Identification of ballast grading for rail track

Yifei Sun
University of Wollongong, Hohai University, ys910@uowmail.edu.au

Chen Chen
Hohai University

Sanjay Nimbalkar
University of Technology Sydney, sanjayn@uow.edu.au

Follow this and additional works at: https://ro.uow.edu.au/eispapers1

Part of the Engineering Commons, and the Science and Technology Studies Commons

Recommended Citation
Sun, Yifei; Chen, Chen; and Nimbalkar, Sanjay, "Identification of ballast grading for rail track" (2017). Faculty of Engineering and Information Sciences - Papers: Part B. 715.
https://ro.uow.edu.au/eispapers1/715

Research Online is the open access institutional repository for the University of Wollongong. For further information contact the UOW Library: research-pubs@uow.edu.au
Identification of ballast grading for rail track

Abstract
Grading has long been recognised to critically influence the mechanical behaviour of ballast. To identify the ballast grading for heavy-haul rail track, monotonic and cyclic triaxial tests are conducted to assess the performances of different gradings. Permanent deformations, aggregates degradation, resilience, shear resistance, maximum and minimum densities are recorded and analysed. The grading is found to affect the behaviour of ballast in that coarser gradings exhibit relatively better strength, resilience and therefore less permanent deformation. However, ballast degradation increases with the overall aggregate size. Therefore, to identify the grading for ballast with different performance objectives, a grey relational theory is used to convert the multi-objective into single-objective, i.e. grey relational grade. A relatively optimal grading that provides the highest grey relational grade is thus suggested for the improved ballast performance.

Disciplines
Engineering | Science and Technology Studies

Publication Details
Identification of ballast grading for rail track

Yifei Sun a, b, *, Chen Chen a, Sanjay Nimbalkar c

Article history:
Received 13 February 2017
Received in revised form 30 March 2017
Accepted 23 April 2017
Available online 10 August 2017

ARTICLE INFO

Grading has long been recognised to critically influence the mechanical behaviour of ballast. To identify the ballast grading for heavy-haul rail track, monotonic and cyclic triaxial tests are conducted to assess the performances of different gradings. Permanent deformations, aggregates degradation, resilience, shear resistance, maximum and minimum densities are recorded and analysed. The grading is found to affect the behaviour of ballast in that coarser gradings exhibit relatively better strength, resilience and therefore less permanent deformation. However, ballast degradation increases with the overall aggregate size. Therefore, to identify the grading for ballast with different performance objectives, a grey relational theory is used to convert the multi-objective into single-objective, i.e. grey relational grade. A relatively optimal grading that provides the highest grey relational grade is thus suggested for the improved ballast performance.

© 2017 Institute of Rock and Soil Mechanics, Chinese Academy of Sciences. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

The foundation of Australian railroad mainly consists of uniformly graded ballast placed between and underneath the sleepers. The purpose of using ballast is to ensure fast drainage and reduce the magnitudes of vertical stress distributed to the underlying weak subgrade from heavy loading trains (Suiker et al., 2005). In recent years, increasingly faster and heavier trains have been used which inevitably compromise the stiffness and drainage capacity of ballast due to significant aggregate degradation and mud pumping (Indraratna et al., 2013). Increased track differential settlements and fouling were thus often encountered in ballast with uniform gradings adopted from the Australian Standard DR 05328 (1996).

To improve the ballast performance for modern high-speed tracks, a variety of methods have been used, such as the application of winged sleepers or intermittent lateral restraints to provide higher restriction of ballast aggregates (Lackenby et al., 2007; Esmaeili et al., 2016), polyurethane polymer reinforcement of ballast to increase track resilience (Kennedy et al., 2013), and the utilisation of geosynthetics to increase particle interlocking within ballast (Yang and Han, 2013; Biabani and Indraratna, 2015). As can be seen, most of these methods were aimed to improve the confinement existing ballast aggregates and substructures. Limited attention (Bian et al., 2016; Indraratna et al., 2016) has been paid to fundamentally modifying the ballast aggregates themselves, i.e. the gradings for better track performance. However, grading has long been recognised as one of the most important characteristics that directly influence the mechanical performance of ballast, e.g. the drainage (Bian et al., 2016), shear resistance (Indraratna et al., 2016), stiffness (Indraratna et al., 2009), deformation (Sevi and Ge, 2012), and particle breakage (Yin et al., 2017). A proper design of the grading for railroad ballast should not only take into account the large voids for providing adequate drainage but also consider the structural stability for track operation. According to Thom and Brown (1988), an increase in the coefficient of uniformity ($C_u$) of granular aggregates would result in increases of the sample density and friction angle; but it would also significantly decrease the drainage capacity and stiffness of the sample. In addition, Indraratna et al. (2016) observed a reduced breakage with the decreasing particle size and increasing $C_u$ while the permanent deformation was shown to have an opposite trend. Therefore, optimisation of ballast grading should not merely rely on a single objective and factor, for instance, particle breakage or coefficient of uniformity. To avoid unfavourable outcomes, it should consider as many main controlling factors (coefficient of uniformity, $C_u$,
maximum and median particle sizes, \(d_m\) and \(d_{50}\); etc.) and objectives (permanent radial and axial strains, \(e_r\) and \(e_l\); particle breakage ratio, \(B_p\); resilient modulus, \(M_r\); shear strength, \(\phi\); density, \(p_d\); etc.) as possible.

