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Abstract

In conventional usage of Data Envelopment Analysis (DEA) for supplier selection, it is assumed that producing more outputs relative to fewer inputs is a criterion of efficiency. However, in the presence of undesirable outputs, suppliers with more good (desirable) outputs and less bad (undesirable) outputs relative to fewer inputs should be recognised as efficient. In addition, to get a complete ranking among suppliers and also eliminate unrealistic weighting schemes among them, this paper proposes a cross-efficiency formulation of DEA, which can treat undesirable outputs. A numerical example demonstrates the application of the proposed model in supplier ranking context. 2012 Inderscience Enterprises Ltd.

Suppliers ranking in the presence of undesirable outputs

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Keywords: suppliers ranking; DEA; data envelopment analysis; cross-efficiency; undesirable outputs.

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1 Introduction

In the current competitive and uncertain environment, the flexibility of supply chain is crucial in satisfying customers' changing needs (Ndubisi et al., 2005). The short-term objective of Supply Chain Management (SCM) is primarily to increase productivity and reduce the entire inventory and the total cycle time, whereas the long-term objective is to increase customer satisfaction, market share and profits for all organisations in the supply chain. To accomplish these objectives, tight coordination among the organisations in supply chain is needed (Lee et al., 2001). Ghodsypour and O'Brien (1998) declare that in manufacturing industries the raw materials and component parts can equal up to 70% of the product cost. In such circumstances, the purchasing department can play a key role in cost reduction by selecting good suppliers. Kuo et al. (2010a) addressed that the supply chain is an extension of logistics, which is mostly focused on related actions of physical products. Theoretically, SCM consists of several connected logistics systems, which integrate the product and service moving into a system and create a continuous and seamless linking. Also, all the actions from raw materials to end customers for merchandises are fully coordinated. Because of such coordination, all the members inside the supply chain will be affected by other chain members either directly or indirectly. For instance, if upstream supplier provides defective raw materials, this will result in producing defective final products for downstream manufacturer. Definitely, this will also reduce the customer satisfaction. Therefore, it is very important to select suitable suppliers to overcome these problems. Regarding the supplier selection, some indicators like production capacity, financial capability, quality, etc., should be taken into account. Otherwise, supplier selection problem may become organisation's crisis. In summary, supplier selection is the process by which suppliers are studied, evaluated and selected to become associated with the supply chain of company (Farzipoor Saen, 2008a; Azadi et al., 2012; Noorizadeh et al., 2011).

This paper proceeds as follows. In Section 2, literature review is presented. Section 3 introduces the proposed method, which is used to rank suppliers. A numerical example and managerial implications are discussed in Sections 4 and 5, respectively. Concluding remarks are given in Section 6.

2 Literature review

Some approaches have been used for supplier selection in the past. Lee et al. (2001) used Analytic Hierarchy Process (AHP) for supplier selection and suggested a methodology leading to effective supplier management processes utilising information obtained from the supplier selection processes. For this methodology, Lee et al. (2001) proposed the Supplier Selection and Management System (SSMS) that includes purchasing strategy system, supplier selection system and supplier management system. Wang et al. (2004) developed an integrated AHP and pre-emptive Goal Programming (GP)-based Multi Criteria Decision Making (MCDM) methodology to select the best set of multiple suppliers to satisfy capacity constraint. Hajidimitriou and Georgiou (2002) presented a quantitative model, based on the GP technique, which uses appropriate criteria to evaluate potential candidates and leads to the selection of the optimal partner (supplier). Sarkis and Talluri (2002) believed that, supplier evaluation factors would influence each other, and the internal interdependency need to be considered in the evaluation process. The authors applied Analytic Network Process (ANP) to evaluate and select the best supplier with respect to organisational factors and strategic performance metrics, which consist of seven evaluating criteria.

Lin (2009) suggested an integrated Fuzzy Analytic Network Process-Multi Objective Linear Programming (FANP-MOLP) approach for identifying top suppliers by considering the effects of interdependence among the selection criteria, as well as to achieve optimal allocation of orders among the selected suppliers. Vinodh et al. (2011) used fuzzy ANP approach for the supplier selection process in an Indian electronics switches manufacturing company. Faez et al. (2009) proposed vendor selection and order allocation using an integrated fuzzy Case-Based Reasoning (CBR) and mixed integer programming model. Kuo et al. (2010b) proposed integration of Particle Swarm Optimisation (PSO)-based Fuzzy Neural Network (FNN) and Artificial Neural Network (ANN) for supplier selection. This study is intended to develop an intelligent supplier decision-support system, which is able to consider both the quantitative and the qualitative factors. It is composed of

- the collection of quantitative data such as profit and productivity
- a PSO-based FNN to derive the rules for qualitative data
- a decision integration model for integrating both the quantitative data and the fuzzy knowledge decision to achieve the optimal decision.

In addition, fuzzy logic approaches are used for supplier selection problem (Lee, 2008; Wanga et al., 2008; Yang et al., 2008). Kuo et al. (2010c) developed a green supplier selection model, which integrates ANN and two Multi-Attribute Decision Analysis (MADA) methods: DEA and ANP. It is called ANN–MADA hybrid method. ANN–MADA hybrid method considers both practicality in traditional supplier selection criteria and environmental regulations.

Amin et al. (2011) proposed a decisional model for supplier selection, which consists of two phases. In the first phase, quantified Strengths, Weaknesses, Opportunities and Threats (SWOT) analysis are applied for evaluating suppliers. The linguistic variables and triangular fuzzy numbers are used to quantify variables. In the second phase, a fuzzy linear programming model is applied to determine the order quantity. Sarkar and

Mohapatra (2006) used the performance and the capability as two major measures in the supplier evaluation and selection problem. The authors used the fuzzy set approach to account for the imprecision involved in numerous subjective characteristics of suppliers. A hypothetical case was adopted to illustrate how the two best suppliers were selected with respect to four performance-based and 10 capability-based factors. Choy et al. (2004) discussed an intelligent supplier relationship management system integrating a company's Customer Relationship Management (CRM) system, supplier rating system and product coding system by the CBR technique to select preferred suppliers during the new product development process. To develop a flexible data access framework, and to support the partner selection activity, the combination of online analytical processing and CBR was proposed by Lau et al. (2005).

Narasimhan and Stoynoff (1986) applied a single objective, mixed integer programming model to a large manufacturing firm in the Midwest to optimise the allocation procurement for a group of suppliers. Mendoza and Ventura (2010) proposed a mixed integer non-linear programming model to determine an optimal inventory policy that coordinates the transfer of items between different stages of a serial supply chain, while properly allocating orders to selected suppliers. Talluri and Baker (2002) presented a multi-phase mathematical programming approach for effective supply chain design. More specifically, they developed and applied a combination of multi-criteria efficiency models, based on game theory concepts, and linear and integer programming methods. Cormican and Cunningham (2007) discovered that reducing the number and improving the quality of suppliers resulted in increased quality, reduced lead time and a reduction in the number of errors and defects, by evaluating supplier performance from a large multinational organisation.

