1-1-2011

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Recommended Citation

Indraratna, Buddhima; Rujikiatkamjorn, Cholachat; and Nimbalkar, Sanjay: Use of geosynthetics in railways including geocomposites and vertical drains 2011, 4733-4742.

<https://ro.uow.edu.au/engpapers/898>

Use of Geosynthetics in Railways including Geocomposites and Vertical Drains

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ABSTRACT

Australia relies heavily on rail for the transportation of bulk commodities and passenger services, and has introduced faster and heavier trains in recent years due to a growing demand. Large cyclic loading from heavy haul and passenger trains often leads to progressive deterioration of the track. The excessive deformations and degradations of the ballast layer and unacceptable differential settlement or pumping of underlying soft and compressible subgrade soils necessitate frequent costly track maintenance works. A proper understanding of load transfer mechanisms and their effects on track deformations are essential prerequisites for minimising maintenance costs. The reinforcement of the track by means of geosynthetics leads to significant reduction in the downward propagation of stresses and assures more resilient long-term performance. The geocomposite serves the functions of reinforcement, drainage and separation, thereby reducing the vertical and lateral deformations. Stabilization of soft subgrade soils by using prefabricated vertical drains (PVDs) is also essential for improving the overall stability of track and to reduce the differential settlement during the operation of trains. The effectiveness of using geocomposite geosynthetic and PVDs has been observed through field measurements and finite element analyses. These have been the first fully instrumented, comprehensive field trials carried out in Australian Railways, and it was very encouraging to see the field observations matching the numerical predictions.

INTRODUCTION

In order to compete with other modes of transportation, rail industries face challenges to minimise track maintenance cost, and to find alternative materials and approaches to improve the performance of rail tracks. The rail track should be designed to withstand large cyclic train loadings to provide protection to subgrade soils against both progressive shear failure and excessive plastic deformation. The track design should also account for the deterioration of ballast due to breakage and subsequent implications on the track deformations. The potential use of geosynthetics in the improvement of track stability and reducing the maintenance cost is well established. The reinforcement of the track by means of geogrids provides confinement to the ballast layer, leads to significant reduction in the vertical and lateral deformations, and assures more resilient long-term performance of the ballast layer (Indraratna and Salim, 2005; Indraratna et al., 2010a). The geocomposite provides reinforcement to the ballast layer, as well as filtration and separation functions

simultaneously. A field trial was conducted on a section of instrumented railway track along the New South Coast in the town of Bulli, NSW Australia, with the specific aims of studying the benefits of a geocomposite (combination of biaxial geogrid and non-woven polypropylene geotextile) installed at the ballast-capping interface, and to evaluate the performance of moderately graded recycled ballast in comparison to traditionally very uniform fresh ballast. In this paper, the field measurements are used for the calibration of constitutive models and successive implementation in the finite elements capturing the elasto-plastic deformation characteristics of ballast and geosynthetic inclusions.

The rail load is usually applied over a relatively narrow width where sufficient ballast and sub-ballast depths are provided. A significant amount of the applied load is sustained within several meters of the soil layer. In this regard, relatively short prefabricated vertical drains (PVDs) between 6 to 8m length can still be adequate to dissipate train induced pore pressures, limit the lateral movements and increase the shear strength and bearing capacity of the soft formation. If significant initial settlement of estuarine deposits cannot be satisfied in terms of maintenance practices (e.g. in new railway tracks where continuous ballast packing may be required), the rate of settlement can still be controlled by optimising the drain spacing and the drain installation pattern. In this way, while the settlements are controlled to an acceptable level, the reduction in lateral strains and gain in shear strength of the soil beneath the track improve its stability significantly.

USE OF GEOCOMPOSITE UNDER RAILWAY TRACK

The comprehensive knowledge of the complex mechanisms associated with track deterioration is essential in the accurate prediction of a typical rail track maintenance cycle. Many simplified analytical and empirical design methods have been used to estimate settlements and stress-transfer between the track layers. However, these design methods are based on the linear elasticity approach, and thus often give crude estimates. Given the complexities of the behavior of the composite track system consisting of rail, sleeper, ballast, sub-ballast and subgrade under repeated traffic loads in the real track environment, the current track design techniques are overly simplified.

