Phase Unwrapping of Self-mixing Signals Observed in Optical Feedback Interferometry for Displacement Measurement

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Phase Unwrapping of Self-mixing Signals Observed in Optical Feedback Interferometry for Displacement Measurement

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Abstract — With an Optical feedback interferometry (OFI), a self-mixing signal (SMS) can be observed and employed to determine parameters of the semiconductor laser and metrological quantities of an object. In the cases of the measurement of the movement trace of an external target, phase unwrapping of the SMS must be performed, which remains a challenging issue. We report a technique for achieving phase unwrapping of the SMS signals as follows. Firstly, the behaviour of the phase equation of the OFI is studied, based on which general laws for laser phase change with respect to external target movement is extracted. Secondly, the correlation between the laser phase change and SMS signal is studied. Finally phase unwrapping algorithm is developed based on the relationship. In order to make sure that the proposed approach works well in the cases of noisy SMS data, pre-processing algorithms are also discussed. The proposed techniques have been tested by both computer simulation and experimental data which show that excellent agreement can be achieved and the movement trace of the external target can be retrieved.

I INTRODUCTION

The optical feedback self-mixing (OFSM) phenomenon has been studied extensively in the past two decades and has seen applications in the measurement of the metrological quantities such as vibration, displacement and velocity of a moving object [1-4]. OFSM occurs when a small portion of the laser beam emitted from a semiconductor laser (SL) is reflected or backscattered by an external target and re-enters the laser cavity, resulting in variance of the lasing field in terms of both frequency and amplitude. Therefore, by intentionally imposing movement to the target and picking up the laser intensity fluctuations with a photodiode at the rear of the SL, the metrological quantities regarding the external target as well as the parameters of the SL can be measured [3, 5].

The target displacement has been measured with a fringe-counting technique in the moderate feedback regime (1< C< 4.6) where the interferometric waveform is sawtooth like and exhibits hysteresis in the work [2] with a resolution of $\lambda/2$. Later efforts were attributed to improve the measuring accuracy of this technique. Addy et al. [6] improved resolution up to $\lambda/4$ by a misalignment of the reflector with $C>1$ and a mirror as a target. Bosch et al. [7] developed an algorithm based on the interpretation of the interferfringe and of the fractional fringe to linearize the measured displacements with a rough target. In [8, 9], a fast modulation of the interferometer phase was generated by means of LD current modulation, resulting in the LD wavelength shift $\Delta \lambda$. By properly sampling the SM signal synchronously with the dither, the resolution is increased to $\lambda/10$.

The present work starts from analysis of the mathematical model of the OFSM system. The target displacement (or target movement track) is computed from the model with the aid of a novel phase unwrapping technique. As is well known, the phase unwrapping in optical interferometry is complicated by the unavoidable noise during the data acquisition process. Our strategy is to remove the noise by means of neural network curve fitting technique before performing phase unwrapping. The rest of the paper is organized as follows: Section II gives the background theory for OFSM system. The noise filtering and phase unwrapping method is proposed in Section III. Section IV and V presents the simulation and experimental result respectively. The conclusion is given in Section VI.

II BACKGROUND THEORY

The widely accepted theoretical model for the OFSM system is known as Lang-Kobayashi equations [10], taking the forms as follows:

$$\phi_F(t) = \phi_0(t) - C \cdot \sin [\phi_F(t) + \arctan(\alpha)]$$

(1)

$$P(t) = P_0 [1 + mF(t)]$$

(2)

$$F(t) = \cos (\phi_F(t))$$

(3)

Eq.(1) represents the phase condition where $\phi_0(t)$ and $\phi_F(t)$ are the laser phases without and with feedback respectively. $t = 2L/c$, is the round trip time between the LD and the external target. $C$ is called feedback factor. Eq.(2) accounts for the laser power fluctuations as a result of the
interferometric phase variations with $P(t)$ and $P_0$ denote the
laser power with and without feedback respectively. $m$
was called modulation index (typically $m=10^5$), $F(t)$ is defined
by Eq. (3) and is a periodic function of period $2\pi$.

