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Displacement Estimation Based on Model Calibration in Weak Feedback Optical Self-Mixing System

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Abstract
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Displacement Estimation Based on Model Calibration in Weak Feedback Optical Self-Mixing System

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Abstract — This paper presents a novel method to estimate displacement of a moving object based on model calibration in a weak feedback laser diode self-mixing system. A hybrid genetic algorithm is employed to locate the global minimum for the cost function which is constructed as the discrepancy between theoretically calculated system output and the experimental data. In turn, displacement measurement is achieved by identifying the optimal parameters for the system model. The proposed method is tested by both simulation and experimental setup with measuring accuracy better than 25nm.

I. INTRODUCTION

Laser diode (LD) self-mixing (SM) interferometry has been studied extensively in recent years due to its compactness, cost efficiency and comparable accuracy with the conventional double-beam interferometer. Many systems based on this effect have been developed to measure displacement, vibration, and velocity etc. of an external moving target [1-6] as well as characteristic parameters of a semiconductor laser such as Linewidth Enhancement Factor (LEF) [7, 8]. Such method is non-contact and thus offers advantages for many applications.

The laser diode self-mixing effect refers to a phenomenon when a small fraction of the light emitted by a semiconductor laser is reflected or backscattered by a remote target and coupled back into the laser active cavity, causing interference with the light already presented in the LD and resulting in modulation of the laser oscillating field in terms of both amplitude and frequency. In the case of a moving target, laser intensity fluctuation can be picked up by a photodiode enclosed at the rear facet of LD package and analyzed to yield information about target movement as well as laser parameters. Based on the strength of reflected signal, three different feedback regimes have been classified by a parameter known as feedback level factor (C). Usually displacement measurement is performed in the moderate feedback regime (1< C<4.6) when the self-mixing signal presents the typical sawtooth like waveforms with each fringe corresponding to \( \lambda_o/2 \) target displacement, where \( \lambda_o \) represents the laser wavelength under free running conditions. By counting the number of fringes with a differentiation circuit, the target displacement is estimated with a resolution of \( \lambda_o/2 \). Developing further from this principle, various approaches have been reported to improve this basic resolution. Addy et al. [9] improved resolution up to \( \lambda_o/4 \) by misalignment of the reflector and using a mirror as a target.

Bosch et al. [10] developed an algorithm based on the interpretation of the interference and of the fractional fringe to linearize the measured displacements with a measuring accuracy of \( \lambda_o/10 \). However, these methods entail additional optical components and preliminary separate measurement of C and LEF which can be very expensive.

In this paper, we present an approach to achieve measurements in weak feedback regime (C<1), where the self-mixing signal is continuous. The proposed approach relies on a model calibration (data-to-model fitting) technique, the principle of which is to search the optimum solutions for the parameters of a physical system’s model so that the difference between model output and experimentally measured input is minimized by means of an optimization algorithm. In particular, Genetic Algorithm (GA) is adopted in this work to search the global minimum of the objective function. This approach inherited the superiorities of the method in [7], which firstly is the immunity to noises incorporated in the observed signal as it analyzes multiple segments of the self-mixing data; secondly, the Lang-Kobayashi model works more pertinently for the weak feedback conditions.

II. THEORY AND ALGORITHM

The mathematical model for a LD self-mixing system was described in [11, 12], taking the form as follows:

\[
\phi_r(t) = \phi_o(t) - C \cdot \sin(\phi_r(t) + \arctan(\alpha)) \tag{1}
\]

\[
P(t) = P_0[1 + m \cdot G(t)] \tag{2}
\]

\[
G(t) = \cos[\phi_r(t)] \tag{3}
\]

Eq. (1) implies that the laser phase with optical feedback \( \phi_r \) varies around the free running laser phase \( \phi_o \) and is influenced by both the feedback level factor (C) and the linewidth enhancement factor (\( \alpha \)). Eq. (2) reveals that the self-mixing laser power \( P \) is a modulation of its non-feedback counterpart \( P_0 \) by a factor of \( 1 + m \cdot G(t) \), where \( m \) is the modulation index (typically \( m = 10^{-3} \)). \( G(t) \) is given in Eq. (3) and is a periodic function of \( 2\pi \cdot \tau = 2L/c \) is the round-

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trip time between LD and external target, where $L$ is the distance between the LD and the external target, $c$ is the speed of light.

