear zone, l = drain length, q_{w} = discharge capacity of the drain, z = depth of horizontal plane under consideration, k_{h} and k_{s} = coefficients of horizontal permeability in the smear zone and undisturbed zone, respectively.

The unit cell analysis can offer the accurate prediction at the embankment centreline where the horizontal movements are zero. In practice, the subsoil where an array of drains installed is usually not uniform, and the process of consolidation does not conform to a one dimensional problem (Indraratna et al., 1992). The numerical techniques such as the finite element method (FEM) are imperative for the analysis and design of multi-drain systems (Hird et al., 1992, Indraratna and Redana 2000, Chai et al., 2001). Various studies including Olson (1998) have indicated that conducting a finite element analysis for multi-drains is often time-consuming, and that it is related to the convergence of solution that inevitably requires large computer memory and a large number of iterations. However, the complexity of the multi-drains (3D) problem can be reduced to an equivalent two-dimensional (2D) situation by transforming the in-situ (3D) soil parameters into equivalent 2D parameters or making other simplifications. For example

Indraratna and Redana (2000) proposed a plane strain (2D) model that considers the effect of smear and well resistance. Most of the previous studies have been dealt with multi-drain systems for embankment strip loading (plane strain). To date, no study has been conducted to model soil consolidation via vertical drains beneath an axisymmetric loaded area.

In this paper, the radial consolidation process below a circular embankment is analysed using the finite element method. An equivalent axisymmetric solution is obtained and validated using a numerical scheme. The FEM code ABAQUS is then employed to analyse the performance of a full-scale test embankment at Skå-Edeby, Stockholm-Arlanda Airport. The numerical predictions are then compared with the available field data.

2 PROPOSED EQUIVALENT MODEL

Figure 1 shows a vertical cross section of a typical vertical drain installation pattern with some field instrumentation beneath a circular embankment at Ska-Edeby (Hansbo, 1960). While the consolidation of soil around an individual vertical drain can be readily analysed as a single unit cell, to analyse a multi-drain system under an axisymmetric condition, one must determine the equivalent soil parameters that give the same time-settlement response in the field. In such a transformation, each drain element should behave as a part of concentric cylindrical drain wall with an increasing perimeter from the centreline (Fig. 2). Figure 3 presents a concept of cylindrical soildrain wall.

The average degree of consolidation vertical drains installed under circular embankment can be given by:

$$U_{\text{h ring}} = 1 - \exp\left(\frac{-8T_{\text{h,ring}}}{\mu_{\text{ring}}}\right) \tag{2}$$

where,

$$\mu_{\text{ring}} = \frac{\alpha^2}{i}\left[\begin{array}{l} 2(i-0.5)^4 \ln\dfrac{i}{i-0.5} \\ +\dfrac{1}{4}(2i-0.5) \\ (-2i^2+3i-0.75) \\ +2(i+0.5)^4 \ln\left(\dfrac{i+0.5}{i}\right) \\ -\dfrac{1}{4}(2i+0.5)(2i^2+3i+0.75) \end{array}\right] \approx \frac{2\alpha^2}{3} \tag{2a}$$

$$S = \alpha d_e \tag{2b}$$

$\alpha = 0.887$ and 0.952 for drains installed in a square pattern and an equilateral triangular pattern, respectively (Holtz et al., 1991). It is of interest to note

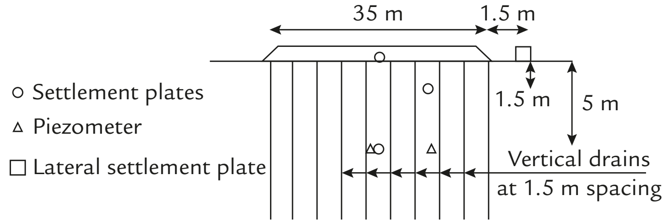

Figure 1 Embankment cross section and locations of instrumentation at Skå-Edeby, Sweden (Indraratna et al. 2007).

