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Abstract

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A new approach for considering a dual-role factor in data envelopment analysis

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Abstract: The conventional data envelopment analysis models deal with dual-role factor as non-discretionary (uncontrollable) factor. However, there might be dual-role factor which is under control of decision-maker. In addition, despite the fact that there are several publications addressing dual-role factors, it seems that their idea of classifying a factor as an input or an output within a single model has a limitation. They also do not consider non-zero input and output slacks and cannot fully measure the inefficiency of decision-making units. To resolve these limitations and to consider dual-role factor as well, this paper proposes a slacks-based measure model which does not consider dual-role factor as a non-discretionary factor. To compare the results of proposed approach with conventional model, a statistical analysis is run. A numerical example demonstrates the application of the proposed method.

Keywords: DEA; data envelopment analysis; dual-role factor; SBM; slacks-based measure; contingency table.

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1 Introduction

Data envelopment analysis (DEA) has been widely applied to address various decision analysis problems due to its usefulness in evaluating multi-criterion systems. DEA is a non-parametric mathematical programming technique that determines an efficient frontier of the most efficient decision-making units (DMUs) and calculates the efficiency of each DMU relative to this efficient frontier based on multiple observed inputs and outputs. An efficiency score of a DMU is generally defined as the weighted sum of outputs divided by the weighted sum of inputs, while weights need to be assigned. To avoid the potential difficulty in assigning these weights among various DMUs, a DEA model computes weights that give the highest possible relative efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights (Liu et al., 2000). In recent years, DEA has been used to measure the efficiency of DMUs in many different settings, such as efficiency and effectiveness in operating management (Goncharuk, 2007; Parkan, 2006), supply chain management (Azadi and Farzipoor Saen, in press; Parkan and Wang, 2007; Wong et al., 2008), sport industry (Boscá et al., 2009; Cooper et al., 2009), construction industry (Baykasoglu et al., 2009; El-Mashaleh et al., 2010), farming industry (Mulwa et al., 2009) and banking industry (Azadeh et al., 2010; Cooper et al., 2008; Paradia et al., 2010).

In the standard use of DEA, it is supposed that one can, given a set of available factors, clearly determine which factors are inputs and which are outputs (Farzipoor Saen, in press). However, there is a strong argument for permitting certain factors to simultaneously play the role of both inputs and outputs in DEA models. For example, Beasley (1990, 1995), in a study of the efficiency of university departments, treated 'research income' on both the input and the output sides as a dual-role factor. Similar arguments can be made regarding the evaluation of suppliers, such as described in Farzipoor Saen (2010b) and Noorizadeh et al. (in press). There, research and development (R&D) cost is treated as dual-role factor. Suppliers R&D cost clearly constitutes an output measure, but at the same time it is an important component of the supplier, hence it is an input. From the perspective of decision-maker who intends to select the best supplier, such measures may play the role of proxy for 'suppliers' innovation'. R&D results in the technology that brings new products and services to the market place or strengthens better processes. Innovation results in high quality jobs, successful businesses, better goods and services and more efficient processes. That is why R&D can reasonably be classified as output. On the other hand, from the perspective of supplier, it can be considered as input that imposes special expenses to the supplier (Noorizadeh et al., in press).

The objective of this paper is to propose an innovative approach for incorporating a dual-role factor in DEA models.

This paper proceeds as follows. In Section 2, literature review is presented. Section 3 introduces the proposed approach. A numerical example and managerial implications are discussed in Sections 4 and 5, respectively. Concluding remarks are given in Section 6.

2 Literature review

Beasley (1990, 1995), in a study of the efficiency of university departments, treated research funding on both the input and the output sides. However, as Cook et al. (2006) addressed, the model proposed by Beasley (1990, 1995) has two limitations. The first limitation is that in the absence of constraints (e.g. assurance region or cone ratio) on the multipliers, each DMU may be 100% efficient. The second limitation is that the dual-role factor is considered differently on the input than on the output side. Cook et al. (2006) developed a new model that has no above-mentioned limitations. Yang et al. (2010) analysed the structure and properties of the production systems and suggested a DEA model which considers dual-role factors. Farzipoor Saen (2010a) proposed a new model for selecting third-party reverse logistics (3PL) providers in the presence of multiple dual-role factors. In this paper, ratings for service-quality experience (EXP) and service-quality credence (CRE) are treated as dual-role factors. EXP and CRE could serve as either inputs or outputs. From the perspective of decision-maker who intends to select the best 3PL providers, such measures may play the role of proxy for 'high quality of services', hence can reasonably be classified as outputs. On the other hand, from the perspective of 3PL provider that intends to supply reverse logistics services, they can be considered as inputs that help the 3PL provider in obtaining more customers.

