Determining the vertical profile of reflectivity using radar observations at long range

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Abstract
The Vertical Profile of Reflectivity (VPR) plays an important role when estimating the rain rate at the surface and has been the subject of radar meteorology research for many years. The VPR can either be sampled directly from observations that are close to the radar where the impact of the convolution with the beam pattern can be ignored, or the parameters for a theoretical form for the VPR are estimated using the available observations or climatology. In either case, a significant difficulty arises when a rain band approaches the radar and quantitative precipitation estimates are required before any detailed observations of the VPR at close range are possible. Long range in this context is the range where the height of the lowest elevation angle in the volume scan is greater than the wet bulb freezing level at that time, and therefore only limited information on the shape of the bright band is available. This paper uses a modified version of the VPR model proposed by Fabry (1997) and evaluates strategies to make optimum use of empirical observations, and how estimates for the model parameters could be updated in time. The technique is demonstrated using case studies of widespread rainfall over Sydney and Brisbane, Australia. Comparing the final technique to both the current short range and long range methods indicates that the parameterised VPR is able to provide similar VPR accuracies as the short range, with great improvement on the current long range method, making it suitable for rainfall corrections.

Keywords
observations, long, vertical, profile, determining, reflectivity, radar, range

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Determining the vertical profile of reflectivity using radar observations at long range

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Abstract The Vertical Profile of Reflectivity (VPR) plays an important role when estimating the rain rate at the surface and has been the subject of radar meteorology research for many years. The VPR can either be sampled directly from observations that are close to the elevation angle in the volume scan is greater than the wet bulb approaches the radar and demonstrated using case studies of widespread observations, available observations or climatology. Determining the vertical profile of reflectivity using radar the surface and has been the subject of radar meteorology research for many years. Key words vertical profile; parameterisation; long-range; beam convolution; Australia

INTRODUCTION

Accurate determinations of surface rainfall by radar are important in providing the spatial distribution of rainfall. This is significant in areas from aviation to flood forecasting. However, radar cannot measure surface rainfall directly, but relies on reflectivity measurements at various altitudes that vary according to the Vertical Profile of Reflectivity (VPR). One of the most significant effects is the increase of reflectivity at the 0°C isotherm, known as the bright band, occurring due to the melting of ice into liquid droplets. Considerable attention has been given to methods of determining the VPR over the past years (Andrieu & Creutin, 1995; Joss & Lee, 1995; Kitchen et al., 1994; Vignal et al., 2000, etc.).

The bright band is a significant increase in radar reflectivity over a relatively small change in altitude, so the profile is strongly dependent on the range of the observation due to the convolution of the Gaussian beam profile and the true vertical profile. The curvature of the Earth implies that the minimum observable height increases with range. These factors combine to limit the amount of information that can be inferred about the VPR from observations that are distant from the radar and illustrates why the VPR for many quantitative precipitation estimation systems is based on observations that are within 65 km of the radar. This paper assumes that the VPR is constant over the area under the radar and provides a method for inferring the VPR from observations at long ranges using a parameterised model. The basic idea that is presented in this paper is to fit VPR model parameters using data from a number of range intervals. Since some parameter estimates are more equal than others (as it were) a range weighted mean of the variable is then calculated.

Previous VPR parameterisations have been performed, in particular that by Fabry (1997):

\[ z(h) = z_0 + 6 \times 10^{-3} h - 6.5 \times 10^{-3} h + 9 \exp \left( -0.6 \frac{(h-h_b)}{d} \right) (h-h_b) \quad h > h_b \]
\[ z(h) = z_0 - 0.5 \times 10^{-3} h + 9 \exp \left( -0.6 \frac{(h-h_b)}{d} \right) (h-h_b) \quad h < h_b \]

where \( d \) is the bright band width, \( z_0 \) the ground reflectivity, \( h \) the height and \( h_b \) the bright band

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height. Fabry (1997) indicated that the VPR was not strongly dependant on rain rate, and such parameterised VPR's may be successful for rainfall correction purposes.

