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Modelling the Dependence Structure between Australian Equity and Real Estate Markets – a Copula Approach

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Keywords

REITS, Dependence Structure, Dynamic Conditional Correlations, Copula Models, Time-Varying Copulas, Risk Analysis

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Abstract

Over the last decades, the Australian market for Real Estate Investment Trusts (REITS) has shown substantial growth rates. We apply conditional copula models in order to investigate the dependence structure between the returns from REITS and equity markets in Australia. The dependence between these assets has a significant impact on the diversification potential and risk for a portfolio of multiple assets and is therefore of great interest to portfolio managers and investors. We compare the suggested copula models to a standard variance-covariance approach and a Dynamic Conditional Correlation (DCC) model. We observe significant positive correlations between the considered series. The level of correlation has also increased during the last decade indicating a limited diversification potential of investments in REITS for Australia markets. We also find tail dependence between the examined series that is best modelled by the Student t copula. Finally, we conduct a back-testing Value-at-Risk study for a portfolio that combines investments in real estate and equity. We find that the conditional copula approach outperforms both a static variance-covariance approach and the DCC model with respect to quantifying the risk of extreme negative returns. Our findings suggest that ignoring the complex and dynamic dependence structure in favour of a simple multivariate normal model leads to a significant underestimation of the actual risk.

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JEL Code(s): C58, G32, G12, R33

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1. Introduction

Markets for Real Estate Investment Trusts (REITS) have shown substantial growth rates globally over the last decades. REITS were originally a tax design instrument for corporations investing in real estate assets in order to reduce or eliminate corporate income tax. In return, REITS are required to distribute 90% of their income, which may be taxable, into the hands of their investors. In general, the REIT structure was designed to provide a similar vehicle for investment in real estate to what mutual funds provide for investment in stocks. Since investments in real estate assets are quite different to those in equity, the correlation between REITS and equity markets and therefore their potential for diversification in a portfolio of multiple assets has traditionally been of great interest, see e.g. Brueggeman et al (1984), Clayton and MacKinnon (2001) or Chen et al (2004). Hudson et al (2003), analysing the reasons for considering real estate investments, suggest reducing the overall risks of the portfolio is the most prominent aspect. This paper contributes to the literature by investigating the relationship between REITS and equity returns in the Australian market using an approach that combines GARCH models with different copula functions for the dependence structure. Our analysis focuses on the dependence structure between these assets from a diversification and risk management perspective for a portfolio of multiple assets and is therefore of great interest to investors and portfolio managers.

Australia has long been a nation with a strong propensity for property investments such that the sector has shown substantial growth rates over the last decades and has played a major role in domestic financial markets. Australian Real Estate Investment Trusts (AREITS) are a unitized portfolio of property assets, listed on the Australian Stock Exchange, which allows investors to purchase a share in a diversified and professionally managed portfolio of real estate. Currently, the sector represents more than 5% of the total market capitalization of the S&P/ASX 200 Index. The market is often classified as offering four different types of REITS, see e.g. Davidson et al (2003): (i) equity trusts where the assets are invested in ownership claims to various types of properties, (ii) mortgage trusts where the assets are invested in claims where interest is the main source of income, for example mortgages, (iii) hybrid trusts that invest in both equity and mortgages and (iv) specialized trusts that invest, for example, in development and construction, or are involved in sale and lease-back arrangements. As pointed out e.g. by Hartzell et al (1999), REITS returns are typically lower than returns of both small and large capitalization stocks. On the other hand, the literature also reports that returns from investment in REITS are less volatile than those of equity investments, see e.g. Clayton and MacKinnon (2001); Ghosh et al (1996); Mueller et al (1994) just to name a few. This is not surprising: due to regulation that 95% of AREITS income must be paid out as dividends such that a lower volatility of the returns could be expected, see e.g. Tien and Sze (2000).

Another focus in the literature is usually set on investigating the correlation between returns from investments in REITS and stock markets, see e.g. Brueggeman et al (1984), Chen and Peiser (1999), Chen et al (2004), Clayton and MacKinnon (2001), Hudson et al (2003) . As mentioned above, such an analysis helps to determine the diversification effects of REITS in portfolios of multiple assets. While earlier research has shown significant negative correlations between REITS and other assets (Brueggeman et al, 1984), several studies conducted in the late 1990s often report weak but positive correlations between investments in REITS and shares (Chen and Peiser, 1999; Clayton and MacKinnon, 2001; Hartzell et al, 1999) that still provide some potential for diversification. More recently, other studies (Case et al, 2012; Cotter and Stevenson, 2006; Glascock et al, 2000; Huang and Zhong, 2006; Yang et al, 2012; Zhou and Bao, 2007) rather indicate strong correlations between REITS and stock returns and suggest a diminishing diversification potential of REITS in multi-asset portfolios.

Chen et al (2004) examine the economic significance of including REITS in an investment portfolio, and show that the mean-variance frontier can be augmented and the investment opportunity set can be enlarged. However, due to changes in the correlation structure, it may be difficult to determine actual diversification effects for a longer time horizon. Glascock et al (2000) identify time-varying correlations and suggest that due to structural changes in the early 1990s REITs behave more like investment in stocks and less like bonds. Cotter and Stevenson (2006) use a VAR-GARCH model to study the daily REIT volatility in the US market. The same methodology is applied in Zhou and Bao (2007) to examine cross-correlation between different types of property indices in Hong Kong. The studies find evidence of strong correlations between the considered markets. Michayluk et al (2006) examine asymmetric volatility, correlation and dynamics of returns using REITS data from US and UK markets. They find significant interaction between the markets considered, as well as asymmetric effects on both the volatility and correlation dynamics between the daily returns.

Inspired by Engle (2002) and his work on Dynamic Conditional Correlation (DCC) models, the changing nature of the dependence between REITS and other investments has also been investigated. Huang and Zhong (2006) apply Engle's model for portfolio construction with REITS, and the authors find that the DCC model outperforms other correlation structures including rolling, historical and constant correlations. A similar approach is used by Case et al (2012) and Yang et al (2012) who apply DCC-GARCH models to investigate the relationship between returns from REITS and other asset classes. The former find significant correlations between publicly traded REITS and non-REIT stocks in the US, however, the level of correlation varies significantly through time. The latter examine index returns of the S&P500, US corporate bonds and real estate markets and find evidence for asymmetric volatilities and correlations.

Recently, there has been some criticism made towards the assumptions underlying the DCC model, in particular with respect to the assumptions of multivariate normality for the joint distribution of asset returns and the use of a covariance matrix as the natural measure of dependence between the assets. As shown in various studies, see e.g. Cherubini and Luciano (2001), Frey and McNeil (2003), Jondeau and Rockinger (2006), Junker et al (2006), Luciano and Marena (2003), the use of correlation does not appropriately describe the dependence structure between financial assets and this could lead to an inadequate measurement of risk. The authors suggest the application of copula methods for modelling the dependence structure of asset returns in order to overcome this problem. For an excellent overview on copula methods in finance, see Cherubini et al (2004), where the application of copulas to portfolio analysis, derivative pricing, interest rates and credit risk analysis is discussed. With respect to analysing the dependence structure between different financial assets, the use of copulas as an alternative to the DCC model has the advantage that it doesn't require assumptions of joint normality for the distributions. Instead it permits joining arbitrary marginal distributions into their multivariate distribution, allowing for a wide range of dependence structures by using different copulas. As such the multivariate joint distribution can be decomposed into marginal distributions and an appropriate functional form for the dependence between the asset returns.

