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## Introduction

Mathematics is often perceived as a subject disconnected from reality. For adult learners who frequently have struggled with and are fearful of maths, this disconnect may even be more pronounced (Jameson & Fusco 2014). More specifically, at two-year tertiary institutions serving diverse student bodies and large numbers of adult learners, high numbers of students are often placed in remedial mathematics, as seen at one large urban institution, which placed a staggering 88% of its incoming students in the fall 2016 semester in such courses ("LaGuardia Community College Institutional Research Facts" 2016). Higher level courses like Calculus 1 accept a small percentage of students each semester because students make their way to the course only after reaching the required skill level, which could involve taking a series of pre-requisites, which may start from remedial mathematics or basic college-level mathematics. This is problematic for many adult learners, as Calculus I is an important course for students in several programs including Engineering, Accounting, and Criminal Justice, and even for some entry-level medical fields. It is the first course where students encounter proofs and concepts that necessitate deeper understanding of the subject, as opposed to mere procedural understanding.

In this paper, we use Holistic Learning Theory to determine how adult community college students use and apply different facets of knowledge to the study of maths, and specifically calculus. We analyse students' essays in a calculus classroom at LaGuardia Community College, New York City and assess to what extent students can use different forms of knowledge at different points during the semester. We begin with a detailed literature review, explain the theoretical and methodological framework and describe the study's data collection and analysis. We then discuss our findings in light of adult learning and the teaching of maths to adults. Finally, we provide concrete suggestions for practitioners in the field.

## Literature review

A recent study by Jameson and Fusco (2014) makes the case that adult learners are an increasingly large percentage of undergraduate populations in both community and four-year institutions across the country. They specifically examined these adult learners and their maths learning in terms of self-efficacy and maths anxiety (Jameson & Fusco 2014), and implications for adult education in maths. There have been a number of studies on adult learners and maths anxiety (Ashcraft 2002; Baloglu & Zelhart 2007; Bessant 1995), and some on how to approach maths instruction with adult learners (McDevitt 2001) and how to democratise maths instruction (Allen 2011). However, little has been written on conceptualising, applying and interpreting maths knowledge with a focus on calculus with adult learners. This pilot study addresses this gap in the literature.
Beyond adult learning and maths, a multitude of factors affect mathematics achievement. For example, gender inequities in mathematics lead some researchers to question gender differences in value beliefs about mathematics (Gaspard et al. 2015). Expectancy-value theory (Eccles et al. 1983) is widely used as a framework to understand students' achievement, using two factors: expectancies for success and task values. Trautwein et al. (2012) examine the power of expectancy and value beliefs to predict achievement in mathematics, finding that both expectancy and value beliefs were statistically associated with mathematics achievement. Mathematics is often perceived as a difficult subject, and US students consistently underperform relative to their international peers on the Program for International Assessment (PISA) exam (National Center for Education Statistics 2018).

Well-documented research found that many students use mathematical procedures with little or no understanding of the concepts behind them (Hiebert & Wearne 1992). Both procedural and conceptual knowledge are needed to achieve a sound understanding of mathematics. According to

Hiebert and Lefevre (1986), while procedural knowledge consists of "step-by-step procedures" and "symbolic representations", conceptual knowledge can be "thought of as a connected web of knowledge, a network in which the linking relationships are prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network" (pp.3-4). Both procedural and conceptual knowledge may be explored to different depths. However, students mainly use procedures in courses before Calculus I, and come with little appreciation for concepts to the calculus classroom. Anderrson, Valero and Meaney (2015) examine four contexts for the study of math – task, situation, school organisation and societal context – which move the study and teaching of maths from procedural to conceptual, and they begin a conversation around the use mathematical argumentation in social justice issues for marginalised populations. Their work on identity narratives, engaging maths with emotions and maths students as historical and social beings is applicable to this study.

Research points to the importance of investing thoughtful pedagogy in a calculus course when students take more-formal upper-level courses where the shift from "procedural" to a "conceptual" content can create problems for students moving towards more proof-oriented courses (Cullinane 2011). Thus, we believe that Holistic Learning Theory is important to teaching mathematics to adults.

