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Using DEA cross-efficiency evaluation for suppliers ranking in the presence of non-discretionary inputs

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Abstract

Supply chain includes all activities associated with the flow and transformation of goods from the raw material stage through to the end user. Supplier selection is one of the most important parts of supply chain management (SCM). For selecting suppliers, data envelopment analysis (DEA), as a multiple criteria decision making tool, has been applied for several times. However, sometimes in supplier selection problem, there may exist some criteria that are beyond the control of a management. These criteria are called non-discretionary or exogenously fixed factors. Since in traditional treatment of non-discretionary inputs in DEA, free reign is given when deciding for each decision making unit (DMU) which outputs and inputs to emphasise, many different avenues are present by which a DMU can appear efficient. Therefore, it is common to have many DMUs that are relatively efficient. In addition, since each DMU has its own set of weights, all of its weight might be put on a single output and input. As a result, the objective of this paper is to propose a cross-efficiency model which is able to consider non-discretionary inputs. A numerical example demonstrates the application of the proposed model in supplier selection context. Copyright 2013 Inderscience Enterprises Ltd.

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Keywords: data envelopment analysis; DEA; cross-efficiency; supplier ranking; non-discretionary inputs.

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1 Introduction

The purpose of supplier selection is to determine the optimal supplier who can offer the best products or services for the customer and become a part of the organisation's supply chain (Ebrahim et al., 2009). As a supplier becomes a part of the established supply chain, it will have a lasting effect on the efficiency and effectiveness of the entire supply chain (Chen et al., 2006). In the past few decades, there have been major changes in the supplier selection practices. The competition has risen and the market has become globally operating. In such a scenario, it has become highly difficult for industries to produce low cost and high quality products successfully without proper suppliers (Weber et al., 1991). Effective supplier evaluation and selection strategies can directly impact

supply chain performance, resulting in organisational productivity and profitability. Ghodsypour and O'Brien (1998) declare that in manufacturing industries the raw materials and component parts can equal up to 70% of the product cost. In such circumstances, the purchasing department can play a key role in cost reduction by selecting good suppliers.

This paper proceeds as follows. In Section 2, literature review is presented. Section 3, introduces the model which selects the suppliers. Numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2 Literature review

Various studies on the supplier selection are briefly summarised in the following subsections.

2.1 Analytic hierarchy process

Xia and Wu (2007) presented an integrated approach of analytic hierarchy process (AHP) improved by rough sets theory and multi-objective mixed integer programming. This model determines the best set of suppliers and their corresponding order quantities. Ghodsypour and O'Brien (1998) proposed an integration of AHP and linear programming to consider both tangible and intangible factors in choosing the best suppliers and placing the optimum order quantities such that the total value of purchasing (TVP) becomes the maximum. Chan et al. (2007) developed an AHP-based multi-criterion decision making approach of supplier selection.

2.2 Analytic network process

Gencer and Gürpınar (2007) suggested an analytic network process (ANP)-based model for an electronic company for supplier evaluation and selection with respect to various evaluating criteria. Sarkis and Talluri (2002) believe that supplier evaluation factors would influence each other, and the internal interdependency need to be considered in the evaluation process. The authors applied ANP to evaluate and select the best supplier with respect to organisational factors and strategic performance metrics, which consist of seven evaluating criteria.

2.3 Fuzzy set theory

Lin (2009) suggested an integrated fuzzy analytic network process multi-objective linear programming (FANP-MOLP) approach for identifying top suppliers by considering the effects of interdependence among the selection criteria, as well as to achieve optimal allocation of orders among the selected suppliers. Vinodh et al. (2011) used fuzzy ANP approach for the supplier selection process in an Indian electronics switches manufacturing company. Kuo et al. (2010) proposed integration of particle swarm optimisation (PSO)-based fuzzy neural network (FNN) and artificial neural network for supplier selection. This study is intended to develop an intelligent supplier decision

support system which is able to consider both the quantitative and qualitative factors. It is composed of:

- 1 the collection of quantitative data such as profit and productivity
- 2 a PSO-based FNN to derive the rules for qualitative data
- 3 a decision integration model for integrating both the quantitative data and fuzzy knowledge decision to achieve the optimal decision.

