Performance of buried HDPE pipes - part I: peaking deflection during initial backfilling process

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Performance of buried HDPE pipes – part I: peaking deflection during initial backfilling process

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ABSTRACT: Peaking deflection caused by compacting the sidefill, referred to as the maximum change of the pipe diameter divided by the undeformed diameter, is an important parameter in the design and safety check of buried pipelines. However, quantitative equations on the deflection useful for engineering practice are very limited. In this paper, a two-dimensional finite element analysis is used to investigate the peaking deflection of high-density polyethylene (HDPE) pipes. In the analyses, the pipe–soil interaction is rationally modeled. A field trial is conducted and the finite-element modeling is evaluated by using the data measured in the field test. Parametric studies are also conducted to investigate the effects of pipe diameter, pipe stiffness, soil modulus, trench width, and compactor type on the peaking deflection of buried HDPE pipes. A new estimating tool is developed that considers the major influencing factors: pipe diameter, pipe stiffness, soil modulus, and compactor type (vibratory plate or rammer) to predict the peaking deflection of HDPE pipes, and the proposed method is finally verified by data reported in published studies. The comparison of the calculated and measured peaking deflections demonstrates a reasonably good prediction of the peaking deflection.

KEYWORDS: Geosynthetics, Peaking deflection, HDPE pipe, Field trial, FE method, Empirical formula


1. INTRODUCTION

Diameters of flexible pipes increase in the vertical direction and decrease in the horizontal direction due to the compaction of the sidefill in the initial backfilling process (i.e. the backfill is lower than the pipe top level), which is referred to as the peaking behavior (Howard et al. 1994; Fleming et al. 1997; McGrath et al. 1999; Shen et al. 2013, 2015; Wang et al. 2015; Witthoeft and Kim 2015). Consequently, the pipes deform to the shape of a vertical ellipse. The diameter change during the initial backfilling process divided by the undeformed pipe diameter is defined as the peaking deflection. The peaking deflection of a flexible pipe can reduce the deflection caused by the weight of the soil cover, which is beneficial for the long-term behaviour of the pipe under loading at the ground surface (Sargand et al. 2002, 2004; Masada and Sargand 2007; Wang et al. 2015). Arockiasamy et al. (2006) concluded that the magnitude of peaking deflection of the high-density polyethylene (HDPE) pipe in their field test is approximately equal to the pipe deflection caused by traffic loading, which indicates the importance and significance of the peaking deflection. This diametrical distortion phenomenon was also found in jacked pipes during construction (Shen et al. 2016), and steel-reinforced HDPE pipes (Han et al. 2015; Khatri et al. 2015), which is controlled by the stress distribution around the pipe and would induce amounts of uncertain pipe strains. Han et al. (2013) proposed a simplified method to calculate the distributed stress on the buried structures based on the Giroud and Han (2004) method. Brachman et al. (2008) indicated that the largest deflections and
strains are found for the HDPE pipe with uncompacted sand backfill placed below the springline. Bathurst et al. (2002) reviewed strain measurement techniques for geosynthetic materials, and indicate that strain gauges and extensometers are effective in estimating strains between 0.02 and 2% and greater than 2%, respectively.

The compaction effect on the performance of flexible pipes has been investigated in previous studies through finite-element analysis (Zhang and Moore 1997; McGrath et al. 1999; Dhar et al. 2004; Masada and Sargand 2007; El-Taher and Moore 2008). McGrath et al. (1999) applied horizontal nodal forces directly onto the pipe in the FE model to simulate the compaction effect on the performance of the pipe. They found that the compactor type can significantly influence the peaking behaviour, and recommend the magnitude of nodal forces for two types of compactors (i.e. vibratory plate and rammer). Dhar et al. (2004) conducted two-dimensional finite-element (2DFE) analysis to investigate the strains and deflections of buried HDPE pipes under vertical pressure from 25 kPa to 500 kPa. They indicated that 2DFE modeling is an effective tool to explore the mechanical responses of the buried HDPE pipes. El-Taher and Moore (2008) employed the 2DFE method to explore the effect of corrosion on the stability of corrugated steel culverts, and proposed a method to model the corrugated pipe as a plain structure based on the concept of equivalent pipe stiffness. Taleb and Moore (1999) proposed a technique by applying horizontal pressure onto the pipe to model the soil compaction effect in conjunction with elastic-plastic soil models to simulate the peaking behaviour. The applied horizontal pressure is equal to the passive earth pressure. They also employed the Mohr-Coulomb model with a linearly varying elastic soil modulus with depth to consider the stress-dependent soil modulus during the pipe installation process. Elshimi and Moore (2013) found that kneading of the soil could lead to its lateral expansion (i.e. plastic strains in the sidefill of the pipe) and consequently increase the vertical deflection of the pipe. They suggested that the values of the applied lateral horizontal pressures should be the passive earth pressure multiplied by the empirical kneading factor, to take the soil kneading into consideration. The values of are suggested as 1.0 and 2.0 for the vibratory plate compactor and the rammer compactor, respectively. In these numerical analyses, the only purpose is to simulate the compaction effect. The influencing factors of the peaking deflection have not been investigated, but are important in understanding the peaking deflection phenomenon.

