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## Rational design of p-type thermoelectric PbTe: Temperature dependent sodium solubility

### Abstract

We develop a solid understanding of the temperature-dependent solubility of sodium p-type PbTe, the most efficient thermoelectric material. The maximum solubility of sodium telluride (NaTe) in PbTe is measured to be 1.4±0.3 at percent and the heat of solution is evaluated and that addresses fundamental issues regarding the formation of nano-precipitates.

### Keywords

thermoelectric, pbte, temperature, dependent, sodium, rational, solubility, design, p, type

### Disciplines

Engineering | Physical Sciences and Mathematics

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We develop a solid understanding of the temperature-dependent solubility of sodium in p-type PbTe, the most efficient thermoelectric material. The maximum solubility of sodium telluride (NaTe) in PbTe is measured to be  $1.4 \pm 0.3$  at% and the heat of solution is evaluated as  $26^{+19}_{-11}$  kJ mol<sup>-1</sup> that addresses fundamental issues regarding the formation of nano-precipitates.

Thermoelectric energy converters are expected to play a significant role in clean and renewable energy, and to reduce the reliance on fossil fuelled heat engines that generate approximately 90% of the world's electrical power at a typical 30–40% efficiency. The maximum efficiency of thermoelectric materials is determined by the dimensionless figure of merit,  $zT = S^2 T \sigma / (\kappa_e + \kappa_L)$ , which is a function of lattice thermal conductivity ( $\kappa_L$ ), electronic thermal conductivity ( $\kappa_e$ ), the Seebeck coefficient ( $S$ ), electrical conductivity ( $\sigma$ ), and absolute temperature.

Although a feasible thermoelectric generator for primary power generation requires thermoelectric materials with  $zT$  above 1.5 at 800 °C operation temperature,<sup>1</sup> the small size, rapid response and robustness of thermoelectric generators have made them rival candidates for several successful applications.<sup>2</sup> Recently, a large number of studies<sup>3–7</sup> have focused on the so-called mid-range temperature (600–900 K) thermoelectric materials, specifically Pb chalcogenides, where the majority of broad-based applications could benefit. PbTe is the premiere mid-range temperature thermoelectric material. Latest studies<sup>3,4,7</sup> have reported significant improvement of its thermoelectric performance through tuning of the electronic structure and/or the nanostructuring of bulk materials. Intrinsic semiconducting lead telluride can be tuned either to the n-type or p-type as it can contain as many as approximately

$1.5 \times 10^{18}$  electrons or  $7.7 \times 10^{18}$  holes per cm<sup>3</sup> due to excess lead or tellurium atoms respectively.<sup>8</sup> These point defects act as donors ( $[V_{Te}^{\prime\prime}]$ ) or acceptors ( $[V_{Pb}^{\bullet\bullet}]$ ). However, precise control of hole concentration in p-type PbTe (ranging from  $\sim 10^{18}$  to  $\sim 10^{20}$  cm<sup>-3</sup>) has been achieved through substitution of monovalent atoms of thallium,<sup>7</sup> sodium<sup>9,10</sup> or potassium<sup>11,12</sup> on the lead sublattice. Alkali metals (Li, Na, K, Rb, Cs), Ag and Tl are likely candidates for p-type doping of PbTe. Nevertheless, atoms with an ionic radius larger than that of K cannot be efficiently doped due to steric interaction, and Tl alters the band structure with resonant states.<sup>7</sup> Interstitial Ag (ref. 13) can also act as an n-type dopant and Ag ionic conductivity in PbTe shows mobility.<sup>14</sup>

Although sodium has been the most viable dopant for PbTe,<sup>9,15,16</sup> its solubility level at various temperatures has not been studied systematically and little attention has been given to the relevant phase diagrams. The band structure of PbTe is typically considered rigid and it is believed that electronic transport properties of N-doped PbTe are affected by heavy effective mass holes, resulting from a high density of states (DOS) near the Fermi level.<sup>17</sup>