Ideally, an optimal grading for railroad ballast should be determined by testing all the potential combinations of different particle fractions. The optimum grading is the one that has proper factors to provide the best balance among different objectives. However, due to the complicated interactions of different particle fractions (Tutumluer et al., 2009) and time-consuming processes through either experimental or numerical approaches, it is unlikely that one can perform a complete study of every possible grading encountered. In fact, this incompleteness of available test results can be regarded as a fundamental characteristic of an unascertained system that can be described by using specific mathematical approaches, e.g. fuzzy mathematics, probability approach, genetic algorithm (Levasseur et al., 2008), and grey system theory (Liu et al., 2012). Fuzzy mathematics concentrates on investigating problems with cognitive uncertainty by making use of experience or membership function. Probability approach studies the phenomena of stochastic uncertainty with emphasis placed on the historical statistical laws. It requires the availability of large number of reliable samples to satisfy a typical form of distribution, which may be unfavourable in geotechnical engineering. The focus of grey system theory is on the uncertainty problems of samples with small discrete data and multi-objective that are difficult for probability or fuzzy sets to handle. No typical distribution of samples is needed. Due to the impossibility for one person to perform a large number of reliable large-scale triaxial tests on ballast with different gradings (\(>30\) for each factor) in a reasonable time, the grey system theory is employed in this study to optimise the grading for railroad ballast. However, it should be noted that the genetic algorithm proposed by Levasseur et al. (2008) and later comprehensively modified by Jin et al. (2016, 2017) can be also applied to optimising the ballasting grading. This method is based on Darwin’s theory of evolution (Yin et al., 2016). It enables the convergence of a set of solutions close to the best one even with a flat or noisy error function. Undoubtedly, this algorithm is more advanced when compared to the grey system theory employed in this study. However, as pointed out by Levasseur et al. (2008), the calculation cost of this method is high. “It is necessary to perform a lot of finite element calculations at the beginning of the optimisation process in order to have a good view of the error function in the search space. This sweep, which is essential for the genetic algorithm efficiency, makes this method very expensive if there is only a few parameters to identify” (Levasseur et al., 2008). Therefore, in this study where three grading parameters \((C_w, d_m, d_{50})\) are evolved for optimisation, the grey system theory is preferred.

This paper is a follow-up study of the authors’ previous work (Sun et al., 2014, 2017; Indraratna et al., 2016). More analyses and discussions on the experimental results by Sun (2017) are presented in 5 sections. In Section 2, laboratory investigation of the influences of different factors on several main objectives is summarised. Section 3 deals with the implementation of the grey relational analysis (GRA) in optimising the grading of railroad ballast. In Section 4, several modifications are suggested for reduced ballast breakage and deformation. Finally, Section 5 concludes the paper.

2. Laboratory investigation

2.1. Test materials

The ballast aggregates were collected from a quarry located in Kiama, New South Wales, Australia with coordinates of 34° 40’15”S and 150° 51’15”E. Its physico-mechanical properties were measured according to ASTM D2434-68 (2006a), ASTM D4254-14 (2014a), ASTM D4253-14 (2014b) and can be found elsewhere in Sun (2017) and Indraratna et al. (1998). Table 1 lists the different grading alternatives used in this study. As can be seen, the grading alternatives can be divided into two different groups, i.e. constant \(C_w\) with varying \(d_m\) and constant \(d_{50}\) with varying \(C_w\).

2.2. Sample preparation and test procedure

The effect of grading on the deformation and degradation behaviour of ballast was studied by using the large-scale triaxial apparatus (Sun, 2017). Aggregates with different particle sizes but similar shapes (Sun et al., 2014) were firstly washed and then sieved separately. They were remixed together according to the target weight being calculated. After that, a 5 mm rubber membrane was lubricated and wrapped inside a split cylindrical mould. Aggregates were then put inside and compacted by four equal layers to the target sample height and diameter (600 mm × 300 mm). Then, the sample was placed inside a cell pressure chamber where water was slowly injected from the base of the cell under a back pressure of 10 kPa and air voids were all removed from the top of the cell to saturate the sample.