Masada and Sargand (2007) proposed an analytical formula to calculate the peaking deflection as follows

\[
\Delta y/D = -\Delta x/D = (4.7P_e + K_0\gamma)/3.874(PS)
\]

where \(\Delta y\) is the pipe diameter change in the vertical direction (m); \(\Delta x\) is the pipe diameter change in the horizontal direction (m); \(D\) is the undeformed diameter of the pipe (m); \(P_e\) is the lateral pressure generated by the compactor (kPa); \(K_0\) is the lateral earth pressure coefficient at rest; \(\gamma\) is the unit weight of the sidefill (kN/m\(^3\)); and \(PS\) is the pipe stiffness (kPa) defined by the following equation

\[
PS = 6.72EI/r^3
\]

where \(E\) is the Young’s modulus of the pipe material (kPa) and \(I\) is the moment of inertia of the pipe wall per unit length (m\(^4\)/m).

It is noted that the friction force at the pipe–soil interface and the modulus of the sidefill are not considered in the Masada and Sargand (2007) method. When these two factors are not considered, different results may be obtained in the redistribution of stresses around the pipe. This will clearly lead to errors in computing the deformation, stress, and strain of the pipe during working states. The effects of pipe–soil interface friction and modulus of the sidefill on the peaking deflection will be discussed in the subsection ‘Comparison of Numerical and Measured Results’ and the section ‘Parametric Studies’, respectively.

This paper is the first of a two-part series investigating the peaking behaviour of buried HPDE pipes during the initial backfilling process by using 2DFE modeling. A field trial is conducted and the test data are used to evaluate the numerical model. Parametric studies are made for analysing crucial factors affecting the peaking behaviour, such as pipe diameter, relative flexure stiffness (i.e. the ratio of the constrained modulus of the sidefill over the pipe stiffness), trench width, and compactor type. An empirical formula is proposed to predict the peaking deflection, and the proposed formula is verified against measured data from this study and published studies. It is seen that the proposed equation improves the accuracy of the Masada and Sargand equation, and gives a reasonably good prediction of the peaking deflection. In Part II (Zhou et al. 2017), 2DFE modeling is employed to yield an empirical formula for the total deflection of buried HDPE pipes (i.e. the pipe deflection at the end of pipe installation) with the consideration of peaking behaviour.

2. BRIEF DESCRIPTION OF FIELD TEST

A field trial test was conducted to investigate the field performance of HDPE pipes during the construction phase in Yixing city, China. Three double-wall corrugated HDPE pipes with a length of 6.0 m and two nominal diameters of 0.3 m and 0.6 m were used in the field tests. The pipe stiffness was 215 kPa. The excavated trench width was 2.0 m and the soil covers were 0.9, 1.0 and 1.9 m in depth. Excavated in-situ soil at its optimum water content (20.6%) was used as backfill material beside the pipe and compacted to the required degrees of compaction (Figure 1) using a vibratory plate compactor, as suggested by China Association for Engineering Construction Standardization (CECS 2004). The sand cone test was adopted to evaluate the soil density and degree of compaction for each zone according to ASTM D1556-07 to check the requirements as listed in Figure 1. Table 1 shows the measured degrees of compaction during the initial backfill process for three pipes to demonstrate

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the effectiveness of the compaction. The groundwater level was 2.5 m below the surface during the construction, which was lower than the trench bottom. Therefore, the groundwater effect was not considered in this study.

Earth pressures around the pipes and the pipe deflections were monitored during the test. Detailed information about the field test can be found in Wang et al. (2015) and Zhou et al. (2015).

3. VALIDATION OF NUMERICAL MODEL

3.1. Numerical model

PLAXIS 2D was employed in this study to investigate the peaking behaviour of HDPE pipes. Figure 2 shows the dimension of the numerical model for the initial backfilling process of the 0.6 m-diameter HDPE pipe. The top surface was set as stress-free to allow vertical and horizontal displacements. The side boundaries were fixed in the horizontal direction, and no constraints were applied on the vertical direction. The bottom boundary was fixed in both the vertical and the horizontal directions. As mentioned in the last section, the groundwater effect was not considered. 15-node triangular elements were adopted for the numerical model.

3.2. Properties of materials

PLAXIS provides a design tool, called ‘tunnel designer’, which can be used to create a pipe model (Shen and Xu 2011; Xu et al. 2012; Shen et al. 2013). In the pipe model, the pipe is treated as an elastic material with plate elements. In this study, the pipe model included 24 plate elements with each element extending for an arc length of 15°. It should be noted that the corrugated pipe needed to be converted to a plane pipe in the numerical modeling. Dhar et al. (2004) suggested converting the corrugated pipe to a plane pipe based on the equivalence of combined axial stiffness (EA) and flexural stiffness (EI), where E is

Table 1. Results of sand cone tests measured during the initial backfilling process

<table>
<thead>
<tr>
<th>Zone</th>
<th>P1 pipe</th>
<th>P2 pipe</th>
<th>P3 pipe</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured degree of compaction (%)</td>
<td>Required degree of compaction (%)</td>
<td>Measured degree of compaction (%)</td>
</tr>
<tr>
<td>Zone I</td>
<td>89</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Zone II</td>
<td>94</td>
<td>95</td>
<td>93</td>
</tr>
<tr>
<td>Zone III</td>
<td>96</td>
<td>95</td>
<td>94</td>
</tr>
</tbody>
</table>

Note: Zone I to Zone III are simulated in this paper, while Zone IV to Zone VI are simulated in the companion paper.