To understand this complex mechanism of track deformations and to study the potential benefits of geosynthetics in rail track, a field trial was conducted. The instrumented track was constructed between two turnouts at Bulli along the coast of New South Wales, north of Wollongong city. The total length of the instrumented track section was 60 m, and it was divided into four sections, each 15 m in length. Two sections were built without the inclusion of a geocomposite layer, while remaining two sections were built by placing a geocomposite layer at the ballast-capping interface (Figure 1). To measure the vertical and horizontal deformations of ballast, settlement pegs and displacement transducers were installed at the sleeper-ballast and ballast-capping interfaces in different track sections. The vertical and horizontal stresses developed in the track bed under repeated wheel loads were measured by pressure cells (230 mm diameter) installed at different locations in section of fresh ballast.

The overall thickness of granular layer was kept as 450 mm including a ballast layer of 300 mm and a capping layer of 150 mm in thickness. The particle size, gradation, and other index properties of ballast used at the Bulli site were in accordance with the Technical Specification TS 3402 (RailCorp, Sydney), which

represents sharp highly angular coarse aggregates of crushed volcanic latite basalt. Recycled ballast was collected from a recycled plant at Chullora yard near Sydney. Table 1 shows the grain size characteristics of fresh ballast, recycled ballast and the sub-ballast materials used in the instrumented track at Bulli. Concrete sleepers were used in the test track. Electrical Friction Cone Penetrometer (EFCP) tests reported that the subgrade soil is a stiff overconsolidated silty clay and had more than sufficient strength to support the train loads. The bedrock is a highly weathered sandstone having weak to medium strength.

Figure 1. Section of ballasted track bed with geocomposite layer (modified after Indraratna et al., 2010b)

Figure 2. Vertical deformation of the ballast layer (modified after Indraratna et al., 2010a).

Table 1. Grain size characteristics of fresh ballast, recycled ballast and capping materials (Indraratna and Salim, 2005; Indraratna et al., 2010a)

Material type	Particle shape	$d_{\max}(\text{mm})$	$d_{\min}(\text{mm})$	$d_{50}(\text{mm})$	C_u	C_c
Fresh Ballast	Highly angular	75.0	19.0	35.0	1.5	1.0
Recycled Ballast	Semi-angular	75.0	9.5	38.0	1.8	1.0
Sub-ballast	Semi-rounded	19.0	0.05	0.26	5.0	1.2

A bi-axial geogrid was placed over the non-woven polypropylene geotextile to serve as the geocomposite layer, which was installed at the ballast-capping interface. The technical specifications of geosynthetic material used at the site can be found elsewhere (Indraratna and Salim, 2005). In order to investigate the overall performance of the ballast layer, the average vertical deformation was considered by deducting the vertical displacements of sleeper-ballast and ballast-capping interfaces. The vertical displacement at each interface was obtained by taking the mean of measurements taken under the rail and the edge of sleeper. The values of average vertical ballast deformation are plotted against the number of load cycles (N) in Figure 2. In the recycled ballast, they are smaller compared to the case of fresh ballast. The better performance of selected moderately-graded recycled ballast can benefit from less breakage as they are often less angular, thereby preventing corner breakage due to high contact stresses. The geocomposite inclusion induces decrease in average vertical deformations of recycled ballast at a large number of cycles. The load distribution capacity of ballast layer is improved by the placement of a flexible and resilient geocomposite layer which results in a substantial reduction of settlement under high cyclic loading.

FINITE ELEMENT ANALYSIS

An elasto-plastic constitutive model of a composite multi-layer track system including rail, sleeper, ballast, sub-ballast and subgrade is proposed. Numerical simulations are performed using a two-dimensional plane-strain finite element analysis PLAXIS to predict the track behaviour with and without geosynthetics. PLAXIS has demonstrated its success in the limit analysis of geotechnical problems. A typical plain strain track model is numerically simulated in a Finite Element discretization as shown in Figure 3a.

Figure 3. (a) Finite element mesh discretisation of a rail track and (b) 15-node continuum soil, 10-node Interface and 5-node geogrid element

The subgrade soil and the track layers are modeled using 15-node linear strain quadrilateral elements 'LSQ'. In representing geogrid elements, 5-node line (tension) elements are used. Since it is also necessary to model the interaction between granular media and geogrid, special interface 10-node elements are adapted. Figure 3b shows details of these elements used in finite element simulations. The 15-node isoparametric element provides a fourth order interpolation for displacements and the numerical integration by Gaussian integration scheme involves twelve Gauss points (stress points). The 4 m high and 8 m wide finite element model is discretised to 794 fifteen-node elements, 26 five-node line elements for geogrid and, 52 five-node elements at the interface. The mesh generation of PLAXIS version 8.0 used here follows a robust triangulation procedure to form 'unstructured meshes', which are considered to be numerically efficient when compared to regular 'structured meshes'.

