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Keywords

demand, liner, transshipment, cargo, deployment, fleet, uncertainty, shipping

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LINER SHIPPING FLEET DEPLOYMENT WITH CARGO TRANSSHIPMENT AND DEMAND UNCERTAINTY

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Abstract: This paper addresses a novel liner shipping fleet deployment problem characterized by cargo transshipment, multiple container routing options and uncertain demand, with the objective of maximizing the expected profit. This problem is formulated as a stochastic program and solved by the sample average approximation method. In this technique the objective function of the stochastic program is approximated by a sample average estimate derived from a random sample, and then the resulting deterministic program is solved. This process is repeated with different samples to obtain a good candidate solution along with the statistical estimate of its optimality gap. We apply the proposed model to a case study inspired from real-world problems faced by a major liner shipping company. Results show that the case is efficiently solved to 1% of relative optimality gap at 95% confidence level.

Key Words: Liner shipping; Fleet deployment; Transshipment; Demand uncertainty; Sample average approximation

1. INTRODUCTION

Fleet deployment decisions arise at the tactical planning level of liner shipping networks. Fundamentally, the liner shipping fleet deployment problem (LSFDP) involves the allocation of ships to service routes for delivering containers. To maximize the total profit, liner shipping companies need to assess the trade-off between the cost and capacity of ships. The LSFDP has three complicating factors due to the unique characteristics of liner shipping services: (i) a certain service frequency or a minimum level of service frequency must be maintained; (ii) more than one routing option is available for delivering containers from origin to destination; (iii) fleet deployment decisions have to be made prior to knowing the exact shipping demand.

Various pure or mixed-integer linear programming models for the LSFDP have been developed to account for different levels of practical constraints arising in liner shipping operations. Perakis and Jaramillo (1991) and Jaramillo and Perakis (1991) contributed the pioneering work for the LSFDP. They proposed a linear programming formulation incorporating the capacity constraint, minimum service frequency requirement and ship chartering. The cargo amount between pairs of ports is given and the aim is to minimize the total fleet operating cost including fuel costs, daily running costs, port charges and canal fees. The aforementioned linear programming model assumes that the number of ships of each type allocated to a route is a continuous rather than an integer variable. To remedy this unrealistic assumption, Powell and Perakis (1997) presented an integer linear programming model for the LSFDP. The above mentioned studies all assume that cargo demand between pairs of ports on each service route is *a priori* known, Cho and Perakis (1996) relaxed this assumption by formulating a model in which the given cargo demand between two specific ports can be served by any route passing through these two ports. Nevertheless, cargo transshipment is still not allowed in this model. To take into account cargo transshipment, Mourão et al. (2001) studied the LSFDP in a simple hub-and-spoke (H&S) network which consists of two routes (a feeder route and a main route) and one origin-destination (O-D) port pair. All containers must be transshipped at the hub port in the feeder route. As the cruising speed of ships have direct implications on the round-trip time and bunker consumption, Gelareh and Meng (2010) presented a mixed-integer programming model by discretizing the cruising speed to integrate the decision of optimal ship speed into the LSFDP. Unlike the above deterministic models, in a recent contribution, Meng and Wang (2010) investigated the uncertain demand in the LSFDP. They assume that a certain level of service for each route has to be maintained. The level of service is defined as the probability that all cargo demand on the route can be satisfied. Thus the uncertain demand is formulated as chance constraints and these chance constraints are then transformed into their equivalent deterministic constraints.

The LSFDP models found in the literature have two main shortcomings when compared to the practical liner shipping operations. First, most of the models assume that the cargo demand is associated with a certain liner route, thus can solely be fulfilled by direct transportation services on the route without transshipment. As a matter of fact, in liner shipping practices, it is prevalent to transfer containers at an intermediate port from one ship to another. Transshipment not only enables the use of large containerships to exploit the economies of scale in the hub-and-spoke networks, but also expands the service scope in that the number of O-D pairs can be much more than the number of direct O-D services operated. Moreover, many routing options for an O-D pair become available due to the possibility of transshipment. Mourão et al. (2001) considered the issue of transshipment in a very special network setting with only one routing option. Cho and Perakis (1996) investigated multiple routing options without transshipment. These two models cannot be adapted to accommodate general cases of both transshipment and multiple routing options. Second, few attempts have been devoted to the inclusion of uncertain demand in the fleet deployment decisions. In contrast to industrial and tramp shipping which operate schedules in response to actual demand, liner shipping is based on a fixed schedule which is generally published up to six months into the future. This means that the fleet deployment decisions are made depending on the demand forecast. The forecast is seldom error-free and hence it is advisable to capture the uncertain nature of the demand. As will be reported in Section 5, compared with simply using the average demand value, the inclusion of the uncertain demand in making the fleet deployment decisions yields superior solutions in terms of the expect profit. The only model with uncertain demand we are aware of is proposed by Meng and Wang (2010). However, this model relies on the assumption that the container numbers in the O-D port pairs are independent and normally distributed. Thus its applicability to real-life situations is rather limited.