All the monotonic compression tests were carried out under the fully drained condition at a constant axial strain rate (0.05 mm/s). By doing this, no excess pore water pressure was observed during the test. Test data were collected by using installed pressure and displacement transducers as shown in Sun et al. (2015), thus not repeated here for simplicity. Triaxial compression was stopped until the axial strain \((e_l)\) arrived at 0.3. Moreover, cyclic tests were carried out by using the fixed minimum and maximum deviatoric stresses, i.e. \(q_{mm} = 45\) kPa and \(q_{max} = 230\) kPa (Sun et al., 2017). A sinusoidal cyclic stress with two different frequencies \((f = 20\) Hz and 30 Hz) was used. Note that the frequency was determined by \(f = V/L\), where \(V\) (≈144 km/h and 212 km/h) was the train speed and \(L\) (≈2.02 m) was the characteristic length between axles (Sun et al., 2015). Cyclic test was suspended until the number of load cycles equal to 500,000 or stopped at \(e_l = 0.3\). Permanent deformation data were recorded at the end of each test. Membrane correction was used when measuring the current stress and strain according to ASTM D4767-11 (2011). Particle breakage before and after the

Table 1 Properties of different samples.

<table>
<thead>
<tr>
<th>Alternative no.</th>
<th>(e_0)</th>
<th>(R_d)</th>
<th>(C_w)</th>
<th>(d_m)</th>
<th>(d_{50})</th>
<th>(d_{10})</th>
<th>(d_{10})</th>
<th>(d_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.75</td>
<td>0.5</td>
<td>1.9</td>
<td>53</td>
<td>52.3</td>
<td>40.8</td>
<td>40.7</td>
<td>22.3</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>0.56</td>
<td>1.95</td>
<td>43.7</td>
<td>34.6</td>
<td>34.5</td>
<td>19.7</td>
<td>9.5</td>
</tr>
<tr>
<td>3</td>
<td>0.75</td>
<td>0.58</td>
<td>1.9</td>
<td>40</td>
<td>37.1</td>
<td>30.5</td>
<td>28.6</td>
<td>17.2</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>0.58</td>
<td>1.95</td>
<td>37.5</td>
<td>32.7</td>
<td>28.4</td>
<td>25.1</td>
<td>16.1</td>
</tr>
<tr>
<td>5</td>
<td>0.75</td>
<td>0.61</td>
<td>1.91</td>
<td>31.5</td>
<td>30.5</td>
<td>23.4</td>
<td>23.4</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>0.78</td>
<td>0.77</td>
<td>1.25</td>
<td>50.3</td>
<td>49.4</td>
<td>46.9</td>
<td>41.9</td>
<td>37.5</td>
</tr>
<tr>
<td>7</td>
<td>0.78</td>
<td>0.63</td>
<td>1.5</td>
<td>47.2</td>
<td>45.3</td>
<td>40.3</td>
<td>31.4</td>
<td>4.75</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
<td>0.52</td>
<td>2</td>
<td>43.4</td>
<td>40.4</td>
<td>33.1</td>
<td>21.6</td>
<td>9.5</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.35</td>
<td>2.5</td>
<td>53</td>
<td>40.7</td>
<td>37.1</td>
<td>28.5</td>
<td>16.3</td>
</tr>
<tr>
<td>10</td>
<td>0.75</td>
<td>0.23</td>
<td>3</td>
<td>38.4</td>
<td>34.2</td>
<td>24.9</td>
<td>12.8</td>
<td>2.36</td>
</tr>
<tr>
<td>11</td>
<td>0.72</td>
<td>0.16</td>
<td>4.5</td>
<td>34.4</td>
<td>29.6</td>
<td>19.5</td>
<td>8.6</td>
<td>2.36</td>
</tr>
<tr>
<td>12</td>
<td>0.7</td>
<td>0.07</td>
<td>4.5</td>
<td>32.4</td>
<td>27.2</td>
<td>17.1</td>
<td>7.2</td>
<td>2.36</td>
</tr>
<tr>
<td>13</td>
<td>0.82</td>
<td>0.63</td>
<td>1.25</td>
<td>50.3</td>
<td>49.4</td>
<td>46.9</td>
<td>41.9</td>
<td>37.5</td>
</tr>
<tr>
<td>14</td>
<td>0.71</td>
<td>0.63</td>
<td>2</td>
<td>53</td>
<td>43.4</td>
<td>40.4</td>
<td>33.1</td>
<td>21.6</td>
</tr>
<tr>
<td>15</td>
<td>0.66</td>
<td>0.63</td>
<td>2.5</td>
<td>53</td>
<td>40.7</td>
<td>37.1</td>
<td>28.5</td>
<td>16.3</td>
</tr>
<tr>
<td>16</td>
<td>0.62</td>
<td>0.63</td>
<td>3</td>
<td>38.4</td>
<td>34.2</td>
<td>24.9</td>
<td>12.8</td>
<td>2.36</td>
</tr>
<tr>
<td>17</td>
<td>0.57</td>
<td>0.63</td>
<td>4.5</td>
<td>34.4</td>
<td>29.6</td>
<td>19.5</td>
<td>8.6</td>
<td>2.36</td>
</tr>
<tr>
<td>18</td>
<td>0.53</td>
<td>0.63</td>
<td>4.5</td>
<td>32.4</td>
<td>27.2</td>
<td>17.1</td>
<td>7.2</td>
<td>2.36</td>
</tr>
</tbody>
</table>

Note: \(e_0\) is the initial void ratio; \(R_d\) is the initial relative density; \(d_{10}, d_{50}\) and \(d_m\) are the aggregate diameters at 60%, 30% and 10% passing, respectively; \(d_m\) is the minimum aggregate diameter.
test was also evaluated (ASTM C136, 2006b). Details of the test procedure can be found in Sun (2017).