Amin et al. (2011) proposed a decisional model for supplier selection which consists of two phases. In the first phase, quantified strengths, weaknesses, opportunities and threats (SWOT) analysis are applied for evaluating suppliers. The linguistic variables and triangular fuzzy numbers are used to quantify variables. In the second phase, a fuzzy linear programming model is applied to determine the order quantity. Sarkar and Mohapatra (2006) used the performance and the capability as two major measures in the supplier evaluation and selection problem. The authors used the fuzzy set approach to account for the imprecision involved in numerous subjective characteristics of suppliers. A hypothetical case was adopted to illustrate how the two best suppliers were selected with respect to four performance-based and ten capability-based factors.

2.4 Mathematical programming

Jayaraman et al. (1999) proposed a mixed integer linear programming model to solve the supplier selection and order quantity allocation problem. Mendoza and Ventura (2010) proposed a mixed integer non-linear programming model to determine an optimal inventory policy that coordinates the transfer of items between different stages of a serial supply chain, while properly allocating orders to selected suppliers. Wadhwa and Ravindran (2007) modelled the supplier selection problem as a multi-objective programming (MOP) problem, in which there are three objective functions (i.e., minimisation of price, lead time, and rejects). Three solution approaches, including weighted objective method, goal programming (GP) method, and compromise programming, were used to compare the solutions.

2.5 Data envelopment analysis

Weber (1996) applied data envelopment analysis (DEA) in supplier evaluation for an individual product and demonstrated the advantages of applying DEA to such a system. In this study, the criteria for selecting suppliers were significant reductions in costs, late deliveries and rejected materials. Weber et al. (2000) also presented an approach for evaluating the number of suppliers to employ in a procurement situation using MOP and DEA. Wu (2009) used DEA, decision trees (DTs) and neural networks to assess supplier performance. The model consists of two modules: Module 1 applies DEA and classifies suppliers into efficient and inefficient clusters based on the resulting efficiency scores. Module 2 utilises firm performance-related data to train DT, neural networks model and apply the trained DT model to new suppliers. Wu et al. (2007) presented a so-called augmented imprecise DEA for supplier selection. The proposed model is able to handle imprecise data and allow for increased discriminatory power (i.e., to discriminate efficient suppliers from poor performing suppliers). Mahdilo et al. (2012) proposed an

algorithm for ranking suppliers in the presence of volume discount offers in terms of multiple criteria in the context of cross-efficiency evaluation.

In this paper, DEA as a non-parametric and multiple criteria decision making tool is used to suppliers' evaluation. DEA was first introduced by Charnes, Cooper, and Rhodes (CCR) in 1978 and it is a linear-programming-based methodology that uses multiple inputs and multiple outputs to calculate efficiency scores. Since then, DEA is used in different settings like efficiency measurement of container-terminals (Bichou, 2011), airports (Chow et al., 2010), and seaports (Kamble et al., 2010).

The efficiency score for each decision making unit (DMU) is defined as a weighted sum of outputs divided by a weighted sum of inputs, where all efficiencies are restricted to a range from 0 to 1. To avoid the potential difficulty in assigning these weights among various DMUs, a DEA model computes weights that give the highest possible relative efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights (Liu et al., 2000). DEA is a robust, standardised and transparent methodology. Furthermore, it possesses additional features which made it suitable to be used as a tool for evaluating the suppliers. The advantages of DEA are as below (Wong and Wong, 2008):

- DEA is an effective tool for evaluating the relative value of DMUs in the presence of multiple performance measures.
- DEA is able to address the complexity arising from the lack of a common scale of measurement.
- In DEA, one need not assume a priori the existence of a particular production function for weighting and aggregating inputs or outputs.
- The objectivity stemming from DEA weighting variables during the optimisation procedure frees the analysis from subjective estimates and randomness.

With the features and inherent characteristics of DEA discussed above, DEA is justified to be used as an evaluation tool for suppliers.