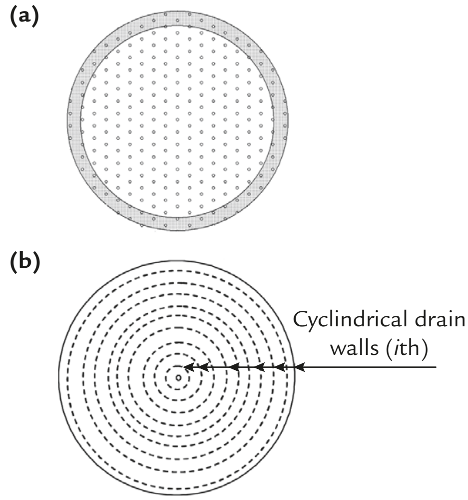

Figure 2 Conversion for multi-drain system under circular loading adopted for analytical solutions (a) Actual field condition and (b) Equivalent axisymmetric condition Indraratna et al. 2007).

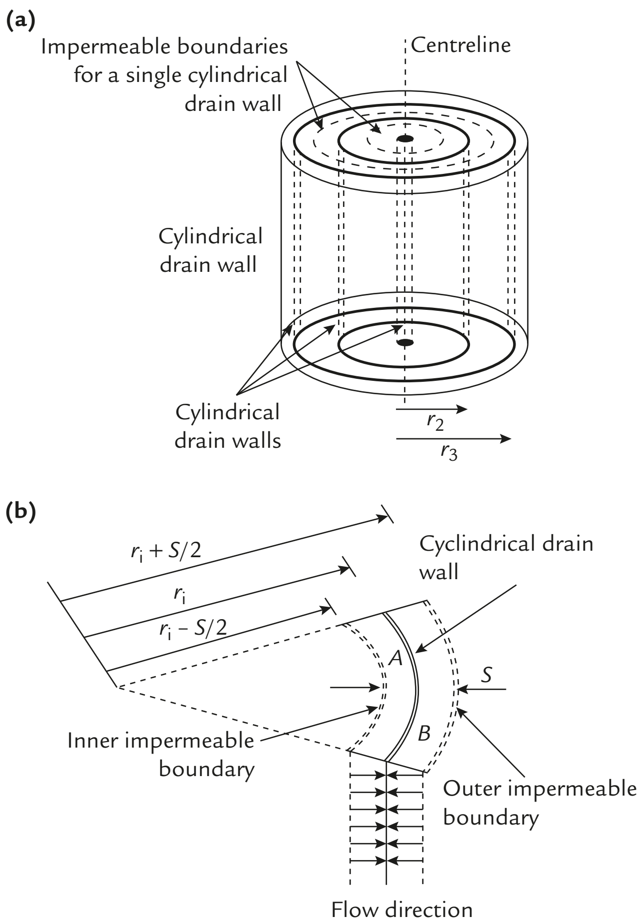

Figure 3 (a) Multi-drain system under axisymmetric condition (b) a single hollow cylinder of soil-drain (Indraratna et al. 2007).

that the value of μ_{ring} converges to $\frac{2\alpha^2}{3}$ for all values of $(i > 1)$.

For circular loading, the 'conversion' procedures to establish an equivalent plane strain condition proposed by Indraratna et al. (2007) can be based on the equivalent average excess pore pressure by maintaining the geometric equivalence (see Fig. 3). At a given time step, the average excess pore pressure for both the unit cell and a unit of ith revolving prism of soil $(i > 1)$ are made the same by equating Equation (1) with Equation (2). The equivalent permeability for the multi-drain under axisymmetric condition can then be expressed as:

$$\frac{k_{h,\text{ring}}}{k_h} = \frac{\frac{2}{3}\alpha^2}{\left[\ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s}\ln(s) - \frac{3}{4}\right]} \qquad (3)$$