The nodes along the bottom boundary of the section are considered as pinned supports, i.e., are restrained in both vertical and horizontal directions (i.e. standard fixities). The left and right boundaries are restrained in the horizontal directions, representing smooth contact vertically. The vertical dynamic wheel load is simulated as a line load representing an axle train load of 25 tons with a dynamic impact factor of 1.4. The gauge length of the track is 1.68 m. The shoulder width of ballast is 0.35 m and the side slope of the rail track embankment is 1:2. The constitutive models and material parameters are given in Table 2. The flow rule adopted in HSM is characterised by a classical linear relation, with the mobilised dilatancy angle given by (Schanz et al., 1999):

$$\sin \psi_m = (\sin \phi_m - \sin \phi_{cv}) / (1 - \sin \phi_m \sin \phi_{cv}) \quad (1)$$

where ϕ_{cv} is a material constant (the friction angle at critical state) and:

$$\sin \phi_m = (\sigma'_1 - \sigma'_3) / (\sigma'_1 + \sigma'_3 - 2c \cot \phi) \quad (2)$$

According to equation (1), ψ_m depends on the values of friction, ϕ and dilatancy angles at failure, ψ which control the quantity ϕ_{cv} . Indraratna and Salim (2002) described the dependence of particle breakage and dilatancy on the friction angle of ballast. A modified flow rule considering the energy consumption due to particle breakage during shearing deformations is given by (Salim and Indraratna, 2004):

$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \frac{dE_B}{pd\varepsilon_s^p} \left(\frac{9 - 3M}{9 + 3M - 2\eta M} \right) \left(\frac{6 + 4M}{6 + M} \right) \quad (3)$$

Table 2. Constitutive model and material parameters adapted in FEM analysis

Material Parameters	Rail Track component					
	Rail	Concrete Sleeper	Ballast	Sub-ballast	Subgrade	Geogrid
Material Model	Linear Elastic	Linear Elastic	Hardening Soil	Mohr-Coulomb	Mohr-Coulomb	Elastic
Material Type	Non-porous	Non-porous	Drained	Drained	Drained	-
E (MPa)	210,000	10,000	-	80	34.2	-
E_{50}^{ref} (MPa)	-	-	21.34	-	-	-
E_{oed}^{ref} (MPa)	-	-	21.34	-	-	-
E_{ur}^{ref} (MPa)	-	-	64.02	-	-	-
EA (kN/m)	-	-	-	-	-	1198
γ (kN/m ³)	78	24	15.6	16.67	18.15	-
ν	0.15	0.15	-	0.35	0.33	-
ν_{ur}	-	-	0.2	-	-	-
c (kN/m ²)	-	-	0	0	5.5	-
ϕ (degrees)	-	-	58.47	35	24	-
ψ (degrees)	-	-	12.95	0	0	-
P_{ref} (kN/m ²)	-	-	50	-	-	-
m	-	-	0.5	-	-	-
k_0^m	-	-	0.3	-	-	-
R_f	-	-	0.9	-	-	-

The experimental values of η , p , M and the computed values of $dE_B/d\varepsilon_s^p$ which are linearly related to the rate of particle breakage $dB_g/d\varepsilon_1$ can be readily used to predict the flow rule. In the present study, a non-associative flow rule with a dilatancy angle $\psi = 12.95^\circ$ is used. The values of stress-dependent stiffness moduli E_{50}^{ref} , E_{oed}^{ref} and E_{ur}^{ref} are obtained from previously published results of large scale drained triaxial compression tests under monotonic loading conditions (Indraratna and Salim, 2005). Figures 4 exhibit the evolution of hardening soil model parameters based on deviator stress response and shear strains respectively. The hardening soil model showed better agreement with the strain-hardening behaviour of ballast observed in large scale triaxial tests representing considerable ballast breakage (Shahin and Indraratna, 2006).

Further details of constitutive models can be found in Brinkgreve (2002). The current formulation of finite element is incapable of conducting postpeak analysis into the strain-softening region however such large strains or large deformations are not permitted in the reality, hence the study is focused on the peak strength.