Berger et al. (2004) considered risks associated with a supplier network, which include catastrophic super events that affect all suppliers, as well as unique events that impact only one single supplier, and then present a Decision-Tree (DT)-based model to help determine the optimal number of suppliers needed for the buying firm.

Weber (1996) applied DEA in supplier evaluation for an individual product and demonstrated the advantages of applying DEA to such a system. In this study, the criteria for selecting suppliers were significant reductions in costs, late deliveries and rejected materials. Weber et al. (2000) also presented an approach for evaluating the number of suppliers to employ in a procurement situation using Multi Objective Programming (MOP) and DEA. Farzipoor Saen (2007) proposed a model for determining relative efficiency of slightly non-homogeneous suppliers, in which some suppliers do not comprehensively have all common inputs or all common outputs. As well, Farzipoor Saen (2008b) proposed an innovative algorithm for ranking suppliers in the presence of volume discount offers, with regard to various criteria, based on the super-efficiency DEA model. Wu (2009) used DEA, DTs and Neural Networks (NN) to assess suppliers' performance. The model consists of two modules: Module 1 applies DEA and classifies suppliers into efficient and inefficient clusters based on the resulting efficiency scores. Module 2 utilises firm performance-related data to train DT, neural networks model and apply the trained DT model to new suppliers. Kang and Lee (2010) suggested a supplier performance evaluation model based on AHP and DEA methods. In their study, DEA is applied first to evaluate quantitative factors, and the results are transformed into pairwise comparison values for AHP analysis. Qualitative factors are also evaluated through AHP analysis, and a final ranking of suppliers obtained by combining the quantitative and qualitative results. Jafari Songhori et al. (2011) presented a structured framework for solving the supplier evaluation and order allocation problem. They used DEA and multi-objective mixed integer programming with two objectives for minimising the total costs and maximising the overall efficiencies subject to a set of capacity, demand, storage and lead time constraints.

In this paper, DEA as a non-parametric and multiple criteria decision-making tool is used for ranking suppliers. DEA was first introduced by Charnes, Cooper and Rhodes (CCR) in 1978 and it is a linear-programming-based methodology that uses multiple inputs and multiple outputs to calculate efficiency scores. The efficiency score for each Decision Making Unit (DMU) is defined as a weighted sum of outputs divided by a weighted sum of inputs, where all efficiencies are restricted to a range from 0 to 1. To avoid the potential difficulty in assigning these weights among various DMUs, a DEA model computes weights that give the highest possible relative efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights (Liu et al., 2000).

Wong and Wong (2008) listed some advantages of DEA as follows:

- DEA is an effective tool for evaluating the relative efficiency of DMUs in the presence of multiple performance measures.
- DEA is able to address the complexity arising from the lack of a common scale of measurement. Business processes often involve quantitative measures (i.e., money, time) as well as qualitative measures (i.e., customer relations and employee commitment). DEA inherits the feature that permits the inclusion of qualitative data in performance analysis. Furthermore, it allows management to analyse simultaneously a relatively large number of inputs and outputs measured on different scales.
- In DEA, one does not need to assume a priori the existence of a particular production function for weighting and aggregating inputs or outputs.
- The objectivity stemming from DEA weighting variables during the optimisation procedure frees the analysis from subjective estimates and randomness. This increases the acceptability of its results by affected parties.

The above-mentioned features of DEA make it suitable and motivated us to use it for supplier selection problem.

However, sometimes in suppliers' evaluation problem, there may exist some criteria that should be considered as undesirable outputs. In accordance with the global environmental conservation awareness, undesirable outputs of productions and social activities, e.g., air pollutants and hazardous wastes, are being increasingly recognised as dangerous and undesirable. Thus, development of technologies with less undesirable outputs is an important subject of concern in every area of production. DEA usually assumes that producing more outputs relative to fewer inputs is a criterion of efficiency. However, in the presence of undesirable outputs, DMUs with more good (desirable) outputs and less bad (undesirable) outputs relative to less inputs should be recognised as efficient (Cooper et al., 2007).

To treat desirable and undesirable outputs simultaneously for efficiency evaluation, Färe et al. (1989) introduced a non-linear programming problem. Scheel (2001) proposed some radial measures, which assume that any change of the output level will involve both undesirable and desirable outputs. Jahanshahloo et al. (2005) presented an approach to treat both undesirable inputs and outputs simultaneously in non-radial DEA models.

Seiford and Zhu (2002) proposed a DEA model, in the presence of undesirable outputs, to improve the performance via increasing the desirable outputs and decreasing the undesirable outputs. Korhonen and Luptacik (2004) used DEA to measure the eco-efficiency of 24 coal-fired power plants in a European country. They treated productions emissions directly as inputs in the sense that they wanted to increase desirable outputs and decrease pollutants and inputs. Yang and Pollitt (2009) incorporated undesirable outputs and non-discretionary inputs simultaneously into a DEA model and analysed the performance of Chinese coal-fired power plants. Recently, Farzipoor Saen (2010b) proposed a model for supplier selection in the presence of both undesirable outputs and imprecise data. In his paper, defective Parts Per Million (PPM) is used as an undesirable output.

Nevertheless, in traditional treatments of undesirable outputs in DEA, while each DMU is free to decide which outputs and inputs to emphasise, it is common to have many DMUs that are relatively efficient. To overcome this problem, this paper proposes a cross-efficiency model, which is able to consider undesirable outputs.

However, none of the above-mentioned references deal with undesirable outputs in a cross-efficiency evaluation context. The above-mentioned discussions make it more reasonable to model the cross-efficiency formulation of DEA to consider undesirable outputs.

3 Proposed method

Cross-efficiency evaluation has been used in various applications, e.g., efficiency evaluations of nursing homes (Sexton et al., 1986), Research and Development (R&D) project selection (Oral et al., 1991), preference voting (Green et al., 1996), ranking of countries at the summer Olympics games (Wu et al., 2009) and customer value analysis (Mahdiloo et al., 2011).