However, sometimes in suppliers' evaluation problem, there may exist some criteria that should be considered as non-discretionary factors. Discretionary models for evaluating the efficiency of suppliers assume that all criteria are discretionary, that is, controlled by the management of each supplier and varied at its discretion. Thus, failure of a supplier to produce maximal output levels with minimal input consumption results in a decreased efficiency score. In any realistic situation, however, there may exist exogenously fixed or non-discretionary criteria that are beyond the control of a management (Farzipoor Saen, 2009b). In an analysis of a network of fast food restaurants, Banker and Morey (1986) illustrated the impact of exogenously determined inputs that are not controllable. In their study, each of the 60 restaurants in the fast food chain consumes six inputs to produce three outputs. The three outputs (all controllable) correspond to breakfast, lunch, and dinner sales. Only two of the six inputs, expenditures for supplies and expenditures for labour, are discretionary. The other four inputs (age of store, advertising level, urban/rural location, and presence/absence of drive-in capability) are beyond the control of the individual restaurant manager. Their analysis clearly demonstrated the value of accounting for the non-discretionary character of these inputs explicitly in the DEA models they employed; the result is identification of a considerably enhanced opportunity for targeted savings in the controllable inputs and targeted

increases in the outputs. Liu et al. (2000) for selecting the best suppliers considered distance and supply variety as non-discretionary input and non-discretionary output, respectively. Farzipoor Saen (2009b) proposed a new DEA model for selecting the best suppliers in the presence of non-discretionary factors and imprecise data. As well, Farzipoor Saen (2009a) proposed a model for ranking suppliers in the presence of weight restrictions, non-discretionary factors, and cardinal and ordinal data. Recently, Noorizadeh et al. (2011) developed a model to consider dual-role factors, non-discretionary inputs, and weight restrictions to select suppliers.

The approaches presented in the works of Banker and Morey (1986), Liu et al. (2000), Farzipoor Saen (2009a, 2009b) and Noorizadeh et al. (2011) had considerable contribution for considering non-discretionary factors in the DEA context. However, their treatment of non-discretionary factors in DEA models suffers from some limitations. Since, in traditional DEA, free reign is given when deciding for each DMU which outputs and inputs to emphasise, many different avenues are present by which a DMU can appear efficient. Therefore, it is common to have many DMUs that are relatively efficient. In addition, since each DMU has its own set of weights, all of its weight might be put on a single output and input. While this is permissible, it may not be realistic. To overcome these problems, we propose to incorporate non-discretionary inputs in the cross-efficiency method introduced by Sexton et al. (1986) and developed by Doyle and Green (1994). The main idea of cross-efficiency is to use DEA in a peer evaluation instead of a self-evaluation mode. Anderson et al. (2002) argue that, cross evaluation proponents often two main advantages to its use. First, it usually creates a unique ordering among the DMUs. With cross evaluation, since each DMU is rated not only by its own weighting scheme but the schemes of the others also, this amalgamation of weighting schemes makes it far more difficult to have ties and, in effect, creates a unique ordering in practice. Second, cross evaluation appears to eliminate unrealistic weighting schemes that might be used by the DMUs. Under a cross evaluation, once the DMU has a chosen weighting scheme which has been applied to all DMUs, the efficiency value given to each DMU is set aside forming a cross-efficiency matrix. Once the matrix is filled, each DMU has not only its own self-evaluation but also the peer evaluations it has received via the other DMUs in the sample. The average across self and peer evaluations represent a DMU's cross-efficiency value. A DMU which has a high cross-efficiency value has, therefore, passed a more rigorous test since it cannot only make itself look good but is considered efficient by the majority of its peers.

The above discussions make it more reasonable to model the cross-efficiency formulation of considering non-discretionary factors in DEA models. However, by modelling the cross-efficiency formulation of non-discretionary inputs, a new problem can be occurred. Consider following function, which is the objective function of the model proposed by Banker and Morey (1986).

$$\max \sum_{r=1}^k u_r y_{ro} - \sum_{i \in I_{ND}} v_{iND} x_{io} \quad (1)$$

Presence of $\sum_{i \in I_{ND}} v_{iND} x_{io}$ with the negative sign in the objective function causes the cross-efficiency score of some DMUs to be negative. To avoid this problem and to

consider non-discretionary inputs as well, we impose a kind of restriction to the model proposed by Banker and Morey (1986).

The objective of this paper is to propose a cross-efficiency model which considers non-discretionary inputs. As well, by imposing a kind of restriction into the model, the positive cross-efficiency score of all DMUs is guaranteed.

To the best of knowledge of authors, there is not any reference that uses cross-efficiency model and non-discretionary inputs simultaneously. Some of the contributions of this paper are as below:

- The proposed model evaluates suppliers in a multi criteria context.
- Supplier selection is a straightforward process carried out by the proposed model.
- The proposed model considers non-discretionary inputs for supplier selection.
- To achieve the peer appraisal of suppliers instead of their self-appraisal, the cross-efficiency model which considers non-discretionary inputs is developed.