## Theoretical and methodological framework

This paper employs Yang's Holistic Learning Theory as both its conceptual and theoretical framework, and uses its knowledge dimensions as an analytic tool. Emerging from the fields of adult learning and human resource development, Yang has developed what he defines as an interconnected holistic adult learning theory that intertwines three dimensions of knowledge and knowledge learning: explicit, implicit and emancipatory (transformative) (Yang 2003). Yang builds his theory on Mesirow's (1996) perspective on knowledge acquisition and adult learning, citing three approaches. The first approach is the Western rational tradition that constitutes an objective paradigm of learning. The objectivist paradigm, sometimes called the empirical analytic paradigm, assumes that reality exists independently of mental representations of the world, and that knowledge is objective...(p.59). The second approach is the interpretivist paradigm, which views knowledge as subjective and constructed from one's experience within the frame of prior interpretation; in this paradigm, learning is a function of life and systems of language. The third approach, critical theory, views learning as a transformational process (Yang 2003, p.107).

Drawing from Mesirow, Yang develops Holistic Learning Theory to embrace and connect all three approaches in three dimensions - explicit, implicit and emancipatory. The first dimension is explicit learning. Explicit learning involves acquiring a technical understanding of theories, formulas, principles and models in a particular discipline while learning the jargon, symbols and rules. It is prescriptive learning: highly cognitive, objective and solution-oriented. The second dimension is implicit learning. Implicit learning includes experienced-based, problem-solving, practical and applied approaches to learning. It is applied to real-life situations, tends to be individualist in orientation ( Yang 2003) and in the case of maths, can be found in employment, personal finance and budgeting applications. The third dimension is emancipatory learning. This dimension embodies critical theory (Habermas 1971,1984) and transformative adult learning (Freire 1972). This kind of learning involves examination of power structures, systems, ethics and values, and often has an affective or spiritual component. In orientation, it is most often collectivist, reflecting on and describing issues of social justice and marginalisation. It sometimes works for and imagines a better world, and critically engages in problem-solving that concerns economic equity, racism, sexism and other social issues.

Yang's Holistic Learning Theory further conceptualises these three dimensions – the explicit, implicit and emancipatory – as "a dynamic dialectic among all three facets" (Yang 2003, p.116). For Yang learning occurs when "knowledge is created, acquired, transformed, converted, or utilized in a different context from its origin." (p.117) In other words, learning occurs when one or more of the knowledge dimensions change or alter. This is neither a continuum, with one dimension being superior to another, nor a progression. In fact, according to Holistic Learning Theory, the three dimensions are intertwined, and all are necessary for deep and robust learning. In this study, we maintain that adult educators need to teach from these three perspectives, and we further discuss how this can happen, but first we present a brief description of the study.

## Data collection and analysis

This is an exploratory, comparative case study – an "in-depth description and analysis of a bounded system." (Merriam & Tisdell 2009, p.37). The "bounded system" in this study is one section of Calculus I in an autumn semester. Using a direct content analysis coding approach (Hsiu-Fang & Shannon 2005), the codes (nodes in NVivo) were determined before analysis and refined during analysis. The codes were the actual dimensions – explicit, implicit and emancipatory – and were analysed by two coders (the authors). Students were assigned three writing assignments at different points during the semester (after the first two weeks, half-way through, and at the end of the semester) in which students were asked to answer questions related to the value and relevance of the course content. Approval was obtained from the Institution's Review Board to share anonymous samples of students' work.

This course included engineering and business major students; it is the first course in a three course-sequence for those who wish to achieve in-depth knowledge of calculus. The course met twice for a total of four hours each week during the semester. The homework consisted of regular textbook problems combined with writing-to-learn activities (Jaafar 2016). Some applications were discussed during class time in context, when applicable.

The benefits of writing tasks in mathematics are numerous: they promote critical thinking and problem-solving skills, and they help students develop conceptual understanding and move them away from an apply-the-formula approach (see, for example, Allen 1992; Borasi & Rose 1989; Burton & Morgan 2000; Countryman 1992; Idris 2009; Kenyon 1989; Porter & Masingila 2000). The three assignments are listed in the appendix. Twenty students turned in all three assignments. A research assistant selected a sample of five students from the 20, representing a range of course achievement levels. Those students' three writing samples were analysed using NVivo and manually to compare how students had used the three facets of knowledge.