Farzipoor Saen (2010b) presented a method for selecting suppliers in the presence of dual-role factors and weight restrictions. In this paper, the R&D cost was considered as both an input and an output. For selecting the best technologies in the presence of dual-role factors, Farzipoor Saen (2010c) considered the 'amount of personnel education hours by the robot supplier (RH)' as dual-role factor. Recently, Mahdiloo et al. (2011) addressed the problem of a factor in supplier selection analysis which may be classified either an input or an output. They demonstrated the validity of the proposed approach via comparing the results with conventional models. Farzipoor Saen (2010d) developed a new model for ranking 3PL providers in the presence of multiple dual-role factors. Cook and Zhu (2007) modified the standard constant returns to scale DEA model to treat dual-role factors. Both an individual DMU and an aggregate model were suggested as methodologies for deriving the most appropriate designations for dual-role factors. However, as Toloo addressed (2009), their method may produce incorrect efficiency scores due to a computational problem as a result of introducing a large positive number to the model. Toloo (2009) developed a model that does not need such a large positive number. Farzipoor Saen (in press) suggested a model for selecting media in the presence of both dual-role factors and imprecise data. This paper depicted the media selection process through a DEA model, while allowing for the incorporation of both dual-role factors and imprecise data. In his work, volume of supplied information to audiences was considered as a dual-role factor.

To the best of knowledge of authors, there is no reference dealing with a dual-role factor in a simplistic and straightforward way.

3 Proposed approach

To evaluate the relative efficiency of the DMUs, there may exist a factor which can be classified as either an input or an output. The quantity of such a factor may influence the efficiency of the DMUs. To consider this dual-role factor, Beasley (1990, 1995) proposed Model (1) which is based on the standard Charnes, Cooper and Rhodes model (Charnes et al., 1978). Consider a situation where members k of a set of K DMUs are to be evaluated in terms of R outputs $Y_k = (y_{rk})_{r=1}^R$ and I inputs $X_k = (x_{ik})_{i=1}^I$. In addition, assume that a particular factor is held by each DMU in the amount w_k and serves as both an input and an output factor.

$$\begin{aligned} & \text{Max} \frac{\left(\sum_{r=1}^R \mu_r y_{ro} + \gamma w_o \right)}{\left(\sum_{i=1}^I v_i x_{io} + \beta w_o \right)} \\ & \text{s.t.} \\ & \sum_{r=1}^R \mu_r y_{rk} + \gamma w_k - \sum_{i=1}^I v_i x_{ik} - \beta w_k \leq 0, \quad k = 1, \dots, K, \\ & \mu_r, v_i, \gamma, \beta \geq 0 \end{aligned} \tag{1}$$

The used nomenclatures in this paper are summarised in Table 1.

Table 1 The nomenclatures

DMU _o :	the decision-making unit under investigation
$k = 1, \dots, K$	collection of DMUs
$r = 1, \dots, R$	the set of outputs
$i = 1, \dots, I$	the set of inputs
x_{io} :	i th input of the DMU _o
v_i :	the weight for i th input
y_{ro} :	r th output of DMU _o
μ_r :	the weight for r th output
w_o :	level of dual-role factor of DMU _o
γ :	the weight for dual-role factor when it is treated on the output side
β :	the weight for dual-role factor when it is treated on the input side
x_{ik} :	the i th input of DMU _k
y_{rk} :	r th output of DMU _k
w_k :	level of dual-role factor of DMU _k
θ :	radial efficiency measure for DMU _o
λ_k :	reference weights associated with DMU _k
θ_1 :	radial efficiency measure for DMU _o when dual-role factor is treated on the input side
θ_2 :	radial efficiency measure for DMU _o when dual-role factor is treated on the output side
θ^* :	$\max(\theta_1, \theta_2)$
ζ :	non-radial efficiency measure for DMU _o
ζ_1 :	non-radial efficiency measure for DMU _o when dual-role factor is treated on the input side
ζ_2 :	non-radial efficiency measure for DMU _o when dual-role factor is treated on the output side
ζ^* :	$\max(\zeta_1, \zeta_2)$

Cook et al. (2006) argue that Beasley's (1990, 1995) treatment of dual-role factor on both the input and the output sides is not entirely appropriate and represents somewhat of a contradiction.

Here, to show how this contradiction appears, consider input-oriented Model (2) which is the dual form of Model (1).

$$\begin{aligned}
 & \text{Min } \theta \\
 & \text{s.t.} \\
 & \sum_{k=1}^K x_{ik} \lambda_k \leq \theta x_{io}, \quad i = 1, \dots, I \\
 & \sum_{k=1}^K w_k \lambda_k \leq \theta w_o \\
 & \sum_{k=1}^K y_{rk} \lambda_k \geq y_{ro}, \quad r = 1, \dots, R \\
 & \sum_{k=1}^K w_k \lambda_k \geq w_o \\
 & \lambda_k \geq 0 \\
 & \theta \text{ free}
 \end{aligned} \tag{2}$$