The only available pseudo-binary phase diagrams of PbTe–NaTe and PbTe–Na<sub>2</sub>Te (ref. 18) do not determine the solubility of sodium in PbTe. It is generally believed that the Na solubility in PbTe is above 0.5% up to 1.75% although some loss of Na may occur during processing. The maximum carrier concentration, according to Hall effect measurements at 77 K, that is observed in Na-doped PbTe is  $2.5 \times 10^{20}$  cm<sup>-3</sup> and it is believed that each Na atom donates about 1 hole when Na concentration is higher than  $\sim 6 \times 10^{19}$  cm<sup>-3</sup>, which corresponds to approximately  $x = 0.017$  in Na<sub>x</sub>Pb<sub>(1-x)</sub>Te.<sup>10</sup> In recent microscopic studies<sup>14,19</sup> on sodium doped (more than 1 at%) PbTe samples, nano-scale precipitates have been observed which are suggested to be sodium-rich lead telluride (Na<sub>x</sub>Pb<sub>(1-x)</sub>Te) precipitates<sup>14</sup> distributed in a slightly doped ( $\sim 0.5\%$ ) lead telluride matrix. However, the temperature history of the sample may play a significant role in the solubility of sodium as dopant solubility changes with temperature, as has been found in several PbTe pseudo-binary systems.<sup>20,21</sup>

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In this report, the lead telluride rich side of the PbTe–NaTe phase diagram is revisited to understand the sodium solubility in samples with various heat treatment histories. We measured the solubility of sodium in PbTe by Wavelength Dispersive Spectroscopy (WDS) in an Electron Probe Micro-Analyser (EPMA) at various annealing temperatures. Furthermore, the maximum solubility of sodium in PbTe has been determined. The heat of solution of precipitates in the solid PbTe matrix is calculated and the effect of sodium addition on the nature and formation of nano-precipitates and the microstructure of polycrystalline  $\text{Na}_x\text{Pb}_{(1-x)}\text{Te}$  are discussed.

Polycrystalline  $\text{Na}_x\text{Pb}_{(1-x)}\text{Te}$  ( $x = 0.02, 0.05$ ) samples were synthesized by mixing appropriate amounts of high purity Pb (99.9999%), Te (99.9999%) and Na (99%), with a total mass of 10 g, sealed in carbon-coated quartz tubes under vacuum and heated to 1050 °C at  $\sim 100$  °C  $\text{h}^{-1}$ . After holding at 1050 °C for about 6 hours, the ampoules were cold-water quenched and either annealed at 507 °C and 300 °C for a week or at 500 °C for 48 hours and then quenched in salt water ( $\sim -15$  °C). The last ingot was ground into a powder and hot pressed in a 12 mm diameter graphite mould under an axial pressure of 40 MPa and argon atmosphere at 500 °C for 1 h. The pellet with a relative density of 98% or higher was used for simultaneous measurements of the resistivity and the Hall coefficient ( $R_{\text{H}}$ ) using the van der Pauw technique under a reversible magnetic field of 2 T.<sup>22</sup>

Two hot-pressed  $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$  samples were sealed in evacuated quartz tubes and heated to 800 °C for 8 hours and then annealed at that temperature for 6 hours. Then they were cooled to 350 °C and 240 °C by slow cooling at the rate of 10 °C  $\text{h}^{-1}$  to avoid the formation of a fine secondary phase due to the high degree of non-equilibrium and then annealed at that temperature for two days and three weeks respectively followed by quenching in iced salt water to provide the fastest quenching rate and avoid inner-grain precipitation.