Figure 4. Hardening Soil Model: stress-strain relationship for ballast

Figure 5. Variation of vertical deformation of ballast with the depth

COMPARISON OF FIELD RESULTS WITH FEM PREDICTIONS

In order to validate findings of the finite element analysis, a comparison is made between the elasto-plastic analysis and the field data. Figure 5 shows the vertical deformation profile predicted by FE simulations and the measured values of vertical deformation underneath the rail seat at unreinforced section of the instrumented track. The vertical deformations were monitored at the sleeper-ballast and ballast-capping interfaces using settlement pegs as mentioned earlier. The values predicted by elasto-plastic analysis show slight deviation in comparison with the measured values of real track behavior. This is due to fact that the real cyclic nature of wheel loading is not considered and is approximately represented by equivalent dynamic plain strain analysis in FE studies. Nevertheless, considering the limitations of elasticity based approaches, this prediction is acceptable for the design practice.

DESIGN PROCESS FOR SHORT PVDs UNDER RAILWAY TRACK

The Sandgate Rail Grade Separation Project is located at Sandgate between Maitland and Newcastle, in the Lower Hunter Valley of New South Wales (Figure 6). The new railway tracks were required to reduce the traffic in the Hunter Valley Coal network. In this section, the rail track stabilised using short PVDs in the soft subgrade soil is presented together with the background of the project, the soil improvement details, design methodology and finite element analysis. The effectiveness of PVDs in improving soil condition was demonstrated by Indraratna et al. (2010). Preliminary site investigations were conducted for mapping the soil condition profile at the track. An in-situ and laboratory testing was carried out to provide relevant soil parameters. Site investigation included 6 boreholes, 14 piezocone (CPTU) tests, 2 in-situ vane shear tests, and 2 test pits. Laboratory testing such as soil index property testing, standard oedometer testing, and vane shear testing were also performed.

A typical soil profile showed that existing soft compressible soil thickness varies from 4m to 30m. The soft residual clay is beneath the soft soil layer and

followed by shale bedrock. The soil properties are shown in Figure 7. The groundwater level is at the ground surface. The moisture contents of the soil layers are the same as their liquid limits. The soil unit weight varied from 14 to 16 kN/m³. The undrained shear strength increased from about 10 to 40 kPa. The clay deposit at this site can be considered as lightly overconsolidated ($OCR \approx 1-1.2$). The horizontal coefficient of consolidation (c_h) is approximately 2-10 times the vertical coefficient of consolidation (c_v). Based on preliminary numerical analysis conducted by Indraratna et al. (2010), PVDs with 8m length were suggested and installed at 2m spacing in a triangular pattern. The field instrumentation which included settlement plates, inclinometers, and vibrating wire piezometers was employed to monitor the rail track responses. The settlement plate was installed above the surface of the subgrade layer to directly provide a measurement of the vertical subgrade settlement. The aims of the field monitoring were to:

- ensure the stability of the tracks;
- validate the design of the new railway stabilised by PVDs;
- examine the accuracy of the numerical analysis through Class A predictions, where the field measurements were unavailable at the time of finite element modeling.

Figure 6. Site location (adopted from Hicks, 2005)

Figure 7. Soil properties at Sandgate Rail Grade Separation Project (Indraratna et al. 2010c)

Preliminary Design

Due to the time constraint, rail tracks were built immediately after installing PVDs. The train load at very low speed was used as the only external surcharge. The equivalent dynamic loading using an impact load factor was used to predict the settlements and associated excess pore pressures. In this analysis, a static pressure of 104 kPa with an impact factor of 1.3 was applied according to the low train speed for axle loads up to 25 tonnes, based on the Australian Standards AS 1085.14-1997. The Soft Soil model and Mohr-Coulomb model were employed in the finite element code, PLAXIS (Brinkgreve 2002). The overconsolidated crust and fill layer was simulated by the Mohr-Coulomb theory, whereas the soft clays were conveniently modelled using the Soft Soil model. Soil formation was separated into 3 layers, namely, ballast and fill, Soft soil-1 and Soft soil-2. The soil parameters were given in Table 3.

A vertical cross-section of mesh discretisation of the formation beneath the rail track is shown in Figure 8. A plane strain finite element analysis employed triangular elements with 6 displacement nodes and 3 pore pressure nodes. Total of 4 PVDs were used in the analysis. An equivalent plane strain analysis with appropriate conversion from axisymmetric to 2-D was adopted to analyse the multi-drain analysis (Indraratna et al., 2005). In this method, the corresponding ratio of the smear zone permeability to the undisturbed zone permeability is obtained by:

$$\frac{k_{s,ps}}{k_{h,ps}} = \frac{\beta}{k_{h,ps}/k_{h,ax} [\ln(n/s) + k_{h,ax}/k_{s,ax} \ln(s) - 0.75] - \alpha} \quad (4)$$

$$\alpha = 0.67(n-s)^3 / n^2(n-1), \quad (4a)$$

$$\beta = 2(s-1)[n(n-s-1) + 0.33(s^2 + s + 1)] / n^2(n-1) \quad (4b)$$

$$n = d_e / d_w \quad (4c)$$

$$s = d_s / d_w \quad (4d)$$

In the above expressions, d_e = the diameter of unit cell soil cylinder, d_s = the diameter of the smear zone, d_w = the equivalent diameter of the drain, k_s = horizontal soil permeability in the smear zone, k_h = horizontal soil permeability in the undisturbed zone and the top of the drain and subscripts 'ax' and 'ps' denote the axisymmetric and plane strain condition, respectively.