In this paper we apply copula models to investigate the dependence structure between returns from AREITS and the Australian stock market, represented by the All Ordinaries Index (AOI). Hereby, we contribute to the literature in several dimensions. To our best knowledge this is one of the first studies to thoroughly investigate the nonlinear relationship between AREITS and returns from the Australian stock market. Further, this is a pioneer study on applying different copula models with time-varying dependence structures in real estate markets. So far only few authors have investigated the use of copula models for property markets. Knight et al (2005) find some evidence for tail dependence, in particular for the lower tail between real estate stock and equity market returns. They conclude that real estate and common equity stocks are more closely related when markets produce highly negative returns. Rong and Trück (2010) suggest that the Student t copula is more suitable than a multivariate normal model to describe the dependence structure between returns from REITS and a stock market index.

Finally, we provide a risk analysis by comparing copula models to alternative approaches, including the standard multivariate normal approach and a bivariate DCC model. We apply these approaches to a portfolio that combines investments in real estate and stock markets and examine in particular the quantification of risks in the tail of the return distribution.

The remainder of the paper is set up as follows. Section 2 reviews different copulas and the DCC model. Section 3 describes the data and provides our empirical analysis by applying the models to returns from an Australian REIT and equity index. We further conduct a risk analysis for an exemplary portfolio consisting of investments in both Australian equity and real estate markets, and compare the applied models with respect to density forecasting for different time horizons. Section 4 concludes and provides suggestions for future work.

2. Dynamic Correlation and Copula Models

This section provides a brief review of approaches that will be used in the empirical analysis to examine the dependence structure between returns from the ASX–REITS and AOI. First we provide a brief overview of copula functions and their application to dependence modelling between random variables. Then we review the DCC model as an alternative way to model the time-varying dependence structure between time series.

2.1 Copula Functions and Estimation

A copula is a function that combines marginal distributions to form a joint multivariate distribution. The concept was initially introduced by Sklar (1959) but has only gained strong popularity for use in modelling financial or economic variables in the last two decades. For an introduction to copulas see e.g. Joe (1997) or Nelsen (2006), for applications to various issues in financial economics and econometrics, see, e.g. Cherubini et al. (2004), Frey and McNeil (2003), Patton (2006), just to name a few. As shown by Cherubini and Luciano (2001), Jondeau and Rockinger (2006), Junker et al (2006) and Luciano and Marena (2003), the use of correlation may not appropriately describe the dependence structure between financial returns. Ang and Chen (2002) and Longin and Solnick (2001) empirically demonstrate that, in general, asset returns are more highly correlated during volatile markets and during market downturns. Dowd (2004) suggests that the strength of the copula framework is attributable to not requiring strong assumptions about the joint distributions of financial assets in a portfolio. Jondeau and Rockinger (2006) and Patton (2006) illustrate that copulas can be applied, not only directly to the observed return series but also, for example, to vectors of innovations after fitting univariate GARCH models to the individual return series. Overall, the use of copulas offers the advantage that the nature of dependence can be modelled in a more general setting than using linear dependence only. Copulas also provide a technique for decomposing a multivariate joint distribution into marginal distributions and an appropriate functional form for modelling the dependence between the asset returns.

In the following paragraphs we will briefly summarize the basic ideas and properties of copulas. For a definition of copulas we refer e.g. to Sklar (1959) or Nelsen (2006). Let $(X_1,X_2,..,X_d)$ be continuous random variables with distribution functions $F_i(x_i) = \Pr(X_i \le x_i)$, i=1,..,d. Following Sklar (1959), there exists a unique function C such that:

$$\Pr(X_1 \le x_1, X_2 \le x_2, ..., X_d \le x_d) = C(F_1(X_1), F_2(X_2), ..., F_d(X_d)). \qquad (2.1)$$

Further setting $F_i(X_i) = U_i$, the function $C(u_1,...,u_d) = \Pr(U_1 \le u_1,...,U_d \le u_d)$ is the distribution of $(U_1,U_2,...,U_d) = (F_1(x_1),\dots,F_d(x_d))$ whose margins are uniform on [0,1]. This function C is called a copula and denotes a joint cumulative density function (CDF) of the *d* independent, U~ [0; 1] distribution functions. Another way to express this is that a copula maps independent uniform distributions U~ [0; 1] into one joint distribution. The copula framework can be generalized for any collection of marginal distributions and joint distributions. In our application we will only consider the bivariate case with a function C(u,v) such that,

$$\mathrm{C}(\mathrm{u},\mathrm{v}) = \mathrm{C}\left[\mathrm{F}(\mathrm{x}),\mathrm{G}(\mathrm{y})\right] \qquad (2.2)$$

The function C(u,v) is defined as a copula function which relates the marginal distribution functions F(x) and G(y) to their joint probability distribution. Moreover, if the marginal distributions F(x) and

G(y) are continuous, the copula function C(x,y) is unique, see Sklar (1959), and the copula is an indicator of the dependence between the variables *X* and *Y*.

The literature suggests a wide range of different copulas, see e.g. Joe (1997) or Nelsen (2006) for an overview of the most common parametric families of copulas. In the following we will limit ourselves to a description of a number of copula families that will be used further on in the empirical analysis. Among the most commonly used copulas in finance are the Gaussian, Student t, Clayton and Gumbel copula.

Probably the most intensively applied copulas in financial applications are the Gaussian and Student t copula. The Gaussian copula is constructed using a multivariate normal distribution and can be denoted by

$$C^N(u_1,...,u_d) = \Phi_\Sigma(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d)) \tag{2.3}$$

Hereby, Φ^{-1} denotes the inverse of the standard normal cumulative distribution function and Φ_Σ is the standard multivariate Normal distribution with correlation matrix Σ. The multivariate normal copula correlates the random variables rather near the mean and not in the tails. Therefore, it fails to incorporate tail dependence which can often be observed in financial data. In order to add more dependence in the tails, alternatively, the Student t-copula can be applied. The Student t-copula is denoted by

$$C_v^t(u_1,...,u_d) = t_{v,\Sigma}(t_v^{-1}(u_1),...,t_v^{-1}(u_d)) \tag{2.4}$$

where t_v^{-1} is the inverse of the Student t cumulative distribution function with v degrees of freedom and $t_{v,\Sigma}$ the multivariate Student t distribution with v degrees of freedom and correlation matrix Σ. While the concepts of a multivariate Gaussian and Student t copula are similar, for the Student *t* copula also the degrees of freedom parameter needs to be determined. Depending on this parameter, the Student t copula can also incorporate tail dependence. Hereby, low values of the parameter v indicate strong tail dependence.

Both the Gaussian and Student t copula are symmetric. However, often financial variables are observed to exhibit tail-dependence in only one of the tails, either the upper right or lower left edge of the data. For example, tail-dependence in the lower left tail indicates that the two variables have a tendency to simultaneously yield high negative returns. However, in situations where returns from one of the variables are highly positive the other financial variable may not be affected to the same extent. To model asymmetric tail-dependence, so-called Archimedean copulas can be used, see e.g. Cherubini et al (2004). Two of the most prominent members of the family of Archimedean copulas, the Clayton and Gumbel copula, will be briefly described in the following paragraphs.