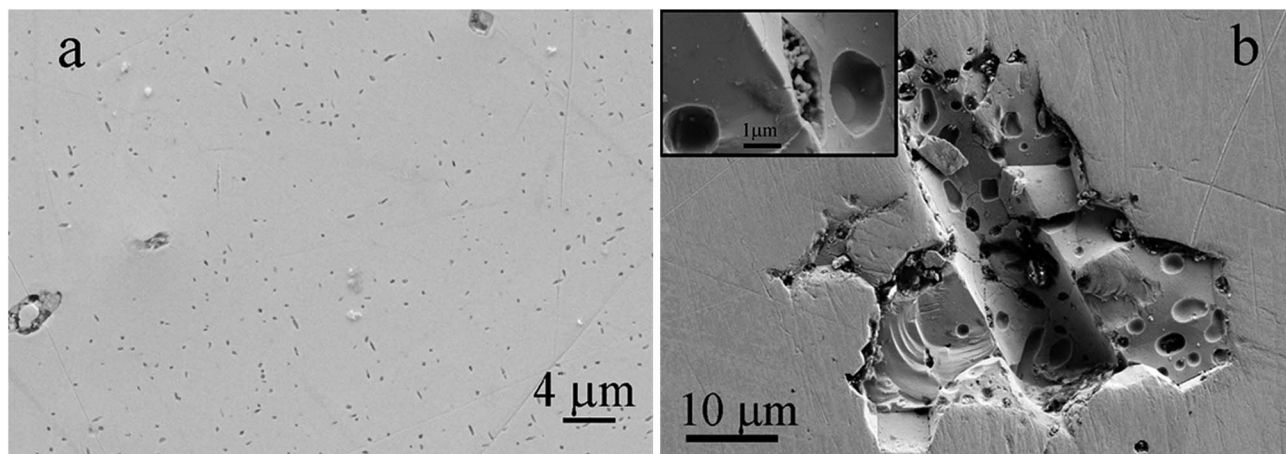
The samples were examined by field emission gun-scanning electron microscopy (SEM), using a Carl Zeiss LEO 1550 VP equipped with an Energy Dispersive X-ray Spectrometer (EDS). Samples were subjected to chemical composition analysis by wavelength-dispersive X-ray spectrometry (WDS) using an electron probe X-ray MicroAnalyser (JXA-JEOL 8200). In the WDS measurements, the intensities of Te  $L_{\alpha}$ , Pb  $M_{\alpha}$ , and Na  $K_{\alpha}$  were measured and corrected by the ZAF method to obtain chemical compositions. PbS, Te and Albite ( $\text{NaAlSi}_3\text{O}_8$ ) standards were used for the ZAF corrections of Te, Pb and Na respectively. The EPMA was operated under a 15 kV accelerating voltage and 25 nA probe current. For regions containing precipitates, the smallest probe diameter was set to measure the matrix compositions.

The optimum p-type carrier concentration in PbTe-based alloys has been achieved at  $\sim 10^{20}$   $\text{cm}^{-3}$  with the addition of approximately 1 at% Na ( $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$ ).<sup>9,15,23</sup> Therefore, in the current study a sample with a nominal composition of  $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$  (1 at% Na) was prepared by quenching the equilibrated liquid at 1050 °C and annealing at 300 °C for 5 days followed by quenching in cold water. The scanning electron microscopy (SEM) image of the polished annealed sample in Fig. 1(a) clearly illustrates the presence of precipitates (up to

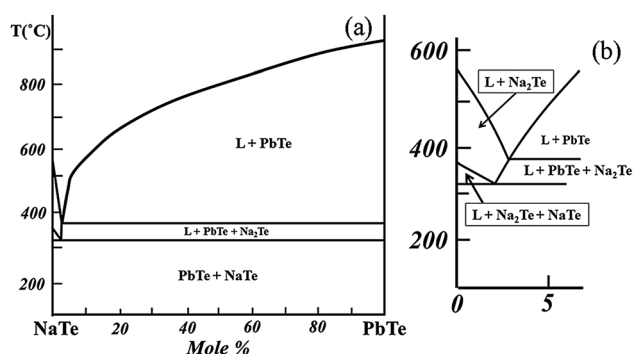
500 nm). Energy Dispersive Spectroscopy (EDS) analysis of precipitates indicates the sodium rich phase. However, the accurate chemical composition of the precipitates cannot be determined due to the large interaction volume of the electron beam with the matrix compared to the precipitate size and the possible oxidation of precipitates whilst it undoubtedly shows that the 1 at% Na is not completely dissolved in the PbTe matrix at 300 °C. Another sample with the same composition ( $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$ ) was annealed at 500 °C for two days, and then ground and hot pressed at the same temperature for an hour. While precipitates are not found in the matrix of the 500 °C annealed sample, Fig. 1(b) shows features at the junction between particles (grain boundaries), where the sintering occurred. The round features on the surface that contain reaction products (Fig. 1(b) inset) suggest the presence of a phase transformation that involves a liquid phase. EDS analysis of these features indicates the presence of sodium and proves the diffusion of sodium to grain boundaries and the formation of a liquid phase in equilibrium with lead telluride at the sintering temperature as suggested by the binary phase diagram of PbTe–NaTe. It is evident that the 1 at% Na is not completely dissolved at 500 °C.