The logic of Model (2) is that in the case where a DMU_o has an efficiency score of θ , then all discretionary inputs, including w_o , are reduced by $1 - \theta$. On the other hand, this factor is also included on the output side where we assume w_o will not be reduced. Thus, Models (1) and (2) treat w_o differently on the input than on the output side. To correct this apparent flaw, Cook et al. (2006) recommend treating w_o as being non-discretionary on the input side. Since, on the output side, variables generally remain fixed in the optimisation process of an input-oriented model, w_o can be viewed as non-discretionary as well. From this perspective, Cook et al. (2006) modified Model (1) as below.

$$\begin{aligned}
 & \text{Max } \sum_{r=1}^R \mu_r y_{ro} + \gamma w_o - \beta w_o \\
 & \text{s.t.} \\
 & \sum_{i=1}^I v_i x_{io} = 1 \\
 & \sum_{r=1}^R \mu_r y_{rk} + \gamma w_k - \beta w_k - \sum_{i=1}^I v_i x_{ik} \leq 0, \quad k = 1, \dots, K, \\
 & \mu_r, v_i, \gamma, \beta \geq 0
 \end{aligned} \tag{3}$$

The inclusion of the dual-role factor on the input side of Model (3) as a non-discretionary input is based on the idea of Banker and Morey (1986). The authors prove that the way to model such inputs is to move them to the output side but with the opposite sign. This idea often arises in situations where there are criteria that are beyond the control of the management but influence the efficiency of DMUs. Thus, in evaluating process, these factors are generally expected to remain at their current level.

Now, one of the three possibilities exists in regard to the sign of $\hat{\gamma} - \hat{\beta}$, where $\hat{\gamma}$ and $\hat{\beta}$ are the optimal values from Model (3); $\hat{\gamma} - \hat{\beta} > 0$, $= 0$ or < 0 (Cook et al., 2006).

Case 1 If $\hat{\gamma} - \hat{\beta} < 0$, then the dual-role factor is ‘behaving like input’. Hence, less of this factor is better and would lead to an increase in efficiency.

Case 2 If $\hat{\gamma} - \hat{\beta} > 0$, then the dual-role factor is ‘behaving like output’. Hence, more of this factor is better and would lead to an increase in efficiency.

Case 3 If $0 \hat{\gamma} - \hat{\beta} = 0$, then dual-role factor is at equilibrium level.

However, Model (3) suffers from three limitations:

- 1 Classifying a factor as an input or an output within a single model cannot guarantee that the production function and the causality relationship between inputs and outputs are verified. That is, considering a factor as an input and an output in a single model means that there is a factor that is used to produce itself.
- 2 They considered dual-role factor as non-discretionary factor. However, in real-world applications, dual-role factor may be as a discretionary factor.
- 3 In DEA, non-zero input and output slacks are very likely to reveal themselves after the radial efficiency score improvement. Often, the non-zero slack values reveal a considerable amount of inefficiency. Therefore, to fully measure the inefficiency in DMU's performance, it is crucial to consider the inefficiency represented by the non-zero slacks in the presence of dual-role factors. However, all the models used in the past, which consider dual-role factor, are radial models and cannot fully measure the inefficiency of DMUs.

At this juncture, we introduce an innovative approach to treat with a dual-role factor and to determine the behaviour of this factor as input, output or equilibrium. This new approach does not suffer from the first limitation of Model (3) discussed above. That is, it considers the causality relationship between inputs and outputs and does not classify a factor as an input and an output within a single model.

Model (4) treats dual-role factor only on the input side and as a non-discretionary input.¹

$$\begin{aligned}
 & \text{Min } \theta_1 \\
 & \text{s.t.} \\
 & \sum_{k=1}^K x_{ik} \lambda_k \leq \theta_1 x_{io}, \quad i = 1, \dots, I \\
 & \sum_{k=1}^K w_k \lambda_k \leq w_o \\
 & \sum_{k=1}^K y_{rk} \lambda_k \geq y_{ro}, \quad r = 1, \dots, R \\
 & \lambda_k \geq 0 \\
 & \theta_1 \text{ free}
 \end{aligned} \tag{4}$$

Model (5) treats dual-role factor only on the output side.

$$\begin{aligned}
 &\text{Min } \theta_2 \\
 &\text{s.t.} \\
 &\sum_{k=1}^K x_{ik} \lambda_k \leq \theta_2 x_{io}, \quad i = 1, \dots, I \\
 &\sum_{k=1}^K y_{rk} \lambda_k \geq y_{ro}, \quad r = 1, \dots, R \\
 &\sum_{k=1}^K w_k \lambda_k \geq w_o \\
 &\lambda_k \geq 0 \\
 &\theta_2 \text{ free}
 \end{aligned} \tag{5}$$

Table 2 illustrates the algorithm of this new approach.