The Clayton copula is an asymmetric Archimedean copula, exhibiting tail dependence in the lower left tail. The multivariate Clayton copula can be denoted by:

$$C(u_1,...,u_d) = \left[\sum_{i=1}^{d} u_i^{-\theta} - d + 1\right]^{-\frac{1}{\theta}} \text{, with } \theta > 0. \tag{2.5}$$

Note that the higher the value of parameter θ, the greater is the degree of dependence between the considered variables, in particular in the lower left tail. The Gumbel copula, on the other hand, exhibits greater dependence in the upper right tail. The multivariate Gumbel copula is denoted by

$$C(u_1,...,u_d) = \exp\left\{-\left[\sum_{i=1}^{d}(-\ln u_i)^{\phi}\right]^{-\frac{1}{\phi}}\right\} \text{ with } \phi > 1 \quad (2.6)$$

The higher the value of parameter ϕ, the higher is also the degree of dependence between the considered variables, in particular in the upper right tail. For further properties and examples of elliptical and Archimedean copulas and the construction of such copulas by using generator functions, we refer to Cherubini et al (2004) or Nelsen (2006).

Note that due to the possible heteroscedastic behaviour of the return series, in the empirical analysis we will not apply copula models to the observed returns directly. Instead the copula functions will be estimated using the vectors of innovations after fitting univariate AR-GARCH models to the individual return series of AREITS and the AOI. This approach has been suggested and successfully applied by e.g. Jondeau and Rockinger (2006) and Patton (2006).

2.2 AR-GARCH-DCC Model

As an alternative approach to the use of copula models, we also apply an AR-GARCH-DCC model in order to capture different regimes of volatility and correlation between the returns series. The use of DCC models for the estimation of conditional correlations has been thoroughly examined in various studies; see e.g. Engle and Sheppard (2001), Engle (2002), Engle, Cappiello and Sheppard (2006), just to name a few. Although the DCC model is not linear, it can be estimated in a straightforward manner using a two-stage maximum likelihood estimation. The DCC model gives more flexibility to modelling conditional correlations as it involves less complicated calculations in comparison to e.g. the GARCH-BEKK model by Engle and Kroner (1995), especially when there are a large number of variables.

For a brief review of the model, we start by decomposing the variance–covariance matrix of asset returns H_t into a conditional correlation matrix R_t and a diagonalized standard deviation matrix D_t:

$$H_t = D_t R_t D_t \quad (2.7)$$

$$D_t = diag(\sqrt{h_{1,t}}, \sqrt{h_{2,t}}, ...)$$

$$r_t = c_1 + c_2 r_{t-1} + \varepsilon_t$$

$$\varepsilon_t = z_t h^{0.5}{}_t$$

$$h_t = w + \alpha\varepsilon^2{}_{t-1} + \beta h_{t-1}$$

Hereby, we assume that the variance of the individual series of asset returns can be modelled by an AR(1)-GARCH(1,1) model. The correlation matrix R_t for the standardized residuals z_t is then assumed to follow the process

$$Q_t = (1-a-b)\overline{Q} + a z_{t-1} z'_{t-1} + b Q_{t-1} \quad (2.8)$$

$$R_t = diag\{Q_t\}^{-0.5} Q_t \, diag\{Q_t\}^{-0.5} \quad ,$$

where a and b are scalar parameters, $\bar{Q}$ is the unconditional variance-covariance and Q_t the conditional variance-covariance matrix of the standardized residuals Z_t. The estimation procedure for the variance-covariance matrix can be divided into two steps: first, estimation of a univariate GARCH model for the individual return series. Then, after filtering out the standardized residuals, in a second step the conditional correlation model (2.8) is estimated.

3. Data Description and Empirical results

In this section we investigate the dependence structure between returns from Australian REITS and the AOI. With respect to Australian REITS we will consider weekly returns from the ASX 200 A-

REIT index, which is an index comprising approximately 70 listed Australian property trusts representing various types of properties under management. The time period extends from June 25, 2001 to August 6, 2012. Data was obtained from DataStream. We consider log-returns that are calculated from the original price series. In the following we will refer to the return series of the All Ordinaries Index as AOI and the return series of the ASX 200 A-REIT index as AREITS. Table 1 provides descriptive statistics for the log-returns of the two series.

Name	Mean	Median	Max	Min	StDev	Skew	Kurtosis
AREITS	-0.03%	0.08 %	6.39%	-9.91%	1.39%	-1.78	15.53
AOI	0.02%	0.14%	3.52%	-7.69%	1.01%	-1.25	10.25

Table 1: Summary statistics of weekly AREITS and AOI log returns for the sample period from 25/6/ 2001 to 6/8/2012

For the selected time period, the sample mean and standard deviation of AREITS is -0.03% and 1.39%, which is larger than the comparable figures for the AOI, yielding a mean of 0.02% and a standard deviation of 1.01%. This somehow contradicts earlier findings by studies that generally report lower returns and standard deviations for REITS than for both small and large equity stocks (Hartzell et al, 1999; Mueller et al, 1994). The table further indicates that both return series exhibit skewness and excess kurtosis. Based on the Jarque-Bera statistics we can easily reject the hypothesis that AREITS and AOI returns follow a normal distribution and conclude that they are asymmetrically distributed and exhibit fat tailed.

3.1 Modelling the Marginals

To estimate the parameters of the considered copula and DCC models, we implement a two-stage procedure: in the first stage, we fit an AR-GARCH model to the univariate return series and obtain the standardized residuals for each series. These residuals are then used in the second stage to estimate dynamic conditional correlations and the different copula functions. Note that one could also model the dependence structure using the original return series. However, due to the heteroscedastic behaviour of financial returns, a conditional approach that models the dependence structure after filtering out autoregressive and heteroscedastic behaviour seems more appropriate as suggested by e.g. Grégoire et al (2008), Jondeau and Rockinger (2006) and Patton (2006).

With respect to the appropriate specification of the AR-GARCH model, a variety of studies in the literature have suggested the use of a GARCH(1,1) process as the 'default' choice for modelling or forecasting volatility of financial returns. For example, Brooks and Persand (2002, 2003) examine various ARCH and GARCH type models with respect to volatility forecasting. They report that, while the forecasting performance of the models depends on the considered data series and time horizon, the overall most preferred model is a simple GARCH(1,1). This is also consistent with other studies such as e.g. Bollerslev *et al*. (1992). Therefore, we decided to apply an AR(1)-GARCH(1,1) first and then examine whether this model yields an appropriate fit to the data.

Estimation results for the fitted AR(1)-GARCH(1,1)-DCC model are provided in Table 2. The maximum likelihood estimation is conducted by assuming that the joint distribution is normal. For the estimated coefficients of the GARCH model, $\alpha + \beta$ are close to 1, indicating that volatility persistence is high for all return series. Therefore, modelling the volatility dynamics with a GARCH process is adequate. In order to examine the approporiateness of the applied AR(1)-GARCH(1,1) for removing the heteroscasticity in the return series, a BDS test is carried out to test the i.i.d property of the standardized residuals. Results are provided in Table 3, with the large p-value indicating that the hypothesis of i.i.d cannot be rejected for both standardized residual series. Therefore, we conclude that using an AR(1)-GARCH(1,1) model is appropriate to to model the dynamics of the return series considered.

As indicated by Table 2, also for the DCC equation, both scalar coefficients are found to be significant at the 5% level, and since the coefficient *b* reflects the 'memory' in the correlation between asset returns, with an estimated value of 0.9699, the impact of previous periods' correlation on the current estimated correlation is very high.