The Hall coefficient ( $R_{\text{H}}$ ) of the sintered sample was measured using the van der Pauw technique whereas the expected carrier concentration for complete solubility of 1 at% Na in PbTe is equal to  $3.0 \times 10^{20}$  based on the assumption that every lead atom which is replaced by sodium donates one hole to the final compound. It should be noted that the Hall concentration of electrical carriers ( $p$ ) is determined from the Hall coefficient ( $R_{\text{H}}$ ) assuming  $p = 1/R_{\text{H}}$ . It is generally understood that lead telluride has two valence bands (a light valence band at  $L$  points and a heavy band at  $\Sigma$  points) which converge at approximately 430 K in highly doped samples<sup>3,24,25</sup> resulting in the maximum Hall coefficient value at that temperature. Therefore, the room temperature ( $\sim 300$  K) carrier concentration is smaller than the actual values and careful measurement of the carrier concentration for p-type PbTe should be performed at temperatures less than 100 K. The carrier concentration value in this study is reported only for the purpose of comparison to previous studies<sup>9,10,14</sup> on Na-doped PbTe.

In order to describe sodium doping in PbTe and gain insight into the nature and composition of the observed precipitates, it is required to know the ternary Na–Pb–Te system, specifically the phase equilibria of the PbTe–NaTe system as shown in Fig. 2(a) (adopted from ref. 18). The phase diagram shows no solubility of NaTe in PbTe although there have been some studies<sup>10,19,26</sup> showing solubility. Differential scanning calorimetry (DSC) and X-ray diffraction analysis techniques were employed by Finogenova *et al.*<sup>18</sup> to investigate the isoplethal PbTe–NaTe section, although these methods are not ideal for measuring solid solubility. Nevertheless, the complicated liquidus of the NaTe rich side of the phase diagram was fully investigated (Fig. 2(b)) and shows a eutectic transformation ( $L = L' + \text{Na}_2\text{Te} + \text{PbTe}$ ) at 370 °C followed by a peritectic isotherm ( $L_3 + \text{Na}_2\text{Te} = \text{NaTe}$ ) at 320 °C. It is anticipated that the maximum solubility of NaTe in PbTe occurs near these transformation temperatures (temperature range of 320–370 °C).



**Fig. 1** SEM micrographs of (a) cast  $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$  alloy which was quenched from the equilibrated melt and annealed at  $300\text{ }^\circ\text{C}$  for a week, (b) sintered  $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$  alloy which was quenched from the equilibrated melt, annealed and sintered at  $500\text{ }^\circ\text{C}$ . The round features on the surface and reaction products suggest the presence of a liquid phase at the sintering temperature.



**Fig. 2** (a) Temperature–composition projection of PbTe–NaTe cross-section of the Na–Pb–Te system, showing no solubility of NaTe in PbTe (adapted from (ref. 18)). (b) The NaTe rich side of the phase diagram is shown in more detail.

In the following section, sodium diffusion in PbTe will be reviewed to elucidate the mechanisms observed in this study. The diffusion coefficient of sodium at  $\sim 10^{18}$  to  $10^{19}\text{ cm}^{-3}$  carrier concentration in a single crystal p-type PbTe follows eqn (1):<sup>27</sup>

$$D = 0.17\exp(-1.91/kT) \quad (1)$$

where  $D$  is the diffusion coefficient ( $\text{cm}^2\text{ s}^{-1}$ ),  $k$  is Boltzmann's constant (in  $\text{eV K}^{-1}$ ) and  $T$  is the absolute temperature (K). The diffusion coefficient is approximately 20 times faster at a higher carrier concentration (in a single crystal). Moreover, diffusion in polycrystalline samples, which is influenced by diffusion along grain boundaries ( $D_b$ ) in addition to volume diffusion ( $D_v$ ) is equal to:<sup>25</sup>