So far, by using the new algorithm, the first limitation of Model (3) has been resolved. However, in this new approach, to treat w_o on both input and output sides in a same manner, dual-role factor is incorporated as a non-discretionary input on the input side. Therefore, the second and third limitations of Model (3) remain. Now, the second limitation is discussed.

As mentioned before, Model (1) treats w_o differently on the input than on the output side. To correct this problem of Model (1), Cook et al. (2006) recommend treating w_o as being non-discretionary on the input side. Although this treatment eliminates the contradiction occurred in Model (1), but this approach itself make a new and apparent contradiction. Consider research income as a non-discretionary factor. It means that this factor is beyond the control of management; hence, we should not blame management of DMU which has a weak performance on this special factor. Here a question arises. Is the dual-role a non-discretionary factor in all situations? The response is that there might be some situations that a dual-role is a discretionary factor. Therefore, we need a model that not only treats w_o on both the input and the output sides in a same manner discussed in Cook et al. (2006), but also considers it as a discretionary factor.

Table 2 Algorithm of the proposed approach

<i>Step 1</i>	Start
<i>Step 2</i>	Treat dual-role factor only on the input side and run Model (4)
<i>Step 3</i>	Treat dual-role factor on the output side and run Model (5)
<i>Step 4</i>	Find $\max(\theta_1, \theta_2) = \theta^*$ and consider it as the efficiency score of DMU_o
<i>Step 5</i>	Now, consider θ_1, θ_2 as an indicator of dual-role factor's behaviour as well. The θ_1, θ_2 are interpreted as below ^a
<i>Case 1</i>	If, $\theta_1 > \theta_2$, then the dual-role factor is 'behaving as input'
<i>Case 2</i>	If $\theta_1 < \theta_2$, then the dual-role factor is 'behaving as output'
<i>Case 3</i>	If $\theta_1 = \theta_2$, then the dual-role factor is at equilibrium level

^aSince DEA computes weights that give the highest possible relative efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights, so we select $\max(\theta_1, \theta_2)$ as the indicator of dual-role factor's behaviour.

The above models are all radial measures which are oriented on the input side (input-oriented) or on the output side (output-oriented) which causes different treatment on the input than on the output side or *vice versa*.

Now, we use slacks-based measure (SBM) model introduced by Tone (2001) and incorporate the concept of dual-role factor in this model. Since SBM is a non-oriented model, so it has the capability of treating w_o on both the input and the output sides in a same manner. Model (6) shows the original SBM model proposed by Tone (2001).

$$\begin{aligned}
 & \text{Max } \xi \\
 & \text{s.t.} \\
 & \xi + \sum_{i=1}^I v_i x_{io} - \sum_{r=1}^R \mu_r y_{ro} = 1 \\
 & - \sum_{i=1}^I v_i x_{ik} + \sum_{r=1}^R \mu_r y_{rk} \leq 0, \quad k = 1, \dots, K \\
 & v_i \geq \frac{1}{I} \left[\frac{1}{x_{io}} \right], \quad i = 1, \dots, I \\
 & \mu_r \geq \frac{\xi}{R} \left[\frac{1}{y_{ro}} \right], \quad r = 1, \dots, R
 \end{aligned} \tag{6}$$

Model (7) is a SBM model which measures the efficiency of DMU_o by incorporating dual-role factor in the input side.

$$\begin{aligned}
 & \text{Max } \xi_1 \\
 & \text{s.t.} \\
 & \xi_1 + \sum_{i=1}^I v_i x_{io} + \beta w_o - \sum_{r=1}^R \mu_r y_{ro} = 1 \\
 & - \left(\sum_{i=1}^I v_i x_{ik} + \beta w_k \right) + \sum_{r=1}^R \mu_r y_{rk} \leq 0, \quad k = 1, \dots, K \\
 & v_i \geq \frac{1}{I+1} \left[\frac{1}{x_{io}} \right], \quad i = 1, \dots, I \\
 & \beta \geq \frac{1}{I+1} \left[\frac{1}{w_o} \right] \\
 & \mu_r \geq \frac{\xi_1}{R} \left[\frac{1}{y_{ro}} \right], \quad r = 1, \dots, R
 \end{aligned} \tag{7}$$

Model (8) is a SBM model which incorporates dual-role factor on the output side.²

$$\begin{aligned}
 & \text{Max } \xi_2 \\
 & \text{s.t.} \\
 & \xi_2 + \sum_{i=1}^I v_i x_{io} - \sum_{r=1}^R \mu_r y_{ro} + \gamma w_o = 1 \\
 & - \sum_{i=1}^I v_i x_{ik} + \sum_{r=1}^R \mu_r y_{rk} + \gamma w_k \leq 0, \quad k = 1, \dots, K \\
 & v_i \geq \frac{1}{I} \left[\frac{1}{x_{io}} \right], \quad i = 1, \dots, I \\
 & \mu_r \geq \frac{\xi_2}{R+1} \left[\frac{1}{y_{ro}} \right], \quad r = 1, \dots, R \\
 & \gamma \geq \frac{1}{R+1} \left[\frac{1}{w_o} \right]
 \end{aligned} \tag{8}$$

Note that, Models (7) and (8) are both non-radial and non-oriented. Therefore, they have two important features:

- 1 they do not need to incorporate dual-role as a non-discretionary factor on the input side
- 2 they can fully measure the inefficiency of DMUs.