Theoretically, the unperturbed laser phase $\phi_{0}(t)$ can be
obtained from solving the three equations, once $P(t)$ is
measured in an OFSM experimental setup. As $\phi_{0}(t)=2\pi\cdot(2L(t)/\lambda_0)$
where $L(t)$ is the target distance and $\lambda_0$ is the unperturbed laser
wavelength, the target displacement (or moving track) can be retrieved by
the relation $\Delta L(t)=L(t)-L_0=\phi_{0}(t)/4\pi-L_0$ with $L_0$
the initial target distance from the L.D. However, when looking
at Eq. (3) in more detail, one can find that $\phi_{0}(t)$ is computed
from an inverse cosine function which always produces
values in the interval $(-\pi, \pi]$. This is a common problem
encountered in the optical interferometry and is termed
wrapped phase. As a consequence, the technique to
reconstruct the true phase map for the wrapped phase is
called phase unwrapping which has been an active research
topic for more than two decades with the outcome of
numerous algorithms. Whereas the existent phase
unwrapping algorithms are not ready to be used in this
case due to the fact that the OFSM waveforms differ
significantly from the others such as in SAR or fringe
interferometry.

The basic mechanism for phase unwrapping is rather
simple. Given the interferometric signal is sampled at a
frequency much higher than the Nyquist rate according to the
sampling theory, there is only small phase difference exists
between any two adjacent points in the wrapped phase
except those points with near $2\pi$ phase jumps. Hence phase
unwrapping can be carried out by successive comparison of
the adjacent wrapped phases. Once a large phase jump is
encountered, an integer multiple of $2\pi$ should be added or

In practice, the SM signals are always contaminated with
random noises and speckles which renders phase
unwrapping non-trivial. Intuitively if the noise is eliminated
effectively, unwrapped phase could be readily achieved. As
can be seen in the following text, a variety of data processing
methods are proved necessary to remove different type of
noises in the experimental data.

III. NOISE FILTERING AND PHASE UNWRAPPING

Our first attempts were made to filter the noise by
traditional spatial and frequency filters. The phase field was
first filtered by median filter to remove the speckles and then
by mean filter to remove the white noise. The tradeoff in
choosing the filter length is the ability to preserve the
characteristics of the signal and the noise removal performance. More filter points will yield smoother signal
waveform which translates into lost signal property. Less
filter points by contrast is capable of maintaining the signal
characteristics with sacrifice of the noise removal performance. The empirical filter length is five in our case.
Whereas in most circumstances the resulting waveform is far
from satisfaction that can be used to perform unwrapping
due to the complex nature of the noises. Recent development
of neural network has recognized itself a powerful tool for
pattern recognition and non-linear system modelling, which
also sheds light on the noise elimination in SMS signal
processing studies.

We explored the application of neural networks in order
to remove the noises. The majority of neural networks are
multilayer structures which consist of a layer of input
neurons, a layer of output neurons and one or more layers of
hidden neurons (so called as both their input and output are
unknown to the user). The output is a nonlinear function of
the input variables [12]. In this work, we employed a neural
network with a single layer of hidden neurons with a sigmoid
activation function and a linear output neuron. The output of
the network is thus given by

$$g(x, w) = \sum_{j} \left[ w_{j} \cdot \tanh(\sum_{j} w_{j} \cdot x_{j}) \right] + w_{0}$$

where $x$ is the input $(n+1)$ vector, and $w$ is the vector of
$(n+1)N_x + (N_x + 1)$ parameters.

Both simulation and experimentation were carried out
using the neural network toolbox for MATLAB. The training
data are median filter- and mean filter-processed
experimental SMS signals. Each training set consists of two
hundred data points which were randomly fed to the network.
The Root Mean Square (RMS) error between the desirable
and actual output was calculated by the package. When the
RMS error reached its convergence threshold which is set up
by the user (in our case 0.01), the training was completed.
We tested the experimental SMS signals at different
feedback and found that the weak feedback curves can be
fitted very well with the proposed network which yields best
result with five neurons at the hidden layer. However, the
situation for moderate feedback is a bit complex as the
network does not perform good fitting for the curves with
jumps. We solved this problem by segmenting the
waveforms at the jumping points and fitted the signal
segments only, satisfactory outcome were produced as a
result. Curve fitted data are then extracted from the network
as noise-filtered SMS data to recover the phase.