In this paper, we take the initiative to investigate the LSFDP with cargo transshipment, multiple routing options and demand uncertainty. The uncertain demand is represented by a multivariate random variable and can have any general probability distribution. This problem is formulated as a stochastic model with the objective of maximizing the expected profit. We use the sample average approximation (SAA) method to solve the model and estimate the statistical optimality gap.

The remainder of this paper is organized as follows: Section 2 is the detailed description of the LSFDP. In Section 3 we first presents the stochastic programming formulation for the basic LSFDP; then we discuss some extensions of the model to adapt to more practical constraints. Section 4 is dedicated to employing the SAA method to address the stochastic model. Section 5 is a numerical example to assess the proposed model. Conclusions are presented in Section 6.

2. PROBLEM DESCRIPTION

Consider a liner shipping company operating a set of routes to transport containers between pairs of ports. The itinerary (sequence of portcalls) of each route is given as the input for the fleet deployment problem. All the routes have a weekly service frequency. The liner shipping company deploys a string of ships of the same type on each route in order to maintain the weekly service frequency and to deliver containers at maximum profit. Liner routes intersect at the common port of call, and thus containers can be transshipped between ships on two liner routes. An illustrative liner shipping network is shown in Figure 1.

In the mathematical description of the problem, the set of routes is represented by R. The number of portcalls in a round-trip of a route $r \in R$ is denoted by N_r. We use the port calling sequence on a route to refer to the port of call because a port may be visited more than once during a round-trip. For example, route r_3 in Figure 1 has 4 portcalls. If SH is defined as the 1st portcall, both the 2nd and the 4th portcalls refer to SG, but they are different portcalls. Similarly, we define the i^{th} leg of a route r, denoted by $r(i)$, as the voyage from the i^{th} portcall to the $(i+1)^{th}$, except that the N_r^{th} leg is from the N_r^{th} portcall to the 1st . We further define $I_r := \{1, 2 \ldots N_r\}$. A corresponding definition for the routes in Figure 1 is presented in Table 1.

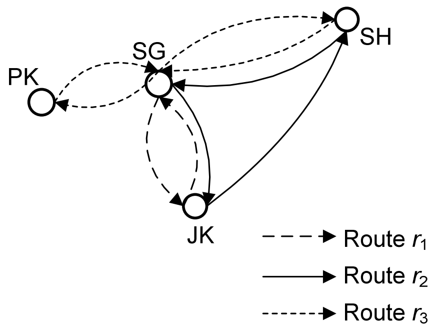

Figure 1. A Liner Shipping Network
SH: Shanghai; SG: Singapore; JK: Jakarta; PK: Port Klang

Table 1. Portcalls and Legs of Routes

Route	Portcalls	Legs
r_1: JK-SG	1st: JK; 2nd: SG	$r_1(1)$: JK-SG; $r_1(2)$: SG-JK
r_2: SH-SG-JK	1st: SH; 2nd: SG; 3rd:JK	$r_2(1)$: SH-SG; $r_2(2)$: SG-JK; $r_2(3)$: JK-SH
r_3: SH-SG-PK-SG	1st: SH; 2nd: SG; 3rd:PK; 4th: SG	$r_3(1)$: SH-SG; $r_3(2)$: SG-PK; $r_3(3)$: PK-SG; $r_3(4)$: SG-SH