$$D_{\text{eff}} = \frac{D_v n_v + D_b n_b}{n_v + n_b} \quad (2)$$

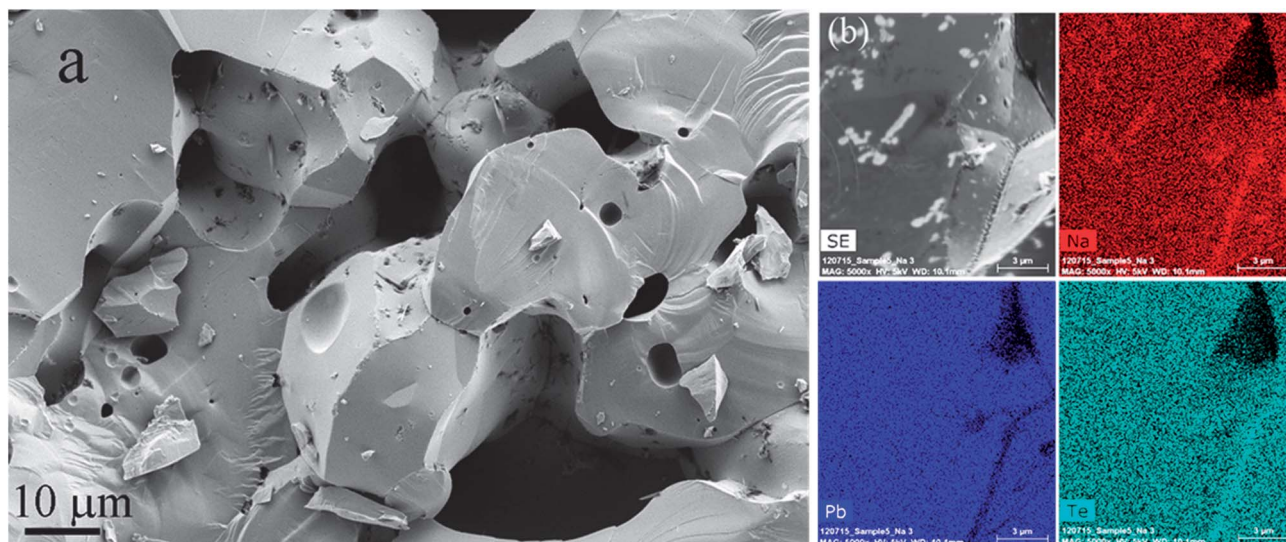
where  $n_v$  and  $n_b$  are the concentrations of diffusing impurities on grain boundaries and in the interior of the crystal respectively. It is generally agreed that at temperatures  $T > (0.8\text{--}0.9)T_m$ ,  $D_v > D_b$  while at temperatures  $T < (0.4\text{--}0.5)T_m$ ,  $D_v < D_b$ ,<sup>25</sup> where  $T_m$  is the melting temperature of the matrix. Therefore, at high

temperatures (over  $0.8 T_m$ ), diffusion within grains is dominant which results in a large diffusion distance and accelerates the phase separation reflecting the equilibrium phase boundary composition and giving a coarse microstructure. Consequently, we chose a temperature of  $800\text{ }^\circ\text{C}$  ( $\sim 0.86 T_m$ ) for the annealing of the sample to promote the diffusion within grains which could result in a coarse microstructure.

The fractured surface of the sample after heat treatment was studied by SEM and EDS analysis prior to chemical composition analysis by WDS as shown in Fig. 3. It is evident that sodium-rich particles which are expected to be the sodium telluride phase form on grain boundaries, as shown in Fig. 3(b) where the X-ray map illustrates sodium and tellurium at a grain boundary. This means that the NaTe concentration in  $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$  surpasses the solubility of PbTe at  $350\text{ }^\circ\text{C}$ . Sodium telluride compounds are air-sensitive and oxidize initially to polytellurides, and finally to metallic Te and sodium oxide residue.

The samples annealed at  $240$ ,  $350$  and  $507\text{ }^\circ\text{C}$  were subjected to chemical composition analysis by WDS using an EPMA in order to carefully measure the sodium content in the matrix. The EPMA was operated under a  $15\text{ kV}$  accelerating voltage. The excitation depth is expected to be around or less than  $1\text{ }\mu\text{m}$ , which is smaller than the spacing between secondary phases, and therefore, the values measured for the lead telluride matrix are precise representatives of the equilibrium phase boundaries at the respective temperatures. The reported values are average values from more than 100 point analysis which was performed close to precipitates within the diffusion distance of sodium in lead telluride.