To the best of the knowledge of authors, there is not any reference that uses cross-efficiency model and undesirable outputs, simultaneously. The input-oriented CCR (Charnes et al., 1978) model evaluates supplier under investigation (DMU$_o$) (o = 1, …, n) by solving the following linear program. The used variables are summarised in nomenclature.

$$
\begin{aligned}
&\text{Max } h_A = \sum_{r=1}^{k} \mu_r y_{ro} \\
&\text{s.t.} \\
&\sum_{i=1}^{m} v_i x_{io} = 1, \\
&\sum_{r=1}^{k} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, \ldots, n \qquad (1) \\
&v_i \ge 0, i = 1, 2, \ldots, m, \\
&\mu_r \ge 0, \quad i = 1, 2, \ldots, k.
\end{aligned}
$$

The dual (envelopment) form of Model (1) is as follows:

$$
\begin{aligned}
&\text{Min } h_B = \theta \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j y_{ij} + s_i^- = \theta x_{io}, \quad i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, 2, \ldots, k, \\
&\lambda_j, \quad s_i^-, \quad s_r^+ \geq 0.
\end{aligned} \tag{2}
$$

Defective PPM is one of the criteria that is used in this paper to evaluate suppliers. The way to treat this factor in DEA is to consider it as an undesirable output. To consider undesirable outputs in an envelopment (dual) form of BCC model, Seiford and Zhu (2002) suggested a linear monotone decreasing transformation, $\overline{y}_{sj}^b = -y_{sj}^b + v > 0,$ where v is a proper translation vector that makes $\overline{y}_{sj}^b > 0.$ To accommodate technologies that exhibit constant returns to scale, we formulate the CCR version of Seiford and Zhu (2002) model as follows:

$$
\begin{aligned}
&\text{Min } h_C = \theta \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, \quad i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, 2, \ldots, k, \\
&\sum_{j=1}^{n} \lambda_j \overline{y}_{sj}^b - s_s^+ = \overline{y}_{SO}^b, \quad s = k+1, \ldots, p \\
&\lambda_j, s_i^-, s_r^+, s_S^+ \geq 0.
\end{aligned} \tag{3}
$$

Table 1 presents a simple numerical example involving 10 DMUs, with a single input, a desirable output and an undesirable output, which reveals a problem in the CCR version of Seiford and Zhu (2002) model. Note that this problem occurs owing to the arbitrariness of v. That is, when we translate the original data of undesirable output with different amounts of v and run Model (3), the classification of the DMUs as weak-efficient or inefficient remains, but the efficiency score of each inefficient unit is distorted. As Zhu and Cook (2007) discussed, the translation invariance property allows the envelopment form of many DEA models to translate inputs or outputs data without any difference between the results of translated data and original data. However, the envelopment form of the input (output)-oriented CCR model is not translation invariant with respect to either outputs or inputs.

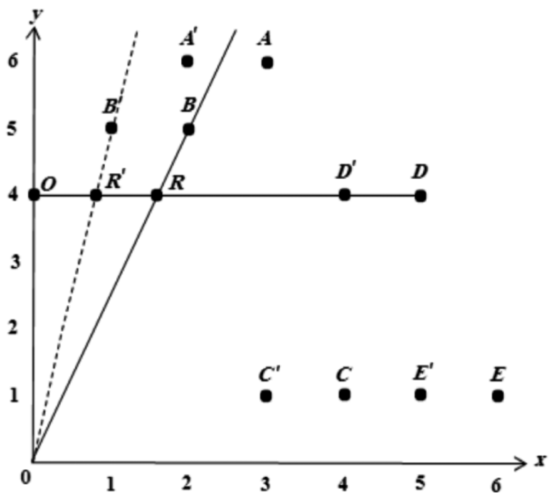
Table 1 Numerical example

DMUs	*X*	y^g	y^b	*Efficiency scores* (v = *15*)	*Efficiency scores* (v = *20*)
1	9	12	10	0.333	0.333
2	8	14	11	0.438	0.438
3	5	13	12	0.650	0.650
4	4	12	5	0.766	0.750
5	2.5	10	7	1	1
6	3	9	3	1	1
7	7	5	5	0.357	0.357
8	11	4	4	0.250	0.236
9	13	3	8	0.135	0.150
10	2	1	7	1	1

The efficiency scores defined in Model (3) with $v = 15$ and $v = 20$ are reported in Table 1. It can be easily seen that the results obtained by $v = 15$ and $v = 20$ are different from each other and it reduces the validity of the model.

Therefore, the strategy of Seiford and Zhu (2002) to change undesirable outputs to desirable outputs has a limitation; i.e., before using any model, the translation invariance property of the model should be viewed first. Figure 1 demonstrates lack of translation invariance property of Model (2) graphically.

Figure 1 Translation in the CCR model



In Figure 1, DMU_D has the input-oriented CCR efficiency OR/OD, which is the distance of DMU_D from the efficiency frontier constructed by efficient unit B. Since OR/OD is equal to the objective function of Model (2), OR/OD = θ = 1.6/5. This ratio is not invariant when we translate input values by deducting a unity from them. Now, efficiency frontier shifts to the left and input-oriented CCR efficiency of $DMU_{D'}$, DMU_D after translation, becomes OR′/OD′ = θ = 0.8/4, which is the distance of $DMU_{D'}$ from the efficiency frontier constructed by efficient unit B′. Since $\theta \neq \theta'$, the input-oriented CCR model is not translation invariant with respect to inputs. Notice that similar process can be done to show Model (3) is not translation invariant with respect to either outputs or inputs. Therefore, the strategy of Seiford and Zhu (2002) to change undesirable outputs to desirable outputs cannot be used in the CCR Model.

Therefore, following Korhonen and Luptacik (2004), Yang and Pollitt (2009) and Mahdiloo et al. (2011, 2012), undesirable outputs are included like inputs into the CCR Model (Model 1), which do not suffer from the above-mentioned problem. Suppose that there are n homogeneous DMUs each consuming m inputs and producing p outputs. The outputs corresponding to indices $1, 2, \ldots, k$ are desirable and the outputs corresponding to indices $k + 1,\ k + 2, \ldots, p$ are undesirable outputs. It is preferred to produce desirable outputs as much as possible and not to produce undesirable ones. Let $x \in R_+^{m\times n}$ and $y \in R_+^{p\times n}$ be the matrices, consisting of non-negative elements, containing the observed inputs and outputs for the DMUs, respectively. The matrix

$$y = \begin{pmatrix} y^g \\ y^b \end{pmatrix}$$

is decomposed where matrix y^g stands for desirable outputs (good) and matrix y^b stands for undesirable outputs (bad). The vector y_j is decomposed into two parts, i.e.,

$$y_j = \begin{pmatrix} y_j^g \\ y_j^b \end{pmatrix}$$

where vectors y_j^g and y_j^b refer to the desirable and undesirable outputs of DMU$_j$, respectively. The vector x_j is the input consumed by DMU$_j$, and x_{ij} stands for the quantity of input i consumed by DMU$_j$.