3 Proposed model

Charnes et al. (1978) proposed CCR model (Model 2) as a mathematical programming procedure for evaluating the relative efficiency of multiple DMUs that involve multiple inputs and multiple outputs. The used nomenclatures in this paper are summarised in Table 1.

Table 1 The nomenclatures

DMU_o	The decision making unit under investigation
$k = 1, \dots, K$	Collection of DMUs (suppliers)
$r = 1, \dots, R$	The set of outputs
$i = 1, \dots, I$	The set of inputs
I_D	The set of discretionary inputs
I_{ND}	The set of non-discretionary inputs
x_{io}	The i^{th} input of the DMU_o
y_{ro}	The r^{th} output of DMU_o
v_{iD}	The weight for i^{th} discretionary input
v_{iND}	The weight for i^{th} non-discretionary input
u_r	The weight for r^{th} output
x_{ik}	The i^{th} input of DMU_k
y_{rk}	The r^{th} output of DMU_k
E_{ok}	Shows the relative efficiency of DMU_k with optimal weights for inputs and outputs of DMU_o
E_{oo}	Is the efficiency score of DMU_o by its optimal weights

The input-oriented CCR model evaluates the suppliers under investigation (DMU_o) ($o = 1, \dots, K$) by solving Model (2).

$$\begin{aligned}
\max \quad & h_A = \sum_{r=1}^R u_r y_{ro} \\
\text{s.t.} \quad & \sum_{i=1}^I v_i x_{io} = 1 \\
& \sum_{r=1}^R u_r y_{rk} - \sum_{i=1}^I v_i x_{ik} \leq 0, \quad k = 1, \dots, K, \\
& v_i \geq 0 \quad i = 1, \dots, I, \\
& u_r \geq 0, \quad r = 1, \dots, R.
\end{aligned} \tag{2}$$

3.1 CCR-non-discretionary inputs

Suppose that the input variables to be partitioned into subsets of discretionary (D) and non-discretionary (N) variables. Thus, $i \{1, \dots, I\} = I_D \cup I_N$, $I_D \cap I_N = \emptyset$.

Banker and Morey (1986) developed Model (3) to consider non-discretionary inputs. This model is called CCR-non-discretionary inputs.

$$\begin{aligned}
\max \quad & E_{oo} = \sum_{r=1}^R u_r y_{ro} - \sum_{i \in I_{ND}} v_{iND} x_{io} \\
\text{s.t.} \quad & \sum_{i \in I_D} v_{iD} x_{io} = 1, \\
& \sum_{r=1}^R u_r y_{rk} - \left(\sum_{i \in I_D} v_{iD} x_{ik} + \sum_{i \in I_{ND}} v_{iND} x_{ik} \right) \leq 0, \quad k = 1, \dots, K, \\
& v_{iND} \geq 0, \quad i \in I_{ND}, \\
& v_{iD} \geq 0, \quad i \in I_D, \\
& u_r \geq 0, \quad r = 1, \dots, R.
\end{aligned} \tag{3}$$

They proved that, the way to model such inputs is to move them to the output side, but with the opposite sign. This idea often arises in situations where there are criteria that are beyond the control of the management but influence the efficiency of DMUs. Thus, in evaluating process, these factors are generally expected to remain at their current level.

3.2 Cross-efficiency formulation of CCR-non-discretionary inputs

At this juncture to create a unique ordering among the efficient DMUs and to eliminate unrealistic weighting schemes in Model (3), we develop the cross-efficiency form of this model. For each DMU_o ($o = 1, \dots, K$), in Model (3), we can obtain a set of optimal weights (multipliers) $(u_r^*, v_{iND}^*, v_{iD}^*)$. Using these set of weights, the cross-efficiency for any DMU_k ($k = 1, \dots, K$), is then calculated as:

$$E_{ok} = \frac{\sum_{r=1}^R u_r^* y_{rk} - \sum_{i \in I_{ND}} v_{ioND}^* x_{ik}}{\sum_{i \in I_D} v_{ioD}^* x_{ik}} \tag{4}$$

where E_{ok} shows the relative efficiency of DMU_k with optimal weights for inputs and outputs of DMU_o . One can compute the average of the efficiencies in each column to get

a measure of how the DMUs associated with the column are rated by the rest of the DMUs. Good operating practices more likely to be exhibited by relatively efficient DMUs offering high average efficiencies in their associated columns in the cross-efficiency matrix. Since Model (3) will be run n times for n DMUs, respectively, each DMU will get n efficiency scores, which construct a $n \times n$ matrix, called cross-efficiency matrix.

