Transition Analysis for Moderate Feedback Self-Mixing Interferometry

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Abstract
We present a theoretical analysis on the locations of transition points in moderate feedback self-mixing signal which is a fundamental issue to be addressed in preprocessing experimentally acquired data. Locations for the start and end points for upward and downward switchings are calculated based on the Lang-Kobayashi model and discussions are given, which provides guidance in achieving more accurate signal normalization.

Keywords
Optical feedback interferometry, self-mixing effect, semiconductor laser

Disciplines
Physical Sciences and Mathematics

Publication Details
TRANSITION ANALYSIS FOR MODERATE FEEDBACK SELF-MIXING INTERFEROMETRY

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ABSTRACT

We present a theoretical analysis on the locations of transition points in moderate feedback self-mixing signal which is a fundamental issue to be addressed in pre-processing experimentally acquired data. Locations for the start and end points for upward and downward switchings are calculated based on the Lang-Kobayashi model and discussions are given, which provides guidance in achieving more accurate signal normalization.

Index Terms— Optical feedback interferometry, self-mixing effect, semiconductor laser

1. INTRODUCTION

In laser diode (LD) self-mixing interferometry, the sawtooth like self-mixing signal carries important information which can be used to estimate the metrological parameters of a moving target as well as those of the laser itself. In practical setup of such system, a small portion of the light emitted from a semiconductor laser is reflected by an external diffusive moving target and re-enters laser cavity. The mixing of reflected light with the light already presented in the laser cavity results in modulation of the laser intensity, referred as self-mixing signal (SMS) and can be monitored by a photo diode and analyzed to yield information about target movement or laser diode itself. Based on the strength of reflected signal, three feedback regimes are marked by a parameter known as feedback level factor (C), among which moderate feedback regime (1<C<4.6) is characterized by the sawtooth-like interferometric waveforms with abrupt transitions at every 2π intervals. This characteristic waveform has been employed for displacement measurement with basic resolution of λ/2 which corresponds to one fringe in the self-mixing signal [1] as well as laser parameter estimation in [2].

The measuring accuracy of such methods relies largely on the quality of acquired self-mixing signals, which usually contains significant noise due to various reasons. Hence we believe an appropriate signal processing procedure which aims at normalizing and denoising the self-mixing signal, has the potential to improve measuring accuracy. This paper present theoretical analysis on the locations of transition points in the sawtooth like self-mixing signal, which plays a key role in achieving better signal normalization. The whole possible parameter range for feedback level factor (C) and linewidth enhancement factor (α) is explored in order to show an overall tendency of the transition occurrence points, which determines the appearance of self-mixing signal accordingly. To our best knowledge, this is the first time that a discussion on this issue is given.

2. THEORETICAL ANALYSIS

The theoretical modal of optical feedback in laser diodes has been described by various authors [3-6] and can be summarized as follows:

\[ \phi_f(t) = \phi_0(t) - C \cdot \sin[\phi_f(t) + \arctan(\alpha)] \] (1)

\[ P(t) = P_0[1 + mF(t)] \] (2)

\[ F(t) = \cos(\phi_f(t)) \] (3)

where \( \phi_0(t) \) and \( \phi_f(t) \) are the laser phases without and with feedback respectively. \( \tau = 2L/c \) is the round trip time between LD and the external target. C is the feedback level factor and \( \alpha \) denotes the linewidth enhancement factor. \( P(t) \) and \( P_0 \) represent the laser output power with and without feedback respectively, and \( m \) the modulation index with typical value of \( 10^3 \). \( F(t) \) is defined by Eq.(3) and is a periodic function of \( \phi_f(t) \) with a period of \( 2\pi \). Obviously, \( P(t) \) and \( F(t) \) have the same shape but different amplitude and \( F(t) \) always falls in the range [-1,1].