2.1 Fleet Deployment

The liner shipping company has a set of ships to deploy on the liner routes. Let S be the set of ship types and the container capacity in terms of twenty-foot-equivalent unit (TEU) of a particular ship type $s \in S$ is denoted by V_s. We assume that there are an unlimited number of ships in each type. When a route r is deployed with ships of type s, we first assume that the number of ships is fixed and we will relax this assumption later on. Consequently, the cost of operating the route, denoted by c_{rs} (\$/week), including the fuel costs, daily running costs, port charges and canal fees as in Perakis and Jaramillo (1991), is also determined. As each route has a weekly service frequency, the round-trip time equals the number of ships deployed multiplied by 7 days. For instance, if the round-trip time is 35 days on a route r, 5 ships are required to ensure the same portcall is visited on the same day every week. The round-trip time consists of the time at sea (the round-trip distance divided by cruising speed), the time for pilotage in and out of ports, and the berth time for container handling. We assume the ship speed and the pilotage time at port are given, thus the maximum berth time T_{rs} (h) for container handling at all portcalls of the route-ship type combination r-s is also determined and must be respected.

2.2 Container Routing Plan

The liner shipping company transport containers from the origin port to the destination. Denote the set of O-D port pairs by D and lowercase letter d refers to a particular O-D. The number of containers for an O-D d is denoted by a random variable ξ_d (TEUs/week). We assume that the value of ξ_d, although unknown in advance, remains constant in each week over the fleet deployment planning period. The joint probability distribution of ξ_d, $d \in D$, is estimated e.g. from historical data and used as input for the fleet deployment decisions. The revenue from delivering one TEU for the O-D d is g_d (dollars/TEU).

Usually there are many routing options for delivering containers from origin to destination. For example, in Figure 1, containers from JK to SH can be transported (i) on route r_2; or (ii) on route r_1 to SG, and then transshipped to ships on route r_3 and then delivered to SH. Each routing option is defined as a *container routing plan* (referred to as *routing plan* for short hereafter). A routing plan, denoted by h, consists of a group of ordered legs to be visited. For example, the above mentioned two routing plans for the demand JK to SH, can be denoted by $r_2(3)$ and $r_1(1) \rightarrow r_3(4)$, respectively. Some routing plans for the liner shipping network in Figure 1 are provided in Table 2.

Table 2. Routing Plans for Different O-Ds

O-D	Routing Plans
d_1: JK-SH	h_1: $r_2(3)$
	h_2: $r_1(1)\rightarrow r_3(4)$
d_2: SH-PK	h_3: $r_3(1)\rightarrow r_3(2)$
d_3: SH-SG	h_4: $r_2(1)$
	h_5: $r_3(1)$

A routing plan contains all information on how containers are transported, such as origin, destination, liner routes involved, and transshipment ports. An O-D d may have several routing plans, the set of which denoted by H_d, whereas a routing plan corresponds to a single O-D. Containers of the same O-D can be split among several routing plans. The set of all the routing plans, denoted by H, is provided by the liner shipping company as an input for the fleet deployment model.

The operating cost of a route is determined once its fleet deployment decision is made. The main variable cost in liner shipping operations is the container handling cost, which consists of loading cost at origins, discharging cost at destinations and transshipment costs at transshipment ports, if any. Two routing plans for the same O-D may incur different handling costs. For example, the routing plan h_1 in Table 2 involves exclusively the loading cost at JK and discharging cost at SH, while the routing plan h_2 is associated with an additional transshipment cost at SG. We denote the handling cost of the routing plan h by c_h (dollars/TEU).

Routing plans also influence the berth time of related routes. Let t_{ps} (h/TEU) be the average container handling time at port p for ship type s. Note that larger ships allow more quay cranes to work simultaneously, thus having a higher handling efficiency. Containers in routing plan h_2 are loaded at JK and discharged for transshipment at SG on route r_1. Hence if route r_1 is deployed with ships of type s, let p=JK and q=SG, then the *incremental* berth time on route r_1 for routing plan h_2 is $t_{ps}+t_{qs}$ (h/TEU).

3. MATHEMATICAL MODEL

In view of the uncertain demand, the liner shipping company aims to maximize its expected profit. We first formulate the expected value model for the basic LSFDP described in Section 2. Then we discuss some extensions of the model to handle more practical constraints.