Therefore, by using Models (7) and (8), all the three limitations of Model (3) are obviated.

3.1 Contingency table

In this section, we apply a statistical technique to determine the association between the results of Cook et al. (2006) and the new suggested approach in determining the behaviour of dual-role factor.

Since the behaviour of dual-role factor in Model (3) and the proposed algorithm is recorded in three sets of mutually exclusive and exhaustive categories, it is possible to construct a cross tabulation or contingency table. In the analysis of categorical data, the cross tabulation is the analogue of the scatter plot. In the case of nominal data, chi-square-based statistics (ϕ , Cramer's V and contingency coefficient) should be used, which are used for determining the presence of an association between qualitative variables. An ideal measure should mimic the correlation coefficient by having a maximum absolute value of 1 for perfect association, and a value of 0 for no association. The choice of the appropriate statistic depends on whether the variables are ordinal or nominal, and whether the contingency (cross tabulation) table is 2×2 (each variable has two categories) or larger. For example, one such statistic is the ϕ coefficient obtained by dividing the value of chi-square by the total frequency and taking the square root. For two-way contingency tables involving variables with more than two categories (such as our case which the behaviour of dual-role factor categorised in k_1 , k_2 and k_3), however, another statistic, known as Cramer's V , is preferred because with more complex tables,

Cramer's measure can still, as in the 2×2 case, achieve its maximum value of unity (Kinnear and Gray, 1994).

In Section 4, a numerical example is presented. In this example, we examine the ability of our proposed approach in determining the efficiency scorers of DMUs and the behaviour of dual-role factor.

4 Numerical example

As discussed in Section 2, considering the dual-role factor as a non-discretionary factor is incorrect. However, to compare our approach with Cook et al. (2006), we first have to consider the dual-role factor as a non-discretionary factor. So before using the final proposed approach (Models (7) and (8)), we use Models (4) and (5) to compare universities as discussed in Beasley (1990, 1995) and Cook et al. (2006). Beasley evaluates the efficiency scores of both Chemistry and Physics Departments at 50 UK universities. Table 3 displays a portion of the data for Physics Department, recreated from Beasley (1990). As well, Cook et al. (2006) used these data to examine the modelling of dual-role factors in their proposed model (Model (3)). Inputs are general expenditure (x_1) and equipment expenditure (x_2). Outputs consist of three student groups (y_1 , y_2 and y_3). And research income (w) simultaneously plays the role of both input and output.

Tables 4 and 5 show the results of the evaluation by Model (3) and our proposed radial approach (Models (4) and (5)), respectively. The results in Table 4 imply that $k_1 = 31$, $k_2 = 16$ and $k_3 = 3$. DMUs in k_1 are those wherein the research income is behaving like an output, and where more of such income would improve the efficiencies of the members of that set. For those DMUs in k_2 , research income is behaving like an input, and less of such factor would increase the efficiency of the members. The three universities in k_3 are in equilibrium.

In Table 5, θ_1 shows the radial efficiency score of DMU_o derived by Model (4), that the research income is treated on the input side and as a non-discretionary input. The θ_2 shows the radial efficiency score of DMU_o by using Model (5), that the research income is treated on the output side. The θ^* is the final radial efficiency score of DMU_o derived by $\max(\theta_1, \theta_2)$.

The results derived by Model (3) and our proposed approach consist of two parts: efficiency scores and behaviour of dual-role factor. Part one is as below.

Since the efficiency scores derived by two methods are exactly the same, this validates our proposed approach as an efficiency measurement tool. As a result, as you see, our proposed approach is a simple and straightforward algorithm to evaluate the efficiency of DMUs in the presence of dual-role factor.

Part two is as follows. To compare the behaviour of dual-role factor, we construct a contingency table. Since we have a two-way contingency table involving variables with three categories (k_1 , k_2 and k_3), so statistic known as Cramer's V is used for determining the association between two groups (as recommended by Kinnear and Gray, 1994). The null hypothesis (H_0) is that there is no association between the behaviour of dual-role factor in Model (3) and our proposed approach. Table 6 depicts the contingency table. The values of the table are frequencies.

Table 7 displays the statistics of contingency table using SPSS software.