Table 2 summarises the triaxial tests performed on ballast. Note that the symbols C and S in Table 2 indicate the test types, i.e. cyclic and static, respectively. As can be seen, the samples were prepared based on two categories, i.e. similar initial void ratio (ε0) and similar initial relative density (Rd). No excess pore pressure accumulation was observed in any of the prepared samples (Indraratna et al., 2016). Therefore, all the gradings used in this study can provide fast and free drainage of water. The test results are discussed in the next section based on the following objectives: (1) permanent strains (e.g. axial and radial strains); (2) resilient modulus; (3) extent of particle breakage; (4) shear strength; and (5) density (e.g. maximum and minimum densities).

2.3. Objectives for optimisation

2.3.1. Permanent strains

Figs. 1–3 show the evolution of the permanent axial (ε1) and radial (ε3) strains of railroad ballast with different gradings under two different loading frequencies. Similar to Sun et al. (2015), higher ε1 and ε3 are expected for higher loading frequency throughout all the test samples. As can be observed in Fig. 1a, ε1 decreases firstly and then progressively increases with an increase in the coefficient of uniformity. A slight decrease followed by a gradual increase with the increasing coefficient of uniformity is observed in ε3, regardless of the sample density. Uniform gradings were found to have a higher settlement when compared with the relatively well graded one (Indraratna et al., 2006). This was attributed to the larger voids and smaller aggregates accumulated in the voids formed by the skeleton aggregates that reduced the interlocking among larger aggregates. Fig. 1b shows the similar test results by Thom and Brown (1988) where samples with high coefficient of uniformity experienced increased plastic deformation. Therefore, for the purpose of reduced permanent deformation of ballast, a proper grading with neither high nor low coefficient of uniformity is suggested.

Fig. 2 represents the influence of the maximum particle size on the ε1 and ε3 of ballast. To eliminate the effect of coefficient of uniformity, a constant Cu value of 1.9 was used. It is found that the absolute values of both the ε1 and ε3 continuously decrease with the increasing particle size. This is due to the improved shear resistance of ballast aggregates (Sevi and Ge, 2012; Indraratna et al., 2016). Similar results on quartz sand can be found in Wichmann et al. (2015). Fig. 3 illustrates the variation of permanent strains with the median particle size. It combines all the above test results of samples with either varying coefficient of uniformity or maximum particle size. It is observed that ε1 firstly decreases and then increases with the increasing median particle size whereas the radial strain exhibits a progressive decreasing trend. Similar findings can be found in Sevi and Ge (2012). This difference between Figs. 2 and 3 is attributed to the cross influences of the different grading factors, e.g. coefficient of uniformity, median and maximum particles. Therefore, to have an improved settlement performance of ballast under cyclic loading, a properly larger particle size should be preferred.

2.3.2. Particle breakage extent

The variation of the extent of particle breakage can be found in Figs. 4–6 by using Marsal's breakage ratio, Bg (Marsal, 1967). As expected, a higher extent of particle breakage is observed in samples tested under higher loading frequency owing to significant corner breakage and fatigue of internal aggregates (Indraratna et al., 2016; Sun, 2017). Moreover, due to the improved interparticle contacts and subsequently reduced stress concentration caused by increasingly broadening the ballast grading, particle breakage extent significantly decreases with the increase in coefficient of uniformity. Therefore, to reduce the particle breakage extent of railroad ballast suffering dynamic loading, a relatively larger value of the coefficient of uniformity than the original specification (DR 05328, 1996) is preferred. However, this value cannot be too large because the plastic deformation would increase as reported in Fig. 1.