Figure 2. Finite-element model for the initial backfilling process (Zone I and II: air-dried sand; Zone III: backfilled excavated in-situ soil; Zone VII: in-situ soil (m))
The elastic modulus of the pipe, $A$ is the cross-sectional area of the pipe wall per unit length, and $I$ is the moment of inertia of the pipe wall per unit length. Although Siddique and Dhar (2015) proposed a viscoplastic model of the HDPE pipe material, the value of $E$ is taken as 520 MPa, representing a secant modulus at the termination of the pipe installation in the field trial (approximately 1 h), according to the time dependent power law model developed by Chua (1986). For the determination of the values of $A$ and $I$ of the pipe wall, double-wall corrugated pipe was idealised as plate elements as per AASHTO (AASHTO 2012). Geometric properties (i.e. the $A$ and $I$) were determined by the integration of the areas of the corrugations. The calculated values of $A$ and $I$ were 8.6 mm$^2$/mm and 221 mm$^3$/m for pipe with 0.3 m diameter, and 14.4 mm$^2$/mm and 1774 mm$^3$/mm for pipe with 0.6 m diameter, respectively. A unit weight of 9.5 kN/m$^3$ and a Poisson's ratio of 0.46 were used for the HDPE pipe, as recommended by CECS (2004) and Elshimi and Moore (2013), respectively.

The soils (i.e. backfilled sand and local soil) were modeled using an elastic-plastic soil model with the Mohr-Coulomb failure criterion. The increase in soil modulus $E$ with depth can be modeled using a linearly varying elastic soil modulus relative to a reference position, i.e., $E = E_0 + E_{\text{increment}}z$, where $E_0$ is the elastic soil modulus at the reference position, which was kept at the top surface of the backfill at each step of backfill placement; $E_{\text{increment}}$ is the increase of soil modulus per unit of depth; $z$ is the depth of backfill (Taleb and Moore 1999). The elastic moduli $E$ of soils were obtained from the stress-dependent values of the constrained soil modulus ($M_c$) and Poisson's ratio ($\nu$) reported by McGrath et al. (1999) using the equation $E = (1 + \nu)(1 - 2\nu)M_c/(1 - \nu)$. Those modulus profiles (i.e. $E$ and corresponding $z$) were then approximated as linear functions to determine the $E_0$ and $E_{\text{increment}}$. An attempt was also made to assume constant soil moduli (i.e. $E_0$ listed in Table 2) for all depths in the FE models to calculate the pipe deflections. The results indicate that the differences in both horizontal and vertical deflection between the cases with and without change of modulus with depth are insignificant (less than 5%) within the range of backfill depth considered in this study. Therefore, the authors decided to use the constant soil modulus in this study for computational efficiencies. The values of the angle of internal friction and cohesion were determined from direct shear tests as per ASTM D3080-11. Strain-controlled direct shear tests were conducted on the compacted remolded native clayey soils under vertical pressures of 100, 200, 300 and 400 kPa with a shearing rate of 0.8 mm/min. The soils were compacted using a Havard compactor under the conditions of controlled water content and dry density. The water content of the samples was 20.6% and the dry density values were 1.53 Mg/m$^3$, 1.62 Mg/m$^3$ and 1.71 Mg/m$^3$, corresponding to the degrees of compaction of 85%, 90% and 95%, respectively. The diameter and the thickness of the soil specimen were 59 and 20 mm, respectively. The internal friction angle of the sand with a compaction degree of 90% and 95% was consistent with those suggested by Elshimi and Moore (2013). The values of the dilation angle of the backfills was determined using the angle of internal friction minus 30° (Bolton 1986). The properties of the sand, backfilled clayey soil, ground soil and the HDPE pipe are listed in Table 2.

### Table 2. Properties of backfills and HDPE pipe used for numerical model of the field test

<table>
<thead>
<tr>
<th>Model material</th>
<th>Degree of compaction (%)</th>
<th>Unit weight (kN/m$^3$)</th>
<th>$E_0$ (MPa)</th>
<th>Angle of internal friction (°)</th>
<th>Dilation angle (°)</th>
<th>Cohesion (kPa)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone I$^a$</td>
<td>90</td>
<td>16</td>
<td>5.45</td>
<td>42</td>
<td>12</td>
<td>1.0</td>
<td>0.35</td>
</tr>
<tr>
<td>Zone II$^a$</td>
<td>95</td>
<td>16.9</td>
<td>6.44</td>
<td>48</td>
<td>18</td>
<td>1.0</td>
<td>0.40</td>
</tr>
<tr>
<td>Zone III$^b$</td>
<td>95</td>
<td>20.2</td>
<td>1.44</td>
<td>28</td>
<td>0</td>
<td>16</td>
<td>0.42</td>
</tr>
<tr>
<td>Zone IV$^b$</td>
<td>90</td>
<td>19.1</td>
<td>1.09</td>
<td>28</td>
<td>0</td>
<td>13</td>
<td>0.35</td>
</tr>
<tr>
<td>Zone V$^b$</td>
<td>85</td>
<td>18</td>
<td>0.67</td>
<td>28</td>
<td>0</td>
<td>11</td>
<td>0.3</td>
</tr>
<tr>
<td>Zone VI$^b$</td>
<td>95</td>
<td>20.2</td>
<td>1.44</td>
<td>28</td>
<td>0</td>
<td>16</td>
<td>0.42</td>
</tr>
<tr>
<td>Zone VII$^b$</td>
<td>—</td>
<td>21</td>
<td>1.25</td>
<td>28</td>
<td>0</td>
<td>23</td>
<td>0.35</td>
</tr>
<tr>
<td>HDPE pipe</td>
<td>NA$^d$</td>
<td>9.5</td>
<td>520</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>0.46</td>
</tr>
</tbody>
</table>