$$E_{OO} = \text{Max } h_D = \sum_{r=1}^{k} \mu_r^g y_{ro}^g$$

s.t.

$$\sum_{i=1}^{m} v_i x_{io} + \sum_{S=k+1}^{p} \mu_S^b y_{SO}^g = 1,$$

$$\sum_{r=1}^{k} \mu_r^g y_{rj}^g - \sum_{s=k+1}^{p} \mu_s^b y_{sj}^b - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, 2, \ldots, n,$$

$$\mu_r^g \ge 0, \quad r = 1, 2, \ldots, k,$$

$$v_i \ge 0, \quad i = 1, 2, \ldots, m,$$

$$\mu_s^b \ge 0, \quad s = k+1, \ldots, p. \tag{4}$$

At this juncture, to create a unique ordering among the efficient DMUs and to eliminate unrealistic weighting schemes in Model (4), we develop the cross-efficiency form of this model. For each DMU$_o$ ($o = 1, \ldots, n$), in Model (4), we can obtain a set of optimal weights (multipliers) $(\mu_r^{*g}, \mu_S^{(*b)}, v_i^*)$. Using these sets of weights, the cross-efficiency for any DMU$_j$ ($j = 1, \ldots, n$) is then calculated as:

$$E_{oj}=\frac{\sum_{r=1}^{k}\mu_{ro}^{*g}y_{rj}^{g}}{\sum_{r=1}^{m}v_{io}^{*}x_{ij}+\sum_{S=k+1}^{p}\mu_{SO}^{*b}y_{sj}^{b}},\quad o,j=1,2,\ldots,n \tag{5}$$

where E_{oj} shows the relative efficiency of DMU_j with optimal weights for inputs and outputs of DMU_o. One can compute the average of the efficiencies in each column to get a measure of how the DMUs associated with the column are rated by the rest of the DMUs. Good operating practices are more likely to be exhibited by relatively efficient DMUs offering high average efficiencies in their associated columns in the cross-efficiency matrix. Since Model (4) will be run n times for n DMUs, respectively, each DMU will get n efficiency scores, which construct an $n\times n$ matrix, called cross-efficiency matrix. For DMU_j ($j = 1, \ldots, n$), the average of all E_{oj} ($o = 1, \ldots, n$), namely

$$\overline{E}_j=\frac{1}{n}\sum_{o,j=1}^{n}E_{oj} \tag{6}$$

can be used as an efficiency measure for DMU_j, and will be referred to as the cross-efficiency score for DMU_j.

The non-uniqueness of the DEA optimal weights possibly reduces the usefulness of the cross-efficiency, which considers undesirable outputs. To overcome this problem, Doyle and Green (1994) suggested the use of aggressive and benevolent cross evaluation. A cross evaluation is aggressive/benevolent in the sense that it selects a set of weights, which not only maximise the efficiency of a particular DMU under evaluation, but also minimise/maximise the efficiencies of all other DMUs in some sense. We develop the aggressive formulation of Model (4) and present it as Model (7). Note that the benevolent formulation has the same set of constraints except that the objective function is maximised.

$$\begin{aligned}
&\text{Min }\ h_E=\mu_r^g\sum_{j\neq 0}y_{rj}^g\\
&\text{s.t.}\\
&v_i\sum_{j\neq 0}x_{ij}+\mu_s^b\sum_{j\neq 0}y_{sj}^b=1,\\
&\sum_{r=1}^{k}\mu_r^g y_{rj}^g-\left(\sum_{i=1}^{m}v_i x_{ij}+\sum_{s=k+1}^{p}\mu_s^b y_{sj}^b\right)\leq 0,\quad j\neq o,\\
&\sum_{r=1}^{k}\mu_r^g y_{ro}^g-E_{oo}\left(\sum_{i=1}^{m}v_i x_{io}+\sum_{s=k+1}^{p}\mu_s^b y_{sj}^b\right)=0,\quad j=1,2,\ldots,n,\\
&\mu_s^g\geq 0,\quad r=1,2,\ldots,k,\\
&v_i\geq 0,\quad i=1,2,\ldots,m,\\
&\mu_s^b\geq 0,\quad s=k+1,\ldots,p,
\end{aligned} \tag{7}$$

where E_{oo} is the efficiency of DMU_o obtained from Model (4).

4 Numerical example

To demonstrate the application of the proposed model in supplier ranking context, the data set for this study is partially taken from Farzipoor Saen (2010a). The inputs for selecting suppliers include Total Cost of shipments (TC) and Number of Shipments per month (NS). The desirable outputs utilised in the study are Number of shipments to arrive On Time (NOT) and Number of Bills received from the supplier without errors (NB), and Defective PPM is considered as an undesirable output. Table 2 shows the data set for 18 suppliers.

Table 2 Data set for 18 suppliers

	Inputs		*Outputs*		
Supplier No. (DMU)	*TC (1000$)* x_{1j}	*NS* x_{2j}	*NOT* y^b_{1j}	*NB* y^b_{2j}	*PPM* y^b_{1j}
1	253	197	187	90	1
2	268	198	194	130	5.3
3	259	229	220	200	4.6
4	180	169	160	100	30
5	257	212	204	173	30
6	248	197	192	170	30
7	272	209	194	60	30
8	330	203	195	145	13.8
9	327	208	200	150	4
10	330	203	171	90	30
11	321	207	174	100	26.4
12	329	234	209	200	25.8
13	281	173	165	163	25.8
14	309	203	199	170	21.9
15	291	193	188	185	9
16	334	177	168	85	7
17	249	185	177	130	6.3
18	216	176	167	160	28.8

Table 3 illustrates the efficiency scores of suppliers, using Model (4), and their ranking results. In this model, each supplier seeks to maximise its efficiency score by choosing a set of optimal weights for all inputs and outputs. In this evaluation, the best suppliers are suppliers 1, 2, 3, 4, 6, 14, 15 and 18, which their efficiency scores equal to unity.

As you see, Model (4) cannot give a complete ranking and there are ties among eight efficient suppliers. Therefore, we used Model (7) to derive the suppliers' cross-efficiency score and their complete ranking. Table 5 shows the cross-efficiency matrix.

Table 4 displays the supplier's final efficiency scores and final rankings derived by cross-efficiency approach.