### Table 2

**Test programs.**

<table>
<thead>
<tr>
<th>Initial state</th>
<th>da (mm)</th>
<th>Cu</th>
<th>f (Hz)</th>
<th>σf (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similar ε0</td>
<td>C53/1.9, C40/1.9, C31.5/1.9, C53/1.2, C53/1.5, C53/2, C53/2.5, C53/3, C53/4.5, C53/6, C53/10</td>
<td>20</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C53/1.9, C45/1.9, C40/1.9, C37.5/1.9, C53/1.5, C53/1.2, C53/1.5, C53/2, C53/2.5, C53/3, C53/4</td>
<td>30</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S53/1.2, S53/2, S53/3, S53/4.5, S53/4.5, S53/1.9, S45/1.9, S40/1.9, S37.5/1.9, S31.5/1.9</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Similar Rd</td>
<td>C53/1.2, C53/2, C53/2.5, C53/3, C53/4.5</td>
<td>20</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C53/1.2, C53/2, C53/2.5, C53/3, C53/4</td>
<td>30</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S53/1.2, S53/2, S53/3, S53/4.5</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>
It can be also observed from Figs. 5 and 6 that particle breakage ratio increases with the increasing maximum and median particle sizes, implying more serious breakage in ballast aggregates with larger size. This is due to the increasing number of larger sized aggregates which have high angularity (Le Pen et al., 2013; Sun et al., 2014) within the samples. To facilitate particle rearrangement, ballast aggregates with larger size experienced serious corner breakage and even split due to tensile strength concentration and fatigue (Lackenby et al., 2007; Sun et al., 2015). Therefore, for the purpose of reducing particle breakage ratio of railroad ballast, relatively smaller sized aggregates may be suggested. However, the particle size should not be too small, as the permanent deformation would increase as reported in Fig. 2.

2.3.3. Resilient modulus

Figs. 7–9 show the influences of the particle sizes and coefficient of uniformity on the resilient modulus of railroad ballast under two different loading frequencies. A lower resilient modulus is observed in samples tested under higher loading frequency. This can be attributed to the higher extent of particle breakage induced by higher frequency (Sun, 2017).

The effect of coefficient of uniformity on the resilient modulus of ballast can be found in Fig. 7. It is observed that the resilient modulus decreases with the increase in coefficient of uniformity. The higher the sample density is, the higher the resilient modulus will be. Similar observations on sand can be found in Wichtmann and Triantafyllidis (2014). Therefore, to have a higher resiliency of the railroad ballast, a relatively small value of the coefficient of uniformity is suggested. However, a small value of the coefficient of uniformity can induce more extent of particle breakage (Fig. 4) which would increase maintenance cost. Thus, a proper balance of these factors should be found.

Figs. 8 and 9 illustrate the variations of the resilient modulus with differing particle sizes. It can be found that the resilient modulus decreases with the increase of the maximum particle size, which is in accord with Sitharam and Nimbkar (2000). However, it decreases and then increases with an increase in the median particle size, as shown in Fig. 9. This difference between Figs. 8 and 9 can be also attributed to the coupling influences among different grading factors. As the minimum particle size was fixed during each test, increasing the maximum particle size is increasing the size span between the minimum and maximum particle sizes, which actually influences the resilient modulus. Relevant mechanistic elaboration can be found in Sun (2017). Therefore, a properly small
deviation between the minimum and maximum particle sizes is suggested.

2.3.4. Shear resistance

Fig. 10 shows the variation of the peak friction angle with differing coefficient of uniformity. It can be observed that the peak friction angle increases and then decreases with the increasing coefficient of uniformity. A better shear resistance is found in samples with higher initial density due to better aggregates interlocking. The shear resistance is also observed to increase with the increasing particle size, as shown in Fig. 11. When the average aggregate size increases in a granular material, greater interlocking is achieved among larger angular aggregates, which would increase the shear resistance of the sample (Honkanadavar and Sharma, 2016). However, it cannot be denied that a greater degree of breakage also takes place in larger aggregates due to higher stress concentration between angular aggregates. The effect of increase in interlocking is to increase the shear resistance, while the effect of aggregate breakage is to reduce the shear resistance. In the case of ballast under low confinement, the effect of increase in interlocking overweights the effect of increase in particle breakage with the increasing particle size. Therefore, the net effect is an increase in the shear resistance with particle size at a given confining pressure.

Similar observations can be found in Wang et al. (2013) and Li et al. (2001).

2.3.5. Density

Figs. 12–14 show the variations of the minimum and maximum dry densities with varying coefficient of uniformity and particle size. As observed in Fig. 12, the minimum and maximum densities generally increase with the increasing coefficient of uniformity, indicating a better packing capacity for broader gradings. However, the densities only exhibit a slight increase with the increasing maximum particle size (Fig. 13), given the same coefficient of uniformity. The overall effect of median particle size can be found in Fig. 14 where the densities generally increase and then decrease with the increasing median particle size.

3. Multi-objective optimisation of ballast

Grey relational grade (GRG) is usually utilised in order to convert the optimisation problem from the complicated multi-objective to a relatively easy single-objective. The purpose of this section is to find the optimal alternative of the controlling grading factors that simultaneously minimise permanent deformation and degradation while increase the resilient modulus, shear resistance, and density.
of railroad ballast. To achieve this target, a mathematical approach called GRA (Deng, 1989; Liu et al., 2012) is used in this study. The basic equations of this theory can be found in the Appendix.