$^a$Air-dried sand.

$^b$Backfilled excavated native soil.

$^c$Foundation soil above groundwater level.

$^d$Not available.

Note: Zone I to Zone III are simulated in this paper, while Zone IV to Zone VI are simulated in the companion paper.

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The value of \( R_{\text{inter}} \) is also set as 0.01 to simulate a scenario where the pipe–soil interface has a zero friction angle in order to address the effect of pipe–soil friction on the peak deflection of the pipes. Interface elements with an \( R_{\text{inter}} \) of 0.9 were employed to simulate the interface between the trench and sidefill as suggested by Brinkgreve (2006). The pipe deflection at the end of construction of Zone III is the peaking deflection. The thickness of each lift was equal to 0.15 m. The compaction effect was simulated by applying horizontal point loads on the lateral sides of the pipe, which was also adopted by Corey et al. (2014). Horizontal point loads were applied to the nodes of the pipe elements with an increment of 15° around the pipe. It is calculated by integrating the horizontal pressure obtained from Equation 3 within the arc length of each pipe element (Corey et al. 2014).

\[
\sigma_h = \sigma_v K_p K_n
\]

where \( \sigma_h \) is the horizontal pressure imposed on the pipe side (kPa), \( \sigma_v \) is the soil layer overburden pressure (kPa), \( K_p \) is the passive earth pressure coefficient, and \( K_n \) is the kneading factor. The value of \( K_n \) is set as 1.0 and 2.0 to simulate the soil compaction using the vibratory plate and rammer compactor, respectively, as suggested by Elshimi and Moore (2013). In the numerical models of this study, \( K_n \) of the first soil layer and second to fourth soil layers at the lateral side of the pipe were 6.79 and 2.77, respectively, determined using the internal friction angles of 48° and 28° (Table 2). The \( K_n \) is set as 1.0, as suggested by Elshimi and Moore (2013), since the vibratory plate was used in the field trial. The tension cut-off technique, with a maximum allowable tensile stress of zero, was activated to remove any possible tensile stresses of the soil. The point loads were not removed in the subsequent steps.

The modeling procedure is summarised as follows.

1. Create the numerical model and set the boundary and initial conditions.
2. Input the properties of materials.
3. Excavate the pipe trench with a width of 2 m and a depth based on the pipe diameter and the thickness of the soil cover.
4. Activate the pipe model.
5. Backfill the first layer material (Zone I in Figure 1) and apply horizontal point loads onto the pipe.
6. Repeat Step 5 until the sidefill reaches the pipe top level.

### 3.4. Comparison of numerical and measured results

A comparison of the measured and calculated peaking deflections of HDPE pipes in both vertical and horizontal directions is shown in Table 4. The calculated peaking deflections of HDPE pipes agree well with the measured peaking deflections, with relative errors of less than 8% for the vertical deflection and less than 5% for the horizontal deflection. The comparison demonstrates that the numerical model adopted in this study is effective in simulating the peaking behaviour of HDPE pipes. In addition, the peaking deflection of the pipe with a smooth pipe–soil interface has a zero friction angle.

<table>
<thead>
<tr>
<th>Pipe ID</th>
<th>Pipe diameter (m)</th>
<th>Sidefill material</th>
<th>Degree of compaction (%)</th>
<th>FE results (with pipe–soil interface friction)</th>
<th>Relative errors (FE results–Field data/Field data × 100%)</th>
<th>FE results (without pipe–soil interface friction)</th>
<th>Relative errors (FE results–Field data/Field data × 100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( d_y ) (%) ( d_x ) (%) ( d_y ) (%) ( d_x ) (%)</td>
<td>( d_y ) (%) ( d_x ) (%) ( d_y ) (%) ( d_x ) (%)</td>
<td>( d_y ) (%) ( d_x ) (%) ( d_y ) (%) ( d_x ) (%)</td>
<td>( d_y ) (%) ( d_x ) (%) ( d_y ) (%) ( d_x ) (%)</td>
</tr>
</tbody>
</table>
interface (i.e. zero friction) was calculated and compared to the field data (Table 4). It is seen that the calculated peaking deflections are 11% to 21% smaller than the measured ones, which indicates that the friction force at the pipe–soil interface has a significant effect on the peaking deflection.