Since we often talk about a moving target in a SM system, for the sake of simplicity, we consider a harmonic vibration as an example, hence the target distance from LD can be written as
\[
L(t) = L_0 + \Delta L \cos(2\pi ft)
\]  
(4)

Where $L_0$ is the equilibrium position of the target vibration, $\Delta L$ represents the target displacement, $f$ is the vibration frequency. As $\phi_0 = 4\Delta L / \lambda_0$, by substituting $L$ with Eq. (4), we can rewrite it as
\[
\phi_0(t) = \phi_0 + \Delta \phi \cos(2\pi ft)
\]  
(5)

with $\phi_0 = 4\Delta L / \lambda_0$, $\Delta \phi = 4\pi \Delta L / \lambda_0$. Combining Eq. (1), (3) and (5), we can calculate the values of $\tilde{G}(\tau)$ theoretically by
\[
\tilde{G}(\tau) = \cos(\phi_0 + \Delta \phi \cos(2\pi ft) - C \cdot \sin[\phi_0(t) \tau(t)] + \arctan(\alpha))
\]  
(6)

On the other hand, the laser intensity $P(\tau)$ is measured experimentally and real $G(\tau)$ values can be worked out according to Eq. (2). Therefore, the problem falls into a paradigm where the unknown parameters of the model of a physical system are to be identified so that the difference between the model computed output and measured values is minimized, which implies that the system parameters are optimal and fits best with real situation. Specifically, $\phi_0, \Delta \phi, C$ and $\alpha$ in Eq. (6) are the unknown variables to be identified. Once $\Delta \phi$ is determined, the target displacement is readily obtained according to $\Delta L = \Delta \phi \cdot \lambda_0 / 4\pi$.

To this end, an objective function (also called cost function) is constructed as follows:
\[
F(\phi_0, \Delta \phi, \tilde{C}, \tilde{\alpha}) = \sum_i \left[ \tilde{G}(\tau(i)) - \tilde{G}(\tau(i), \phi_0, \Delta \phi, \tilde{C}, \tilde{\alpha}) \right]^2
\]  
(7)

Where $N$ is the data length of $G(\tau(t))$ used for parameter estimation, $\tilde{G}(\tau(t), \phi_0, \Delta \phi, \tilde{C}, \tilde{\alpha})$ is the calculated values based on the theoretical model, $G(\tau(t))$ is the experimentally measured data. Clearly the best data-to-model fitting occurs when $F(\phi_0, \Delta \phi, \tilde{C}, \tilde{\alpha})$ presents its minimum value. In [7], a gradient-based approach was adopted to solve this optimization problem as the cost function analysis reveals only one local minimum existed for the entire parameter space. However, since we are considering a more practical condition in this paper where $\Delta L$ is also unknown, we will re-investigate the surface shape of the cost function under this circumstance to explore the viability of using gradient-based approach to search the minimum in cost function. As there are four variables to be identified in the cost function and it is not possible to show the surface shape in one plot, we look at this issue in two steps. Firstly we examine the surface shape of the cost function when $\phi_0$ and $\Delta \phi$ are taken as unknown variables and $C$ and $\alpha$ take their true values by computer simulation. For this purpose, a self-mixing signal was generated based on Eq. (1)-(3) using a set of specific parameter values and is taken as the experimentally observed data. In order to fully reveal the characteristics of the cost function, all possible values of $\phi_0$ and $\Delta \phi$ should be evaluated. As is seen in Eq. (6), for any integer value of $k$, $\phi_0 + 2k\pi$ will yield the same result due to cosine function. In other word, $\phi_0$ can only been identified at the interval of $(0,2\pi)$, which is also the range of interest in our simulation.