In assignments 1 and 2, students were asked about the meaning of conceptualising maths, how mathematics is used in everyday life and the definition of calculus. In the last assignment, students were asked to summarise the entire course while explaining the value and the relevance of what they had learned. Emancipatory knowledge is related to motivations for learning, personal goals and objectives and larger issues of ethics and moral implications of applying maths in a globalised world. In the holistic theory, the three facets of knowledge interact with each other. The knowledge base represents explicit knowledge such as theorems, but implicit knowledge is needed for students to generalise and transfer learning to other situations. To adapt these facets to our assignments, we used the guidelines listed in Table 1.

**Table 1. Coding of students' responses**

| Explicit | Implicit | Emancipatory |
|---|---|---|
| • Students provided two basic examples such as money and finance.<br>• Students understood that maths (specifically, calculus) is about understanding and solving problems.<br>• Students may have provided meaningless facts or disconnected bites of information.<br>• The provided information was technical.<br>• "Theories, models, and formulas in textbooks" fit this category (Yang 2003). | • Students provided in-depth examples from fields beyond money and finance.<br>• Students could connect maths to other disciplines.<br>• Students could explain how maths may simplify life and how it can help with the "why" of things.<br>• Students understood the purpose of theorems.<br>• Students could explain and illustrate a concept (practical).<br>• Students provided context-specific information (Yang 2003)<br>• Students could transfer information from one situation to another. | • Students could address "Why we do calculus": their responses included vision, world connection, aspiration, ideals (could be ethics and moral standards).<br>• Students could use everyday language to relate concepts to everyday life.<br>• Students could discuss whether calculus was important to their identity.<br>• Students linked their responses to social justice.<br>• Students seemed to analyse how maths and society were intertwined in historical context.<br>• Students made meaning of their learning and detailed the value of calculus. |

## Reliability and validity

This study employed two types of data-collection and analysis triangulation methods: data triangulation and investigator triangulation (Denzin 1978). Data triangulation occurred with the comparison of five calculus students' case studies with three reflective written responses over the span of a college semester. Investigator triangulation employed two researchers (the authors) coding the same data separately using a direct content analysis approach. One researcher used manual coding; the other used NVivo qualitative software. Once the coding results were completed, the two researchers compared, discussed and re-examined the data during the analysis process.

Determining intercoder reliability and agreement was important. Intercoder reliability, as defined by Campbell, Quincy, Osserman and Pedersen (2013, p.297), "requires that two or more equally capable coders operating in isolation from each other select the same code for the same unit of text". In contrast, intercoder agreement "requires that two or more coders are able to reconcile through discussion whatever coding discrepancies they may have for the same unit of text" (Campbell, et al. p.297.) Generally speaking, the issue of intercoder reliability was difficult to establish, as the two coders, although analysing precisely the same used different data-coding approaches. While consistently using the three pre-determined codes, the first researcher coded each of the 15 (three for each student participant) samples with one overarching categorisation or code, while the second coded discrete sentences and paragraphs within each sample. For intercoder agreement, with one exception, both coders agreed on an overarching code for each survey sample. In this case, the researchers then reconciled their coding discrepancy through discussion. After coding the samples, one of the researchers (who taught the class) reread the remaining assignments, which had not been included in the coding, to make sure that the chosen sample provided sufficient evidence of students' learning in terms of Holistic Learning Theory.

## Data analysis

This section gives the results of content-analysis coding that the two researchers conducted independently for each of the five students. Table 1 shows how the scale was adapted for the mathematics class. The assignments are listed in the appendix. The students' scores are listed in the tables, followed by an explanation.

### *Student 1*

The first researcher categorised all three assignments as predominantly implicit. Using NVivo, the second researcher obtained the following for the three assignments respectively: 1) explicit 19.17%, implicit 60.47%; 2) explicit 15%, implicit 42.18%, emancipatory 13.08%; 3) explicit 48.43%, implicit 44.67% (percentages represent content text that was coded in each dimension; text that did not code for any of the three dimensions is not reflected). This excerpt from assignment to is an example of content reflecting the emancipatory dimension:

> Today's technology is the result of math. We are using internet, computers, different kinds of software's, transportation and etc. these are given by math. In my eyes, I want to say today's world we got because of revolution of mathematics.