For DMU_k ($k = 1, \dots, K$), the average of all E_{ok} ($o = 1, \dots, K$), can be used as an efficiency measure for DMU_k , and will be referred to as the cross-efficiency score for DMU_k . The average is calculated as below.

$$\bar{E}_k = \frac{1}{n} \sum_{o=1}^K E_{ok} \quad (5)$$

The non-uniqueness of the DEA optimal weights possibly reduces the usefulness of the cross-efficiency. To overcome this problem, Doyle and Green (1994) suggested the use of aggressive and benevolent cross evaluation. A cross evaluation is aggressive/benevolent in the sense that it selects a set of weights which not only maximise the efficiency of a particular DMU under evaluation, but also minimise/maximise the efficiencies of all other DMUs in some sense. We develop the aggressive formulation of Model (3) and represent it as Model (6). Note that the benevolent formulation has the same set of constraints except that the objective function is maximised.

$$\begin{aligned} \min \quad & h_B = u_r \sum_{k \neq o} y_{rk} - v_{iND} \sum_{k \neq o} x_{ik} \\ \text{s.t.} \quad & v_{iD} \sum_{k \neq o} x_{ik} = 1, \\ & \sum_{r=1}^R u_r y_{rk} - \left(\sum_{i \in I_D} v_{iD} x_{ik} + \sum_{i \in I_{ND}} v_{iND} x_{ik} \right) \leq 0, \quad k \neq o, \\ & \left(\sum_{r=1}^R u_r y_{ro} - \sum_{i \in I_{ND}} v_{iND} x_{io} \right) - E_{oo} \left(\sum_{i \in I_D} v_{iD} x_{io} \right) = 0, \quad k = 1, \dots, K, \\ & v_{iND} \geq 0, \quad i \in I_{ND}, \\ & v_{iD} \geq 0, \quad i \in I_D, \\ & u_r \geq 0, \quad r = 1, \dots, R. \end{aligned} \quad (6)$$

where E_{oo} is the efficiency of DMU_o obtained from Model (3).

3.3 Modified CCR-non-discretionary inputs

The E_{ok} for some DMUs can be negative. The negative E_{ok} score is due to $\sum_{r=1}^R u_r y_{rk} - \sum_{i \in I_{ND}} v_{iND} x_{ik} < 0$ for some DMU_k , i.e., some DMU_k will have negative efficiency ratios when they use a set of optimal weights obtained when DMU_o is under evaluation.¹ Normally, we want every output-input efficiency ratio to be positive regardless of the chosen weights. To avoid negative efficiency ratios problem, Wu et al. (2009) proposed adding $\sum_{r=1}^k u_r y_{rj} - w_d \geq 0$ into variable return to scale (VRS) model when calculating the cross-efficiency scores. Therefore, we suggest adding

$\sum_{r=1}^R u_r y_{rk} - \sum_{i \in I_{ND}} v_{iND} x_{ik} \geq 0$ into the Model (3) to prevent negative efficiency ratios.

This idea leads to the Model (7) which is called the modified CCR-non-discretionary inputs model.

$$\begin{aligned}
 \max \quad & h_C = \sum_{r=1}^R u_r y_{ro} - \sum_{i \in I_{ND}} v_{iND} x_{io} \\
 \text{s.t.} \quad & \sum_{i \in I_D} v_{iD} x_{io} = 1, \\
 & \sum_{r=1}^R u_r y_{rk} - \left(\sum_{i \in I_D} v_{iD} x_{ik} + \sum_{i \in I_{ND}} v_{iND} x_{ik} \right) \leq 0, \quad k = 1, \dots, K, \\
 & \sum_{r=1}^R u_r y_{rk} - \sum_{i \in I_{ND}} v_{iND} x_{ik} \geq 0, \\
 & v_{iND} \geq 0, \quad i \in I_{ND}, \\
 & v_{iD} \geq 0, \quad i \in I_D, \\
 & u_r \geq 0, \quad r = 1, \dots, R.
 \end{aligned} \tag{7}$$