Table 3 Data for Physics Departments from Beasley (1990)

<i>DMUs</i>	x_1	x_2	y_1	y_2	y_3	W
University 1	528	64	145	0	26	254
University 2	2,605	301	381	16	54	1,485
University 3	304	23	44	3	3	45
University 4	1,620	485	287	0	48	940
University 5	490	90	91	8	22	106
University 6	2,675	767	352	4	166	2,967
University 7	422	0	70	12	19	298
University 8	986	126	203	0	32	776
University 9	523	32	60	0	17	39
University 10	585	87	80	17	27	353
University 11	931	161	191	0	20	293
University 12	1,060	91	139	0	37	781
University 13	500	109	104	0	19	215
University 14	714	77	132	0	24	269
University 15	923	121	135	10	31	392
University 16	1,267	128	169	0	31	546
University 17	891	116	125	0	24	925
University 18	1,395	571	176	14	27	764
University 19	990	83	28	36	57	615
University 20	3,512	267	511	23	153	3,182
University 21	1,451	226	198	0	53	791
University 22	1,018	81	161	5	29	741
University 23	1,115	450	148	4	32	347
University 24	2,055	112	207	1	47	2,945
University 25	440	74	115	0	9	453
University 26	3,897	841	353	28	65	2,331
University 27	836	81	129	0	37	695
University 28	1,007	50	174	7	23	98
University 29	1,188	170	253	0	38	879
University 30	4,630	628	544	0	217	4,838
University 31	977	77	94	26	26	490
University 32	829	61	128	17	25	291
University 33	898	39	190	1	18	327
University 34	901	131	168	9	50	956
University 35	924	119	119	37	48	512
University 36	1,251	62	193	13	43	563

Table 3 Data for Physics Departments from Beasley (1990) (continued)

<i>DMUs</i>	x_1	x_2	y_1	y_2	y_3	W
University 37	1,011	235	217	0	36	714
University 38	732	94	151	3	23	297
University 39	444	46	49	2	19	277
University 40	308	28	57	0	7	154
University 41	483	40	117	0	23	531
University 42	515	68	79	7	23	305
University 43	593	82	101	1	9	85
University 44	570	26	71	20	11	130
University 45	1,317	123	293	1	39	1,043
University 46	2,013	149	403	2	51	1,523
University 47	992	89	161	1	30	743
University 48	1,038	82	151	13	47	513
University 49	206	1	16	0	6	72
University 50	1,193	95	240	0	32	485

Table 4 Efficiency scores and output/input behaviour using Model (3)

<i>DMUs</i>	<i>Efficiency</i>	<i>Behaviour</i>
University 1	1.000	k_2
University 2	0.640	k_1
University 3	0.810	k_2
University 4	0.686	k_1
University 5	1.000	k_2
University 6	1.000	k_1
University 7	1.000	k_1
University 8	0.812	k_1
University 9	1.000	k_2
University 10	0.907	k_1
University 11	0.828	k_2
University 12	0.709	k_1
University 13	0.772	k_1
University 14	0.703	k_2
University 15	0.688	k_1
University 16	0.520	k_1
University 17	0.819	k_1
University 18	0.628	k_1

Table 4 Efficiency scores and output/input behaviour using Model (3) (continued)

<i>DMUs</i>	<i>Efficiency</i>	<i>Behaviour</i>
University 19	1.000	k_1
University 20	0.898	k_1
University 21	0.674	k_2
University 22	0.717	k_1
University 23	0.563	k_2
University 24	1.000	k_1
University 25	1.000	k_1
University 26	0.565	k_1
University 27	0.855	k_1
University 28	1.000	k_2
University 29	0.825	k_1
University 30	0.930	k_1
University 31	0.776	k_1
University 32	0.867	k_2
University 33	1.000	k_3
University 34	1.000	k_1
University 35	1.000	k_1
University 36	0.737	k_2
University 37	0.831	k_1
University 38	0.806	k_1
University 39	0.790	k_2
University 40	0.741	k_1
University 41	1.000	k_1
University 42	0.841	k_2
University 43	0.900	k_2
University 44	1.000	k_3
University 45	0.889	k_1
University 46	0.851	k_3
University 47	0.688	k_1
University 48	0.909	k_2
University 49	1.000	k_2
University 50	0.835	k_1

Table 5 Efficiency scores and output/input behaviour using Models (4) and (5)