Figures 1 and 2 provide a plot of the conditional variance as well as the conditional correlation between AREITS and the AOI. Figure 1 illustrates that the conditional variance of AREITS is observed to be larger than the conditional variance of the AOI, especially during the GFC period. As indicated by Figure 2, the conditional correlation between the two assets remains positive for the entire sample period. We also find that based on the estimated DCC model we observe an increase in correlation during the considered time period: starting from a level around 0.45 in 2001, the overall level of correlation increases to a level of around 0.65 since 2007. Interestingly, the highest estimated correlation (0.74) is recorded on 15/9/2008, the day Lehman Brothers filed for bankruptcy. Overall, our results contradict earlier studies by Brueggeman et al (1984), Chen and Peiser (1999), Clayton and MacKinnon (2001) and Hartzell et al (1999) who report significant negative or only weak positive correlations between REITS and other assets, in particular equity investments. On the other hand, our results confirm more recent studies by e.g. Case et al (2012), Cotter and Stevenson (2006), Huang and Zhong (2006), Yang et al (2012), who indicate rather strong correlations between REITS and stock returns and suggest a diminishing diversification potential of REITS in multi-asset portfolios.

Figure 3 provides a kernel density plot of the standardized innovation series, indicating that the time series of innovations still exhibits some skewness and excess kurtosis. Recall, however that the carried out BDS test does not reject the i.i.d. property of the innovation series. In the following section we will continue to model the dependence structure between the conditional distributions of the standardized innovations using the Gaussian, Student t, Clayton and Gumbel copula.

Parameters	AREITS	AOI	AREITS-AOI
c_1	0.0007* (0.0003)	0.0008* (0.0003)	
c_2	-0.1619* (0.0485)	0.0379 (0.0453)	
W	0.00001 (0.00002)	0.00001* (0.000002)	
α	0.1030* (0.0208)	0.2310* (0.0357)	
β	0.8953* (0.0163)	0.7323* (0.0442)	
a			0.0176* (0.0002)
b			0.9699* (0.0086)
LLF	1765	1853	435.5

*Table 2: Estimation results for AR(1)-GARCH(1,1)-DCC model. The standard errors are in parenthesis. * indicates significance at the 5% level.*

BDS Test	Dimension	BDS Statistic	Std error	Z-stat	P-value
AREITS	2	-0.0011	0.0021	-0.5261	0.5988
AOI	2	0.0018	0.0022	1.6627	0.3802

Table 3: Results of BDS test for i.i.d property of standardized residuals filtered by the AR(1)-GARCH(1,1) model.

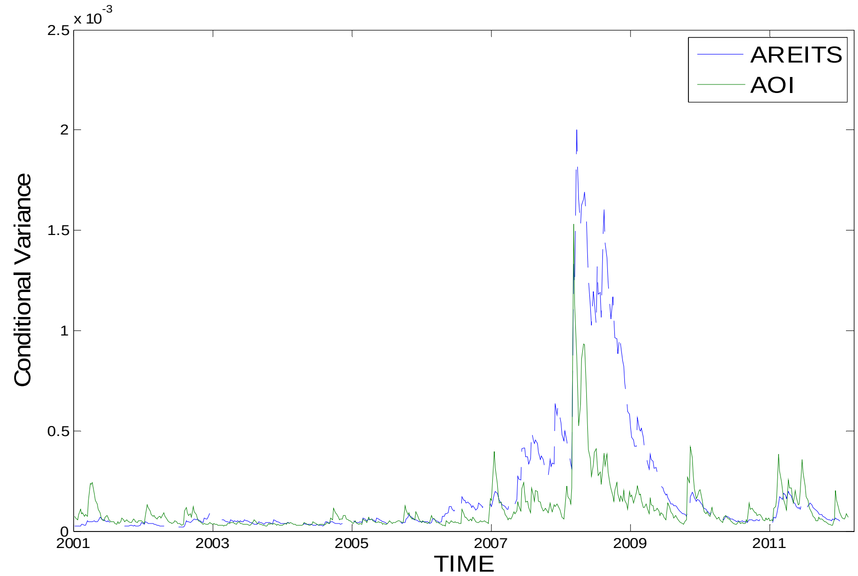

Figure 1: Conditional variance for AREITS and the AOI index based on the estimated AR-GARCH model for the considered time period 25/6/ 2001 to 6/8/2012.

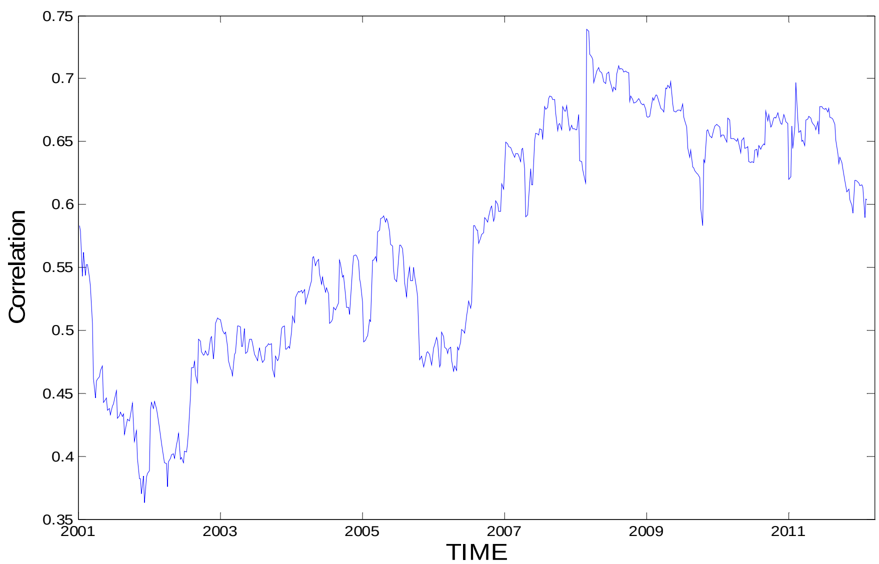

Figure 2: Conditional correlation between AREITS and the AOI based on the estimated DCC model for the considered time period 25/6/2001 to 6/8/2012. Figure 3: A Gaussian kernel smoothed density plot for standardized innovations for AREITS (upper panel), and the AOI (lower panel). The density plot indicates that both series are not normally distributed.

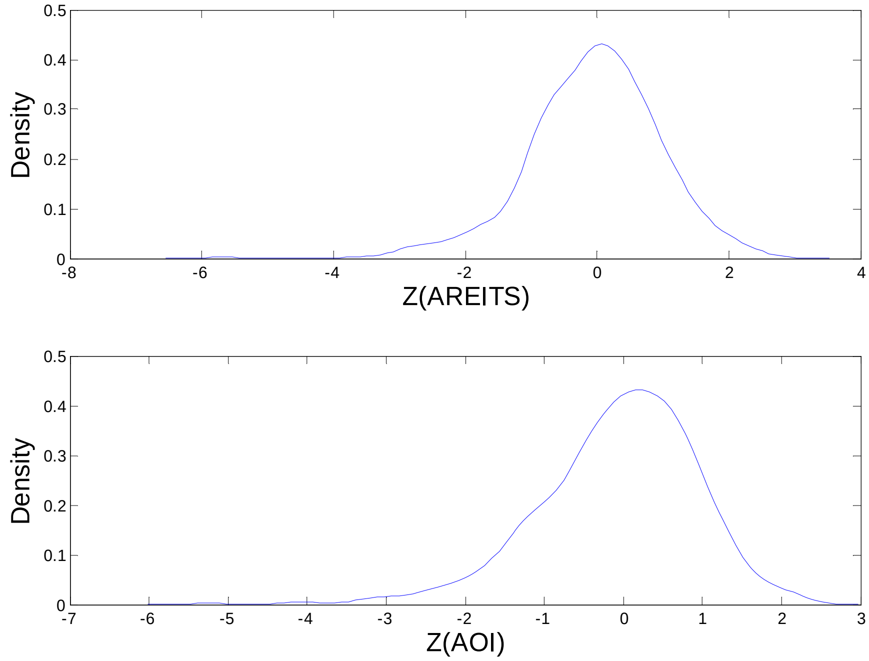

Figure 3: A Gaussian kernel smoothed density plot for standardized innovations for AREITS (upper panel), and the AOI (lower panel). The density plot indicates that both series are not normally distributed.