3 MODEL VALIDATION

Finite element program ABAQUS (Hibbitt et al., 2006) based on Biot's consolidation theory was employed with a particular soil properties to validate the proposed analytical solution. An elastic analysis was conducted with mv = 10^{-3} m^2/kN and simulating the condition of no lateral strain ($\nu = 0$) for validating the proposed solution (Eqn. 3). A total of 3200 axisymmetric elements (8-node bi-quadratic displacement and bilinear pore pressure) were discretised in to simulate a 10 m long vertical drain with a spacing of 1.5 m (Figures 4a and 4b). A c_h value of 0.32 m^2/yr and d_e of 1.575 m (i.e. equivalent to $S = 1.5$ m for triangular drain pattern) were used. The aspect ratio of the finite elements was kept below 3. The horizontal undisturbed soil permeability (k_h) was taken as 10^{-10} m/s, and the ratio of the undisturbed permeability to the smear zone permeability (k_h/k_s) and the value of (d_s/d_w) ratio were assumed to be 4.0 and 2.0, respectively. The equivalent plane strain permeability was determined based on Equation (2). The top, bottom and outer boundaries were set as no flow boundaries (Figure 4b). The vertical loading pressure (50 kPa) was applied at the top of the cell, and only vertical movement was allowed. Rigid elements were included at the top soil surface to ensure the equal strain condition.

Figure 5 illustrates the comparison between the analytical and numerical models before and after conversion for $i = 2$ and 15. It is noted that the in-situ permeability was used for the case "before conversion" and the equivalent permeability via Equation (3) was used for the case "after conversion". It can be seen from Equation (3) that the degree of consolidation of a particular soil-drain wall is independent on the value of i, hence, the numerical results

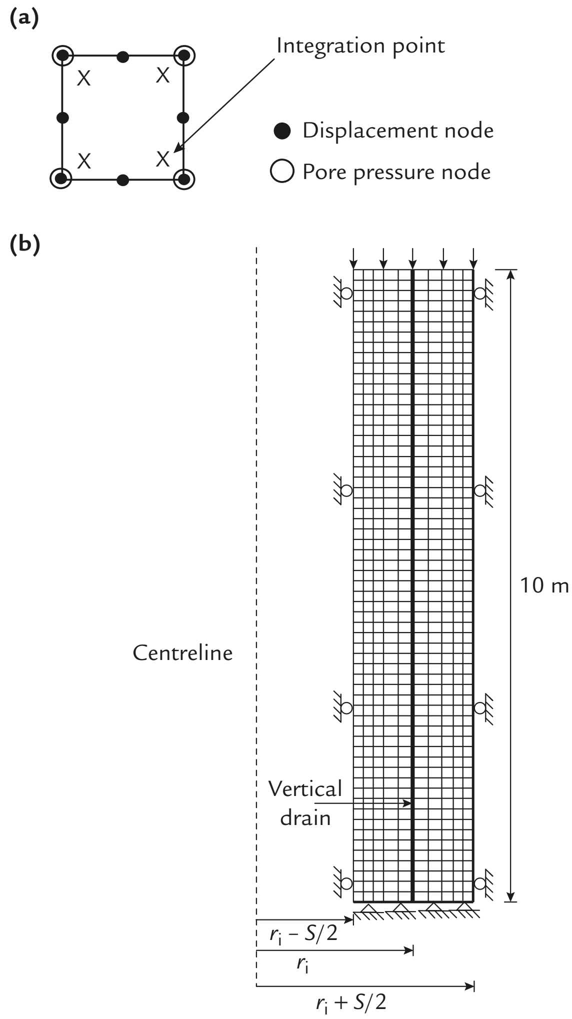

Figure 4 (a) Nodes and integration points for a single 8-node bi-quadratic displacement, bilinear pore pressure axisymmetric element (b) Mesh discretization for a single hollow cylinder of soil-drain (not to scale) (Indraratna et al., 2007).

for $i = 2$ and 15 are plotted separately in Figures 5a and 5b, respectively. Before incorporating the equivalent soil permeability from Equation (1), a good agreement between the numerical and analytical results was obtained by Equation (2). Small deviations are noted for the range $300 < t < 1000$ days with a maximum error of about 5% in the degree of consolidation. It can be seen after incorporating Equation (3) in the numerical model that the consolidation response agrees with Equation (1). In general, the above matching procedure provides the reliability of the proposed procedure.