3.4 Modified cross-efficiency formulation of CCR-non-discretionary inputs

Where suppliers use the best possible set of weights for their inputs and outputs, traditional DEA can suffer from unrealistic weighting scheme of some suppliers. These suppliers try to disregard those criteria on which they have weakly performed by giving them a zero weight. It follows that these suppliers might be identified as efficient, while this efficiency score of 1 is only due to the type of weights that they have used. In addition, when the number of inputs and outputs increases, the discrimination power of traditional DEA models may decrease. Consequently, these cannot distinguish among those suppliers on the efficiency frontier. To overcome these kinds of problems, a modified cross-efficiency evaluation model which can consider non-discretionary inputs is developed. The proposed model is as below.

$$\begin{aligned}
 \min \quad & h_D = u_r \sum_{k \neq o} y_{rk} - v_{iND} \sum_{k \neq o} x_{ik} \\
 \text{s.t.} \quad & v_{iD} \sum_{k \neq o} x_{ik} = 1, \\
 & \sum_{r=1}^R u_r y_{rk} - \left(\sum_{i \in I_D} v_{iD} x_{ik} + \sum_{i \in I_{ND}} v_{iND} x_{ik} \right) \leq 0, \quad k \neq o, \\
 & \sum_{r=1}^R u_r y_{rk} - \sum_{i \in I_{ND}} v_{iND} x_{ik} \geq 0, \quad k = 1, \dots, K, \\
 & \left(\sum_{r=1}^R u_r y_{ro} - \sum_{i \in I_{ND}} v_{iND} x_{io} \right) - E_{oo} \left(\sum_{i \in I_D} v_{iD} x_{io} \right) = 0, \quad k = 1, \dots, K, \\
 & v_{iND} \geq 0, \quad i \in I_{ND}, \\
 & v_{iD} \geq 0, \quad i \in I_D, \\
 & u_r \geq 0, \quad r = 1, \dots, R.
 \end{aligned} \tag{8}$$

4 Numerical example

In order to demonstrate the application of the proposed model in supplier selection context, the dataset is partially taken from Farzipoor Saen (2009a). The inputs for selecting suppliers include total cost of shipments (TC), and distance (D). The outputs utilised in the study are number of shipments to arrive on time (NOT), number of bills received from the supplier without errors (NB), and number of parts (supply variety, SV) that a supplier supplies. Distance (D) is considered as a non-discretionary input. We divide non-discretionary factors into two parts of temporary and permanent. The temporary refers to those factors that DMU can control them after spending time and energy. Suppliers distance can be considered as an example of this type. On the other hand, Permanent non-discretionary factors refer to those factors that DMU cannot control them at all. For example, in the efficiency evaluation of farming lands, amount of rains as an input, is out of the control permanently.

Table 2 displays the dataset for 18 suppliers.

Table 2 Dataset for 18 suppliers

Supplier (DMU)	Inputs		Outputs		
	TC (1,000\$)	D(km)	NOT	NB	SV
	x_{1k}	x_{2k}	y_{1k}	y_{2k}	y_{3k}
1	253	249	187	90	2
2	268	643	194	130	13
3	259	714	220	200	3
4	180	1,809	160	100	3
5	257	238	204	173	24
6	248	241	192	170	28
7	272	1,404	194	60	1
8	330	984	195	145	24
9	327	641	200	150	11
10	330	588	171	90	53
11	321	241	174	100	10
12	329	567	209	200	7
13	281	567	165	163	19
14	309	967	199	170	12
15	291	635	188	185	33
16	334	795	168	85	2
17	249	689	177	130	34
18	216	913	167	160	9

Table 3 Results of evaluation using Model (3)

<i>Supplier (DMU)</i>	<i>Efficiency score</i>	<i>Rank</i>
1	0.926	9
2	0.877	10
3	1	1
4	1	1
5	1	1
6	1	1
7	0.821	12
8	0.746	15
9	0.737	16
10	1	1
11	0.678	17
12	0.821	11
13	0.816	13
14	0.778	14
15	0.968	8
16	0.587	18
17	1	1
18	0.993	7

Table 4 Efficiency scores where 'D' is treated as a discretionary input

<i>Supplier (DMU)</i>	<i>Efficiency score</i>	<i>Rank</i>
1	0.929	9
2	0.879	10
3	1	1
4	1	1
5	1	1
6	1	1
7	0.827	13
8	0.746	16
9	0.741	17
10	1	1
11	0.842	11
12	0.841	12
13	0.816	14
14	0.778	15
15	0.968	8
16	0.600	18
17	1	1
18	0.993	7

Table 3 illustrates the efficiency scores of suppliers, using Model (3), and their ranking results. In this model, each supplier seeks to maximise its efficiency score by choosing a set of optimal weights for all inputs and outputs. In this evaluation, the best suppliers are suppliers 3, 4, 5, 6, 10, and 17 which their efficiency scores equal to unity.