<i>DMUs</i>	θ_1	θ_2	θ^*	<i>Behaviour</i>
University 1	1.000	1.000	1.000	k_3
University 2	0.615	0.640	0.640	k_1
University 3	0.810	0.663	0.810	k_2
University 4	0.645	0.686	0.686	k_1
University 5	1.000	0.893	1.000	k_2
University 6	1.000	1.000	1.000	k_3
University 7	1.000	1.000	1.000	k_3
University 8	0.750	0.812	0.812	k_1
University 9	1.000	0.658	1.000	k_2
University 10	0.892	0.907	0.907	k_1
University 11	0.828	0.747	0.828	k_2
University 12	0.690	0.709	0.709	k_1
University 13	0.767	0.772	0.772	k_1
University 14	0.703	0.702	0.703	k_2
University 15	0.676	0.688	0.688	k_1
University 16	0.518	0.520	0.520	k_1
University 17	0.536	0.819	0.819	k_1
University 18	0.593	0.628	0.628	k_1
University 19	1.000	1.000	1.000	k_3
University 20	0.858	0.898	0.898	k_1
University 21	0.674	0.669	0.674	k_2
University 22	0.664	0.717	0.717	k_1
University 23	0.563	0.560	0.563	k_2
University 24	0.484	1.000	1.000	k_1
University 25	0.952	1.000	1.000	k_1
University 26	0.425	0.565	0.565	k_1
University 27	0.852	0.855	0.855	k_1
University 28	1.000	0.809	1.000	k_2
University 29	0.775	0.825	0.825	k_1
University 30	0.831	0.930	0.930	k_1
University 31	0.728	0.776	0.776	k_1
University 32	0.867	0.841	0.867	k_2
University 33	1.000	1.000	1.000	k_3
University 34	1.000	1.000	1.000	k_3
University 35	1.000	1.000	1.000	k_3
University 36	0.737	0.735	0.737	k_2

Table 5 Efficiency scores and output/input behaviour using Models (4) and (5) (continued)

<i>DMUs</i>	θ_1	θ_2	θ^*	<i>Behaviour</i>
University 37	0.782	0.831	0.831	k_1
University 38	0.806	0.806	0.806	k_3
University 39	0.790	0.789	0.790	k_2
University 40	0.740	0.741	0.741	k_1
University 41	1.000	1.000	1.000	k_3
University 42	0.841	0.835	0.841	k_2
University 43	0.900	0.643	0.900	k_2
University 44	1.000	1.000	1.000	k_3
University 45	0.883	0.889	0.889	k_1
University 46	0.848	0.851	0.851	k_1
University 47	0.655	0.688	0.688	k_1
University 48	0.909	0.883	0.909	k_2
University 49	1.000	0.637	1.000	k_2
University 50	0.835	0.835	0.835	k_3

Table 6 Contingency table

<i>Proposed approach</i>	<i>Model (3)</i>			
	k_1	k_2	k_3	<i>Summation</i>
k_1	23	0	8	31
k_2	0	15	1	16
k_3	1	0	2	3
Summation	24	15	11	50

Table 7 Statistics of contingency table

	<i>Value</i>	<i>Approximate significance</i>
Cramer's V	0.699	0.000
No. of valid cases	50	

It can be concluded that there is a significant association between the results of two models in determining the behaviour of dual-role factor, as shown by the last column of Table 7 (<0.01). Cramer's V coefficient provides a measure of the strength of the association.

Now, after demonstrating real nature of the proposed approach statistically, the final models (Models (7) and (8)) are used to evaluate the efficiency score of universities. The results are shown in Table 8. In this table, the ξ_1 shows the non-radial efficiency score of DMU_o derived by Model (7), where research income is treated as the input. The ξ_2 indicates the non-radial efficiency score of DMU_o derived by Model (8), where research income is treated as the output. The ξ^* is the final non-radial efficiency score of DMU_o determined by $\max(\xi_1, \xi_2)$.³

By comparing the efficiency scores derived by radials Models (3)–(5) with non-radial Models (7) and (8), it can be easily seen that radial models including conventional models, which consider dual-role factors, are not able to fully measure the inefficiencies of inefficient DMUs.

The visual computational results obtained by Models (7) and (8) and the final efficiency score of DMUs (ξ^*) are presented in Figure 1. This figure has two coordinates (university and efficiency) and shows the efficiency of the 50 universities under consideration. As shown, efficiency scores for the efficient universities are one and the efficiency scores of all the remaining universities are less than one.

Table 8 Efficiency scores and output/input behaviour using Models (7) and (8)

<i>DMUs</i>	ξ_1	ξ_2	ξ^*
University 1	1	1	1
University 2	0.2625	0.218	0.2625
University 3	0.5188	0.1497	0.5188
University 4	0.0005	0.0005	0.0005
University 5	1	0.3172	1
University 6	1	1	1
University 7	1	1	1
University 8	0.0012	0.0018	0.0018
University 9	1	0.0013	1
University 10	0.7384	0.4907	0.7384
University 11	0.0021	0.0015	0.0021
University 12	0.0006	0.0007	0.0007
University 13	0.0029	0.0028	0.0029
University 14	0.0017	0.0014	0.0017
University 15	0.4871	0.2958	0.4871
University 16	0.0005	0.0006	0.0006
University 17	0.0006	0.0018	0.0018
University 18	0.2804	0.2576	0.2804
University 19	1	1	1
University 20	0.2731	0.4911	0.4911
University 21	0.0004	0.0005	0.0005
University 22	0.2312	0.2216	0.2312
University 23	0.1861	0.1537	0.1861
University 24	0.0241	1	1
University 25	0.0134	1	1
University 26	0.2148	0.2069	0.2148
University 27	0.0008	0.0013	0.0013
University 28	1	0.1608	1