4 APPLICATION TO A CASE HISTORY

In 1957, the Swedish Geotechnical Institute (SGI) and the Swedish Road Broad constructed four circular test embankments to investigate consolidation behaviour of the soft clay and to obtain relevant information for new airport construction (Hansbo, 1960). The site was on an island about 25 km west of Stockholm, Sweden. Sand drains were installed at three test embankments. This study considers circular

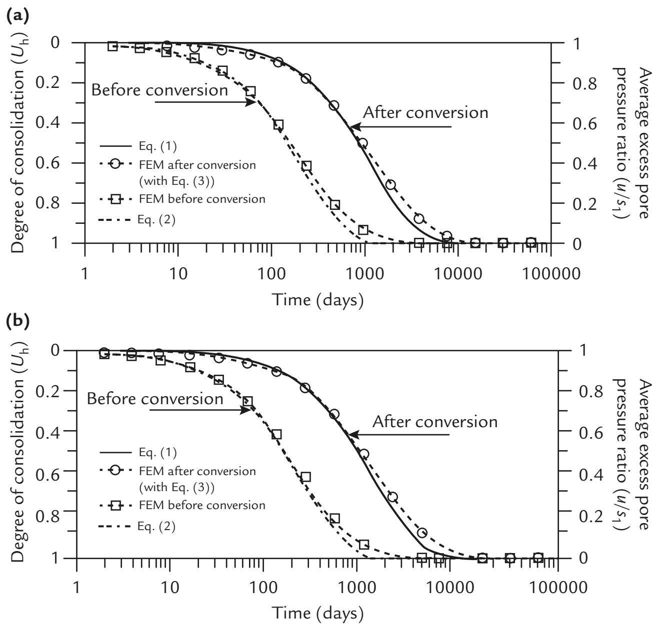

Figure 5 Comparison of analytical model with the equivalent model before and after conversion (a) $i = 2$, (b) $i = 15$ (Indraratna et al., 2007).

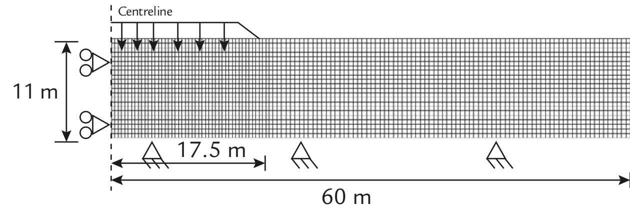

Figure 6 Finite element mesh (Indraratna et al., 2007).

Table 1 Soil properties used in the analysis (Hansbo, 1960)

	Layer 1	Layer 2	Layer 3	Layer 4
Depth (m)	0.00–1.00	1.00–3.00	3.00–6.00	6.00–11.00
Unit weight (kN/ m^3)	14.2	14.5	15.6	16
Pre-consolidation pressure (kPa)	27	24	35	48
k_v and k_s (m/year)	0.0075	0.0064	0.005	0.008
k_h (m/year)	0.03	0.026	0.02	0.032
$k_{h\ ring}$ (m/year) (Eq. 3)	0.003	0.003	0.002	0.003
λ		0.93	1.55	1.54
κ		0.093	0.155	0.154
Undrained shear strength (kPa)		12	8	10
c' (kPa)	30			
ϕ'	30			
E (MPa)	2.7			

test embankment (area II) which was built between June and July 1957, with a base diameter of 35 m, slope of 1.5H:1V and height of 1.5 m. Gravel of unit weight of 17.9 kN/m^3 was adopted as the embankment surcharge after removing 0.25 m of the top soil. Sand drains of 0.18 m diameter were installed in a triangular pattern @ a spacing of 1.5 m. The water table was located 1.0 m below the ground surface. The settlement plates, piezometers and horizontal settlement gauges were installed to measure the vertical displacement, pore water pressure and lateral displacements, respectively. The locations of the instrumentations were shown earlier in Figure 1.

4.1 Site Geology and Geotechnical Properties

The site geology and geotechnical properties of soil were described in details by Hansbo (1960) and have been presented in several publications (Holtz and Broms, 1972 and Hansbo, 2005). Only a summary is given here. The geologic data of the site area can be described as composed of very recent glacial and postglacial clay deposits. The ground surface is about 2.5 m above the mean level of the Baltic Sea. The deepest sediments contain glacial clays, (7500 years old) while the upper postglacial soils were deposited during the past 4500 years. These deposits can be considered as soft, normally consolidated clay. Site investigations showed that the subsoil conditions are relatively uniform, consisting of a weathered crust with a total thickness of 1 m, overlying the soft clay layer extending to approximately 8–10 m below the surface, followed by bedrock or very dense glacial deposits at a depth of 10 to 12 m. The groundwater table is about 1 m below the ground surface. The bulk unit weight generally increased from 15 kN/m^3 near the ground surface to 17 kN/m^3 at the bottom of the soil profile. The compression and consolidation characteristics of the clay were determined using oedometer test. Surface and subsurface settlement plates, hydraulic piezometers, a concrete block and inclinometer were installed to monitor the embankment behaviour (Fig. 1). However, due to malfunction of the inclinometer, only the surface lateral displacement measured by the concrete block is available (Holtz and Broms, 1972).