3.2 Modelling the Dependence Structure using Copulas

One possible way to derive the dependence structure between two time series via a copula is to examine the dependence between the marginal distributions of the standardized residuals. As illustrated in Figure 3, both standardized innovation series exhibit excess skewness and kurtosis. In order to appropriately capture this phenomenon, instead of using a normal distribution for the innovation series we use a non-parametric estimate of the distribution. Hereby, a Gaussian kernel is applied in order to determine the nonparametric estimate of the marginal CDF, with

$$U_t = F(z_t) = \int_{-\infty}^{z_t} \frac{1}{nh} \sum_{i=1}^{n} \frac{1}{2\pi} e^{-\frac{1}{2}(\frac{z_t - z_i}{h})} dz \,, \tag{3.1}$$

where n denotes the sample size and $h>0$ is the bandwidth. Empirically, the optimal bandwidth h^* of a Gaussian kernel can be determined by

$$h^* = 1.06\overline{\sigma} n^{-\frac{1}{5}} \tag{3.2}$$

where $\overline{\sigma}$ denotes the sample standard deviation (Bowman and Azzalini, 1997).

To specify the dynamics of the copula dependence parameter, we follow Patton (2006) and propose observation-driven copula models where the time-varying dependence parameter is a parametric function of transformations of the lagged data and an autoregressive term. Then, using the marginal distribution of the standardized residuals $U_{1,t}$ and $U_{2,t}$, the dynamics of the parameters for the Gaussian, Student t, Gumbel and Clayton copula can be specified. For the dynamics of the correlation for the Gaussian copula, we apply the following model:

$$\rho_t = \Lambda_1\{\omega + \beta\Lambda_1^{-1}(\rho_{t-1}) + a\frac{1}{20}\sum_{j=1}^{20}\Phi^{-1}(U_{1,t-j})\Phi^{-1}(U_{2,t-j})\} \quad (3.3)$$

In a similar manner, the model for the Student t copula can be specified as:

$$\rho_t = \Lambda_1\{\omega + \beta\Lambda_1^{-1}(\rho_{t-1}) + a\frac{1}{20}\sum_{j=1}^{20}t_v^{-1}(U_{1,t-j})t_v^{-1}(U_{2,t-j})\}$$
$$\Lambda_1(x) = \frac{1-\exp^{-x}}{1+\exp^{x}} \quad (3.4)$$

Note that in this specification, ρ_{t-1} is used as a regressor to capture the persistence in the dependence parameter, while the mean of the last 20 observations of the transformed variables $\Phi^{-1}(U_{1,t-j})$ and $\Phi^{-1}(U_{2,t-j})$, respectively $t_v^{-1}(U_{1,t-j})$ and $t_v^{-1}(U_{2,t-j})$ is used to capture any variation in dependence between the innovation series. The link function $\Lambda_1(.)$ denotes a transformation which ensures that the correlation parameter ρ_t will always be in the interval (-1,1). For the Clayton and Gumbel copula, the model for the evolution of the dependence parameter is given by

$$\theta_t = \Lambda_2\{\omega + \beta\Lambda_2^{-1}(\theta_{t-1}) + a\frac{1}{20}\sum_{j=1}^{20}|U_{1,t-1} - U_{2,t-1}|\} \ . \quad (3.5)$$

Note that in this model the mean absolute difference between $U_{1,t}$ and $U_{2,t}$ over the previous 20 observations is used as forcing variable to capture any variation in dependence. Similar to equation (3.3) and (3.4), also for the Clayton and Gumbel copula an appropriate transformation function $\Lambda_2(.)$ is applied in order to ensure that the copula parameter remains in its domain. Recall that for the Clayton copula the parameter domain is $\theta > 0$, while for the Gumbel copula $\phi > 1$. Therefore, Patton (2006) suggests the transformation function $\Lambda_2(x) = \exp^x$ for the Clayton copula and $\Lambda_2(x) = \exp^x + 1$ for the Gumbel copula. The suggested dynamic copula models can then be estimated using maximum likelihood.

	Gaussian	**Student t**	**Clayton**	**Gumbel**
ω	-0.0490 (0.1265)	-0.0685 (0.1328)	-2.5546 (0.3618)	-0.1751* (0.0713)
β	2.1593* (0.2638)	2.2162* (0.4915)	0.7855* (0.0910)	0.6089* (0.0354)
a	0.2727* (0.1490)	0.2287 (0.1623)	8.6318* (1.8790)	-0.1450 (0.1045)
v		23.8790* (13.1913)		
LLF	108.31	**111.31**	97.48	89.94
AIC	-211.12	**-214.50**	-188.02	-173.09
BIC	-198.15	**-198.82**	-175.01	-160.93

*Table 4: Estimation results for the process describing the dynamics of the dependence parameter for the considered Gaussian, Student t, Clayton, and Gumbel copula. Standard errors are reported in parenthesis, * indicates significance at the 5% level. The best results with respect to loglikelihood, AIC and BIC model selection criteria are reported in bold letters.*

Estimation results for the time-varying copula models are reported in Table 4. We find that for all models the persistence parameter β is significant, while only for the Gaussian and Clayton copula also α is significant. We are particularly interested in finding the copula model yielding the fit to the dependence structure between the innovation series. Therefore, Table 4 also reports the Logliklihood for the estimated models as well as results for the Akaike information criterion (AIC) and Bayesian information criterion (BIC). We obtain the highest likelihood and smallest values for the AIC and BIC for the Student t copula, indicating that it provides the best fit to the dependence structure between the standardized innovation series. These results also suggest that there is symmetric tail dependence tail dependence between the considered return series.

A plot of the time-variation of the dependence parameter for the Student t copula according to the estimated model (see Table 4) is provided in Figure 4, while Figure 5 displays the evolution of the time-varying dependence parameter for the dynamic Clayton copula model.

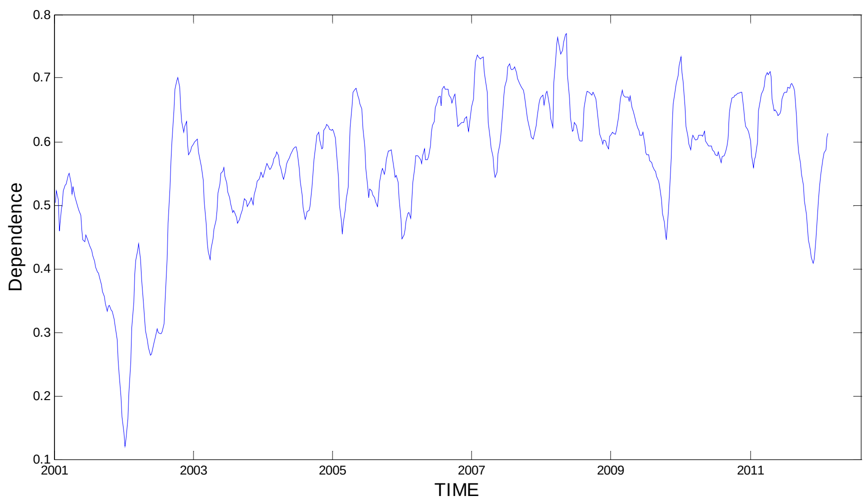

Figure 4: Plot of time varying Student t copula parameter for the considered sample period 25/6/2001 to 6/8/2012.