4. PARAMETRIC STUDIES

The peaking behaviour of flexible pipes is influenced by several factors, such as pipe diameter, pipe stiffness, soil modulus, trench width, and compactor type (Marston and Anderson 1913; Sargand et al. 2001, 2002; Elshimi and Moore 2013; Wang et al. 2015). Fifteen cases with variations of the above mentioned parameters were conducted in the parametric studies. The variation of the geometrical and mechanical parameters is summarised in Table 5. Linear soil properties with an elastic modulus of 7 MPa, Poisson’s ratio of 0.4 and unit weight of 18.7 kN/m$^3$ reported by McGrath et al. (1999) for undisturbed clayey soil were employed for the trench soil. The ranges of pipe diameter (0.3 m to 1.2 m) and pipe stiffness (107 kPa to 860 kPa) are consistent with those outlined by CECS (2004). Sand and gravel are adopted as the backfill materials for buried pipes as recommended by CECS (2004) and ASTM D2321-11. However, sand and gravel may not be available in some areas where local soil with low plasticity can also be used as a backfill material, as suggested by CECS (2004). In addition, compaction is another essential factor for the soil modulus. Soil moduli vary significantly with degree of compaction (McGrath et al. 1999; Sargand et al. 2001, 2002). Therefore, the soil modulus is considered to be dependent on the type of material and the degree of compaction. Three types of backfill materials suggested by ASTM D2321-11 and CECS (2004) were adopted in the parametric studies, namely SW85 (i.e. well-graded sand with a degree of compaction of 85%), SW95 (i.e. well-graded sand with a degree of compaction of 95%), and CL95 (i.e. low plasticity clay with a degree of compaction of 95%). The soil properties of SW85, SW95 and CL95 are tabulated in Table 6.

Relative flexure stiffness ($S_f$), a function of pipe stiffness and the constrained modulus of the sidefill defined by McGrath et al. (2002), is widely used to analyse the combined effects of pipe stiffness and the soil modulus as follows

$$S_f = \frac{6.72 M_s}{PS}$$

where $M_s$ is the constrained soil modulus (kPa); and $PS$ is the pipe stiffness (kPa).

The values of $S_f$ were calculated as 101, 431, and 115 for SW85, SW95, and CL 95, respectively. Two types of compactors (i.e. vibratory plate compactor and rammer compactor) were used to investigate the effect of the compactor type on the peaking behaviour, which is represented by the kneading factor. The kneading factor

<table>
<thead>
<tr>
<th>Case number</th>
<th>Pipe diameter (m)</th>
<th>Pipe stiffness (kPa)</th>
<th>Sidefill material</th>
<th>Soil modulus (MPa)</th>
<th>Trench width (m)</th>
<th>Kneading factor$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>0.3</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>1.2</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>107</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>6</td>
<td>0.3</td>
<td>339</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>430</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>8</td>
<td>0.3</td>
<td>672</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>9</td>
<td>0.3</td>
<td>860</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>0.3</td>
<td>215</td>
<td>SW95</td>
<td>6.44</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>11</td>
<td>0.3</td>
<td>215</td>
<td>CL95</td>
<td>1.44</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>12</td>
<td>0.3</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>13</td>
<td>0.3</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>14</td>
<td>0.3</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>15</td>
<td>0.3</td>
<td>215</td>
<td>SW85</td>
<td>2.56</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

*baseline case.

$^a$SW85, well-graded sand with degree of compaction of 85%; SW95, well-graded sand with degree of compaction of 95%; CL95, low plasticity clay with degree of compaction of 95%.

$^b$Kneading factor: 1.0 = vibratory plate compactor; 2.0 = rammer.

<table>
<thead>
<tr>
<th>Model material</th>
<th>Unit weight (kN/m$^3$)</th>
<th>Degree of compaction (%)</th>
<th>$E_0$ (MPa)</th>
<th>Angle of internal friction (°)</th>
<th>Dilatation angle (°)</th>
<th>Cohesion (kPa)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW85</td>
<td>15</td>
<td>85</td>
<td>2.65</td>
<td>38</td>
<td>8</td>
<td>1.0</td>
<td>0.26</td>
</tr>
<tr>
<td>SW95</td>
<td>16.9</td>
<td>95</td>
<td>6.44</td>
<td>48</td>
<td>12</td>
<td>1.0</td>
<td>0.40</td>
</tr>
<tr>
<td>CL95</td>
<td>20.2</td>
<td>95</td>
<td>1.44</td>
<td>28</td>
<td>0</td>
<td>15</td>
<td>0.42</td>
</tr>
</tbody>
</table>
was assumed to be 1.0 for the vibratory plate compactor, and 2.0 for the rammer.

Figures 3a and 3b show the effect of pipe diameter on the pipe deflection during the initial backfilling. A pipe stiffness of 215 kPa and sidefill material of SW85 were employed to investigate the effect of the pipe diameter on the pipe deflection during the initial backfilling process. It is seen from Figure 3a that the peaking deflections in the vertical and horizontal directions decreased significantly with an increase of the pipe diameter. However, the effect of pipe diameter on the pipe deflection can be eliminated by using a normalised sidefill thickness (i.e. $H'/D$), as shown in Figure 3b. It is seen that both the vertical and the horizontal deformation are essentially the same with $H'/D$, with less than 0.5% difference. For the sake of analysis, $H'/D$ is used as the horizontal coordinate in Figures 4–7.