With regard to $\Delta \phi$, estimation with $\lambda/2$ resolution can be achieved by fringe counting which corresponds to $6.2832 rad$ in phase. In consequence, we should allow a range of $[\Delta \phi, -6.2832, \Delta \phi + 6.2832]$ (when $\Delta \phi = 6.2832 > 0$) or $[0,\Delta \phi + 6.2832]$ (when $\Delta \phi = -6.2832 < 0$) for $\Delta \phi$ when performing optimization, where $\Delta \phi$ represents the estimated value by fringe counting. As an example, Fig 1. (a) shows the surface shape of the cost function when the SMS was generated with parameters $C = 0.6$, $\alpha = 4$, $L_0 = 30km$ (i.e. $\phi_0 = 1.321rad$), $\Delta L = 1.8\mu m$ (i.e. $\Delta \phi = 28.8146rad$), $f = 200Hz$ and $\lambda_0 = 785nm$. The estimated target displacement by fringe counting is $1.77\mu m$ (thus $\Delta \phi = 28.27 rad$). Consequently, the parameter range was selected as $\phi_0 \in [0,2\pi]$ and $\Delta \phi \in [21,99,34,55]$.

Fig 1 (b) shows the surface shape when $C = 0.3$, $\alpha = 3$, $L_0 = 30km$, $\Delta L = 1.0\mu m$ with parameter range of $\phi_0 \in [0,2\pi]$ and $\Delta \phi \in [12,56,25,13]$.

Secondly, we visualize the surface shape when $C$ and $\alpha$ are taken as unknown variables and $\phi_0$ and $\Delta \phi$ take their true values. We use the same set of parameters as used in Fig 1 to generate the SMS with $C$ varied in the range $(0.1,0.9)$ and $\alpha \in [19]$. The surface shapes are plotted in Fig. 2 (a) and (b).
Figure 1. Surface shape of cost function with varied $\phi_0$ and $\Delta \phi$ when $C$ and $\alpha$ take their true values
(a) $C=0.6$, $\alpha=4$, $L_0=30\text{cm}$, $\Delta L=1.8\mu\text{m}$ (b) $C=0.3$, $\alpha=3$, $L_0=30\text{cm}$, $\Delta L=1.0\mu\text{m}$

Figure 2. Surface shape of cost function with varied $C$ and $\alpha$ when $\phi_0$ and $\Delta \phi$ take their true values
(a) $C=0.6$, $\alpha=4$, $L_0=30\text{cm}$, $\Delta L=1.8\mu\text{m}$ (b) $C=0.3$, $\alpha=3$, $L_0=30\text{cm}$, $\Delta L=1.0\mu\text{m}$
It is seen from Fig. 1 that the cost function has more than one local minimum and only one global minimum which corresponds to the real $\phi_0$ and $\Delta\phi$ values given that $C$ and $\alpha$ take their true values. On the other hand, it is revealed from Fig. 2 that within a large range around the true $C$ and $\alpha$ values, the cost function presents similar values which is also the local minimum when $\phi_0$ and $\Delta\phi$ take their true values. Therefore, it is evident to infer that with the aid of an appropriate optimization algorithm that is capable of locating the global minimum of the cost function, we are able to identify the optimal parameters for the system model. However, conventional optimization techniques such as gradient-based algorithm will have difficulty finding the global minimum unless the starting point is within the immediate vicinity of this minimum. In other word, a more competent algorithm is entailed to solve this global optimization problem.

III. HYBRID GA-MGA ALGORITHM

Genetic algorithm is one of the popular global optimization (GO) approaches. It belongs to a family of evolutionary algorithm (EA) that is based on the mechanism of genes and natural selection and adopted the terminology from biology and genetics. The evolution usually starts from a population of randomly generated individuals and repeats in generations. In each generation, the fitness (i.e., cost function) of each individual in the population is evaluated, based on which multiple individuals are selected from the current population, and modified through mating and mutation to form a new population. The new population is then used in the next iteration of the algorithm.

In conventional GA, each variable is encoded as a binary string called a gene. The gene size is determined primarily by the desirable accuracy of the solution. A chromosome is the collection of all genes. For instance, for an optimization problem of four variables each of which is encoded into 10 bit binary string, a chromosome length will be 40 bits as a result. The initial chromosome population size is somewhat arbitrary. The tradeoff is low selection rate limits the genes that are passed on to the offsprings and high selection rate allows the bad performers to continue their traits to the next generation. The GA operators include mating and mutation which introduces new genes not contained in the initial chromosome population. The parents are firstly paired according to roulette wheel weighting, i.e., a chromosome with the highest cost is appointed the lowest probability of mating while a chromosome with the lowest cost has the greatest probability of mating. And offsprings are created by swapping part of the chromosome between the parents. In particular, we employed ten-point crossover and bit mutation in our experiments. The best fitted chromosome in a generation is generally not mutated and is designated as elite solution destined to propagate unchanged to the following generation. GA ends like other optimization algorithms too, by testing for convergence. The convergence criterion could be whether an acceptable solution has been reached or a set number of iterations is exceeded.