Table 3 Efficiency scores and ranking using Model (4)

Supplier No. (*DMU*)	*Efficiency scores*	*Rank*
1	1	1
2	1	1
3	1	1
4	1	1
5	0.992	10
6	1	1
7	0.950	15
8	0.980	12
9	0.997	9
10	0.859	17
11	0.857	18
12	0.919	16
13	0.983	11
14	1	1
15	1	1
16	0.969	14
17	0.979	13
18	1	1

Table 4 Results of evaluation via cross-efficiency approach

Supplier No. (*DMU*)	*Cross-efficiency score*	*Rank*
1	0.882	3
2	0.866	5
3	0.946	1
4	0.780	14
5	0.834	10
6	0.845	7
7	0.695	16
8	0.813	12
9	0.870	4
10	0.651	18
11	0.665	17
12	0.790	13
13	0.822	11
14	0.840	8
15	0.900	2
16	0.758	15
17	0.852	6
18	0.836	9

Table 5 Matrix of cross-efficiency

Supplier No. (DMU)	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
1	**1.000**	0.196	0.256	0.029	0.036	0.034	0.035	0.076	0.267	0.030	0.035	0.043	0.034	0.049	0.112	0.128	0.150	0.031
2	1.000	**1.000**	1.000	0.766*	0.825	0.824	0.779	0.918	0.997	0.708	0.729	0.805	0.812	0.883	0.972	0.943	0.966	0.791
3	1.000	0.582	**1.000**	0.106	0.179	0.176	0.062	0.293	0.758	0.092	0.115	0.234	0.193	0.232	0.544	0.293	0.522	0.174
4	0.832	0.814	0.956	**1.000**	0.893	0.871	0.802	0.665	0.688	0.583	0.610	0.715	0.661	0.725	0.727	0.566	0.800	0.870
5	0.971	0.995	1.000	0.993	**0.992**	1.000	0.948	0.947	0.953	0.830	0.835	0.902	0.940	0.977	0.972	0.910	0.972	0.977
6	0.895	0.956	1.000	0.935	0.984	**1.000**	0.842	0.932	0.938	0.781	0.792	0.919	0.971	0.980	1.000	0.860	0.946	0.990
7	0.973	1.000	0.993	0.982	0.990	1.000	**0.950**	0.966	0.970	0.847	0.849	0.909	0.959	0.991	0.986	0.941	0.977	0.975
8	0.969	1.000	0.981	0.965	0.981	0.994	0.947	**0.980**	0.981	0.859	0.857	0.911	0.973	1.000	0.994	0.969	0.976	0.968
9	1.000	1.000	1.000	0.766	0.825	0.824	0.779	0.918	**0.997**	0.708	0.729	0.805	0.812	0.883	0.972	0.943	0.966	0.791
10	0.968	0.999	0.980	0.966	0.982	0.994	0.947	0.980	0.981	**0.859**	0.857	0.911	0.973	1.000	0.994	0.968	0.976	0.968
11	0.968	0.999	0.980	0.966	0.982	0.994	0.947	0.980	0.981	0.859	**0.857**	0.911	0.973	1.000	0.994	0.968	0.976	0.968
12	0.895	0.956	1.000	0.935	0.984	1.000	0.842	0.932	0.938	0.781	0.792	**0.919**	0.971	0.980	1.000	0.860	0.946	0.990
13	0.477	0.685	0.911	0.617	0.851	0.900	0.299	0.745	0.752	0.463	0.504	0.892	**0.983**	0.874	1.000	0.501	0.733	0.948
14	0.951	0.990	0.983	0.955	0.981	0.996	0.922	0.975	0.976	0.845	0.846	0.916	0.979	**1.000**	1.000	0.952	0.970	0.973
15	0.554	0.734	1.000	0.429	0.646	0.665	0.226	0.695	0.828	0.346	0.397	0.735	0.732	0.725	**1.000**	0.513	0.765	0.681
16	0.969	1.000	0.981	0.965	0.981	0.994	0.947	0.980	0.981	0.859	0.857	0.911	0.973	1.000	0.994	**0.969**	0.976	0.968
17	0.965	1.000	1.000	0.967	0.988	1.000	0.928	0.970	0.976	0.839	0.842	0.916	0.969	0.997	1.000	0.941	**0.979**	0.977
18	0.491	0.687	1.000	0.697	0.903	0.937	0.307	0.676	0.695	0.419	0.469	0.876	0.892	0.822	0.946	0.416	0.737	**1.000**

*0.766 represents the cross-efficiency score of supplier #4 in terms of optimal weights of supplier #2.

**Bold numbers in the leading diagonal are the simple efficiencies derived by Model (2).

As the last column of Table 4 depicts, supplier 3 is the most efficient supplier and is the first candidate for selection.

To demonstrate how Models (4) and (7) are run, samples of these models for supplier #1 have been presented in Appendix 1.

5 Managerial implications

The selection and maintenance of an effective supply base is one of the most important objectives in SCM (Azadeh and Alem, 2010). However, sometimes in suppliers' evaluation problem, there may exist some criteria that should be considered as undesirable outputs. In accordance with the global environmental conservation awareness, undesirable outputs of productions and social activities, e.g., air pollutants and hazardous wastes, are being increasingly recognised as dangerous and undesirable (Cooper et al., 2007). In performance evaluation of the suppliers' problem in which some outputs are undesirable, classical DEA models cannot be used because of the requirement that inputs have to be minimised and outputs have to be maximised (Mahdiloo et al., 2011, 2012). In addition, to get a complete ranking among suppliers and also eliminate unrealistic weighting schemes among them, this paper proposes a cross-efficiency formulation of DEA, which can treat undesirable outputs. Thus, the proposed model can help managers or decision-makers make a more accurate judgement. This is the advantage that the traditional methods cannot have.

6 Concluding remarks

Firms are using effective SCM to support their multiple manufacturing goals such as flexibility, cost, quality and delivery (Wacker, 1996). Supplier selection is used to describe various phenomena in SCM. The purpose of supplier selection is to determine the optimal supplier who can offer the best products or services for the customer and become a part of the organisation's supply chain (Ebrahim et al., 2009). In this paper, DEA as a multiple criteria decision-making tool is used to evaluate suppliers. In applying DEA, we discussed about a particular situation in which some factors play the role of undesirable outputs. To derive a complete ranking of suppliers and eliminate unrealistic weighting schemes among DMUs, the cross-efficiency formulation of undesirable output was developed. Some of the contributions of this paper are as follows:

- the proposed model evaluates suppliers in a multi criteria context
- supplier selection is a straightforward process carried out by the proposed model
- the proposed model considers undesirable outputs for supplier selection
- to achieve the peer appraisal of suppliers instead of their self-appraisal, the cross-efficiency model, which considers undesirable outputs, is developed.

However, the limitation of suggested model in this paper is radial assumption of the model. In DEA, non-zero input and output slacks are more likely to reveal themselves after the radial efficiency score improvement. Often, the non-zero slack values reveal a considerable amount of inefficiency. Consequently, to fully measure the inefficiency in DMUs performance, it is essential to consider the inefficiency represented by the non-zero slacks in the presence of undesirable outputs with regard to cross-efficiency method.