In assignment 1, the student provided examples from insurance, population change, velocity, analysis and prediction, and oil prices. The student did not relate learning mathematics to learning languages. In assignment 3, the student provided historical context for the material learned, illustrated concepts with examples and showed a good understanding of the importance of conceptualising mathematics. The student never related mathematics to identity.

### *Student 2*

The second student's three assignments were categorised as predominantly explicit, implicit and implicit, respectively. The percentages of each aspect for each assignment, determined using NVivo, were 1) explicit 75%, emancipatory 0.38%; 2) explicit 39.72%, implicit 48.73%; 3) explicit 51.38%, implicit 18.09%. This student related calculus to physics and other fields without giving specific examples on the first assignment; thus the response was judged to be predominantly explicit. The only emancipatory dimension was detected when the student used "carbon footprint" as an illustration. Implicit knowledge was demonstrated in this excerpt: "[C]alculus can be applied in medicine. I want to become a veterinarian, so I could possibly apply calculus to my job.... In biology, I could possibly find formulas to calculate growth rates and things of that matter."

In the second assignment, a major shift occurred: the student showed how the subject was related to money, everyday life, the sciences, its use in making models and predicting future values and the student's own major. Thus, the student showed both implicit and explicit knowledge throughout the essay.

The third assignment ranged between explicit and implicit, with the explicit predominant; this student focused on the technical and mechanical aspects of calculus as opposed to the applied relevance of calculus to life. There were some minor references to emancipatory knowledge.

### *Student 3*

The third student's assignments were categorised as explicit, implicit and implicit, respectively. Using NVivo, the researcher obtained the following percentages for the assignments: 1) explicit: 42.16%, implicit 62.23%, emerging emancipatory 2.29% (overlap is present as some text reflects both implicit and explicit elements); 2) explicit 12.38%, implicit 62.47%; 3) explicit 27.70%, implicit 64.38%. The emerging emancipatory dimension is reflected in the student's statement: "Absolute abstraction of mathematics makes us more intelligent, so that we have the ability to think deeply." The student provided a shallow contextualisation, as demonstrated by this statement:

> Descriptive statistics is a method of summarizing, sorting, and reflecting the distribution characteristics in a visual image form by statistical methods such as grouping, tabulation, and drawing by means of investigation and experimentation. Statistics, polyline statistics, fan-shaped statistical charts are all descriptive statistics. Probability statistics are derived from a large number of experimental results, such as tossing coins and dice.

This is in contrast with the second assignment, where the student was able to contextualise and conceptualise calculus. This is demonstrated in the statement:

> Calculus in the engineering field, calculus initially developed for better navigation system. Engineers use calculus for building skyscrapers, bridges. In robotics, calculus is used how robotic parts will work on given command. Electrical and Computer engineers use calculus for system design. Calculus is used to improve safety of vehicles. Calculus in Biology.... Calculus in Science.... Calculus in Economics.... Calculus in other fields, business and politicians often conduct surveys with the help of calculus. Investment plans do not pass before mathematicians approves. Doctors often use calculus in the estimation of the progression of the illness. Global mapping is done with the help of calculus. Calculus also used to solve paradoxes.

In the last assignment, the student continued to use a predominantly implicit approach while incorporating some explicit facets. The student incorporated historical facts and articulated the difference between getting the "results" of a calculation and acquiring problem-solving skills that are relevant beyond calculus. Finally, the student used a more explicit facet in the third assignment, integrating some history, relating calculus to several science fields and recognising the need to use it as "more of a problem-solving tool".

### *Student 4*

The first researcher categorised the three assignments as predominantly 1) explicit, 2) a combination of explicit and implicit and 3) explicit, respectively. The results from NVivo were 1) explicit 37,14%, implicit 51.68%, emerging emancipatory 12.80%; 2) explicit 45.64%, implicit 56.20%; 3) explicit 52.40%, implicit 34.12%, emerging emancipatory 8.27%. This student predominantly demonstrated the implicit facet of knowledge and occasionally used emancipatory language: "...calculus by giving engineers and you the ability to model and control systems gives them extraordinary power over the material world". The student failed to elaborate on half the questions in the assignments, but related mathematics to everyday life and recognised that “students are more actively engaged in learning math and group problem solving”.