Table 3 Selected parameters for soft soil layer used in the FEM (Indraratna et al. 2010c)

Soil layer	Depth of layer (m)	Model	c (kPa)	ϕ	e_0	$\lambda / (1+e_0)$	$\kappa / (1+e_0)$	k_v ($\times 10^{-4}$ m/day)	k_h ($\times 10^{-4}$ m/day)
Soft soil-1	1.0-10.0	Soft Soil	10	25	2.26	0.131	0.020	0.70	1.4
Soft soil-2	10.0-20.0	Soft Soil	15	20	2.04	0.141	0.017	0.75	1.5

Note: ϕ Back-calculated from Cam-clay M value

The ratio of equivalent plane strain to axisymmetric permeability in the undisturbed zone can be attained as,

$$k_{h,ps}/k_{h,ax} = 0.67(n-1)^2 / [n^2[\ln(n) - 0.75]] \quad (5)$$

In the above equation, the equivalent permeability in the smear and undisturbed zone vary with the drain spacing.

Figure 8. Vertical cross section of rail track and foundation (Indraratna et al. 2010).

COMPARISON OF FIELD RESULTS WITH *CLASS A* FEM PREDICTIONS

The field results were released to the Authors by the track owner (Australian Rail Track Corporation) after a year subsequent to the analysis. Therefore all prediction can be categorized as Class A (Lambe, 1973). A spacing of 2m was adopted for Mebra (MD88) vertical drains of 8m in length, based on the Authors' analysis. The field observation data together with the numerical predictions are compared and discussed. The calculated and observed consolidation settlements at the centre line are presented in Figure 9. The predicted settlement matches very well with the field data for *Class A* prediction. The insitu lateral displacement at 180 days at the rail embankment toe is illustrated in Figure 10. As expected, the maximum displacements are measured within the top clay layer, i.e., the softest soil below the 1m crust. As expected, the lateral displacement is restricted to the topmost compacted fill (0-1m deep). The *Class A* predictions of lateral displacements are also in very good agreement with the field behaviour. The effectiveness of wick drains in reducing the effects of undrained cyclic loading through the reduction in lateral movement is evident.

Figure 9. Predicted and measured at the centre line of rail tracks (after Indraratna et al. 2010c)

Figure 10. Measured and predicted lateral displacement at the embankment toe at 180 days (after Indraratna et al. 2010c)

CONCLUSION

The performance of ballasted rail tracks with geosynthetic reinforcement has been described through numerical and field studies. The study confirms that geocomposite layer improves the track performance through reduced vertical deformations when ideally placed at the ballast-capping interface. It is further shown that predictions of elasto-plastic finite element analysis are in close agreement with the field data. The implications of ballast breakage on the friction angle and dilatancy of ballast layer are discussed in the context of finite element simulations. The advantage of using a comprehensive elasto-plastic constitutive model for track layers as compared to conventional (simplified) analytical methods that are primarily based on linear elasticity approach is elucidated. The finite element analysis of the track behaviour conducted prior to track construction was considered as a *Class A* prediction, with field monitoring since the time of construction and even as of today. PVDs significantly decrease the buildup of excess pore water pressure during cyclic loading, and also continue to dissipate excess pore water pressure during the rest

period. The dissipation of the pore water pressure during the rest period makes the track more stable for the next loading stage. Even with the relative short PVDs, both the predictions and field data prove that the lateral displacement can be curtailed. The equivalent plane strain finite element analysis is adequate to predict the behaviour of track improved by short PVDs.

ACKNOWLEDGEMENT

The authors wish to thank the Australia Research Council (Australia), CRC for Rail Innovation (Australia), Railcorp (Sydney), ARTC, ARUP, John Holland, for their continuous support. The authors are most grateful for the continuous support and invaluable advice of David Christie, formerly Senior Geotechnical Consultant, Rail Infrastructure Corporation (NSW).

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