We have measured the solubility of Na at  $240\text{ }^\circ\text{C}$  to be approximately  $0.24 \pm 0.14\text{ at}\%$ , which is equal to  $0.48\text{ mole percent NaTe}$  in PbTe. Although the measurement error is higher at a low concentration of sodium in the matrix, the sodium shows considerable reduction in its solubility over  $100\text{ K}$  temperature reductions. The measured values at  $240\text{ }^\circ\text{C}$ ,  $350\text{ }^\circ\text{C}$  and  $507\text{ }^\circ\text{C}$  are presented in Fig. 4. Therefore, we suggest that the solubility of NaTe in PbTe at  $350\text{ }^\circ\text{C}$ , which is



**Fig. 3** (a) SEM micrographs of a fracture surface of the heat-treated sintered  $\text{Na}_{0.02}\text{Pb}_{0.98}\text{Te}$  sample which was heated to  $800\text{ }^\circ\text{C}$  in 8 hours, annealed for 6 hours and cooled to  $350\text{ }^\circ\text{C}$  at  $10\text{ }^\circ\text{C h}^{-1}$ , followed by annealing at that temperature for two days and quenching in iced salt water, (b) EDS mapping of an area containing a grain boundary, showing particles rich in sodium and tellurium at the grain boundary.

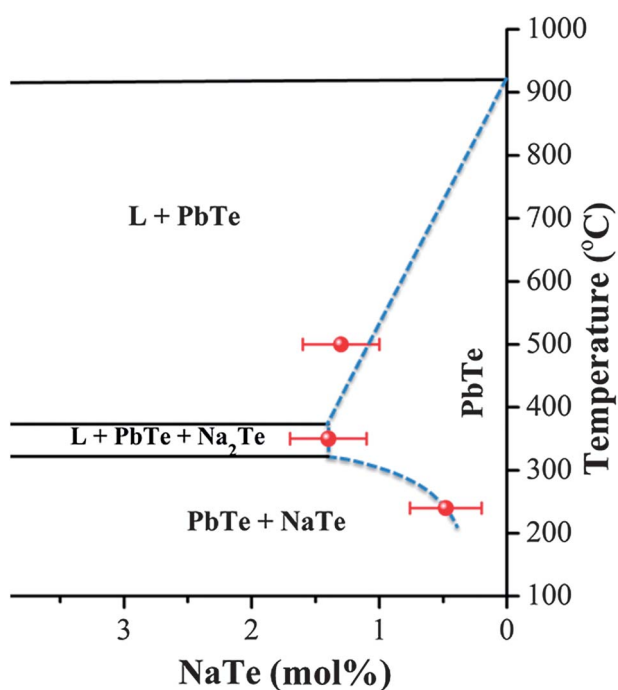
anticipated to be close to the maximum solubility, for  $\text{Na}_x\text{Pb}_{(1-x)}\text{Te}$  is  $x = 0.014 \pm 0.003$ .

Crocker<sup>10</sup> attempted to measure the solubility of sodium in a sliced single crystal of sodium doped PbTe and reported a maximum of  $2.5 \times 10^{20}$  Na atoms per  $\text{cm}^3$  which correspond to  $x = 0.017$  in the  $\text{Na}_x\text{Pb}_{(1-x)}\text{Te}$  alloy. This conclusion was drawn from a graph in which sodium concentration *versus* the Hall carrier concentration at 77 K was plotted. At sodium concentrations above  $2.5 \times 10^{20}$ , the measured Hall carrier

concentrations were scattered and remain almost constant. Therefore, this value was considered as the maximum sodium solubility. However, there are no experimental data available between approximately  $7 \times 10^{19}$  and  $2.5 \times 10^{20}$  per  $\text{cm}^3$  sodium concentration. Moreover, the electrical carrier concentration was determined *via*  $p = 1/(eR_H)$  while the Hall coefficient is related to the carrier concentration through the Hall factor,  $r_H$ , in the relationship  $p = r_H/(eR_H)$  which has strong dependence on the doping level.<sup>31</sup> Although further investigation is required to explore the relationship between the sodium concentration and the carrier concentration, it is evident that this graph cannot be employed to determine the maximum sodium solubility. The recent high resolution transmission electron microscopy (TEM)<sup>14</sup> and atom probe tomography (APT)<sup>19</sup> analyses on similar samples indicate approximately 0.5 at% sodium solubility in the PbTe matrix which is consistent with the current study findings below  $300\text{ }^\circ\text{C}$ .