In the second assignment, the coders agreed about the facets of knowledge used, and the student was able to use both facets almost equally. Finally, the last assignment had predominantly explicit language accompanied, to a lesser degree, by both implicit and emancipatory, as demonstrated by this statement: “We use calculus to solve mathematical problems that cannot be solved by other means, and that in turn allows us to make guesses about the behavior of real-world systems that we could not differently make.”

### *Student 5*

The fifth student’s assignments were categorised as some explicit, explicit and implicit, respectively. The NVivo results categorised the assignments as1) explicit 16.72%, implicit 12.22%; 2) explicit 30.25%, implicit 46.37%, emerging emancipatory 5.83%; 3) explicit 55.18%, implicit 10.01%.

The fifth student was very brief in the first assignment and used limited explicit and implicit language. In the second assignment, the student provided detailed examples of how calculus is used in several fields, showing an ability to conceptualise the subject. The emerging emancipatory dimension was detected in the statement: “To make a time to manage or set things in our life we all should think mathematically. A long time’s plan can change our life.” Finally, in the third assignment the student reverted to the explicit facet: the student showed an understanding of the main ideas in the course but did not give examples of applications.

## Findings and implications: how can adult educators teach holistically?

The major finding was that students used both explicit and implicit dimensions of knowledge in their understanding of calculus concurrently, but seldom demonstrated emancipatory or transformational learning, regardless of their academic achievement level. Four out of the five students used the implicit facet more frequently in the second and third assignments. The results show that students are not able to frame calculus or their prior learning of mathematics using an emancipatory form of knowledge.

### *Teaching implicitly and explicitly*

Students have a natural tendency to use instrumental understanding in mathematics (Idris 2009): they apply a recipe to a given problem, without asking why the formula applies in that context. Most of the rules learned from instrumental understanding are short-lived. Therefore, it is essential to move students away from instrumental understanding towards a conceptual understanding. The use of explicit knowledge alone is not sufficient to achieve a “complete picture” (Yang 2003). Implicit knowledge is “context-specific”, emerging from experience only when coupled with learning and familiarity. This echoes with how mathematics professors may use contextualisation of calculus with examples to help students achieve a better conceptual understanding (El Gaidi & Ekholm 2015).

The professor has designed the course materials to direct students towards conceptual understanding as opposed to a mere procedural ability – in other words, implicit as opposed to explicit understanding. Some examples of those questions have been previously published (Jaafar 2016). For instance, when studying the relationship between velocity and acceleration, students typically learn that acceleration at time t is the instantaneous rate of change of the velocity. Students are asked in class whether it is possible for an object to have zero velocity, but a non-zero acceleration. The instructor refrained from giving any answer but encouraged students to discuss in pairs whether such a situation is plausible. One student finally explained to his/her peers that the scenario is possible with an object thrown upward.

Other classroom discussions required students to explain the difference between acceleration and speeding up. Students had to imagine situations and explain them in writing. When studying integrals, students encountered the challenge of differentiating between distance and displacement. They were given an example worked out by a student in a previous semester where the student obtained zero for distance travelled. Then students had to pinpoint the conceptual misunderstanding and explain the difference between distance and displacement. In the same chapter, students also discussed the concept of integrals as accumulation, using it to calculate the amount by which certain physical quantities increase (such as temperature) over a certain time interval. Students were then encouraged to find an example using the same method but a different context. Students were able to see how this method could be generalised to calculate data transmission during certain time periods, and to calculate the increase in costs and population change or growth.

When studying applications of derivatives and how to find extreme values, students discussed how the shape of a can affects the cost of its manufacturing. Students then computed the values for the height and the radius of the can that minimised the cost. Then they discussed other factors that can produce different results, including the shape of human hands, and the shape of the food inside the can. Discussions of that kind could push students towards implicit forms of knowledge.

This is a cultural breakdown from students' earlier habits of memorising and applying formulas. This raises an important question: since value beliefs are known to be associated with mathematics achievement (Trautwein et al. 2012), the authors strongly feel that a conceptual approach to calculus helps students develop a more pronounced implicit facet of knowledge, with the hope that the emancipatory aspect emerges at some point. This point is further validated by a study involving 303 calculus students taught by eight instructors using either a procedure-based or a concept-based instructional environment. The results showed that students taught using concept-based learning achieved better results than those taught using a procedure-based learning environment (Chappell & Killpatrick 2003). Thus, the concurrent use by students in this study of both explicit and implicit dimensions is encouraging. However, the study suggests that conceptual understanding cannot be assumed to trigger the transformative or emancipatory form of knowledge.