Figure 4 presents the effect of pipe stiffness on the pipe deflection during the initial backfilling process. This figure shows that higher pipe stiffness results in smaller

![Figure 3. Effect of pipe diameter ($D$) on the pipe deflections via FE simulations](image)

![Figure 4. Effect of pipe stiffness (PS) on the pipe deflections via FE simulations](image)
Figure 5. Effects of the sidefill type and the degree of compaction on the pipe deflections via FE simulations (CL, lean clay; SW, well-graded sand)

Figure 6. Effect of trench width on the pipe deflections via FE simulations

Figure 7. Effect of compactor type on the pipe deflections via FE simulations

Figure 8. Comparison of the measured and simulated vertical peaking deflections for the vibratory plate compactor via Equations 1 and 5
pipe deflection at a given thickness of sidefill in spite of the constant $K_p$ value. The pipe deflection in the vertical direction was approximately equal to that in the horizontal direction with the same pipe stiffness and sidefill thickness. The peaking deflection for the case with a pipe stiffness of 107 kPa was 0.45%, and that for the case with a pipe stiffness of 860 kPa was 0.35%. The peaking deflection decreased by 22% when the pipe stiffness increased from 107 kPa to 860 kPa. It is seen that the pipe stiffness significantly influences the peaking deflection.

Figure 5 illustrates the effect of the sidefill type and the degree of compaction on the pipe deflection during the initial backfilling process. For the two cases where sidefill materials had similar relative flexure stiffness (i.e. SW85 ($S_f = 101$) and CL95 ($S_f = 115$)), the peaking deflections were essentially the same, with less than 0.5% difference, regardless of considerably different $K_p$ value (4.2 for SW85 and 2.77 for CL95). However, the peaking deflection for SW95 ($S_f = 431$) was 25% higher than the value for SW85 or CL95. As seen in Table 6, the Young's moduli were 2.64, 1.64 and 6.44 MPa for SW85, CL95 and SW95, respectively. It can be seen that the modulus of SW95 was approximately three times that of SW85 or CL95. Therefore, it is concluded that the soil modulus is an essential parameter for the peaking deflection.

Figure 6 presents the effect of the trench width on the pipe deflection during the initial backfilling process. When the trench width increased from 0.8 to 2.0 m, the differences for both the vertical and horizontal deflections were less 5%. Consequently, it is concluded that the trench width has a minor effect on the peaking deflection.

Figure 7 shows the effect of the compactor type on the pipe deflection during the initial backfilling. As suggested by Elshimi and Moore (2013), the kneading factors, $K_n$, were 1.0 and 2.0 for vibratory plate and rammer compactors, respectively. In other words, the magnitude of the applied horizontal point loads for the case using a rammer compactor was two times that for the case using the vibratory plate compactor. It is seen that the peaking deflection for the rammer compactor was 0.9%, while that for the vibratory compactor was 0.4%. The average ratio of pipe deflections generated by the rammer to those by

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ (m)</td>
<td>0.6</td>
</tr>
<tr>
<td>PS (kPa)</td>
<td>215</td>
</tr>
<tr>
<td>$M^*$ (MPa)</td>
<td>3.68</td>
</tr>
<tr>
<td>$P_c$ (MPa)</td>
<td>0.207</td>
</tr>
<tr>
<td>$K_n$</td>
<td>0.47</td>
</tr>
<tr>
<td>$\gamma$ (kN/m$^3$)</td>
<td>20.2</td>
</tr>
</tbody>
</table>

*Recommended by McGrath et al. (1999).
the vibratory plate compactor is 2.4. Therefore, the compactor type has a significant effect on the peaking deflection.

5. PROPOSED EMPIRICAL FORMULA AND ITS VALIDATION

5.1. Proposed empirical formula

Based on the numerical results, the pipe diameter, the pipe stiffness, the soil modulus and the compactor type are found to have more significant effects on the peaking deflection compared to the trench width. The effects of pipe stiffness and soil modulus can be represented by the relative flexure stiffness. In addition, the magnitude of pipe deflections in both the vertical and the horizontal directions during the initial backfilling increases approximately linearly with the thickness of the sidefill, as shown in Figure 3 through Figure 7.

Based on linear regression, an empirical equation is obtained by linking pipe deflection with the relative flexure stiffness, the type of the compactor and the thickness of the sidefill as follows

\[ \Delta y / D = -\Delta x / D = (0.05S_f + 33)\eta H / (10000D) \]  

(5)

where \( \Delta y / D \) and \( \Delta x / D \) are the peaking deflections in the vertical and the horizontal directions, respectively; \( S_f \) is the relative flexure stiffness; \( \eta \) is an empirical constant related to the compactor type, which is 1.0 and 2.4 for the vibratory plate and the rammer compaction methods, respectively; \( H \) is the height of the soil fill placed at the lateral side of the pipe from the pipe invert (m); and all other parameters and their units are the same as those defined in Equation 1. The coefficient of determination, \( R^2 \), is 0.96. It is concluded from Figures 4 and 5 that the relative flexure stiffness \( (S_f) \) has a more significant effect on the peaking deflection than \( K_p \). Therefore, the \( S_f \) is included in Equation 5 rather than the \( K_p \).