4.2 Finite Element Analysis of Circular Embankment

The finite element mesh consisted of 28160 rectangular CAX8RP elements (8-node biquadratic displacement, bilinear pore pressure) as shown in Figure 6. Only half of the embankment was modelled to reduce calculation time. The soil parameters of 4 subsoil layers are tabulated in Table 1 based on the laboratory test results from Hansbo (1960). For the over-consolidated crust, the Mohr-Coulomb model was deemed appropriate. Based on Hansbo (1997), it was assumed that the diameter of the smear zone (d_s) was 0.36 m and that both the permeability of the smear zone (k_s) and the vertical soil permeability (k_v) are 0.25 times the horizontal undisturbed soil permeability

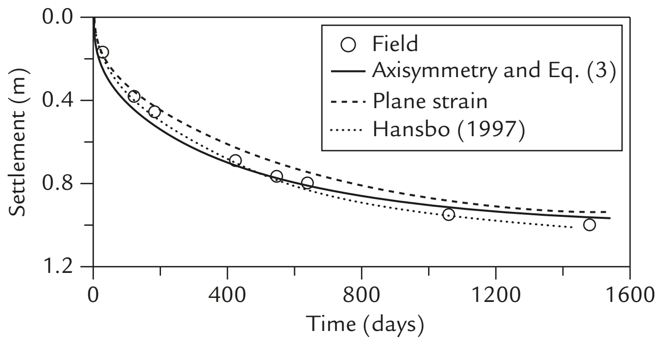

Figure 7 Surface settlement at the centerline (Indraratna et al., 2007).

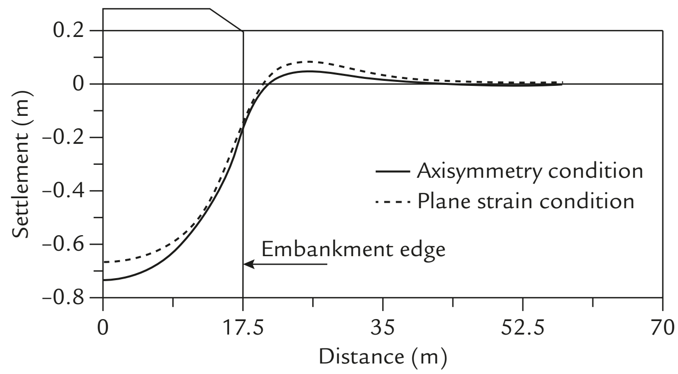

Figure 9 Surface settlement profile after 452 days (Indraratna et al., 2007).

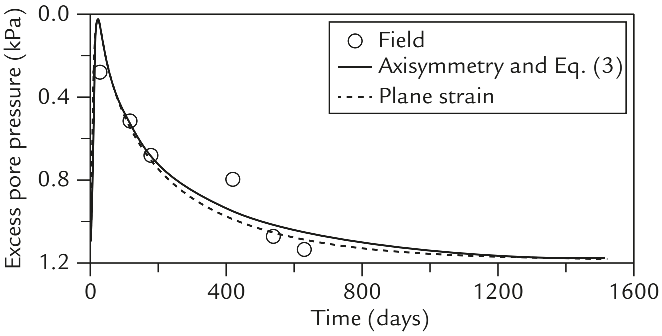

Figure 8 Excess pore pressure at 5 m depth and 0.75 m from the centreline (Indraratna et al., 2007).