The problem considered in this study is at the initial stage of investigation and further researches can be done based on the results of this paper. Some of them are as follows:

- Similar research can be repeated in the presence of imprecise data and fuzzy data.
- Similar research can be repeated in the presence of stochastic data.
- This study used the proposed model for supplier ranking context. It seems that more fields (e.g., technology ranking, personnel ranking, market ranking, etc.) can be applied.

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References

Amin, S.H., Razmi, J. and Zhang, G. (2011) 'Supplier selection and order allocation based on fuzzy SWOT analysis and fuzzy linear programming', *Expert Systems with Applications*, Vol. 38, No. 1, pp.334–342.

Azadeh, A. and Alem, S.M. (2010) 'A flexible deterministic, stochastic and fuzzy data envelopment analysis approach for supply chain risk and vendor selection problem: simulation analysis', *Expert Systems with Applications*, Vol. 37, No. 12, pp.7438–7448.

Azadi, M., Farzipoor Saen, R. and Tavana, M. (2012), Supplier selection using chance-constrained data envelopment analysis with nondiscretionary factors and stochastic data', *International Journal of Industrial and Systems Engineering*, Vol. 10, No. 2, pp. 167-196.

Berger P.D., Gerstenfeld, A. and Zeng, A.Z. (2004) 'How many suppliers are best? A decision-analysis approach', *Omega*, Vol. 32, No. 1, pp.9–15.

Charnes, A., Cooper, W.W. and Rhodes, E. (1978) 'Measuring the efficiency of decision making units', *European Journal of Operational Research*, Vol. 2, No. 6, pp.429–444.

Choy, K.L., Lee, W.B. and Lo, V. (2004) 'Development of a case based intelligent supplier relationship management system-linking supplier rating system and product coding system', *Supply Chain Management: International Journal*, Vol. 9, No. 1, pp.86–101.

Cooper, W.W., Seiford, L.M. and Tone, K. (2007) *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, 2nd ed., Springer, New York.

Cormican, K. and Cunningham, M. (2007) 'Supplier performance evaluation: lessons from a large multinational organization', *Journal of Manufacturing Technology Management*, Vol. 18, No. 4, pp.352–366.

Doyle, J. and Green, R. (1994) 'Efficiency and cross efficiency in DEA: derivations, meanings and the uses', *Journal of the Operational Research Society*, Vol. 45, No. 5, pp.567–578.

Ebrahim, R.M., Razmi, J. and Haleh, H. (2009) 'Scatter search algorithm for supplier selection and order lot sizing under multiple price discount environment', *Advances in Engineering Software*, Vol. 40, No. 9, pp.766–776.

Faez, F., Ghodsypour, S.H. and O'Brien, C. (2009) 'Vendor selection and order allocation using an integrated fuzzy case-based reasoning and mathematical programming model', *International Journal of Production Economics*, Vol. 121, No. 2, pp.395–408.

Färe, R., Grosskopf, S., Lovell, K. and Pasurka, C. (1989) 'Multilateral productivity comparisons when some outputs are undesirable: a nonparametric approach', *The Review of Economics and Statistics*, Vol. 71, No. 1, pp.90–98.

Farzipoor Saen, R. (2007) 'A new mathematical approach for suppliers selection: accounting for non-homogeneity is important', *Applied Mathematics and Computation*, Vol. 185, No. 1, pp.84–95.

Farzipoor Saen, R. (2008a) 'Supplier selection by the new AR-IDEA model', *International Journal of Advanced Manufacturing Technology*, Vol. 39, Nos. 11–12, pp.1061–1070.

Farzipoor Saen, R. (2008b) 'Using super-efficiency analysis for ranking suppliers in the presence of volume discount offers', *International Journal of Physical Distribution & Logistics Management*, Vol. 38, No. 8, pp.637–651.

Farzipoor Saen, R. (2010a) 'Restricting weights in supplier selection decisions in the presence of dual-role factors', *Applied Mathematical Modelling*, Vol. 34, No. 10, pp.2820–2830.

Farzipoor Saen, R. (2010b) 'Developing a new data envelopment analysis methodology for supplier selection in the presence of both undesirable outputs and imprecise data', *International Journal of Advanced Manufacturing Technology*, Vol. 51, Nos. 9–12, pp.1243–1250.

Ghodsypour, S.H. and O'Brien, C. (1998) 'A decision support system for supplier selection using an integrated analytic hierarchy process and linear programming', *International Journal of Production Economics*, Vols. 56–57, No. 1, pp.199–212.

Green, R., Doyle, J. and Cook, W. (1996) 'Preference voting and project ranking using DEA and cross-evaluation', *European Journal of Operational Research*, Vol. 90, No. 3, pp.461–472.

Hajidimitriou, Y.A. and Georgiou, A.C. (2002) 'A goal programming model for partner selection decisions in international joint ventures', *European Journal of Operational Research*, Vol. 138, No. 3, pp.649–662.

Jafari Songhori, M., Tavana, M., Azadeh. A. and Khakbaz, M.H. (2011) 'A supplier selection and order allocation model with multiple transportation alternatives', *International Journal of Advanced Manufacturing Technology*, Vol. 52, Nos. 1–4 , pp.365–376.

Jahanshahloo, G.R., Lotfi, F.H., Shoja, N., Tohidi, G. and Razavyan, S. (2005) 'Undesirable inputs and outputs in DEA models', *Applied Mathematics and Computation*, Vol. 169, No. 2, pp.917–925.

Kang, H. and Lee, A. (2010) 'A new supplier performance evaluation model; a case study of integrated circuit packaging companies', *Kybernetes*, Vol. 39, No. 1, pp.37–54.

Korhonen, P.J. and Luptacik, M. (2004) 'Eco-efficiency analysis of power plants: an extension of data envelopment analysis', *European Journal of Operational Research*, Vol. 154, No. 2, pp.437–446.

Kuo, R.J., Hong, S.Y. and Huang, Y.C. (2010b) 'Integration of particle swarm optimization-based fuzzy neural network and artificial neural network for supplier selection', *Applied Mathematical Modelling*, Vol. 34, No. 12, pp.3976–3990.

Kuo, R.J., Lee, L.Y. and Hu, T. (2010a) 'Developing a supplier selection system through integrating fuzzy AHP and fuzzy DEA: a case study on an auto lighting system company in Taiwan', *Production Planning & Control*, Vol. 21, No. 5, pp.468–484.

Kuo, R.J., Wang, Y.C. and Tien, F.C. (2010c) 'Integration of artificial neural network and MADA methods for green supplier selection', *Journal of Cleaner Production*, Vol. 18, No. 12, pp.1161–1170.