In current study, alternatives \( (d_i) \) (Table 1) that include the following controlling factors for optimisation of ballast gradation are used:

\[
\begin{align*}
\frac{1}{2}d_1 & \quad \frac{1}{2}d_2 & \quad d_3 & \quad d_4 & \quad M_r & \quad \varepsilon_0 & \quad C_u \\
T & = & & & & & \\
\end{align*}
\]

(1)

The corresponding optimisation objectives \( (x_{ij}) \) are considered to be

\[
\begin{align*}
[ & x_{i1} \; x_{i2} \; x_{i3} \; x_{i4} \; x_{i5} \; x_{i6} \; x_{i7} ]^T \\
= & \begin{bmatrix} \varepsilon_1 & \varepsilon_3 & B_8 & M_r & \phi_p & \rho_{dM} & \rho_{dm} \end{bmatrix}^T \quad (i = 1, 2, ..., m)
\end{align*}
\]

(2)

where the objective number \( n = 7 \), according to Eq. (2). Therefore, the order sequences \( j (= 1, 2, ..., 7) \) correspond to the permanent axial strain, permanent radial strain, breakage ratio, resilient modulus, peak friction angle, maximum and minimum densities, respectively. To comprehensively optimise the ballast grading, three different scenarios are investigated.

![Fig. 10. Effect of coefficient of uniformity on the peak friction angle of ballast.](image1)

![Fig. 11. Effect of particle size on the peak friction angle of ballast.](image2)

![Fig. 12. Effect of coefficient of uniformity on the dry density of ballast.](image3)

![Fig. 13. Effect of the maximum particle size on the dry density of ballast.](image4)

The first is that the importance of all objectives is assumed to be equal, which means that the following values of \( w_j \) will be used:

\[
w_j = \frac{1}{7} \quad (j = 1, 2, ..., 7)
\]

(3)

where \( w_j \) is the weight of objective \( j \) which depends on the judgement of the decision maker or the structure of the proposed problem. The second is to assume that the importance of permanent deformation \( (j = 1 \text{ and } 2) \) and degradation \( (j = 3) \) of ballast is superior to the rest objectives, which means

\[
w_j = \frac{3}{13} \quad (j = 1, 2, 3)
\]

(4)

The third is to assume that the permanent deformation \( (j = 1 \text{ and } 2) \) and resilient modulus \( (j = 6) \) of ballast are more important than the rest objectives, which indicates

\[
w_j = \frac{1}{13} \quad (j = 4, 5, 6, 7)
\]

(5)
The GRG indicates the degree of similarity between the comparability sequence (CS) and reference sequence (RS) (Kuo et al., 2008). As can be found in the Appendix, the RS represents the best performance that could be achieved. Therefore, if a CS for an alternative gets the highest GRG with the RS, it is indicated that the alternative is the optimum ballast gradation.

Based on the above procedure, a relatively optimal alternative considering multiple performance objectives can be suggested. Fig. 15 shows the final GRG of alternatives evaluated under different scenarios where grade differences can be observed in some alternatives. For example, alternatives Nos. 6 and 7 have better performances under scenario No. 3 than those under scenarios Nos. 1 and 2, if the ballast resiliency was considered to be more critical than the rest objectives. However, even if the weighting values of different objectives were set as different, the general evolution trend is observed to be similar. The highest GRG is reported by alternative No. 15 with $C_u = 2.5$ and $d_M = 53$ mm for tests under relatively low frequency of 20 Hz, while alternative No. 14 with $C_u = 2$ and $d_M = 53$ mm performs better under high frequency of 30 Hz.

\[ w_j = \frac{3}{13} \quad (j = 1, 2, 6) \]  

(6)

\[ w_j = \frac{1}{13} \quad (j = 3, 4, 5, 7) \]  

(7)
Further analysis of the effect of different grading factors \((C_u, d_M, d_S0)\) can be found in Figs. 16–18. The effect of coefficient of uniformity on multi-objective performances is analysed by calculating the corresponding GRG, as shown in Fig. 16. The GRG generally increases and then decreases with the increase in coefficient of uniformity. An optimal coefficient is found to be around 2.5 for low speed track (20 Hz) and 2 for relatively high speed track (30 Hz) to reduce permanent deformation and aggregates degradation while provide sufficient aggregates resilience and shear resistance. This is in accord with the results by Indraratna et al. (2006) who conducted a series of large triaxial tests on ballast under the cyclic frequency of 20 Hz. It was found that ballast degradation and deformation were significantly reduced by slightly increasing the value of \(C_u\) and therefore they suggested a modified grading with \(C_u\) ranging from 2.3 to 2.6, which agrees well with the current multi-objective analysis.

Figs. 17 and 18 show the effects of particle sizes on the multi-objective performances. It is observed from Fig. 17 that the GRG increases with an increase in the maximum particle size, irrespective of the analysing scenarios. This implies that even if the ballast degradation could be increased when increasing the ballast particle size, larger sized ballast is still suggested, considering the improved deformation performance within ballast layer. The variation of GRG with median particle size can be found in Fig. 18. To have a high GRG, the median particle size should be neither small nor large. A high GRG can be achieved by using a proper value of the median particle size (36–41 mm).