3.1 Stochastic Model for the Basic LSFDP

To formulate the stochastic model of the basic LSFDP, we need further notations. Let the binary coefficient δ_{hri} indicate whether routing plan h contains the i^{th} leg of route r (δ_{hri}=1) or not (δ_{hri}=0). Denote by t_{hrs} (h/TEU) the incremental berth time incurred by one TEU in routing plan h on route r deployed with ships of type s. t_{hrs}=0 if the routing plan h does not use any leg of the route r. The decision variables are as follows: f_h is the number of TEUs transported in routing plan h; x_{rs} is a binary variable which takes the value 1 if and only if ships of type s are deployed on route r. The expected value model for the basic LSFDP is:

[EVM]
$$z^* = \max \mathbf{E}[\sum_{d\in D}\sum_{h\in H_d}(g_d - c_h)f_h - \sum_{r\in R}\sum_{s\in S}c_{rs}x_{rs}] \tag{1}$$

s.t.
$$\sum_{h\in H_d} f_h \le \xi_d, \forall d\in D \tag{2}$$

$$\sum_{h\in H}\delta_{hri}f_h - \sum_{s\in S}V_s x_{rs} \le 0, \forall r\in R, \forall i\in I_r \tag{3}$$

$$\sum_{h\in H} f_h \sum_{s\in S} t_{hrs}x_{rs} \le T_{rs}x_{rs}, \forall r\in R \tag{4}$$

$$\sum_{s\in S} x_{rs} = 1, \forall r \in R \tag{5}$$

$$x_{rs} \in \{0,1\}, \forall r \in R, \forall s \in S \tag{6}$$

$$f_h \geq 0, \forall h \in H \tag{7}$$

The objective function (1) maximizes the expected profit. The first term is the revenue less the handling cost from shipping containers; the second term is the operating cost of the routes. Constraints (2) require that the number of containers transported do not exceed the demand. Constraints (3) are the capacity constraints imposed on each leg of the routes. Constraints (4) ensure that the maximum berth time is respected. Constraints (5) enforce that exactly one type of ship is deployed on each route. Constraints (6) denote x_{rs} as binary variables and constraints (7) denote f_h as nonnegative continuous variables.

3.2 Model Extensions

EVM has some simplifying assumption and now we extend **EVM** so that it can accommodate more practical constraints.

We have assumed that there are an unlimited number of ships in each type. If this assumption is violated, let n_s be the maximum number of ships of type s available and n_{rs} be the number of ships required if ships of type s are deployed on route r, then we can add the following constraints:

$$\sum_{r\in R} n_{rs} x_{rs} \leq n_s, \forall s \in S \tag{8}$$

It is very probable that the liner shipping company not only services the spot market where the cargo demand is optional, but also has some contracted cargo which must be transported. Denote by m_d (TEUs) the number of contracted cargo for O-D d, the following constraints hold:

$$\sum_{h\in H_d} f_h \geq m_d, \forall d \in D \tag{9}$$

Practical liner routes normally provide weekly services. Otherwise we may let t_r be the service frequency (number of days between two arrivals at a portcall) of route r, and constraints (3) and (4) can be restated as:

$$\frac{t_r}{7}\sum_{h\in H} \delta_{hri} f_h - \sum_{s\in S} V_s x_{rs} \leq 0, \forall r \in R, \forall i \in I_r \tag{10}$$

$$\frac{t_r}{7}\sum_{h\in H} f_h \sum_{s\in S} t_{hrs} x_{rs} \leq T_{rs} x_{rs}, \forall r \in R \tag{11}$$

Some types of ship might not be compatible with all the routes, e.g., due to physical restrictions at ports. For any r-s of the incompatible route-ship combinations, we simply define that x_{rs}=0.

When a liner route r is deployed with ships of type s, we have assumed that the number of ships is determined. If the liner shipping company decides to add one more ship in order to provide more berth time (here one week for a weekly service) for container handling at ports, we can simply consider this deployment as having n_{rs}+1 ships of type s_1 which have exactly the same properties as ships of type s.