Now, we analyse the effects of considering 'D' as a non-discretionary input on the results. Therefore, we re-solve the problem by considering 'D' as a discretionary factor. The results are shown in Table 4. By comparing the results, it can be seen that the ranking of some suppliers by two strategies (considering distance as a non-discretionary and discretionary factor) is different. A remarkable difference is found in supplier 11, whose efficiency score increased by 0.164. As a result, supplier 11 promotes its rank from 17 to 11. This is due to the fact that when we consider distance as a discretionary input, weight of this factor increases from 0.000124 to 0.004149. The rationale behind this is that supplier 11 is the second best supplier regarding this factor.

Table 5 Results of evaluation via Model (7) and Model (8)

<i>Supplier (DMU)</i>	<i>Efficiency score obtained by Model (7)</i>	<i>Rank</i>	<i>Modified cross-efficiency obtained by Model (8)</i>	<i>Rank</i>
1	0.926	9	0.622	13
2	0.877	10	0.704	7
3	1	1	0.854	3
4	1	1	0.618	14
5	1	1	0.940	2
6	1	1	0.963	1
7	0.821	11	0.440	17
8	0.746	15	0.623	12
9	0.737	16	0.608	15
10	1	1	0.661	10
11	0.678	17	0.518	16
12	0.818	12	0.681	9
13	0.816	13	0.702	8
14	0.778	14	0.654	11
15	0.968	8	0.832	5
16	0.587	18	0.391	18
17	1	1	0.841	4
18	0.993	6	0.805	6

The problem now becomes selecting a supplier from efficient suppliers (when distance is considered as a non-discretionary input). As is seen, Model (3) cannot give a complete ranking and there are ties among six efficient suppliers. Therefore, Model (6) is used to derive the suppliers' cross-efficiency score and their complete ranking. However, by using Model (6) which is the aggressive formulation of Model (3), the cross-efficiency scores of some DMUs are negative. For example, the cross-efficiency scores of suppliers 1, 2, 3, 4, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, and 18 have negative efficiency ratios when they use a set of optimal weights obtained when DMU₁₀ is under evaluation. Therefore, to guarantee the non-negativity of all suppliers cross-efficiency, Models (7) and (8) are

used. The new cross-efficiency matrix is shown in Table 6 in the Appendix. Table 5 shows the suppliers final efficiency scores obtained by Model (7) and their cross-efficiency scores. Also, suppliers' final ranking derived by Model (7) and cross-efficiency approach is presented in this table. Cross-efficiency ranking is used as the final ranking of suppliers.

As the last column of Table 5 presents, supplier #6 is the most efficient supplier and is the first candidate for selection.

5 Concluding remarks

Supply chain focuses on the improvement of customer service, profitability and business performance. Strategic partnership with better suppliers needs to be formed to improve quality, flexibility as well as to reduce lead time. The problem of supplier selection is a multi-criteria decision making (MCDM) problem in the presence of many criteria. A decision maker needs to make use one of the MCDM methods (Ayag and Ozdemir, 2009). To this end, we used DEA as a multiple criteria decision making tool to evaluate suppliers. To rank suppliers and to consider non-discretionary inputs as well, cross-efficiency formulation of the model proposed by Banker and Morey (1986) is developed.

The problem considered in this study is at the initial stage of investigation and further researches can be done based on the results of this paper. Some of them are as below.

- Similar research can be repeated in the presence of imprecise data and fuzzy data.
- One of the assumptions of all the classical models of DEA is based on complete homogeneity of DMUs (suppliers), where as this assumption in many real applications cannot be generalised. In other words, some inputs and/or outputs are not common for all the DMUs occasionally. Therefore, there is a need for a model that deals with these conditions.
- Similar research can be repeated for suppliers ranking in the presence of dual-role factors. Dual-role factors refer to those factors which could serve as either inputs or outputs.