$\phi_{0}(t)$ was reconstructed from the above pre-processed
self-mixing waveforms based on the following algorithm:

$$\phi_{0}(t) = (-1)^{M_1} \arccos(F(t)) + M_2 \cdot 2\pi$$

Where $M_1$ accumulated by one when the self-mixing signal
reaches peak and valley points and $M_2$ incremented by one
when the SMS reaches the valleys. This is illustrated in Fig.
1. One fringe of the SMS corresponds to a $2\pi$ changes in $\phi_{0}$.

The starting point is selected where the target reaches the
farthest position toward or from the L.D, It is worth noting
that in the proposed method the target moving law should be
a known priori.

Fig. 1 Target moving track (a) $\phi_{i}$ (b) $\phi_{0}$ and SMS waveform (c).
IV SIMULATIONS

The proposed phase unwrapping method was tested firstly with computer simulations. The external target is assumed to be subject to a harmonic vibration which can be represented as \( L(t) = L_0 + \Delta L \cos(2\pi f t) \), where \( L_0 \) is the initial distance between the laser front facet and the target, \( f \) is the vibration frequency, \( t \) is time variable. The laser phase without feedback is then calculated as

\[
\phi_0(t) = \frac{4\pi L(t)}{\lambda_0} = \frac{4\pi L_0}{\lambda_0} + \frac{4\pi \Delta L}{\lambda_0} \cos(2\pi f t)
\]  

(6)

If we assume \( f = 20 \) Hz, \( L_0/\lambda_0 = 20000 \) and \( \Delta L/\lambda_0 = 2 \), the self-mixing signal can be generated using (1)–(3) and (6) with \( C = 0.8 \), \( \alpha = 4 \) for weak feedback and \( C = 3 \), \( \alpha = 4 \) for moderate feedback. A small white noise is also added with signal-to-noise ratio (SNR) of 20 dB to emulate the practical situation as shown in Fig. 2. The curve fitted waveforms are plotted in Fig. 3. In order to evaluate the accuracy of our method, we compare the reconstructed moving track with its real counterpart and the error is plotted in Fig. 4. It can be seen that accuracy of our proposed method can reach \( \lambda/25 \) in the case of weak feedback and \( \lambda/20 \) where moderate feedback is presented.

![Fig. 2 Simulated SMS signals with SNR = 20dB(a) moving track of external target (b) SMS with C = 0.8 and α = 4 (c) SMS with C = 3 and α = 4](image1)

![Fig. 3 Curve fitted SMS signals (a) Weak feedback (b) Moderate feedback](image2)

V EXPERIMENT RESULTS

In the OFSM experimental setup, the SL is biased with a dc current. A metal plate is used as the target, which is made to vibrate harmonically by placing it close to a loudspeaker driven by a sinusoidal signal. The SMS is detected by the monitor photodiode (PD) and is amplified by a trans-impedance amplifier, as shown in Fig. 5 [5].

![Fig. 5 OFSM experimental setup](image3)

The neural network training took approximately 1000 presentations of data to reach the convergence criterion. It was shown that testing waveforms are fitted very well with the neural network and the target moving tracks are recovered correctly based on the unwrapped phases. The waveforms of the SMS in weak feedback (up) and moderate feedback (down) are shown in Fig. 6. Speckles and white noise are significant in both signals.

Fig. 7-a, b show the median filter and mean filter processed one period SMS signal of the harmonic movement of the target in weak feedback and moderate feedback respectively. It can be found that the random noise present in the moderate feedback SMS is still significant. Fig. 7-c, d shows further neural network processed SMS signals for weak feedback and moderate feedback circumstances respectively. It is clearly seen that the noises have been effectively eliminated.

Fig. 8 shows the target moving track recovered from SMS waveforms. It is approximately harmonic, which reveals a good recovery of the real movement of the target. The speckles along the recovered moving track for moderate feedback is due to the broadened jumping sections in the measured SMS waveforms.

![Fig. 4 The error between recovered target movement track and the real track (a) weak feedback (b) moderate feedback](image4)
A phase-unwrapping based displacement measurement approach is proposed in this paper. Neural network curve fitting method has been employed to remove the noises in SMS signals after median and mean filtering. The fitting results show effective elimination of noises while maintain good coordination between the input data and the fitted curve. With the proposed algorithm, the phases with optical feedback have been unwrapped and the target moving trajectory has recovered with good accuracy of \(\lambda/25\) for weak feedback and \(\lambda/20\) for moderate feedback according to computer simulation.

**References:**