4. Recommendations

The size of ballast was selected using trial-and-error method with different sizes over the years. It seems that smaller sized ballast (≤20 mm) was preferred in past due to easy and efficient maintenance of the track by manual means. However, with the increase of modern track speed and the introduction of mechanised on-track tamping, ballast size has been increased significantly even up to 80 mm in French high-speed railway (Claisse and Calla, 2006). Indraratna et al. (2006) suggested the use of slightly graded ballast \((C_u = 2.3–2.6)\) instead of the conventional uniform grading (DR 05328, 1996) with \(C_u\) around 1.5 to reduce ballast deformation and degradation (grading A, Table 3). This would work well for trains running at current low speed according to this study. However, with the increase of track speed, the overall ballast sizes should not remain small; the median particle size should be increased to 36–41 mm for a maximum particle size of 53 mm. To ensure a high resilient modulus and meanwhile a reduced ballast deformation and degradation, the coefficient of uniformity should also not be large; a proper value around 2 can be used. Therefore, a modified grading for railroad ballast is recommended, as shown in Table 3. Similar grading with \(C_u = 1.9\) (grading B, Table 3) was also suggested by Claisse and Calla (2006) based on experiences. The advantages of the proposed specification would be the reduced maintenance cost of ballasted rail track and increased life of ballast aggregates.

5. Conclusions

Ballast is the largest component of the railroad track, which shares a significant part of the entire railroad budget for purchasing, distributing, and maintenance. Ballast grading fundamentally determines its physical and mechanical characteristics. The size of ballast was usually determined by experience with different sizes in the past. However, from the mechanical point of view, there are several important objectives for selecting a possible grading of ballast, for example, higher shear resistance, lower deformation and degradation. The coefficient of uniformity and particle size of a grading had complex effects on these objectives and sometimes even had cross impact. In light of the research into these influences by using experimental and grey relational study, some interesting facts have come to light with regard to the aggregate size and distribution for ballast. Several main findings of this study can be summarised as follows:

1. Ballast with uniform gradings \((C_u < 2)\) exhibited larger permanent deformations than that of the slightly broad grading \((C_u = 2–2.5)\). Further broader gradings \((C_u \geq 3)\) would bring in smaller aggregates which weakened the interlock between larger aggregates and therefore resulted in increased permanent deformation, if samples were not compacted well before. Increasing the aggregate size and density would improve the shear resistance of ballast, which resulted in reduced permanent deformation.

2. Ballast degradation was significantly reduced by increasing the coefficient of uniformity of ballast aggregates. But it increased when the overall aggregate size was increased due to the significant corner breakage and attrition occurring in larger sized aggregates.

3. The resiliency of ballast increased with the increasing particle size. However, it decreased with the increasing coefficient of uniformity due to more aggregates taking part in the stress carrying queue which made the sample less stiff.

4. By using GRA, the multi-objective problem was converted to a single-objective problem, i.e. searching for the highest GRG. It was found that the GRG generally increased with the increasing maximum aggregate size. However, an increase followed by a decrease in the GRG with the median aggregate size and coefficient of uniformity was observed.

5. A relatively optimal grading for ballast was proposed to improve its deformation and degradation performances under cyclic loading with high frequency, which would further increase the lifecycle of ballast and therefore reduce the maintenance cost of ballasted rail track.

### Table 3: Recommended gradings.

<table>
<thead>
<tr>
<th>Size (mm)</th>
<th>Percentage passing (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>100–100</td>
</tr>
<tr>
<td>53</td>
<td>100–64</td>
</tr>
<tr>
<td>37.5</td>
<td>43–25</td>
</tr>
<tr>
<td>26.5</td>
<td>17.5–7</td>
</tr>
<tr>
<td>19</td>
<td>7–0</td>
</tr>
<tr>
<td>13.2</td>
<td>2–0</td>
</tr>
<tr>
<td>9.5</td>
<td>0–0</td>
</tr>
<tr>
<td>2.36</td>
<td>0–0</td>
</tr>
</tbody>
</table>

Further analysis of the effect of different grading factors \((C_u, d_M, d_S0)\) can be found in Figs. 16–18.
Conflict of interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Acknowledgements

The experimental work co-sponsored by the Centre for Geomechanics & Railway Engineering of the University of Wollongong is appreciated. The financial support provided by the Fundamental Research Funds for the Central Universities (Grant No. 2017B05214) and the Priority Academic Program Development of Jiangsu Higher Education Institutions are also appreciated.