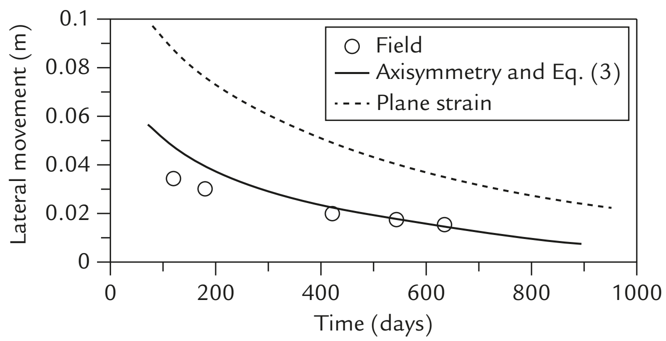

Figure 10 Lateral displacements at the surface at 1.5 m from the embankment edge (Indraratna et al., 2007).

(k_h). The well resistance is igmored due to sufficient drainage capacity of the drains of more than 150 m^3/year. For the equivalent multi-drain analysis, the soil permeability for each layer under the embankment loading was calculated based on Equation (3). Embankment surcharge loading of 27 kPa was applied and increased linearly to the upper boundary for 30 days, followed by rest period.

4.3 Discussions of the Results

In this section, the predictions under both axisymmetric and plane strain conditions are compared with the field measurements. Equation (3) was incorporated under the axisymmetric condition, whereas the equivalent plane strain approach proposed by Indraratna et al., (2007) was adopted for the plane strain condition. Figure 7 shows the comparison between the predicted and recorded field settlements at the centreline of the embankment at the ground surface. It can be seen that the predicted settlements from the axisymmetric conditions using Equation (3) agree with the measured results as well as with those predicted by Hansbo (1997), whereas the plane strain analysis under-predicts the field results but gives the same ultimate settlement. Undoubtedly, the excess pore pressure predictions from both cases at 5 m depth and at a lateral distance of 0.75 m away from the embankment centreline also agree with the field measurements (Fig. 8). Figure 9 illustrates the surface settlement profile after 452 days, which shows that plane strain condition gives less settlement at the centreline and more heave at the embankment toe.

The comparisons between the measured and predicted lateral movements at 1.5 m from the embankment toe are shown in Figure 10. The predictions from the axisymmetric condition agree well with the measured results, whereas the plane strain analysis over-predicts the measurements. In general, for a circular embankment improved by vertical drains, the axisymmetric analysis with an appropriate conversion procedure is essential to obtain accurate predictions, in terms of settlement, excess pore pressure and lateral displacement.

5 CONCLUSIONS

The paper presented a new technique to model consolidation by vertical drains below a circular loaded area where the system of vertical drains in the field was transformed by a series of equivalent concentric cylindrical drain walls. A rigorous solution for radial drainage towards these equivalent walls was derived by matching the degree of consolidation of Hansbo's unit cell model (Hansbo, 1997). An equivalent value for the coefficient of horizontal permeability could be obtained to analyse the multi-drain problem under circular loading.

A multi-drain analysis based on the proposed conversion was adopted to evaluate the performance of a selected full-scale circular embankment at Skå-Edeby, Sweden, using the finite element code, ABAQUS.

The effect of smear associated with the sand drains was also considered in the analysis. Unlike unit cell analysis, the settlements, excess pore water pressures and lateral movements elsewhere in the embankment were analysed and compared with the available field data by employing the proposed conversion procedure. Comparisons made with the corresponding predictions using the proposed conversion and Hansbo's solution showed that the predicted results from proposed conversion are more accurate than those from Hansbo's solution apart from the centreline of the embankment. The multi-drain analysis under equivalent axisymmetric condition using the proposed conversion gives more accurate prediction, whereas the equivalent plane strain FEM analysis tends to under-predict settlements and excess pore pressure by approximately 20%. Moreover, lateral displacement prediction under plane strain condition is almost two times that from the axisymmetric condition. The predictions of soft soil improved by vertical drains under circular loading are more accurate when using multi-drain analysis under equivalent axisymmetric condition.

6 ACKNOWLEDGEMENT

The Authors gratefully acknowledge the financial assistance from Australian Research Council and the Endeavour program Department of Education, Employment and Workplace Relations. More elaborate details of the contents discussed in this paper can be found in previous publication in Journal of Geomechanics, ASCE.

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