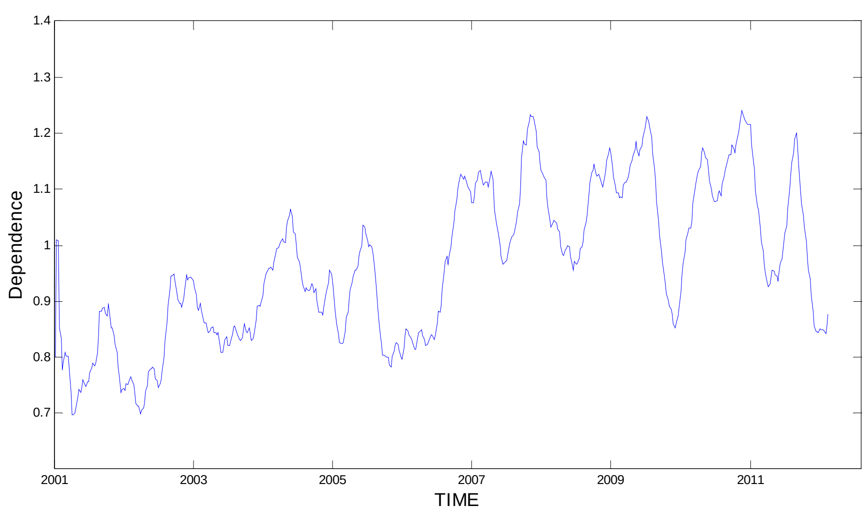

Figure 5: Plot of time varying Clayton dependence parameter for the considered sample period 25/6/2001 to 6/8/2012.

Examining these plots, we find that between 2001 and 2007 correlation between the considered real estate and equity index has increased from approximately 0.45 to an overall level of around 0.6. Since then, as indicated by Figure 4, correlations have remained at a higher level for most of the time between 2007 and 2012. Also the estimated dependence parameter for the Clayton copula shows a tendency to increase during the considered time period. Overall, the estimated time-varying copula models confirms our results for the DCC model and suggest an increasing level of dependence between Australian equity and real estate returns during the considered time period. These results also point towards the reduced diversification potential of investments in AREITS during the last decade.

3.3 Risk Analysis

After fitting appropriate time series models to the bivariate series, we now conduct a risk analysis for a portfolio consisting of AREITS and AOI. In our analysis, the estimated time-varying copulas as well as the DCC model are compared to a static multivariate normal approach as a benchmark for risk quantification. In particular, we will look at the performance of the models with respect to quantifying higher distributional quantiles for the constructed portfolio. Value-at-Risk (VaR) has first been suggested as a standard measure of risk in the 1990s, see e.g. JP Morgan (1996). Since then, it has become the probably most popular tool for internal capital allocations, determining regulatory capital and reporting in risk management of financial institutions. Generally, VaR is denoted by

$$\mathrm{VaR}\left(\mathrm{a}\right) = \mathrm{F}_p\left(\mathrm{x}\right) > \mathrm{a} \ , \tag{3.6}$$

where F_p is the probability distribution of the portfolio returns X, measured against some threshold probability level *a*, which usually refers to a probability of e.g. 0.1%, 1% or 5%. Traditionally, financial returns are assumed to follow a normal distribution such that the dependence structure between the different assets in the portfolio is completely described by a correlation matrix. However, a static variance-covariance approach may have the following issues: (i) non-normality of the marginal distributions, (ii) misspecifying the actual dependence structure through the use of correlation as the only measure of dependence and (iii) ignoring the dynamic dependence structure between the considered return series.

In the following we illustrate how the applied copula and DCC approach may be used appropriately to describe the dependence structure and risk profile for a portfolio consisting of investments in AREITS and the AOI. For a more detailed description of this process we refer to e.g. Engle (2002), Frees and Valdez (1998), Genest and MacKay (1986), Lee (1993) or Marshall and Olkin (1988). Due to the superior fit of the Student t copula in comparison to the other copulas, in the following we provide results for the Student t copula model only. Results for the Gaussian, Clayton and Gumbel copula are available upon request to the authors.

To quantify the risk for a portfolio of investments in AREITS and the AOI, in a first step, we simulate pairs of bivariate uniformly distributed random variables ($U_{1,t}$, $U_{2,t}$) from a Student t copula. Hereby, at each time step we use the estimated time-varying dependence parameter, see equation (3.4), for the simulation. The random numbers obtained are then used to calculate the inverse of the nonparametric CDF for the standardized residuals. By using the inverse of the estimated Kernel density in (3.1), $z_t = F^{-1}(U_t)$, a pair of dependent innovations is then obtained. Figure 6 shows an exemplary bivariate plot for 5000 simulated standardized innovations using the t copula with a unconditional dependence parameter $\theta = 0.5645$ and 23.879 degrees of freedom. At each point in time, the simulated innovations in combination with the conditional volatility estimates from the univariate AR-GARCH models can then be used to determine the distribution of the portfolio returns. A similar procedure is conducted to obtain the portfolio return distribution for the estimated DCC model. In this case, the Cholesky decomposition of the covariance matrix Q_t for time t can be used to simulate dependent random variables for the standardized innovations, see e.g. Cherubini et al (2004).

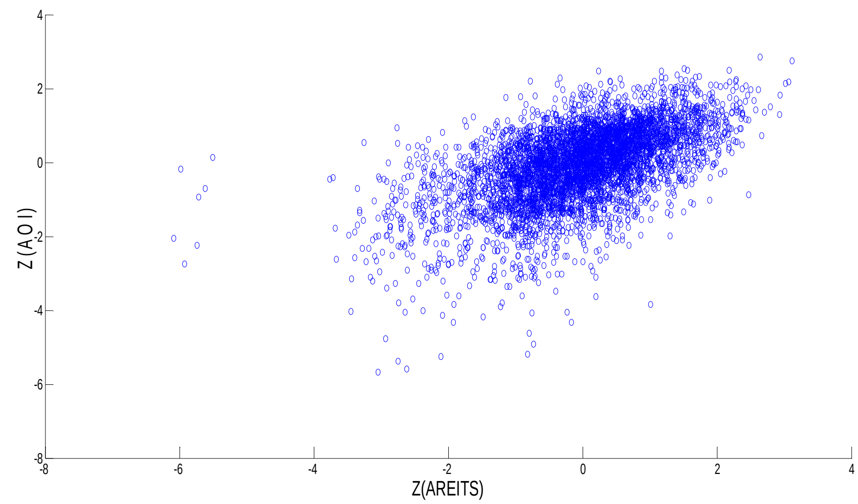

Figure 6: 5000 simulated standardized innovations based on probability integral transform of uniformly distributed random variables ($u_{1,t}$, $u_{2,t}$) from a t copula with dependence parameter $\theta = 0.5645$.

For an exemplary portfolio we choose the weights as 50% each for investment in AREITS and the AOI. By applying the corresponding portfolio weights and the estimated parameters of the AR-GARCH model, it is straightforward to determine a probability distribution of portfolio returns. Using the 10%, 5% and 1% quantile of this distribution, we can then calculate the corresponding 90%, 95% and 99% Value-at-Risk figures for each point in time.