The major drawback of GA is that although they are efficient in locating the basins of the optima, they are often unable to explore these basins effectively and quickly in order to find the exact global optima with a high degree of accuracy [13]. This is in good accordance with our preliminary simulation results. As an alternative, we employed a hybrid GA scheme which performs a mirogenetic (MGA) search to find the exact global optima with a high degree of accuracy. As an alternative, we employed a hybrid GA scheme which performs a mirogenetic (MGA) search to find the exact global optima with a high degree of accuracy. As an alternative, we employed a hybrid GA scheme which performs a mirogenetic (MGA) search to find the exact global optima with a high degree of accuracy.

To summarize, the proposed algorithm can be indicated by steps as shown in Table I.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Estimate target displacement by fringe counting</td>
</tr>
<tr>
<td>2.</td>
<td>Decide the parameter ranges for each variable</td>
</tr>
<tr>
<td>3.</td>
<td>Select gene sizes for the variables</td>
</tr>
<tr>
<td>4.</td>
<td>Generate initial chromosome population for GA</td>
</tr>
<tr>
<td>5.</td>
<td>Produce offsprings through Pairing, Mating and Mutation</td>
</tr>
<tr>
<td>6.</td>
<td>Recover the population size by replacing the less fitted parents with offsprings created in Step5</td>
</tr>
<tr>
<td>7.</td>
<td>Repeat step5 &amp;6 until the convergence criterion is met</td>
</tr>
<tr>
<td>8.</td>
<td>Perform MGA to refine the solutions</td>
</tr>
</tbody>
</table>

IV. SIMULATION

In order to evaluate the proposed approach, we run computer simulation first by generating the self-mixing signals as discussed in Section II and taking them as the experimental data. Based on the parameter range discussed in the previous sections, we first select the gene size for main GA as 10bits for $C, \phi_0$ and 12 bits for $\alpha, \Delta\phi$. Consequently, the quantization errors are $9.7656 \times 10^{-4}$, $2.4 \times 10^{-3}$, $6.1 \times 10^{-3}$ and $3.1 \times 10^{-7}$ for $C, \alpha, \phi_0, \Delta\phi$ respectively. As a result, each chromosome will be 44 bit long. The initial population of chromosome was selected as 36 based on our trials. The
selection and mutate rates were used as 0.5, 0.35 respectively and running limit of 1000 generations. The configuration for the first iteration of MGA was the following: 5bits for $C, \phi_0, \Delta \phi$, 10bits for $\alpha$ with the total chromosome length of 25bits, an initial population of 16 and running limit of 200 generations. The second iteration of MGA used 2bits for $C, \phi_0, \Delta \phi$, 5bits for $\alpha$ with the total chromosome length of 11bits, initial population of 8 and running limit of 50 generations.

The simulation results are summarized in Table II, in which the estimated values are based on the 15 times of data fitting using the same set of SMS data. We calculate the deviation from its true value for each variable, i.e., $\delta_\delta = \hat{C} - C$ to reveal the measuring accuracy of our proposed approach. It is seen that the measuring accuracy for target displacement is very high and the measurement for $C$ and $\Delta \phi$ also shows promising improvement compared with other approaches. Note that the measuring accuracy for small displacement which results $\Delta \phi$ to present a value close to the other variables presents deterioration which is around 10 times higher than others. This might due to the algorithm deploys the same set of gene sizes and operator parameters for different situations. Therefore, better solutions are expected by refining the algorithm parameters.

In order to investigate the impact of noise on the proposed approach, simulations with different added noise levels were also carried out. The actual parameter values were used as $C = 0.6, \alpha = 4, L_0 = 0.2cm(thus \phi_0 = 0.8804), \Delta L = 0.8 \mu m (thus \Delta \phi = 12.8065)$. The simulation result is summarized in Table III. It is observed that the accuracy is satisfactory when SNR is better than 20 dB. It is also found that noise impairs the performance of MGA proportionally and MGA hardly improves the result any further when the SNR is less than 10dB. Note that a good fact is the displacement can always been measured with good accuracy despite of the presence of significant noise as high as 10 dB of SNR.