In constraints (3), we have used the vessel capacity V_s as the leg capacity for all voyage legs. In practice, ships may not be allowed to fully load on some legs because of restricted water. Therefore the capacity might vary on different legs of a route. We can let V_{rsi} be the capacity on the i^{th} leg of route r when it is deployed with ships of type s, and constraints (3) can be restated as:

$$\sum_{h\in H} \delta_{hri} f_h - \sum_{s\in S} V_{rsi} x_{rs} \leq 0, \forall r \in R, \forall i \in I_r \tag{12}$$

4. SOLUTION ALGORITHM

In this section we present the solution algorithm for the basic model of the LSFDP. The extensions of the basic model can be addressed similarly. The basic model is difficult in that it is a stochastic one with nonlinear constraints (4). In Section 4.1, we use the sample average approximation method to transform the stochastic model into an approximating deterministic model. Section 4.2 is dedicated to the statistical analysis of the solution quality. An equivalent linear formulation of constraints (4) is presented in Section 4.3.

4.1 Sample Average Approximation

Unless the uncertain demand ξ_d, $d\in D$, has a small number of possible realizations (scenarios), it is usually impossible to solve **EVM** exactly. One approach for approximately solving **EVM** is the sample average approximation (SAA) method (Verweij et al., 2003). The SAA method is an approach for solving stochastic optimization problems by using Monto Carlo simulation. In this technique the objective function of the stochastic program is approximated by a sample average estimate derived from a random sample. The resulting sample average approximating problem is then solved by deterministic optimization approaches. This process is repeated with different samples to obtain a good candidate solution along with the statistical estimate of its optimality gap.

To solve **EVM**, we first uses the Monte Carlo procedure to generate N independent and identically distributed observations ξ_d^n, n=1, 2… N, according to the joint probability distribution of the demand. Then the expected value function is approximated by the sample average function. Let I_N:={1, 2… N}, and the approximating deterministic model is:

[ADM]
$$z_N = \max \frac{1}{N}\sum_{n\in I_N}\sum_{d\in D}\sum_{h\in H_d}(g_d - c_h) f_h^n - \sum_{r\in R}\sum_{s\in S} c_{rs} x_{rs} \tag{13}$$

s.t.
$$\sum_{h\in H_d} f_h^n \le \xi_d^n, \forall d\in D, \forall n\in I_N \tag{14}$$

$$\sum_{h\in H}\delta_{hri} f_h^n - \sum_{s\in S} V_s x_{rs} \le 0, \forall r\in R, \forall i\in I_r, \forall n\in I_N \tag{15}$$

$$\sum_{h\in H} f_h^n \sum_{s\in S} t_{hrs} x_{rs} \le T_{rs} x_{rs}, \forall r\in R, \forall n\in I_N \tag{16}$$

$$\sum_{s\in S} x_{rs} = 1, \forall r\in R \tag{17}$$

$$x_{rs}\in\{0,1\}, \forall r\in R, \forall s\in S \tag{18}$$

$$f_h^n \ge 0, \forall h\in H, \forall n\in I_N \tag{19}$$

4.2 Statistical Analysis of Solution Quality

ADM is a deterministic optimization problem and its optimal objective value z_N and optimal fleet deployment decisions, denoted by the vector $\mathbf{x}_N$, can be used as an estimate of their counterparts z^* and $\mathbf{x}^*$ in **EVM**. This approach is justified by the epi-convergence theory: under mild assumptions, $\{z_N\}_{N=1}^{\infty}$ converges to z^* with probability 1, and accumulation points $\{\mathbf{x}_N\}_{N=1}^{\infty}$ are optimal solutions to **EVM** with probability 1 (Shapiro, 1991).

As **ADM** is an approximating model, we need to assess the quality of the solution provided by **ADM**. To this end, a common way is to estimate the lower and upper bounds of z^* in **EVM**. We use the method proposed by Mak et al. (1999) to estimate the statistical upper and lower bounds of z^*. To keep this paper relatively self-contained, we briefly introduce this method.

From Mak et al. (1999), the expected value of z_N has the property $\mathbf{E}z_N \geq \mathbf{E}z_{N+1} \geq z^*$ in the maximization problem **ADM**. Consequently, if we generate *M* independent samples of the uncertain demand, each of size *N*, and obtain *M* optimal objective values z_N^k and the optimal solution $\mathbf{x}_N^k$ for **ADM**, $k = 1,2,\ldots,M$, then a statistical upper bound for z^* can be estimated by $\overline{U} = \sum_{k=1}^{M} z_N^k / M$. Let S_U be the sample variance of z_N^k, $k = 1,2,\ldots,M$, then the statistic $\varepsilon_U = (\overline{U} - \mathbf{E}z_N) / \sqrt{S_U / M}$ has a *t*-distribution with $M-1$ degrees of freedom. Let $t_{\alpha,M-1}$ satisfy $P\{T \leq t_{\alpha,M-1}\} = 1-\alpha$, where the random variable *T* has a *t*-distribution with $M-1$ degrees of freedom, and $0 \leq \alpha \leq 1$, then $P\{Ez_N \leq \overline{U} + t_{\alpha,M-1}\sqrt{S_U / M}\} = 1-\alpha$ and hence $\overline{U} + t_{\alpha,M-1}\sqrt{S_U / M}$ is an upper bound of z^* with at least $1-\alpha$ level of confidence.