The maximum solubility of NaTe in PbTe,  $\sim 0.7$  at% Na, is smaller, compared to similar systems of  $\text{PbTe-Sb}_2\text{Te}_3$  (6 at% Sb)<sup>29</sup> or  $\text{PbTe-Ag}_2\text{Te}$  (15 at% Ag).<sup>28</sup> The solubility of sodium in the lead telluride matrix is reduced significantly upon reducing the temperature below  $320\text{ }^\circ\text{C}$ , resulting in precipitation of NaTe when the sample is annealed at low temperatures (as shown in Fig. 1) or with continuous cooling of solid solution samples to room temperature.<sup>14,19</sup> In the following section, the relationship between the thermodynamic properties, solubility and precipitate size will be briefly discussed.

Isolethal sections of the phase diagrams of  $\text{PbTe-NaTe}$ ,  $\text{PbTe-Ag}_2\text{Te}$  and  $\text{PbTe-Sb}_2\text{Te}_3$  reveal markedly low solubility of NaTe, very limited solubility of  $\text{Ag}_2\text{Te}$  and comparatively larger solubility of  $\text{Sb}_2\text{Te}_3$  in the PbTe matrix. Here, we try to explain this fact by reviewing the Hume-Rothery rules<sup>32</sup> that introduce similarity in the atomic radius of the solute to that of the solvent atoms, similar valence and electronegativity and matching crystal structure to identify the degree of dissolution in solid



**Fig. 4** Temperature-composition projection of  $\text{PbTe-NaTe}$  cross-section of the  $\text{Na-Pb-Te}$  system, showing the solubility of NaTe in PbTe.

solution alloys of substitutional elements. The ionic radii of  $\text{Ag}^+$  (0.115 nm) and  $\text{Na}^+$  (0.102 nm) are comparable with the ionic radius of  $\text{Pb}^{2+}$  (0.119 nm), while they are considerably larger than that of  $\text{Sb}^{3+}$  (0.076 nm).<sup>33</sup> Therefore, atomic strain cannot be responsible for the poor solubility of  $\text{Ag}^+$  and  $\text{Na}^+$  in the PbTe matrix. However, the electronegativity of Pb, Te, Sb, Ag and Na is equal to 3.08, 3.59, 3.34, 2.59, and 0.70, respectively, by Sanderson's definition.<sup>34</sup> The difference in electronegativity of the anion, Te, from that of the cations in the matrix phase is more pronounced in the case of Ag and Na than that of Sb, indicating greater attraction of Ag and Na atoms for bonding electrons of telluride in the chemical bonds compared with Pb and Sb. Moreover, NaTe crystallizes in the orthorhombic structure<sup>35</sup> and  $\text{Ag}_2\text{Te}$  in the monoclinic while  $\text{Sb}_2\text{Te}_3$  has a hexagonal structure that is more similar to the FCC structure of the lead telluride matrix.

Nucleation of precipitates in the matrix remains to be assessed. The driving force of precipitate nucleation ( $\Delta G_{\text{chem}}$ ) is proportional to the heat of solution as<sup>36</sup>

$$\Delta G_{\text{chem}} = -\Delta H_s \frac{\Delta T}{T_s} \quad (3)$$

where  $\Delta H_s$  is the heat of solution,  $\Delta T$  is the degree of supercooling, and  $T_s$  is the solvus temperature. Thus, in general, the larger the heat of solution, the higher is the chemical driving force, which leads to a higher nucleation rate and hence a finer precipitate microstructure.

For a dilute solution in a eutectic binary system consisting of elements A and B where A and B form  $\alpha$  and  $\beta$  phases, respectively, with different crystal structures, B is soluble in the  $\alpha$  phase, and the solubility of A in the  $\beta$  phase is negligible; the solubility of B in the  $\alpha$  phase  $x_B^\alpha$  is expressed as:

$$x_B^\alpha = A \exp\left(-\frac{\Delta H_s}{RT}\right) \quad (4)$$

where  $A$  is a constant,  $\Delta H_s$  is the heat of solution,  $R$  is the gas constant, and  $T$  is the absolute temperature.<sup>37</sup> Fig. 5 shows the temperature dependent solubility of NaTe in PbTe compared with those of  $\text{Ag}_2\text{Te}$  (ref. 28) and  $\text{Sb}_2\text{Te}_3$ .<sup>29</sup> Here, the atomic fractions of NaTe,  $\text{Ag}_2\text{Te}$ , or  $\text{Sb}_2\text{Te}_3$  in PbTe are used for  $x_B^\alpha$ . The heat of solution for NaTe in PbTe is calculated as equal to  $26_{-11}^{+19}$   $\text{kJ mol}^{-1}$ , where the error bar corresponds to the error range for the solubility at 240 °C. This value is comparable to  $22 \pm 1$   $\text{kJ mol}^{-1}$  for  $\text{Sb}_2\text{Te}_3$  or  $42 \pm 11$   $\text{kJ mol}^{-1}$  for  $\text{Ag}_2\text{Te}$ . This means that the dissolution of NaTe in PbTe is energetically unfavorable in a similar degree to that of  $\text{Sb}_2\text{Te}_3$  or  $\text{Ag}_2\text{Te}$ . Both  $\text{Ag}_2\text{Te}$  (ref. 20) and  $\text{Sb}_2\text{Te}_3$  (ref. 38) precipitates form in the nanoscale size in PbTe matrices. In light of the heats of solution in PbTe matrices, the driving force for nucleation is increased considerably by decreasing the temperature according to eqn (4) and the large number of defects in the cast material provides heterogeneous nucleation sites for precipitation. This is highly desirable for forming nano-precipitates. Therefore, it is expected that NaTe precipitates on the nanoscale, similar to  $\text{Ag}_2\text{Te}$  and  $\text{Sb}_2\text{Te}_3$  through solid state precipitation. This evidently describes the nature of the sodium-rich nano-precipitates observed in a previous study.<sup>14</sup> The observed nano-precipitates

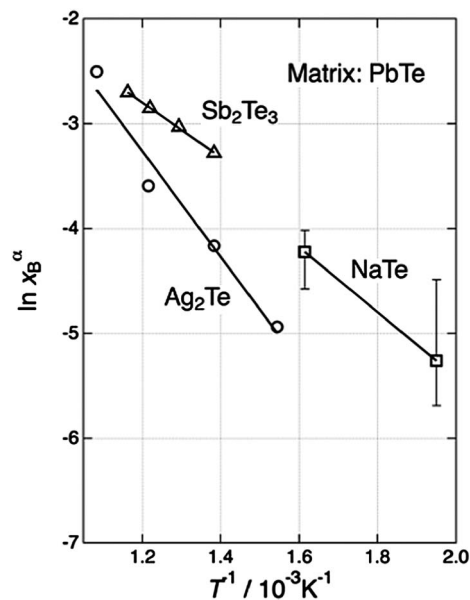


Fig. 5 Temperature dependence of the solubility of NaTe shown together with those of  $\text{Ag}_2\text{Te}$  (ref. 28) and  $\text{Sb}_2\text{Te}_3$ .<sup>29</sup> The solubility of  $\text{Ag}_2\text{Te}$  is corrected taking into account the effect of mutual solubilities.<sup>30</sup>

in the PbTe matrix are thin NaTe precipitates that formed due to the significantly decreased sodium solubility at temperatures less than 320 °C. These thin solid state nano-precipitates are embedded in the PbTe matrix and it is anticipated that they dissolve in the matrix up to the solubility limit when the temperature is increased. The dissolved sodium should then contribute to the electronic properties of PbTe.

## Conclusions

In summary, we have measured the maximum solubility of sodium telluride in lead telluride to be  $1.4 \pm 0.3$  mol% at 350 °C. We have shown that the addition of sodium to PbTe beyond this solubility limit results in the formation of a sodium-rich liquid phase at temperatures above 360 °C at grain boundaries, which may affect the mechanical properties of the alloy when cooled. On reducing the temperature below the maximum solubility temperature, solid sodium telluride precipitates in the PbTe matrix. We believe that the nano-precipitates which were observed in previous studies are solid sodium telluride which precipitates from the Na saturated lead telluride during cooling to room temperature as the solubility decreases with temperature. Furthermore, we expect that these nano-precipitates will be re-dissolved in the matrix by heating during application at high temperatures such as in measurements of high temperature thermoelectric properties.

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## Notes and references

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