As a start, calculus instructors at a community college who are teaching predominantly adult learners who started from remedial math should focus on transforming their traditional teaching of few key concepts. This is a start to help students relate their knowledge of calculus to the outside world, and to make connections between their knowledge of the outside world and the mathematical knowledge achieved in class. This is particularly important for adult learners: as Jameson and Fusco (2014) argue, "Mastery experiences in the classroom should be related to other fields of study…." This can also provide evidence of students' learning because "learning occurs as learners relate concepts descriptive of the new knowledge to previous knowledge within their cognitive structure" (Yang 2003). For example, instantaneous velocity is often explained as the slope of the tangent in a position-versus-time diagram. Students should instead be asked before class to bring values for their car speed and whether they drove over the speed limit (if not, they can imagine a scenario). In class, working in pairs, students can be asked what they would do if they spotted a police car, using those numbers to make a decision. From this point, the instructor could transition to the idea of average velocity. The instructor could them modify the scenario to include a police car that records the car's velocity at a discrete time interval (e,g. five or 10 seconds). They could then transition into the idea of instantaneous velocity and how it relates to the limit.

Another example that lends itself to the use of both facets of knowledge is the Mean Value Theorem, which is usually introduced as (Stewart, 2014, p. 288):

> Let f be a function that satisfies the following hypotheses:
> 1. $f$ is continuous on the closed interval $[a, b]$.
> 2. $f$ is continuous on the open interval $(a, b)$.
>
> Then there is a number $c$ in $(a, b)$ such that
>
> $$f'(c) = \frac{f(b) - f(a)}{b - a}$$
>
> Or, equivalently $f(b) - f(a) = f'(c)(b - a)$

Typically, the Mean Value Theorem is introduced, and students are later expected to use it in problems involving functions. Instead, the instructor could start by asking students to think about a distance they traveled in some particular time (e.g. 100 miles in two hours), then they could brainstorm possible values of the speedometer at different points in time. It is quite possible that a student would give 50 miles/hour. Students could then deduce that there is a number at which the instantaneous velocity is equal to the average velocity. This could be further reduced to the fact that the derivative gives information about the function itself. Finally, the formal definition could be introduced.

These examples illustrate a starting point, but instructors could also survey their students and determine their fields of study (for example, whether it is engineering or business majors) to design examples that could help their students transform their knowledge on maths.

### *Teaching to emancipation or transformation*

As mentioned previously, Andersson, Valero and Meaney's (2015) work begins to examine mathematical argumentation and the use of mathematical thinking in reflection and problem-solving related to social justice for marginalised groups and in a variety of societal and historical contexts (2015). It considers the engagement of emotion, identity and agency in larger contexts toward transformational learning. This kind of emancipatory or transformational learning harks back to Paulo Freire and his literacy work in Brazil (1972). This pilot study begins to explore what teaching regarding social-justice issues might encompass and how. For example, when students study differential equations in other calculus courses, they often encounter problems involving a first-order differential equation applied to an abstract problem about population growth. Such problems can be used in an emancipatory way to study population growth among inmates convicted of different crime categories. The data is publicly available ("Mass incarceration in America explained in 22 charts and maps" 2016), and those examples can be used to model and calculate rate of change in population (from 1994-2013) among those convicted of violent crimes, weapons-related crimes, immigration-related crimes and minor drug offenses at the federal level, for example. Students may also access the cost of mass incarceration and compare it with the cost of educating inmates (Education vs prison costs 2018).

Other problems may be open-ended, such as the rate at which the population of Germany has been changing. This could lead to questions such as how many refugees Germany should take to solve the problems associated with an aging population (see, for example, DeSilver 2015). Immigration, climate change, inequality of education opportunity, income disparity, the cost of war – all have mathematical components that can engage students in conceptual processes as well as be tools for emancipatory maths engagement, which has the potential to engage the emotions, identity and intellect toward agency and action in the world. This study just begins to broach this kind of maths learning and instruction.