Figure 7 and 8 provide a plot of the actual returns for the exemplary portfolio and the 99% VaR estimates for the applied DCC as well as for the conditional copula model. At first glance we find that both models provide similar results with respect to capturing the dynamics of VaR of the portfolio return. Results on the number of exceedances for the determined 90%, 95% and 99%-VaR figures are provided in Table 5.

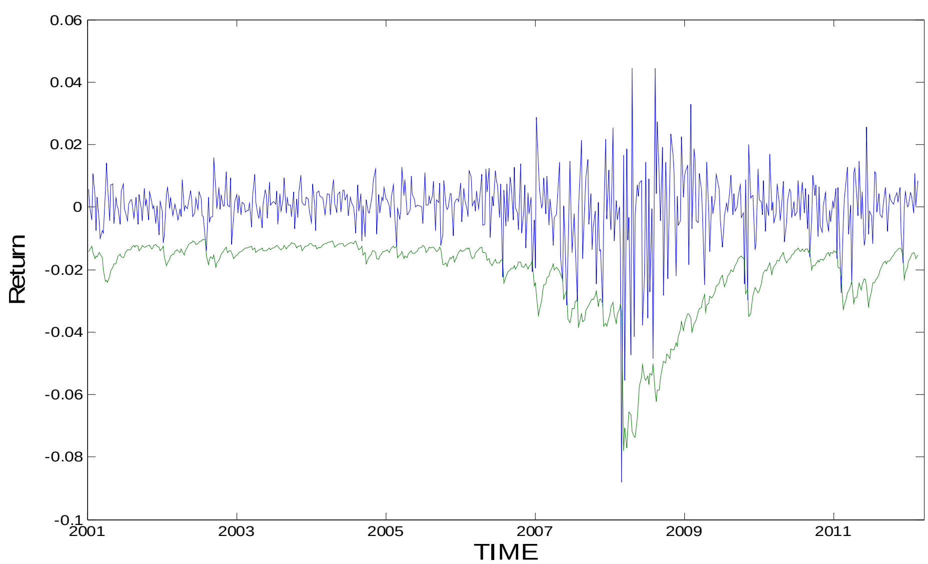

Figure 7: Actual returns and estimated 99% VAR for a portfolio with weights 50% each in AREITS and the AOI based on the estimated DCC model.

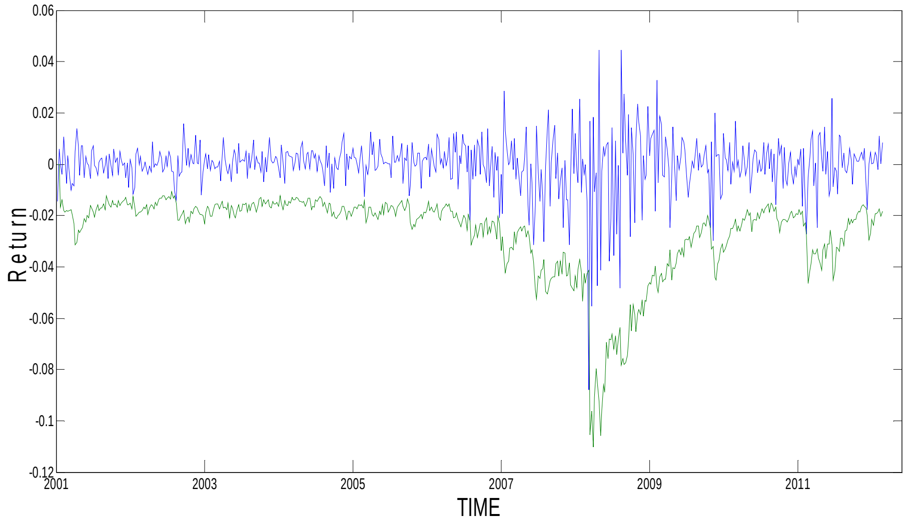

Figure 8: Actual returns and estimated 99% VAR based on a dependence structure modelled using the t copula for a portfolio with weighting 50% each in AREITS and the AOI.

Model	90% level	95% level	99% level
Static Bivariate Normal p-value	56/ (56) 0.4645	27 / (28) 0.4729	14 / (6)* 0.0007
Dynamic Student t copula p-value	45/ (56) 0.0965	26/ (28) 0.3740	2 / (6) 0.0780
DCC model p-value	47/ (56) 0.1138	32 / (28) 0.1894	13/ (6)* 0.0019

*Table 5: Actual and expected number (in brackets) of VaR exceptions for a (i) static bivariate normal, (ii) DCC, and, (iii) dynamic Student t copula model. The table also provides p-values of conducted tests for appropriate risk quantification. The null hypothesis of the test is that the model provides the correct number of VaR exceptions at the considered 90%, 95% or 99% confidence level. * indicates rejection of the model at the 1% level.*

We compare the number of actually observed exceedances to the expected ones in order to evaluate the model's performance with respect to an appropriate quantification of the risk, see e.g. Christoffersen (1998). The null hypothesis of the conducted tests is that the model provides the correct number of VaR exceptions at each confidence level. Thus, for each model and confidence level, reported p-values indicate the significance level at which the appropriateness of the model is rejected. For example, for the 99% VaR, we find that both the static bivariate normal and the DCC model are rejected at any reasonable level of significance while the dynamic Student t copula model is not rejected at the 1% and 5% level of significance. Overall, we find that the static variance-covariance approach provides a reasonable quantification of VaR at the 90% and 95% level. On the other hand, it underestimates the risk at the 99% confidence level where 14 instead of the expected 6 VaR exceptions are observed. Similar results are obtained for the DCC model, where an appropriate quantification of the 99% VaR is also rejected. On the other hand, the lowest number of exceptions is generated by the Student t copula model that cannot be rejected for any of the considered confidence levels. We conclude that among the applied models, the dynamic Student t copula is the only one to appropriately describe the behaviour of the portfolio returns also in the tails of the distribution. The applied approach takes into account important features of the series such as heteroscedasticity, non-normality of the standardized residuals, and changes in the dependence structure through time.

3.4 Density forecast

In a last step we compare the different models with respect to generating density forecasts at future points in time. In particular we derive density forecasts for weekly returns of the considered portfolio made at August 6, 2012, the last day of the considered sample period.. We derive a forecast of the weekly return distribution 1 week, 6 weeks and 12 weeks ahead. Since the static variance-covariance approach provided rather inappropriate results for modelling the dynamic changes in correlation and volatility regimes, we decided to exclude it from this part of the study. Instead, we compare the following three approaches: forecasts based on (i) univariate AR-GARCH models, i.e. assuming independence; (ii) the estimated DCC model; (iii) univariate AR-GARCH models for the marginal return series in combination with the estimated Student t copula model for the dependence structure; and,. The analysis will provide insights into the effect of model choice on volatility and density forecasts for different time horizons.

In a first step, we use the estimation results of the AR-GARCH and DCC models to derive volatility and correlation forecasts for the considered time horizons. The analytical expressions for the predicted variance of the marginal series or the variance-covariance can be obtained in Hull (2009). In a second step, we simulate portfolio returns based on variance forecasts from the univariate AR-GARCH model where no dependence is assumed, and the forecasted variance-covariance matrix from the DCC model under a bivariate normal distribution. For the dynamic Student t copula model, we use simulated standardized innovations, assuming that parameter θ follows equation (3.4), in combination with variance forecasts from the AR-GARCH model to calculate the density forecasts. A plot of the density forecast for the return distribution 12 weeks ahead for all three models is provided in Figure 9. The figure illustrates that in particular the Student t, but also the DCC model yield higher probabilities for more substantial negative portfolio returns. On the other hand, as indicated by Figure 1 as well as Figure 7 and 8, current estimates for volatility of the individual assets, and therefore also for the considered portfolio are quite low such that also the perceived risk of extreme negative returns is quite low.