Appendix

The GRA is a quantitative mathematical approach to evaluate the similarity and dissimilarity among alternatives in developing dynamic process (Liu et al., 2012). It measures the correlation degree between different alternatives and objectives; the more the similarities develop, the more they correlate. Due to the fact that the performance of different alternatives may have different dimensions and units, the most important step of GRA is the so-called grey relational generating where the performances of all alternatives will be transformed into a CS. After that, a RS that provides the potentially best performance is defined. Then, the grey relational coefficient (GRC) between all the CSs and the corresponding RS is calculated. Finally, based on these GRCs, the GRG between the RS and every CS is calculated. If a CS translated from an alternative has the highest GRG between the RS and itself, that alternative will be the optimum choice. Detailed steps of GRA are described as follows.

(1) Grey relational generating

The first step of GRA is to transform the performance values for each factor into a sequence by normalisation. This is intended to make each factor and objective comparable. When the units of the performance are different for various objectives, the influence of the corresponding factors could be ignored (Kuo et al., 2008). Furthermore, if the functions of these factors are distinct, for example, the higher Cw results in unfavourably lower resilient modulus but favourably reduced ballast degradation, it will also cause unfavourable results during the analysis. Therefore, the experimental data should be normalised according to the type of cause unfavourable results during the analysis. Therefore, the modulus but favourably reduced ballast degradation, it will also.

(2) Reference sequence definition

For an objective j of alternative i, if the value \( x_{ij} = 1 \), or is closer to 1 than the value of any other alternative after the procedure of grey relational generating, it implies that the alternative i performs the best for the objective j. Therefore, only the one with all of its performance values closest or equal to 1 can be regarded as the best choice. However, this type of alternative usually does not exist. Instead, an alternative that compromises all these objectives is often suggested. This section then defines a RS \( X_0 \) to find the alternative whose CS is closest to the RS:

\[
(X_{01}, X_{02}, \ldots, X_{0j}, \ldots, X_{0n}) = (1, 1, \ldots, 1, \ldots, 1) \quad (A3)
\]

(3) Grey relational coefficient

To assess how close \( x_{ij} \) is to \( x_{0j} \), the GRC is thus used. The larger the GRC, the closer \( x_{ij} \) and \( x_{0j} \) are. The GRC can be determined by

\[
\gamma(x_{0j}, x_{ij}) = \frac{D_{\text{min}} + \xi D_{\text{max}}}{D_{ij} + \xi D_{\text{max}}} \quad (i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n) \quad (A4)
\]

where

\[
D_{ij} = |x_{0j} - x_{ij}| \quad (i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n) \quad (A5)
\]

\[
D_{\text{min}} = \min \{D_{ij}\} \quad (i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n) \quad (A6)
\]

\[
D_{\text{max}} = \max \{D_{ij}\} \quad (i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n) \quad (A7)
\]

where \( \gamma(x_{0j}, x_{ij}) \) is the GRC between \( x_{ij} \) and \( x_{0j} \); \( \xi \in [0, 1] \) is the distinguishing coefficient (DC), used to adjust the difference of the GRC (Deng, 1989). Different values of \( \gamma(x_{0j}, x_{ij}) \) can be observed if different DCS are adopted. But whatever the DC is, the ranking order of \( \gamma(x_{0j}, x_{ij}) \) does not change (Kuo et al., 2008). Thus, the DC is set as 0.5 for simplicity.

(4) Grey relational grade

After determining all the GRC \( \gamma(x_{0j}, x_{ij}) \), the GRG can be obtained by

\[
\Gamma(X_0, X_i) = \sum_{j=1}^{n} w_j \gamma(x_{0j}, x_{ij}) \quad (i = 1, 2, \ldots, m) \quad (A8)
\]

where \( \Gamma(X_0, X_i) \) is the GRG between \( X_0 \) and \( X_i \). It represents the level of correlation between the RS and the CS. The weight can be expressed as

\[
\sum_{j=1}^{n} w_j = 1 \quad (A9)
\]

For example, if there are \( n \) objectives that are considered to have equal importance, then the weight \( w_j \) should be equally taken as 1/n. However, if the permanent deformation and degradation of ballast are considered to be more critical than other objectives, then
the weight of the permanent deformation and degradation could have higher values than 1/n.

References


Sun Y. Effect of particle angularity and size distribution on the deformation and degradation of ballast under cyclic loading, PhD Thesis. Wollongong, Australia: School of Civil, Mining, and Environmental Engineering, University of Wollongong; 2017.

Yifei Sun obtained his BSc degree in Engineering Mechanics and MSc degree in Geotechnical Engineering from Hohai University, China, in 2010 and 2013, respectively, and his PhD in Geotechnical Engineering from University of Wollongong, Australia in 2017. He is now affiliated as Associate Professor in the Key Laboratory of Ministry of Education for Geomechanics and Embankment Engineering, Hohai University. His research interests include (1) characterisation and constitutive modelling of granular soils, (2) application of fractional calculus, including fractional plasticity, fractional viscoelasticity, fractional non-Fourier heat conduction. He has received several Chinese national projects.