To estimate the lower bound, let $k^* := \arg\max z_N^k$, $k = 1,2,\ldots,M$, and define $\mathbf{x}_N^* := \mathbf{x}_N^{k^*}$. Denote by z' the optimal objective value of **EVM** when the values of x_{rs} are fixed at $\mathbf{x}_N^*$. As $\mathbf{x}_N^*$ is a feasible solution to **EVM**, z' is a lower bound of z^*. Generate another N' ($N' \gg N$) observations $\boldsymbol{\xi}^j$, $j = 1,2,\ldots,N'$, according to the joint probability distribution of the demand. These N' observations are independent from the previous ones generated for the estimation of the statistical upper bound. Let z'_j be the optimal objective value of **EVM** when the values of x_{rs} are fixed at $\mathbf{x}_N^*$, and the random variables ξ_d are replaced by the observation $\boldsymbol{\xi}^j$, $j = 1,2,\ldots,N'$. It should be noticed that now **EVM** has become a deterministic optimization problem with simply the container routing variables f_h. Consequently $\overline{L} = \sum_{j=1}^{N'} z'_j / N'$ can be used to estimate z'. Similarly, denote by S_L the sample variance of z'_j, $j = 1,2,\ldots,N'$, then the statistic $\varepsilon_L = (\overline{L} - z') / \sqrt{S_L / N'}$ has a *t*-distribution with $N'-1$ degrees of freedom. Hence $\overline{L} - t_{\alpha,N'-1}\sqrt{S_L / N'}$ is a lower bound of z^* with at least $1-\alpha$ level of confidence.

Hence the optimal objective value of **EVM** z^* lies in the interval $[\overline{L}, \overline{U}]$ with at least $1-2\alpha$ level of confidence. To assess the quality of the candidate solution $\mathbf{x}_N^*$, we can use the relative optimality gap which is defined as $|(\overline{U} - \overline{L}) / \overline{L}|$.

4.3 Linearization

There are still difficulties in solving **ADM** as the constraints (4) are nonlinear. However, they can be replaced with the following equivalent linear constraints:

$$\sum_{h \in H} f_h t_{hrs} \leq T_{rs} + M_H (1 - x_{rs}), \forall r \in R, \forall s \in S \tag{20}$$

In constraints (20), M_H is a huge number which ensures that when $x_{rs} = 0$, no constraints on the f_h variables are imposed. M_H can be set as the maximum container handling time during a round-trip of any route with any type of ship. To represent the value of M_H mathematically, let t_{rsi} denote the average container handling efficiency (h/TEU) at the i^{th} port of route r for ships of type s, then we can set $M_H = \max\{2V_s \sum_{i\in I_r} t_{rsi} \mid \forall r \in R, \forall s \in S\}$, in which $2V_s \sum_{i\in I_r} t_{rsi}$ means the berth time of route r with ships of type s, when full shipload of containers are discharged and another full shipload loaded at all portcalls.

5. CASE STUDY

We test our model and algorithm on a case inspired from real-world problems faced by a major liner shipping company. This test case has a network of 46 ports as shown in Figure 2. There are a total of 3 types of ship, 1500 TEUs, 3000 TEUs and 5000 TEUs, to be deployed on 11 routes that connect the 46 ports. A total of 652 O-D port pairs have non-zero container shipping demand. For each O-D d, we assume that there are three possible scenarios of the container number, μ_d, $\mu_d(1+\kappa)$, and $\mu_d(1-\kappa)$, $0<\kappa<1$, each having a probability of 1/3. The container numbers of different O-D pairs are independent. We test three cases of demand variability with $\kappa=0.2$, $\kappa=0.5$ and $\kappa=0.9$. In each case, the total number of possible realizations of the uncertain demand is 3^{652}.