## Implications for administrators

In general, administrators at community colleges realise the need to support students in upper-level courses, given students' prior maths trajectory. The help of a teaching assistant or a resource centre to provide tutoring is essential. In the class taught by one of the authors, a peer tutor helped students outside of class. If this idea is expanded, problem-based instructions may be introduced in some topics, especially those related to social justice. Several textbooks provide such examples, which can be used without increasing faculty workload or altering the course syllabus (see, for example, the "How to Build a Better Roller Coaster" project for use when studying maxima and minima, Stewart 2014, p. 182).

Transformative learning is sometimes an individual endeavor, but educators and administrators can alter mathematics instruction and provide support services for adults to encourage critical and collective transformative or emancipatory learning (Johnson-Bailey 2012). For example, they can jointly brainstorm ideas on how maths instruction can teach in the service of social-justice goals and taught with emphasis upon an emancipatory agenda that supports long-emphasised aims of adult education. Learning communities at community colleges can build on this integration of maths with criminal-justice courses, human services or other subjects in a capstone setting where students can integrate their learning from different disciplines to help them achieve an emancipatory facet of knowledge. This may necessitate the design of professional-development seminars to provide support to ensure the initiative's success.

## Conclusion and future directions

This pilot study raises questions about the language of and approaches to mathematics instruction when teaching adults in relation to the conscious emphasis adult educators can place on various dimensions of mathematical knowledge. Mathematics instruction traditionally emphasises the explicit nature of the discipline; this approach does not consider the diverse background of students in the community-college classroom, and might therefore contribute to greater achievement gaps (Nelson 1996). Therefore, an approach is needed that tackles application, relevance and the value of maths to jobs, real-world activities, finance and financial literacy – the implicit dimension, as examined in this study. But beyond the need to teach to implicit learning, this pilot study raises the following question: What would a calculus or other adult-education mathematics class look like if emancipatory language and approaches were incorporated more intentionally? For example, how would calculus be conceptualised if taught within the context of global income inequality, carbon footprint, mass incarceration in America, global poverty, women in STEM, economics and immigration? How might mathematics instruction begin to look if the dimensions of personal and collective explicit, implicit and emancipatory dimensions were incorporated?

Future studies examining calculus instruction and holistic adult learning theory are planned beyond this pilot study, but early indications are that instructors' intentional teaching that incorporates the knowledge dimensions in course design, assignments and face-to-face instruction plays a role in whether or not all three knowledge dimensions – explicit, implicit and emancipatory – are operating in an interconnected way.

## Appendix

### *Written Assignment 1*

1. Of what value is calculus for your life right now (not for a career, job, GPA, etc.)
2. What does it mean to you to conceptualize mathematics?
3. What is a mathematics concept?
4. For those of you who speak a second or third (fourth) language, is learning calculus anything like that?
5. How do you see yourself applying calculus to your life?
6. How is calculus meaningful to you?

### *Written Assignment 2*

1. Give concrete, specific examples of how math is used in your life.
2. Define what calculus is about.
3. How does the teaching of mathematics in your country of origin differ from what you are experiencing in your classrooms in America?
4. Do you see the world mathematically? If so, how? Be specific.

### *Written Assignment 3*

The goal of this assignment is to write a two-page summary of your calculus course, explaining the historical development of the course. You are also expected to spell out the value of the course and its relevance academically and to everyday life, as well as advice on study habits.

As the semester comes to an end, you have a good sense of what calculus is about. Imagine you meet John, who is currently taking a pre-calculus class. John is excited about taking calculus and wants to know what calculus is about. You want to impress John by explaining to him in brief the two branches of calculus and the relationship between them.

Write a two-page essay (1,000 to 2,000 words), TYPED, summarising each branch of calculus, and explaining the relationship between the two branches. You must also give a historical background of the subject and cite the fight between Leibniz and Newton.

In your letter, you must also explain to John the value of calculus to his life and how he might apply it in the future in an academic or non-academic context. How is it relevant to his everyday life? How would you instruct him to prepare, make a study plan and approach the concepts in calculus? (Illustrate by explaining a concept/theorem that is observed in everyday life and that you have seen in the course.)

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