PARAMETERISATION

The parameterisation was initially defined as being similar to equation (1) using five parameters now with \( a \), the slope in reflectivity below the bright band, \( b \), the slope above the bright band and, \( c \), the bright band intensity while an added parameter \( e \) was found to be required so as to account for the offset of reflectivity of rain above the bright band to ice below.

\[
z(h) = z_0 + ah + b(h - h_b) + c\exp\left(-\frac{(h - h_b)}{d'}\right) \quad h > h_b
\]

\[
z(h) = z_0 + ah + b(h - h_b) + e\exp\left(-\frac{(h - h_b)}{d'}\right) \quad h \leq h_b
\]

The VPR model can also be compared to other inverse methods such as that proposed by Kistetter et al. (2010). This method however, requires the use of an a priori VPR within close range, making it unsuitable when only long-range data are available. It also has a time update of 1 h rather than 6 minutes, as in the VPR model proposed in this paper. However, unlike the model presented in this paper, which is restricted to long-range data, Kistetter et al. (2010) allows for improved VPR determination and application over the entire radar domain.

ANALYSIS OF RAINFALL DATA METHOD

The two sets of rainfall data analysed include: (i) 122 six-minute profiles from Sydney (Terry Hills), New South Wales, Australia 25 May 2010, and (ii) 240 six-minute profiles from Brisbane (Mt. St. Mary's), Queensland, Australia 19 August 2010. The profiles were calculated from the volume scan data in 0.2 km intervals up to 9.8 km above the radar and 10 km range intervals up to 140 km from the radar. Typical or initial values for each parameter are taken as an overall estimate based on previous studies (Battan, 1973; Kitchen et al., 1994; Fabry & Zawadzki, 1995; Matrosov et al., 2007; Scovell et al., 2008) and observed profiles from the case studies. The parameter estimates are then \( a = -0.85 \text{ dBZ km}^{-1} \), \( b = -3 \text{ dBZ km}^{-1} \), \( c = 7 \text{ dBZ} \), \( d = 0.35 \text{ km} \), \( e = -2.1 \text{ dBZ} \) and \( h_b = 2 \text{ km} \).

With the initial parameters, the determined VPR is convolved with the beam profile so as to simulate the radar observations of the VPR at a particular range. The sensitivity of the VPR to each parameter is then determined as a function of range by calculating the RMSE variation between the initial model and the VPR obtained when one of the parameters is varied to indicate the relative error in that parameter (results for 10% error given in Fig. 1). Low sensitivity in the RMSE to perturbations in the value of the parameter implies a limited ability to determine the parameter from observations and allows for consideration of appropriate methods for determining each parameter inversely from the measured observations.

Once methods to determine the parameters are found, weighting with range must be applied to determine the final parameter values. The weighting is also performed so that parameter values may still be obtained even if data were only available at the furthest ranges, and these estimates are updated as data closer to the radar becomes available. Initially standard deviation is considered, however it is found for poorly measured profiles that outlier terms with low standard deviations occur, providing unsuitable weighting.

Slope of the VPR above the bright band, \( b \)

From Fig. 1(a), the RMSE for \( b \) is uniformly large for all ranges and therefore it should be possible to estimate the value of \( b \) reliably. Also, measurements above the bright band are well defined at long ranges, and a distinct value of \( b \) is attainable directly from the data using a linear regression
of points measured 700 m above the wet bulb freezing level. This range is chosen as Scovell et al. (2008) found that values 500 m above the bright band peak allowed the measured slope to provide an accurate indication of the VPR slope regardless of range, and 700 m allows for errors in the estimate. Then, due to the uniform sensitivity, the mean \( b \) determined over the various ranges provides a robust estimate.

![Image](image_url)

**Fig. 1** (a) RMSE between the initial VPR convolved with the beam profile to the relevant range and the VPR when there is a 10% error in one parameter and (b) RMSE (Error c) for positive and negative errors of c about 7 dBZ versus the minimum height at a particular range minus the bright band height.