Recall from Section 4.2 that the SAA method requires the solution of **ADM** M times, each including N sampled scenarios. In this case study we set M=21 and N=30. The number of observations N' for calculating the lower bound is set to be 201. We use the open source code lp_solve version 5.5 (Lp_solve, 2010) and develop the algorithm with C++ on Visual Studio 2005. All computations are carried out on a computer with an Intel Duo 3.20 GHz processor and 3 GB of RAM.

Computational results show that it takes less than 9 minutes to calculate one instance of **ADM**. The statistical lower bound, upper bound, confidence interval and relative optimality gap for different demand variability are presented in Table 3. In all the three cases of demand variability, the standard deviations of z'_j and z^k_N are less than 4% of their respective average value. Consequently the 95% confidence interval is very tight, and the relative optimality gap is less than 1%. Moreover, the variance of the statistical lower bound, upper bound, the width of the confidence interval and the relative optimality gap increase with the variability of the demand.

To justify the efforts for solving the stochastic fleet deployment model, we also calculate the expected profit of fleet deployment decisions without considering demand uncertainty with the following procedure: firstly, solve the deterministic optimization problem **EVM** by replacing the random variable ξ_d with their mean value μ_d in constraints (2), and get the optimal fleet deployment result; then use different realization of ξ_d to calculate the expected value of the profit similar to the calculation of lower bound in Section 4.2. The comparison result under different demand variability is presented in Figure 3. In all the demand variability cases, the expected profit with the inclusion of demand uncertainty in fleet deployment decisions is higher.

Figure 2 A Liner Shipping Network of 46 Ports

Table 3. Statistical Analysis of Solution Quality

Demand Variability κ	z'_j		z_N^k		95% Confidence Interval		Relative Optimality Gap
	Average	Std. Dev.	Average	Std. Dev.	$\overline{L}$	$\overline{U}$	
0.2	26.58	0.201	26.59	0.035	26.58	26.60	0.1%
0.5	26.58	0.506	26.60	0.086	26.51	26.64	0.5%
0.9	26.26	0.988	26.28	0.162	26.12	26.35	0.9%

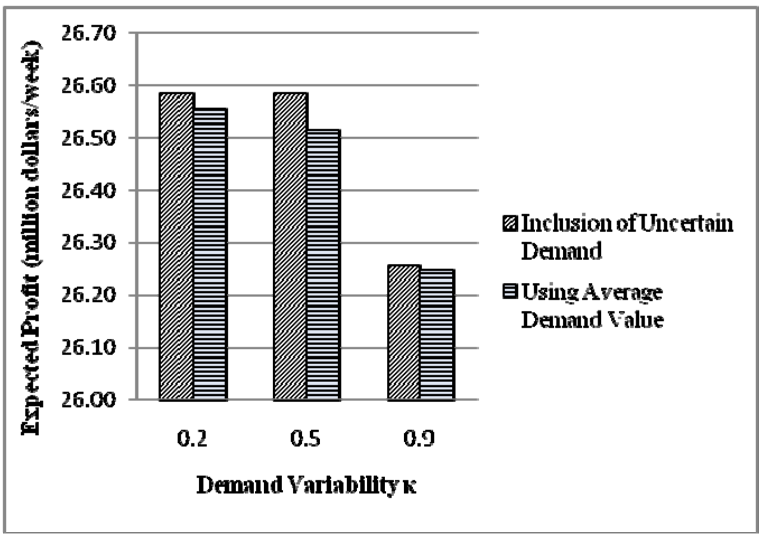

Figure 3 Comparison of Expected Profit with and without Considering Demand Uncertainty

6. CONCLUSIONS

In this paper, we have investigated a novel LSFDP arising in the liner shipping industry. Many practical constraints in liner shipping operations, such as container handling time, weekly service frequency, transshipment, multiple routing plans, are incorporated. In view of the uncertain demand, we optimize the fleet deployment decisions by maximizing the expected profit. The case study based on realistic problems shows that the problem can be solved efficiently with the sample average approximation method and the statistical relative optimality gap is very tight. Results also suggest that the inclusion of uncertain demand into the model is preferable compared with using the deterministic average demand value.

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