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Notes

- In the fractional programming of Model (3), $\sum_{r=1}^R u_r y_{ro} - \sum_{i \in I_{ND}} v_{iND} x_{io} / \sum_{i \in I_D} v_{iD} x_{io}$ can be regarded as the aggregated output to input ratio.

Appendix

Table 6 Matrix of cross-efficiency

Supplier (DMU)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0.926	0.850	1.000	0.762	1.000	0.973	0.727	0.653	0.721	0.606	0.678	0.761	0.687	0.717	0.757	0.562	0.819	0.842
2	0.878	0.877	1.000	1.000	1.000	0.989	0.816	0.731	0.737	0.710	0.661	0.758	0.730	0.771	0.829	0.586	0.917	0.919
3	0.479	0.585*	1.000	0.193	0.968	0.984	0.000	0.477	0.573	0.300	0.427	0.816	0.755	0.636	0.829	0.233	0.617	0.856
4	0.832	0.814	0.956	1.000	0.893	0.871	0.802	0.665	0.688	0.583	0.610	0.715	0.661	0.725	0.727	0.566	0.800	0.870
5	0.623	0.616	0.950	0.000	1.000	1.000	0.046	0.447	0.580	0.353	0.510	0.777	0.696	0.577	0.766	0.266	0.613	0.733
6	0.289	0.489	0.412	0.000	0.895	1.000	0.000	0.546	0.396	1.000	0.369	0.406	0.622	0.410	0.897	0.121	0.961	0.474
7	0.879	0.852	1.000	1.000	0.945	0.921	0.821	0.689	0.720	0.609	0.645	0.750	0.691	0.751	0.760	0.587	0.834	0.899
8	0.824	0.860	0.950	1.000	0.996	1.000	0.790	0.746	0.717	0.781	0.637	0.727	0.735	0.760	0.860	0.561	0.961	0.905
9	0.878	0.877	1.000	1.000	1.000	0.989	0.816	0.731	0.737	0.710	0.661	0.758	0.730	0.771	0.829	0.586	0.917	0.919
10	0.045	0.294	0.060	0.060	0.582	0.704	0.000	0.443	0.202	1.000	0.192	0.126	0.415	0.230	0.702	0.027	0.845	0.243
11	0.926	0.850	1.000	0.762	1.000	0.973	0.727	0.653	0.721	0.606	0.678	0.761	0.687	0.717	0.757	0.562	0.819	0.842
12	0.480	0.591	1.000	0.194	0.981	1.000	0.000	0.487	0.577	0.325	0.431	0.818	0.764	0.641	0.845	0.234	0.637	0.861
13	0.463	0.673	1.000	0.729	0.963	1.000	0.286	0.641	0.623	0.524	0.432	0.801	0.816	0.746	0.937	0.332	0.817	0.993
14	0.805	0.852	1.000	1.000	0.999	1.000	0.749	0.734	0.723	0.709	0.623	0.765	0.754	0.778	0.865	0.551	0.926	0.946
15	0.280	0.560	0.581	0.458	0.900	1.000	0.168	0.643	0.472	0.943	0.359	0.515	0.715	0.558	0.968	0.202	1.000	0.702
16	0.879	0.852	1.000	1.000	0.945	0.921	0.821	0.689	0.720	0.609	0.645	0.750	0.691	0.751	0.760	0.587	0.834	0.899
17	0.236	0.508	0.471	0.243	0.888	1.000	0.060	0.602	0.419	1.000	0.340	0.445	0.674	0.481	0.952	0.141	1.000	0.590
18	0.463	0.673	1.000	0.729	0.963	1.000	0.286	0.641	0.623	0.524	0.432	0.801	0.816	0.746	0.937	0.332	0.817	0.993
Average	0.622	0.704	0.854	0.618	0.940	0.963	0.440	0.623	0.608	0.661	0.518	0.681	0.702	0.654	0.832	0.391	0.841	0.805
Rank	13	7	3	14	2	1	17	12	15	10	16	9	8	11	5	18	4	6

Notes: *0.585 represents the cross-efficiency score of supplier #2 in terms of optimal weights of supplier #3. **Italic numbers in the leading diagonal are the efficiency scores obtained by Model (3).