5.2. Validation of proposed formulas

McGrath et al. (1999), Arockiasamy et al. (2006), Masada and Sargand (2007) and Corey et al. (2014) reported field and laboratory investigations into the peaking deflection of HDPE pipes. Their data were used to evaluate the proposed method (i.e. Equation 5). In Figures 8–11, comparisons are presented of the predicted peaking deflections of the pipe between the Masada and Sargand (2007) method (Equation 1) and the proposed method using the data reported in the literature. The values of input parameters used in the Masada and Sargand (2007) and the proposed method in this study are shown in Tables 7–11. Krizek et al. (1971) found that the constrained soil modulus \( M_s \) could be 0.7 to 1.5 times the soil reaction modulus \( E' \). Hartley and Duncan (1987) and McGrath (1998) pointed out \( E' \) can be treated as equal to \( M_s \). In this study, \( E' = M_s \) is adopted to simplify the analysis.

Figure 8 shows that the peaking deflections in the vertical direction calculated using Equation 1 proposed by Masada and Sargand (2007) agree well with the measured data reported by McGrath et al. (1999) (tests 4, 7 and 11), Arockiasamy et al. (2006) and Masada and Sargand (2007). The range of relative error is from 9% to 20%. However, values of pipe deflection predicted by Equation 1 are six to eight times those measured in tests 13 and 14 in McGrath et al. (1999) (data points circled by a dashed line). The peaking deflections predicted by the proposed method, that is, Equation 5, in this study, match well with all the data reported by McGrath et al. (1999), Arockiasamy et al. (2006) and Masada and Sargand (2007) with relative errors of less than 10%.

Figure 9 presents the peaking deflections in the horizontal direction calculated using Equation 1 proposed by Masada and Sargand (2007) which are in good agreement with those reported by McGrath et al. (1999) (tests 4, 7, and 11) and Masada and Sargand (2007) with a range of relative error of 5% to 15% and 10% to 21%, respectively. However, the predicted values are five to six times those measured in tests 13 and 14 reported by McGrath et al. (1999) (data points circled by dashed line).

### Table 8. Input parameters used for calculation of peaking deflections in tests reported by McGrath et al. (1999)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D ) (m)</td>
<td>1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.7 1.7</td>
</tr>
<tr>
<td>PS (kPa)</td>
<td>390 390 390 390 390 390 390 390 390 390</td>
</tr>
<tr>
<td>( M_s ) (MPa)</td>
<td>13.06 13.06 7.58 13.8 8.78 13.8 13.06 7.58 7.58 7.58</td>
</tr>
<tr>
<td>Compactor</td>
<td>Rammer Rammer Vibratory plate Rammer Vibratory plate Vibratory plate Vibratory plate Vibratory plate Vibratory plate Vibratory plate</td>
</tr>
<tr>
<td>( P_s ) (kPa)</td>
<td>5.51 5.51 0.4 2.69 0.207 2.69 5.51 0.4 0.4 0.207</td>
</tr>
<tr>
<td>( K_o ) (kN/m³)</td>
<td>0.41 0.41 0.41 0.53 0.53 0.53 0.41 0.41 0.41 0.53</td>
</tr>
<tr>
<td>( \gamma ) (kN/m³)</td>
<td>20.7 20.7 19.3 15.4 14.3 15.4 20.7 19.3 19.3 14.3</td>
</tr>
</tbody>
</table>

Note: HDPE pipes are used in all the test cases 1 to 14, but data for test cases 2, 5, 10 and 12 are not available.

### Table 9. Input parameters used for calculation of peaking deflections in tests reported by Arockiasamy et al. (2006)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS (kPa)</td>
<td>238</td>
</tr>
<tr>
<td>( M_s^* ) (MPa)</td>
<td>9.76</td>
</tr>
<tr>
<td>( D ) (m)</td>
<td>1.34</td>
</tr>
<tr>
<td>( \gamma ) (kN/m³)</td>
<td>20</td>
</tr>
<tr>
<td>( K_o )</td>
<td>0.34</td>
</tr>
<tr>
<td>( P_s^* ) (kPa)</td>
<td>0.207</td>
</tr>
</tbody>
</table>

\( ^* \) Recommended by McGrath et al. (1999).

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The peaking deflection in the horizontal direction calculated by Equation 5 matches well with all the measured data reported by McGrath et al. (1999) and Masada and Sargand (2007), with a range of lower relative error of 5% to 10% and 3% to 8%, respectively. The vertical and horizontal peaking deflections of the pipe calculated by the Masada and Sargand (2007) method, that is, Equation 1, do not represent well the data for the tests reported by McGrath et al. (1999), as shown in Figures 8 and 9. A possible reason is that the sidefill modulus and the pipe–soil interface friction are not considered in Equation 1.

Figure 10 illustrates that the peaking deflections in the vertical direction predicted using the proposed method agree well with the measured data reported by Corey et al. (2014), with a range of relative error of 4% to 20%. The values calculated using both methods for tests reported by McGrath et al. (1999) do not match well with the measured data. Nevertheless, the proposed method presents a better quantitative guideline than the Masada and Sargand (2007) method. Further research on this aspect is needed. Figure 11 presents similar comparisons for the peaking deflections in the horizontal direction.