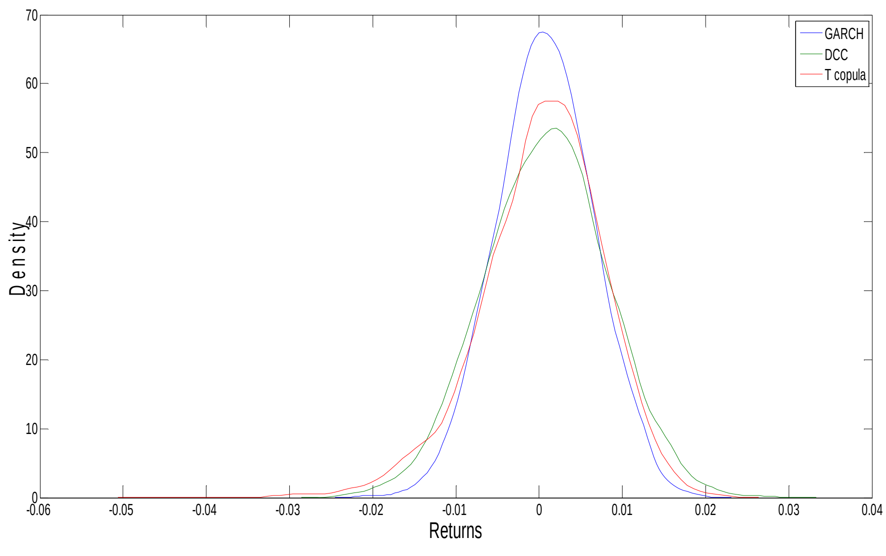

Figure 9: Results for density forecasts of weekly returns for the exemplary portfolio (50% in AREITs and 50% in the AOI) made on August 6, 2012 for a point in time 12 week-ahead. Results are provided for the estimated univariate GARCH models ignoring the dependence structure (blue), the DCC model (green) and the Student t copula model (red).

Univariate GARCH Model			
	1 week	6 week	12 week
90% VAR	-0.60%	-0.63%	-0.67%
95% VAR	-0.78%	-0.82%	-0.87%
99% VAR	-1.13%	-1.18%	-1.30%
DCC model			
	1 week	6 week	12 week
90% VAR	-0.80%	-0.82%	-0.87%
95% VAR	-1.03%	-1.08%	-1.16%
99% VAR	-1.45%	-1.54%	-1.65%
Dynamic Student t copula			
	1 week	6 week	12 week
90% VAR	-0.78%	-0.86%	-0.91%
95% VAR	-1.13%	-1.19%	-1.31%
99% VAR	-1.87%	-1.96%	-2.04%

Table 6: Predicted Value-at-risk figures based on 1-, 6-, and 12-week density forecasts for the exemplary portfolio. Reported are 90%, 95% and 99%-VaR figures for the univariate GARCH models, the bivariate DCC model, and the dynamic Clayton copula model.

Table 6 reports 90%, 95% and 99% VaR figures based on density forecasts for the weekly portfolio returns in 1 week, 6 weeks and 12 weeks. Due to the estimated low level of volatility on August 6, 2012 and the mean-reverting behaviour of volatility for an AR-GARCH model, we find that all models suggest a higher volatility for weekly portfolio returns in upcoming periods. Therefore, the quantified risk is higher for the provided density forecast 12 weeks ahead in comparison to 6 weeks or 1 week ahead. In the extreme tail, i.e. for the 99% VaR due to the tail dependence exhibited by the Student t copula and the heavy tail of the residuals in the univariate GARCH models, we find significantly higher risk figures, in particular for the copula model. While the 99% VaR for the univariate GARCH model and DCC model is between 1.13% and 1.65%, the figures for the same levels of confidence are higher for the Student t copula model, ranging from 1.87% to 2.04%. Overall, in order to characterize future risks for a portfolio, an understanding of the current level of variance as well as an appropriate specification of the dependence structure between individual asset returns, are crucial components.

4. Conclusion

This paper provides an investigation of the dependence structure between weekly returns from Australian Real Estate Investment Trusts (AREITS) and the All Ordinaries Index (AOI) for the time period 2001 to 2012. Australia has long been a nation with a strong tradition of property investment such that over the last decades investments in REITS have shown substantial growth rates. The literature argues that real estate investments generally enable investors to further diversify their portfolio. In this study we apply Dynamic Conditional Correlation models as well as univariate GARCH models in combination with different copula specifications for the dependence structure to examine the joint dynamics of the considered assets. In particular, we apply the Clayton, Gumbel, Gaussian and Student t copula to investigate the dependence structure between Australian real estate and equity returns. To our best knowledge this is one of the first studies to apply and test conditional copula models in these markets.

We find significant correlations between the returns of an AREIT index and the AOI. These findings somehow contradict earlier studies by e.g. Brueggeman et al (1984), Clayton and MacKinnon (2001) and Hartzell et al (1999) reporting significant negative or only weak positive correlations between REITS and equity investments. On the other hand, our results are in line with more recent studies on overseas property markets by Cotter and Stevenson (2006), Case et al (2011), Huang and Zhong

(2006), Yang et al (2011). These authors observe rather strong correlations between REITS and stock returns, suggesting a diminishing diversification potential of REITS in portfolios of multiple asset classes. We find that similar to other major markets, the diversification potential of investments in REITS in the Australian market has also diminished during the last decade. An explanation for this could be that the overvaluation of Australian real estate markets over the past decade was driven by wealth effects and portfolio shocks from Australian equity markets, see e.g. Fry et al (2010). This explains not only the positive relationship between real estate and equity returns but also joint negative returns when both markets simultaneously return to their market fundamental levels as e.g. during the financial crisis.

With respect to modelling the dependence structure between the considered assets, we examine DCC and copula models. For the latter, we follow Patton (2006) and estimate dynamic time-varying dependence parameters for the applied copulas. We find that the Student t copula provides the best fit to the dependence structure between the considered return series. Thus, we conclude that the return series exhibit dependence particularly in the tails suggesting that Australian real estate and common equity stocks are more closely related when markets are volatile. This also confirms results of a previous study on copulas in property markets by Knight et al (2005), where the authors report tail dependence, when investigating the relationship between UK and global public real estate stocks with equity markets.

We also provide a risk analysis, using the different approaches to quantify the Value-at-Risk of a portfolio combining investments in real estate and equities. Our results show that the applied copula model yields the best results for risk quantification, in particular in the tail of the portfolio distribution. We also find that dynamic models clearly provide a more appropriate measurement of portfolio risk in comparison to a static variance-covariance approach. Our findings further suggest that ignoring (i) heteroscedasticity, (ii) the actual distribution of the marginals, and, (iii) the complex dependence structure between the considered return series, leads to a significant underestimation of the actual risk. Finally, with respect to forecasting portfolio returns we find that the model choice might also have significant implications for the shape and term structure of density and VaR forecasts through time.

Extensions of the conducted work could examine the impacts of the detected dependence structure on optimal portfolio construction, e.g. by including asymmetric dependence or tail dependence in the asset allocation decision, see e.g. Hatherley and Alcock (2007). Furthermore, since this study is limited to a bivariate setting, future research should extend the analysis to the multivariate case including various other asset classes next to real estate and equity returns. Also the set of considered copula functions could be extended using alternative copulas like e.g. Frank, SJC or mixture specifications of copulas.

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