When 1<C<4.6 (denoted as moderate feedback), there are some time intervals where three solutions exist to Eq. (1), two of which are stable and can occur in practice. As an example, Fig. 1 gives plot of \( F(t) \) versus \( \phi_0(t) \) for \( C=3, \alpha=4 \) by solving Eq. (1)-(3). When \( \phi_0(t) \) increases from point O, \( F(t) \) evolves along the theoretical waveform until
reaches point A, where a downward transition to point A’ occurs. Then as $\phi_h(\tau)$ increases again, the next transition will occur at point C. Vice versa, when $\phi_h(\tau)$ decreases from point C’ and reaches point B, an upward transition to B’ takes place. This accounts for the unresemblance between the positive and negative fringes of a self-mixing signal.

![Figure 1](image.png)

Figure 1. Theoretical plot of $F(\tau)$ versus $\phi_h(\tau)$ for $C=3$, $\alpha=4$

Developing this point further, when a self-mixing waveform is observed in an experimental setup and data acquired by means of an analog/digital (A/D) conversion device, some form of data processing has to be performed due to the noises introduced in the acquisition process as well as to normalize the signal in the interval [-1,1] before further measurements can be done. Usually the positive portion of signal is normalized with the maximum of 1 and negative portion normalized with the minimum of -1. However, a quick glance at Fig.1, where the downward transitions occur at around $F(\tau)=0.8$, reveals the limitations and inaccuracy of such a simple operation. Apart from this, the position of A’ and B’ varies in significant range with different C and $\alpha$ values, whose correct locating is also of great importance for better measuring accuracy. This can be more clearly shown in Fig. 2 where SMS with different C and $\alpha$ values are plotted. The theoretical locating of these points will also help get rid of amplitude drift (i.e. waveform envelope) by normalizing them to their real values.

For the purpose of calculating the exact locations where the transitions occur, we first look at the relationship between $\phi_f(\tau)$ and $\phi_p(\tau)$ as depicted in Eq. (1) and plotted in Fig. 3. As $\phi_f(\tau)$ always varies monotonically with $\phi_h(\tau)$, when $\phi_h(\tau)$ increases and reaches point A where $\frac{d\phi_f(\tau)}{d\phi_h(\tau)} = \infty$, any further increase in $\phi_h(\tau)$ will cause a discontinuous jump to point A’ in $\phi_f(\tau)$ which is the largest solution due to mode competition. On the other hand, when $\phi_h(\tau)$ decreases and reaches point B, further decrease will cause a drop to B’ which is the smallest solution. Hence a monotonic mapping between $\phi_h(\tau)$ and $\phi_f(\tau)$ can be established and $F(\tau)$ is obtained according to Eq. (3).

![Figure 2](image.png)

Figure 2. Plot of SMS (a) $C=3$, $\alpha=4$ (b) $C=4.2$, $\alpha=2$ (c) $C=1.6$, $\alpha=6$

![Figure 3](image.png)

Figure 3. Relationship between $\phi_f(\tau)$ and $\phi_h(\tau)$

An overall view on the locations of downward and upward transition points with regard to different C and $\alpha$ values are shown in Fig. 4 and 5 for increasing and decreasing $\phi_h(\tau)$ respectively. The behavior of the downward transitions when $\phi_h(\tau)$ increases can be summarized as:

(i) System behaves very differently for small LEF such as $\alpha=1$. Reverse switching exists as C increases over 3.2 when the SM presents a coil-like waveform as shown in Fig 6.

(ii) The transition start points varies in relatively large ranges for smaller feedbacks, i.e. 1<$C$<$2.2$ and present similar values for larger feedbacks. For example, when $\alpha=3$ C=1.2, transition start at $F(\tau)=0.3$. Comparatively, when $\alpha=3$ C=2.4, transition start at $F(\tau)=0.7$ and for C=4.2, transition start at $F(\tau)=0.9$.

(iii) The transition end points increases from -1 with increasing feedback level until above zero, which means no zero-crossing point will be observed in positive fringes of a self-mixing signal. In general, the larger the LEF ($\alpha$), the zero-crossing point will disappear at stronger feedback level. For instance, when $\alpha=1$, the zero-crossing point doesn’t exist.
for larger feedback level than \( C=2.2 \); when \( \alpha=9 \), zero-crossing point disappears only for \( C>4 \).