Table 8 Efficiency scores and output/input behaviour using Models (7) and (8) (continued)

DMUs	ξ_1	ξ_2	ξ^*
University 29	0.0011	0.0016	0.0016
University 30	0.0001	0.0005	0.0005
University 31	0.5534	0.3395	0.5534
University 32	0.7852	0.3354	0.7852
University 33	1	1	1
University 34	1	1	1
University 35	1	1	1
University 36	0.5649	0.2991	0.5649
University 37	0.0013	0.0017	0.0017
University 38	0.4262	0.3224	0.4262
University 39	0.2398	0.2	0.2398
University 40	0.0037	0.0034	0.0037
University 41	1	1	1
University 42	0.5528	0.3722	0.5528
University 43	0.2506	0.0828	0.2506
University 44	1	1	1
University 45	0.1176	0.1909	0.1909
University 46	0.1051	0.1402	0.1402
University 47	0.0619	0.0692	0.0692
University 48	0.7108	0.3506	0.7108
University 49	1	0.0034	1
University 50	0.0014	0.0013	0.0014

Figure 1 Visual results of the computations

5 Managerial implications

All conventional DEA models which consider dual-role factors are radial models. In DEA, non-zero input and output slacks are very likely to reveal themselves after the radial efficiency score improvement. Often, the non-zero slack values reveal a considerable amount of inefficiency. Therefore, to fully measure the inefficiency in performance of DMUs, it is crucial to consider the inefficiency represented by the non-zero slacks in the presence of dual-role factors. Dual-role factors play the role of both inputs and outputs in DEA models, simultaneously. In addition, conventional DEA models deal with dual-role factors as non-discretionary factors. However, there might be dual-role factors which are completely discretionary. Decision variable is an important element in DEA framework, and the cognition of decision variable improves the progress of DEA-based theories and approaches (Yang et al., 2010). Farzipoor Saen (in press) addresses that, in traditional DEA, the decision-maker decides which criteria are inputs and which are outputs. However, in the dual-role factor context, the decision-maker is wavered. In other words, the decision-maker does not know whether this dual-role factor is an input or an output. Therefore, there is a need for a model that determines the status of the dual-role factor for each DMU, separately. After running the model, the decision-maker will find the status of the dual-role factor.

6 Concluding remarks

This paper addressed the problem of a factor in a DEA analysis which may be classified as either an input or an output. The quantity of such a factor may influence the relative efficiency of the DMUs. Such problems have been discussed in the literature before. However, the main contribution of this paper is to consider a dual-role factor as a discretionary variable and to analyse it in the context of a SBM. The validity of the proposed approach is demonstrated via comparing the results with results of previous model using a contingency table. A two-way contingency table and a statistic known as Cramer's V were applied to check the significance association between the results of proposed approach and Cook et al. (2006). The approach presented in this paper has some distinctive contributions, as follows:

- the proposed approach is very simple and straightforward
- the proposed approach can be easily used in each kind of DEA models without any effort to combine concept of dual-role factor with these models
- the proposed approach does not deal with dual-role factor as a non-discretionary factor
- this is the first time that dual-role factor and SBM are discussed simultaneously
- this is the first time that results of DEA analysis are compared by means of a statistical technique, i.e. contingency table.

The problem considered in this study is at initial stage of investigation and further researches can be done based on the results of this paper. Some of them are as below:

- Since in the proposed approach each DMU is free to decide which outputs and inputs to emphasise, there may exist many efficient DMUs. Therefore, the proposed approach should be developed to rank efficient DMUs.
- The proposed algorithm can be used in the presence of imprecise data.
- In any realistic situation there may exist some criteria that are beyond the control of the management. These factors are called non-discretionary or exogenously fixed factors. Similar research can be repeated for the evaluation of DMUs in the presence of non-discretionary factors.
- Preferences of decision-maker can be incorporated into the proposed algorithm by restricting the feasible region of the inputs and outputs' weights.

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Notes

- ¹To treat w_o on both input and output sides in a same manner, dual-role factor is incorporated as a non-discretionary input on the input side.
- ²The presence of dual-role factor caused $1/I$ and ξ/R in Model (6) to be changed to $1/(I+1)$ and $\xi_2/(R+1)$ in Models (7) and (8).
- ³SBM model cannot be solved when there are zero values in the data set. However, the data used by Beasley (1990, 1995) and Cook et al. (2006) have some zero values in x_2 and y_2 . In this case, we set x_{2o} and y_{2o} when they are zero to a small positive number ε and continue the usual process. The value of ε is proposed by Tone (2002) to be set, e.g. to: $\varepsilon = (\text{the smallest positive } x_2 \text{ and } y_2 \text{ value in the data set})/100$.