<table>
<thead>
<tr>
<th>Preset values</th>
<th>$\delta_\delta / C$</th>
<th>$\delta_\alpha / \phi_0$</th>
<th>$\delta_{\Delta \phi} / \Delta \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 4</td>
<td>0.8804</td>
<td>12.8065</td>
<td>0.42%</td>
</tr>
<tr>
<td>0.6 3</td>
<td>1.3207</td>
<td>28.81</td>
<td>0.31%</td>
</tr>
<tr>
<td>0.7 8</td>
<td>1.7609</td>
<td>19.2097</td>
<td>0.71%</td>
</tr>
<tr>
<td>0.5 3</td>
<td>1.3207</td>
<td>6.4032</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>$C$</th>
<th>$\delta_\delta / C$</th>
<th>$\alpha$</th>
<th>$\delta_\alpha / \alpha$</th>
<th>$\Delta \phi$</th>
<th>$\delta_{\Delta \phi} / \Delta \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.5987</td>
<td>0.22%</td>
<td>3.9952</td>
<td>0.12%</td>
<td>12.8052</td>
<td>0.01%</td>
</tr>
<tr>
<td>30</td>
<td>0.596</td>
<td>0.67%</td>
<td>3.9688</td>
<td>0.78%</td>
<td>12.804</td>
<td>0.019%</td>
</tr>
<tr>
<td>20</td>
<td>0.61</td>
<td>1.67%</td>
<td>4.1105</td>
<td>2.76%</td>
<td>12.8105</td>
<td>0.024%</td>
</tr>
<tr>
<td>10</td>
<td>0.5848</td>
<td>2.54%</td>
<td>4.5029</td>
<td>12.57%</td>
<td>12.8034</td>
<td>0.03%</td>
</tr>
<tr>
<td>5</td>
<td>0.6429</td>
<td>7.15%</td>
<td>3.5899</td>
<td>10.25%</td>
<td>12.7998</td>
<td>0.052%</td>
</tr>
</tbody>
</table>

V. EXPERIMENTAL RESULT

The proposed approach was tested with the OFISM setup as shown in Figure 3. A laser diode (LD) HL7851 with the wavelength of 785 nm is used in the experiment. The LD is biased with a dc current of 80mA and operates at single mode. A metal plate is used as the target and is made to vibrate harmonically with the frequency of 195Hz and the amplitude of 1.0μm by placing it close to a loudspeaker. The temperature is maintained at $25^\circ C \pm 0.1^\circ C$. The experiment condition is kept the same for acquiring all the SMS specimens. However, the target position is adjusted slightly in order to get different C levels, as a result, we obtained four blocks of SMSs which correspond to four different p-p amplitudes. For each block of SMS, we chose ten segments, each corresponding to a vibration period of the target. The proposed approach is applied over the ten segments and averaged to yield the estimated parameter values for each feedback level. We calculate the standard deviation of the estimated results to reveal the consistency of the measurement as shown in Table IV. It is observed the measuring results for $\alpha$ are close for all the blocks which is reasonable as the same LD is used for all the experiments. It is also seen that the feedback level factor is estimated in good accordance with [7] as the same set of SMS are used for both experiments. In order to better evaluate the measuring accuracy for target displacement, we compute their deviation from the nominal value of 1.0μm and a remarkable improvement is found with an accuracy of under 25nm.
VI. CONCLUSIONS

This paper presented a method capable of estimating target displacement in an optical feedback self-mixing system where the target is made on a simple harmonic vibration. The method is based on model calibration paradigm which is extremely simple in terms of system setup compared with other approaches. In particular, it employed the genetic algorithm in optimizing the system modal parameters to achieve the best matching between experimental data and the theoretical model of the laser diode self-mixing system. Computer simulation shows measurement of target displacement is immune to noises with good accuracy when SNR is as bad as 5dB. Experimental data is also used to test the proposed approach and reveals good accordance with computer simulation. A very promising measuring accuracy for target displacement is achieved as 25nm.

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