Some innovative numerical approaches for pricing American options

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Some innovative numerical approaches
for pricing American options

A thesis submitted in (partial) fulfillment of the
requirements for the award of the degree of

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by

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I, Jin Zhang, declare that this Thesis, submitted in fulfilment of the requirements for the award of Master of Science, in the School of Mathematics and Applied Statistics, University of Wollongong. This Thesis is my own work unless otherwise referenced. The document has not been submitted for a higher degree to any other University or Institution.

Jin Zhang

March, 2007
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ABSTRACT

With the well-known model of lognormal asset price, the option valuation problems can be implemented by using the Black-Scholes partial differential equation approach. However, for American option pricing problems, it is hard to find an analytical formula due to the moving boundary feature [23]. This thesis presents two innovative numerical methods [38, 39] to value American put options in terms of solving the Black-Scholes partial differential equation with a set of appropriate boundary conditions.

The first method is the Laplace Transform Method, which extends the pseudo-steady-state approximation idea for the American option pricing problems in non-dividend yield case [35] to the one in constant dividend yield case. The approach transfers the original partial differential equations system to an ordinary differential equations system, to derive the solutions of the option prices and the optimal exercise boundary in the Laplace space respectively. After that, numerical inversions are performed to restore their corresponding values in the original time space.

The second method promotes a new predictor-corrector idea that uses a hybrid finite difference scheme to tackle the nonlinear nature of American option pricing problems, which is explicitly exposed after applying the front-fixing technique [21] to the original Black-Scholes partial differential equation. The new predictor-corrector scheme implements the computation of the option prices and the optimal exercise boundary through solving a set of linearized difference equations at each time step, to achieve high computational efficiency and numerical accuracy.

Through the comparison with Zhu’s analytical solution [34], we found that, the Laplace Transform Method is highly efficient since numerical calculations are only
performed for the inversion part, whereas the calculations of the Laplace transform are done analytically. Although the Laplace Transform Method slightly undervalues the optimal exercise boundary due to the pseudo-steady-state approximation introduced to allow the Laplace transform to be performed on the moving boundary. The loss of the accuracy in this regard is greatly compensated by its high computational speed. For the second method, we have shown that the numerical results obtained from the predictor-corrector scheme converge uniformly to Zhu’s exact optimal exercise boundary and option values [34], provided a convergence criterion is imposed. Furthermore, the agreement between the numerical solutions from the second method, and those from the Grid Stretching Method [24] that is a fourth-order scheme for both the asset price and time discretizations, not only validates the second method once again but also demonstrates its accuracy in that a lower-order scheme has virtually achieved the same level of accuracy as a higher-order scheme does.
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