### Intensity of the bright band, \( c \)

While a fairly low RMSE occurs in Fig. 1(a) for \( c \), the intensity of the bright band, changes in \( c \) only affect the VPR around the bright band meaning \( c \) may be determined from the measured data if the elevation of the base scan is below the height of the bright band. The relative error was assumed to be a function of the difference of the minimum observable height at a particular range and the bright band height as observed by the decreasing trend in Fig. 1(a). To determine this dependence \( c \) was varied about the “true” value of 7 dBZ in the positive direction up to 14 dBZ and downward to 0 dBZ in 0.05 dBZ increments using \( h_n = 1.5 \) km, 2 km and 2.5 km. The linear trend of RMSE versus error in \( c \) was then calculated with the corresponding RMSE/Error \( c \) values given in Fig. 1(b).

From Fig. 1(b), it is shown that uniformly accurate results occur for ranges with a minimum observable height 100 m above the bright band or less, and hence are provided equal weighting. For ranges beyond this, an exponentially decreasing weighting is applied:

\[
\begin{align*}
\omega_i &= 1 \\
\omega_i &= -\exp(h_n - (fl - 0.5)) + 1.15, \quad \text{for } 0.1 < h_n - (fl - 0.5) < 2.1 \\
\omega_i &= 0, \quad \text{for } h_n - (fl - 0.5) > 2.1
\end{align*}
\]

(3)

where \( \omega_i \) is the weighting factor \( 0 < \omega_i < 1 \), \( h_n \) is the minimum observable height at range \( i \) and \( fl \) is the wet bulb freezing level forecast from a Numerical Weather Prediction model (NWP). The final \( c \) is then calculated as the weighted mean using \( \omega_i^2 \).

### Height of the bright band, \( h_b \)

In Fig. 1(a), the RMSE shows large sensitivity to perturbations in \( h_n \) the height of the bright band. However, this ability decreases with range and it may be expedient to use the wet bulb freezing level as an estimate of \( h_b \) beyond some range. The combination of results should allow \( h_b \) to be known within a 200 m accuracy as required for a suitable VPR (Mittermaier & Fillungworth, 2002).
The Nelder-Mead downhill minimisation method (Press, 2002) was used to estimate values of \( c, h_b \) and \( z_0 \) for each range interval. The function minimised was the absolute difference between the parameterised VPR, convolved to the relevant range, and the measured VPR at that range. Weighting for \( h_b \) is then applied where the wet bulb freezing level estimates provides the theoretical estimate of \( h_b \). While standard deviation alone does not provide a suitable indication of accuracy, it may be compared to the RMSE of the wet bulb freezing level (\( RMS_{wbf} \)) and allow results to be weighted with the NWP estimate of the height of the bright band. In this case, let the bright band height determined from the data be \( h_{bm} \) with standard deviation \( ST_{m} \). Then the weighting will depend on the \( ST_{m} \) compared to \( RMS_{wbf} \) in a linear fashion giving the final value of \( h_b \) for \( N \) available ranges as:

\[
h_b = \frac{1}{N} \sum_{i=1}^{N} w_{i} h_{wm} + (1 - w_{i}) (\beta - 0.35) \text{ where } w_{i} = \frac{RMS_{wbf}}{ST_{m} + RMS_{wbf}}
\]  

(4)

Slope of the VPR below the bright band, \( a \)

From Fig. 1(a), a relatively small RMSE occurs for \( a \). At long ranges changes in \( a \) effectively act to shift the VPR meaning changes are spread over the entire profile and VPRs with different \( a \) values will be hardly discernible. Taking into account ground clutter, it may then be most suitable to fix \( a \) based on climatological and orographic trends in the area. These values are approximately 0.8 dBZ km\(^{-1}\) for Terry Hills and 0.2 dBZ km\(^{-1}\) for Mt Stagylion based on fitting parameters to the mean of the short range VPR’s over the entire period and simply provide an idea of how a fixed value affects the results given some idea of trends of low level rainfall.

Rain-snow offset, \( e \)

Conclusions of fixing \( e \) are similarly determined since measuring \( e \) requires information below the bright band. Also, \( e \) is based on hydrometeor properties and has little dependence on precipitation intensity (Fafy & Zawadzki, 1995), hence it was initially fixed to -2 dBZ.