### 6. CONCLUSIONS

2DFE modeling was conducted to investigate the peaking deflections of buried HDPE pipes during the initial backfilling process. The results were evaluated using the field test data. Parametric studies were conducted to investigate the effects of pipe diameter, pipe stiffness, soil modulus, trench width, and compactor type on the peaking deflections of the pipes. It is seen that pipe diameter, pipe stiffness, soil modulus and compactor type have a significant effect on the peaking deflection and their influences need to be included into deflection computations. Meanwhile, the influence of the trench width was found to be insignificant and can be omitted.

An empirical formula is proposed for estimating the peaking deflection of buried HDPE pipes

\[ \Delta y/D = -\Delta x/D = (0.055_{\gamma} + 33)\eta H'/(10000D) \]

The proposed equation is verified against field data from the published studies. Compared with the method proposed by Masada and Sargand (2007), valuable improvement in prediction is achieved and the proposed equation can provide a useful tool for geotechnical engineering practice.

### ACKNOWLEDGEMENTS

The authors are grateful for the financial support from the National Natural Science Foundation of China (Grant No. 51108078, 51278100 and 41472258), Natural Science Foundation of Jiangsu Province (Grant No. BK20131294 and BK2012022), the Fundamental Research Funds for the Central Universities, Colleges and Universities in Jiangsu Province Plans to Graduate Research and Innovation (Grant No. KYLX_0144), and the Scientific Research Foundation of Graduate School of Southeast University (Grant No. YBJJ1632). The authors also express their gratitude to graduate students X. G., Qin, Q. You, D. D. Dong, and P. C. Wang at Southeast University for their assistance in conducting the field test.

### NOTATION

Basic SI units are given in parentheses.

- $A$: cross-sectional area of the pipe wall per unit length ($m^2/m$)
- $D$: undeformed diameter of the pipe (m)
- $dx$: horizontal diameter change/un-deformed pipe diameter (dimensionless)
- $dy$: vertical diameter change/un-deformed pipe diameter (dimensionless)

---

**Table 10. Input parameters used for peaking deflections in tests reported by Masada and Sargand (2007)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS (kPa)</td>
<td>294</td>
</tr>
<tr>
<td>$M^*_a$ (MPa)</td>
<td>13.8</td>
</tr>
<tr>
<td>$P^*_b$ (kPa)</td>
<td>0.207</td>
</tr>
<tr>
<td>$r$ (m)</td>
<td>0.41</td>
</tr>
<tr>
<td>$\gamma$ (kN/m$^3$)</td>
<td>18.95</td>
</tr>
<tr>
<td>$K_o$</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Note: steel-reinforced HDPE pipe was used in the laboratory model test. Recommended by McGrath et al. (1999).

**Table 11. Input parameters used for calculation of peaking deflections in tests reported by Corey et al. (2014)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS (kPa)</td>
<td>294</td>
</tr>
<tr>
<td>$M^*_a$ (MPa)</td>
<td>13.8</td>
</tr>
<tr>
<td>$D$ (m)</td>
<td>0.642</td>
</tr>
<tr>
<td>$\gamma$ (kN/m$^3$)</td>
<td>18.1</td>
</tr>
<tr>
<td>$K_o$</td>
<td>0.34</td>
</tr>
<tr>
<td>$P^*_b$ (kPa)</td>
<td>2.69</td>
</tr>
</tbody>
</table>

Note: steel-reinforced HDPE pipe was used in the laboratory model test. Recommended by McGrath et al. (1999).
$E$ Young’s modulus of pipe material (Pa)
$E_0$ elastic soil modulus at the reference position (Pa)
$E_{\text{increment}}$ increase of soil modulus per unit of depth (Pa)
$H'$ sidefill thickness (m)
$I$ moment of inertia of pipe wall per unit length (m$^4$/m)
$K_0$ lateral earth pressure coefficient at rest (dimensionless)
$K_n$ kneading factor (dimensionless)
$K_p$ passive earth pressure coefficient (dimensionless)
$M_s$ constrained soil modulus (kPa)
$P_c$ lateral pressure generated by the compactor (Pa)
$P_S$ pipe stiffness (kPa)
$R^2$ coefficient of determination (dimensionless)
$R_{\text{inter}}$ strength reduction factor (dimensionless)
$r$ radius of the undeformed pipe (m)
$S_f$ relative flexure stiffness (dimensionless)
$z$ depth of backfill (m)
$\gamma$ unit weight of the sidefill (N/m$^3$)
$\Delta x$ pipe diameter change in the horizontal direction (m)
$\Delta y$ pipe diameter change in the vertical direction (m)
$\eta$ an empirical constant related to the compactor type (dimensionless)
$\sigma_h$ horizontal pressure imposed on the pipe side (Pa)
$\sigma_v$ soil layer overburden pressure (Pa)
$v$ Poisson’s ratio (dimensionless)

**ABBREVIATIONS**

2DFF two-dimensional finite-element
AASHTO American Association of State Highway and Transportation Officials
ASTM American Society for Testing and Materials
CECS China Association for Engineering Construction Standardization
CL95 low plasticity clay with a degree of compaction of 95%
HDPE high-density polyethylene
SW85 well-graded sand with a degree of compaction of 85%
SW95 well-graded sand with a degree of compaction of 95%

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