Lau, H., Lau, P., Fung, R., Chan, F. and Ip, R. (2005) 'A virtual case benchmarking scheme for vendors, performance assessment', *Benchmarking: International Journal*, Vol. 12, No. 1, pp.61–80.

Lee, A.H.I. (2008) 'A fuzzy supplier selection model with the consideration of benefits, opportunities, costs and risks', *Expert Systems with Applications*, Vol. 36, No. 2, pp.2879–2893.

Lee, E., Ha, S. and Kim, S. (2001) 'Supplier selection and management system considering relationships in supply chain management', *IEEE Transactions on Engineering Management*, Vol. 48, No. 3, pp.307–318.

Lin, R.H. (2009) 'An integrated FANP-MOLP for supplier evaluation and order allocation', *Applied Mathematical Modelling*, Vol. 33, No. 6, pp.2730–2736.

Liu, J., Ding, F.Y. and Lall, V. (2000) 'Using data envelopment analysis to compare suppliers for supplier selection and performance improvement', *Supply Chain Management: An International Journal*, Vol. 5, No. 3, pp.143–150.

Mahdiloo, M., Farzipoor Saen, R. and Tavana, M. (2012) 'A novel Data Envelopment Analysis model for solving supplier selection problems with undesirable outputs and lack of inputs', *Int. J. Logistics Systems and Management*, Vol. 11, No. 3, pp.285–305.

Mahdiloo, M., Noorizadeh, A. and Farzipoor Saen, R. (2011) 'Developing a new data envelopment analysis model for customer value analysis', *Journal of Industrial and Management Optimization*, Vol. 7, No. 3, pp.531–558.

Mendoza, A. and Ventura, J.A. (2010) 'A serial inventory system with supplier selection and order quantity allocation', *European Journal of Operational Research*, Vol. 207, No. 3, pp.1304–1315.

Narasimhan, R. and Stoynoff, K. (1986) 'Optimizing aggregate procurement allocation decisions', *Journal of Purchasing and Materials Management*, Vol. 22, No. 1, pp.23–30.

Ndubisi, N.O., Jantan, M., Hing, L.C. and Ayub, M.S. (2005) 'Supplier selection and management strategies and manufacturing flexibility', *The Journal of Enterprise Information Management*, Vol. 18, No. 3, pp.330–349.

Noorizadeh, A., Mahdiloo, M. and Farzipoor Saen, R. (2011) 'Supplier selection in the presence of dual-role factors, nondiscretionary inputs, and weight restrictions', *International Journal of Productivity and Quality Management*, Vol. 8, No.2, pp.134–152.

Oral, M., Kettani, O. and Lang, P. (1991) 'A methodology for collective evaluation and selection of industrial R&D projects', *Management Science*, Vol. 37, No. 7, pp.871–883.

Sarkar, A. and Mohapatra, P.K.J. (2006) 'Evaluation of supplier capability and performance: a method for supply base reduction', *Journal of Purchasing and Supply Management*, Vol. 12, No. 3, pp.148–163.

Sarkis, J. and Talluri, S. (2002) 'A model for strategic supplier selection', *Journal of Supply Chain*, Vol. 38, No. 1, pp.18–28.

Scheel, H. (2001) 'Undesirable outputs in efficiency valuations', *European Journal of Operational Research*, Vol. 132, No. 2, pp.400–410.

Seiford, L.M. and Zhu, J. (2002) 'Modeling undesirable factors in efficiency evaluation', *European Journal of Operational Research*, Vol. 142, No. 1, pp.16–20.

Sexton, T.R., Silkman, R.H. and Hogan, A.J. (1986) 'Data envelopment analysis: critique and extensions', in Silkman, R.H. (Ed.): *Measuring Efficiency: An Assessment of Data Envelopment Analysis*, Jossey-Bass, San Francisco, Chapter 4, pp.73–105.

Talluri, S. and Baker, R.C. (2002) 'A multi-phase mathematical programming approach for effective supply chain design', *European Journal of Operational Research*, Vol. 141, No. 3, pp.544–558.

Vinodh, S., Anesh Ramiya, R. and Gautham, S.G. (2011) 'Application of fuzzy analytic network process for supplier selection in a manufacturing organisation', *Expert Systems with Applications*, Vol. 38, No. 1, pp.272–280.

Wacker, J.G. (1996) 'A theoretical model of manufacturing lead times and their relationship to a manufacturing goal hierarchy', *Decision Sciences*, Vol. 27, No. 3, pp.483–517.

Wang, G., Huang, S.H. and Dismukes, J.P. (2004) 'Product-driven supply chain selection using integrated multi-criteria decision-making methodology', *International Journal of Production Economics*, Vol. 91, No. 1, pp.1–15.

Wanga, J., Zhaoa, R. and Tang, W. (2008) 'Fuzzy programming models for vendor selection problem in a supply chain', *Tsinghua Science and Technology*, Vol. 13, No. 1, pp.106–111.

Weber, C.A. (1996) 'A data envelopment analysis approach to measuring vendor performance', *Supply Chain Management: An International Journal*, Vol. 1, No. 1, pp.28–39.

Weber, C.A., Current, J. and Desai, A. (2000) 'An optimization approach to determining the number of vendors to employ', *Supply Chain Management: An International Journal*, Vol. 5, No. 2, pp.90–98.

Wong, W.P. and Wong, K.Y. (2008) 'A review on benchmarking of supply chain performance measures', *Benchmarking: An International Journal*, Vol. 15, No. 1, pp.25–51.

Wu, D. (2009) 'Supplier selection: a hybrid model using DEA, decision tree and neural network', *Expert Systems with Applications*, Vol. 36, No. 5, pp.9105–9112.

Wu, J., Liang, L. and Yang, F. (2009) 'Achievement and benchmarking of countries at the summer Olympics using cross efficiency evaluation method', *European Journal of Operational Research*, Vol. 197, No. 2, pp.722–730.

Yang, H. and Pollitt, M. (2009) 'Incorporating both undesirable outputs and uncontrollable variables into DEA: the performance of Chinese coal-fired power plants', *European Journal of Operational Research*, Vol. 197, No. 3, pp.1095–1105.

Yang, J.L., Chiu, H.N. and Tzeng, G.H. (2008) 'Vendor selection by integrated fuzzy MCDM techniques with independence and interdependence', *Information Sciences*, Vol. 178, No. 21, pp.4166–4183.

Zhu, J. and Cook, W.D. (2007) *Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis, A Problem-Solving Handbook*, Springer, New York, USA.