Initial VPR results, however, showed that it would be advantageous to measure \( e \) in some cases. For example, calculating the total mean profile over the period at Terry Hills 60 km from the radar and determining the parameterisation for fixed \( e \), the results of the VPR below the bright band predict values near 20 dBZ while the measured values are closer to 22 dBZ. The deviation indicates that determining \( e \) would be advantageous, particularly in cases of higher bright bands. Yet as close as 90 km, the standard deviations in \( e \) means all results are within the limit -2 dBZ. Hence, determining \( e \) conditionally with range is most appropriate.

Width of the bright band, \( d \)

The \( d \) parameter provides the lowest overall sensitivity in Fig. 1, meaning a distinct value will be difficult to estimate. Since less information will be available at longer ranges and \( d \) is dependent on the bright band height, the most suitable approach is to fix \( d \) to 0.35 km, the standard profile bright band width used in studies such as Kitchen et al. (1994).

Updating the model parameters in time

The VPR parameters for the current time step are used to update the model parameters using:

\[
s_i = (1 - r) x_{i-1} + r s_{i-1}
\]

where \( s_i \) is the smoothed parameter at time \( i \), \( x_t \) is the estimate of the parameter at time \( t \), and \( r \) is the lag 1 autocorrelation of the \( x_t \) time series.

COMPARISON TO CURRENT METHOD

The current method of determining the VPR in Australia relies on data being within 65 km of the radar. Within this range, the VPR for each time step is calculated using the mean over range of the
data at each height, no temporal smoothing is performed. This VPR is then convolved to ranges greater than 65 km so as to obtain the expected VPR at the longer ranges. If data are not available within 65 km, the VPR is assumed constant with height. The VPR from the parameterised method devised is compared to both these forms of VPR by calculating the RMSE between the determined VPR in each case and the measured data. This is performed for all 240 Mt Stopyton profiles at each range and the mean of the RMSE for each range taken and compared in Fig. 2(a) (similar results for Terry Hills).

![Graph showing RMSE comparisons](image)

**Fig. 2** (a) Comparing RMSE of VPR methods of mean 0–65 km, mean 40–65 km, constant long-range and parameterised profiles with minimum ranges of 60 km and 120 km for Mt Stopyton, and (b) VPR comparison of the parameterised 60–140 km and 0–65 km mean profile at Mt Stopyton 12:42 10 August 2010.

In Fig. 2(a), it is shown that the difference between the parameterised profiles, whether using data from 60–140 km or 120–140 km, is negligible. In addition, the magnitude of the RMSE of the parameterised profiles, when compared to the measured data at the short ranges, is of the same order of magnitude as the mean VPR indicating the parameterised VPR provides close approximations to the short range mean VPR. The RMSE for the parameterised VPR is far lower than the RMSE for the constant VPR, which is currently in use when there is no short range data available to calculate the mean VPR.

As another comparison between the 60–140 km parameterised VPR and the 0–65 km mean VPR, the RMSE between the profiles themselves at each 6-minute interval is calculated. Figure 2(b) shows the 60–140 km parameterised VPR for a typical time in the data and the 0–65 km mean VPR. The standard deviations between the mean VPR profiles were calculated and are shown as error bars around the mean VPR. Results show some variation below the bright band, however even with this slight deviation, it is indicated that a typical parameterised VPR result will be within the error range of the mean 0–65 km VPR and hence provides accuracies comparable to that of the 0–65 km mean VPR.

**CONCLUSION**

The sensitivity of each parameter was used to provide an indication of how accurately that parameter can be determined and an idea of suitable methods of measuring the parameters. Only the height of the bright band and the slope of the VPR above the bright band can be determined reliably using long-range data, and at some point even the height of the bright band becomes difficult to estimate. When there is little data below the bright band, NWP forecasts of wet bulb freezing levels and climatological values must be used, otherwise linear regression and
minimisation techniques allow for suitable values to be obtained. Smoothing over time allows for the range weighted results to be updated with time and the final parameterised VPR obtained. These parameterized VPRs provide comparably accurate VPRs regardless of the minimum observable range and a typical profile comparison indicates that these profiles are within the error range of the mean 0–65 km profile. Hence, the method proposed is able to determine an approximate VPR from long-range data only and is suitable for rainfall corrections.

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