(iv) As the feedback level increases, the SMS fringes become shorter with more remarkable hysteresis.

(v) The system performs similarly when \( 5<\alpha<9 \) for different feedback levels while behaves very differently for small \( \alpha \).

The similar trends hold true when \( \phi_{n}(\tau) \) decreases except that negative fringes of self-mixing signal always present zero-crossing points. It is also worth noting that the downward and upward transitions are asymmetric which renders the SM signals with remarkably different positive and negative extremes, particularly in the case of lower feedback levels (\( C<2 \)) and small LEF \( \alpha \) such as \( \alpha=1 \) or 3. This implies the even normalization between -1 and 1 for all self-mixing signals is inappropriate under these circumstances. For instance, a self-mixing signal obtained at \( C=1.8 \), \( \alpha=3 \) ought to be normalized with maximum of 0.6 and minimum of -0.9 instead.

![Figure 4. Trend of transition locations when \( \omega_{01} \) increases](image)

Upper: Transition start location; Lower: Transition end location

![Figure 5. Trend of transition locations when \( \omega_{01} \) decreases](image)

Lower: Transition start location; Upper: Transition end location

![Figure 6. The theoretical (thin line) and real (thick line) self-mixing signal waveform as \( \alpha_{r} \) increases](image)

(a) \( C=1.4 \), \( \alpha=1 \) (b) \( C=3.2 \), \( \alpha=1 \) (c) \( C=4.2 \), \( \alpha=1 \)

### 3. EXPERIMENTS

We verify the above observations obtained from theoretical analysis by using experimental setup as shown in Fig. 7 [7]. An SL with the wavelength of 785nm was biased with a dc current of 80mA. A metal plate was used as target, which is made to vibrate harmonically by placing it close to a loudspeaker driven by a sinusoidal signal of 195Hz with peak-to-peak (p-p) amplitude of 10V. Temperature is controlled at 25±0.1°C. The SMS was detected by the monitor photodiode (PD) and was amplified by a trans-impedance amplifier.

![Figure 7. OFSM experimental setup](image)

The target position was adjusted slightly to get different feedback levels until the typical sawtooth like moderate feedback waveforms were observed. We acquired a few sets of data and estimated the \( C \) and \( \alpha \) values using the approach proposed in [2], the corresponding waveforms are shown in Fig. 8. Enlarged plot of one period of each signal after removing impulsive noises (sparkles) with 5 point median filter is presented in Fig. 9. We then normalized the filtered signal to the extremes as indicated in Fig. 4 & 5, i.e. (a) \( C=2 \), \( \alpha=3.1 \) the normalization is performed between maximum \( N_{\text{max}}=0.6635 \) and minimum \( N_{\text{min}}=-0.9797 \) (b)
C=3.3, α=3.1, N_{max}=0.81, N_{min}=-0.9998. The resulting waveforms are shown in Fig. 10, whose characteristics are in good accordance with the observations in the previous section. The waveform fringes when C=3.3 is much shorter than that of C=2, particularly the positive fringes for C=3.3 do not have zero-crossing point which complies with the result in Fig. (4).

![Figure 8. Experimental Self-mixing Signals](image)

(a) C=2, α=3.1 (b) C=3.3, α=3.1

![Figure 9. Enlarged Plot of Experimental Self-mixing Signal](image)

(a) C=2, α=3.1 (b) C=3.3, α=3.1

5. REFERENCES


4. CONCLUSION

We have presented theoretical analysis on the transition points and characteristics of moderate feedback laser diode self-mixing signals based on Lang-Kobayashi equations. Consistent trend is found for the appearance of self-mixing waveforms. The behavior of interferometric system for smaller LEF (α) varies significantly from that with larger LEF. The result is useful in achieving better normalization which is an important subtask in pre-processing deformed and noisy self-mixing signals. Our observations are in good accordance with experimental results.