Nomenclature

DMU_o	The Decision Making Unit under investigation
n	The set of DMUs (suppliers)
$j=1, \ldots, n$	Collection of DMUs
$r=1, \ldots, k$	The set of desirable outputs
$i=1, \ldots, m$	The set of inputs
$s=k+1, \ldots, p$	The set of undesirable outputs
y_{ro}^{g}	rth desirable output of the DMU_o
x_{io}	ith input of the DMU_o
y_{so}^{b}	sth undesirable output of the DMU_o
μ_r^g	The weight for rth desirable output
v_i	The weight for ith input
μ_s^b	The weight for sth undesirable output
$y_{r_j}^{g}$	The rth desirable output of DMU_j
x_{ij}	The ith input of DMU_j
$y_{s_j}^{b}$	The sth undesirable output of DMU_j
θ	Efficiency measure for DMU_o
s_r^+	Shortages in rth desirable output
s_i^-	Excesses in ith input
s_s^+	Excesses in sth undesirable output

$\overline{y}_{s_j}^b$	The $y_{s_j}^b$ which is translated into desirable output
λ_j	Reference weights (benchmarks) associated with DMU_j
E_{oj}	Shows the relative efficiency of DMU_j with the set of optimal weights for inputs and outputs of DMU_o
E_{oo}	The efficiency score of DMU_o by its own set of optimal weights

Appendix 1

Model (4) for supplier #1:

$$\text{Max } 187\mu_1^g + 90\mu_2^g$$

s.t.

$$253v_1 + 197v_2 + 1\mu_1^b = 1,$$

$$187\mu_1^g + 90\mu_2^g - (253v_1 + 197v_2 + 1\mu_1^b) \le 0,$$

$$194\mu_1^g + 130\mu_2^g - (268v_1 + 198v_2 + 5.3\mu_1^b) \le 0,$$

$$220\mu_1^g + 200\mu_2^g - (259v_1 + 229v_2 + \mu_1^b) \le 0,$$

$$160\mu_1^g + 100\mu_2^g - (180v_1 + 169v_2 + 30\mu_1^b) \le 0,$$

$$204\mu_1^g + 173\mu_2^g - (257v_1 + 212v_2 + 30\mu_1^b) \le 0,$$

$$192\mu_1^g + 170\mu_2^g - (248v_1 + 197v_2 + 30\mu_1^b) \le 0,$$

$$194\mu_1^g + 60\mu_2^g - (272v_1 + 209v_2 + 30\mu_1^b) \le 0,$$

$$195\mu_1^g + 145\mu_2^g - (330v_1 + 203v_2 + 13.8\mu_1^b) \le 0,$$

$$200\mu_1^g + 150\mu_2^g - (327v_1 + 208v_2 + 4\mu_1^B) \le 0,$$

$$171\mu_1^g + 90\mu_2^g - (330v_1 + 203v_2 + 30\mu_1^b) \le 0,$$

$$174\mu_1^g + 100\mu_2^g - (321v_1 + 207v_2 + 26.4\mu_1^b) \le 0,$$

$$209\mu_1^g + 200\mu_2^g - (329v_1 + 234v_2 + 25.8\mu_1^b) \le 0,$$

$$165\mu_1^g + 163\mu_2^g - (281v_1 + 173v_2 + 25.8\mu_1^b) \le 0,$$

$$199\mu_1^g + 170\mu_2^g - (309v_1 + 203v_2 + 21.9\mu_1^b) \le 0,$$

$$188\mu_1^g + 185\mu_2^g - (291v_1 + 193v_2 + 9\mu_1^b) \le 0,$$

$$168\mu_1^g + 85\mu_2^g - (334v_1 + 177v_2 + 7\mu_1^b) \le 0,$$

$177\mu_1^g + 130\mu_2^g - (249v_1 + 185v_2 + 6.3\mu_1^b) \le 0,$

$167\mu_1^g + 160\mu_2^g - (216v_1 + 176v_2 + 28.8\mu_1^b) \le 0,$

$\mu_1^g \ge 0,$

$\mu_2^g \ge 0,$

$v_1 \ge 0,$

$v_2 \ge 0,$

$\mu_1^b \ge 0.$

Model (7) for supplier #1:

Min $3177\mu_1^g + 2411\mu_2^g,$

s.t.

$4801v_1 + 3376v_2 + 328.7\mu_1^b = 1,$

$187\mu_1^g + 90\mu_2^g - 1(253v_1 + 197v_2 + 1\mu_1^b) = 0,$

$194\mu_1^g + 130\mu_2^g - (268v_1 + 198v_2 + 5.3\mu_1^b) \le 0,$

$220\mu_1^g + 200\mu_2^g - (259v_1 + 229v_2 + \mu_1^b) \le 0,$

$160\mu_1^g + 100\mu_2^g - (180v_1 + 169v_2 + 30\mu_1^b) \le 0,$

$204\mu_1^g + 173\mu_2^g - (257v_1 + 212v_2 + 30\mu_1^b) \le 0,$

$192\mu_1^g + 170\mu_2^g - (248v_1 + 197v_2 + 30\mu_1^b) \le 0,$

$194\mu_1^g + 60\mu_2^g - (272v_1 + 209v_2 + 30\mu_1^b) \le 0,$

$195\mu_1^g + 145\mu_2^g - (330v_1 + 203v_2 + 13.8\mu_1^b) \le 0,$

$200\mu_1^g + 150\mu_2^g - (327v_1 + 208v_2 + 4\mu_1^b) \le 0,$

$171\mu_1^g + 90\mu_2^g - (330v_1 + 203v_2 + 30\mu_1^b) \le 0,$

$174\mu_1^g + 100\mu_2^g - (321v_1 + 207v_2 + 26.4\mu_1^b) \le 0,$

$209\mu_1^g + 200\mu_2^g - (329v_1 + 234v_2 + 25.8\mu_1^b) \le 0,$

$165\mu_1^g + 163\mu_2^g - (281v_1 + 173v_2 + 25.8\mu_1^b) \le 0,$

$199\mu_1^g + 170\mu_2^g - (309v_1 + 203v_2 + 21.9\mu_1^b) \le 0,$

$$188\mu_1^g + 185\mu_2^g - (291v_1 + 193v_2 + 9\mu_1^b) \le 0,$$

$$168\mu_1^g + 85\mu_2^g - (334v_1 + 177v_2 + 7\mu_1^b) \le 0,$$

$$177\mu_1^g + 130\mu_2^g - (249v_1 + 185v_2 + 6.3\mu_1^b) \le 0,$$

$$167\mu_1^g + 160\mu_2^g - (216v_1 + 176v_2 + 28.8\mu_1^b) \le 0,$$

$$\mu_1^g \ge 0,$$

$$\mu_2^g \ge 0,$$

$$v_1 \ge 0,$$

$$v_2 \ge 0,$$

$$\mu_1^b \ge 0.$$