Dynamic Responses of Marine Risers/Pipes Transporting Fluid Subject to Top End Excitations

J. Leklong  
*Technip Engineering, Thailand*

S. Chucheepsakul  
*King Mongkut’s University of Technology, Thailand*

S. Kaewunruen  
*University of Wollongong, sakdirat@hotmail.com*

http://ro.uow.edu.au/engpapers/468
Dynamic Responses of Marine Risers/Pipes Transporting Fluid Subject to Top End Excitations

Jirapol Leklong*, Somchai Chucheepsakul**, and Sakdirat Kaewunruen***

*Technip Engineering (Thailand) Ltd, Bangkok, Thailand
**King Mongkut’s University of Technology, Trungkru, Bangkok, Thailand
***University of Wollongong, Wollongong, NSW, Australia

ABSTRACT

This paper deals with the dynamic responses to top end excitation of marine risers/pipes conveying internal fluid. The marine riser is often used as a flexible link between undersea bore head and subsurface offshore platform. The tidal waves and the changes of sea level consistently excite its top end connected to a floating vessel. In order to carry out the performance-based design of the marine risers, the evaluation of their dynamic responses to top end excitations is imperative. In this study, the marine riser is simulated using two-dimensional beam elements. Energy functional of the marine risers conveying fluids is derived from variational principle. Nonlinear equations of motion influenced by the nonlinear Morison waveform are obtained through Hamilton’s principle. Investigation of the dynamic responses of marine risers to top end excitation is achieved using the finite element method and Newmark Average Acceleration Method. Interestingly, either beating or resonant phenomenon can be observed from the responses. It is also found that the top tension plays a major role in the increment of undamped frequencies of marine risers, while either the internal flow rate or the external hydrodynamic drag force remarkably affects the displacement amplitudes of the marine risers’ dynamic responses.

KEY WORDS: Marine risers; pipes; dynamic response; top end excitation; internal fluid; finite elements; direct integration.

INTRODUCTION

There are a number of different types of offshore platforms, which are depending on the functional uses. All of the offshore structures are generally designed against the dynamic load actions due to the severe wind and wave interactions. Marine risers/pipes are part of the offshore platform, generally attached to the platform structures or floating vessels as a transportation means for the hydrocarbon resources underneath the sea bed e.g. crude oil, natural gas, and so on. Generally, the dynamic excitations acting on the marine risers/pipes are due to the ocean waves resulting in the complicated movements of the vessels. Sen (2006) described the movements, which comprise of high frequency response to the tidal waves and low frequency excursion (slow drift). In particular, these forces are acting at the top end of the marine risers in both horizontal and vertical directions, while the hydrodynamic drag force acts along the riser. Most vulnerable to excessive stresses are the flexible pipelines at the dynamic mode shape inflexion points subject to the corresponding resonances (Kaewunruen et al., 2005). It is widely accepted that even a small damage of the offshore structural components could lead to significantly detrimental effect to environment, human lives, and assets. It is thus imperative for structural offshore engineers to carry out the analytical simulations to determine the effects of those top end excitations on the dynamic responses and behaviour of the marine risers/pipes transporting the internal fluid.

The marine risers have been first used since 1949 in Mohole Project in the US. In general design perspective, the marine risers are considered to carry their own weight and subjected to a static offset. They are also designed to confront the nonlinear hydrodynamic drag forces, internal and external fluid pressures, surface waves, undersea turbulences, and vortex induced vibrations. It is found that although there are a number of investigations related to the dynamic responses of the marine risers or pipes, the dynamic effect due to top end excitation has not been addressed yet. The previous research topics of interest have included both linear and nonlinear problems. Those key parametric studies have covered the effects of internal flow, bending rigidity, axial extensibility, top tension, Poisson’s ratio, and even radial deformation.

A recent step stone in riser static analysis has been carried out by Athisakul et al. (2002) whereas the large strain and large sag (higher order geometric nonlinearity) have been considered using a variational approach. To obtain the analytical results, the shooting method has been employed and the results were compared against the nonlinear finite element analyses. It was found that the large extensible strain and the bending rigidity have little effect on the static equilibrium. However, a highlight is that the bending moment envelope of the risers increases as the static offsets (Monprapassorn, 2002; Chucheepsakul et al., 2003).

In relation to dynamic analysis of marine risers/pipes, Irani et al. (1987) used energy and finite element methods to investigate the effects of internal flow and nutation damping on the dynamic responses of the marine risers to linear hydrodynamics. It was found that the internal
flow speed decreases the bending stiffness of the risers. This study has been confirmed by Moe and Chucheepsakul (1988) that the higher the internal flow speed, the lower the natural frequency of the marine risers. Chen and Lin (1989) compared the dynamic responses of marine risers to harmonic wave and linear hydrodynamic drag forces using Newmark integration and Fourier Expansion methods. The results were in good agreement.

Wu and Lou (1991) developed a mathematical model to investigate the effects of bending rigidity and internal flow on the dynamic responses of the marine risers. This study made use of perturbation method to tackle the problem. Interestingly, the results show that the bending rigidity plays a substantial role on the dynamic responses when the internal flow speed is very fast. Atadan et al. (1997) considered shear action on the marine risers in three dimensions on the basis of nonlinear riser extensibility. Butenin method (1965) was applied to obtain the analytical solutions. The results reveal that the hydrodynamic damping tends to streamline the dynamic responses of the marine risers onto steady states. The findings have later been confirmed by Bar-Avi (2000) who studied the dynamic responses of marine risers conveying fluid, considering geometric nonlinearity and hydrodynamic drag force. In his study, finite difference method was adopted. The structural damping effect has been evaluated by Park and Jung (2002). Rayleigh damping approximation method was employed in order to general the finite element matrices. Although, in their investigation, the internal flow has not been taken into account, it was found that the riser responses to the superposition of total excitations are slightly different to those due to the harmonic excitation. More recently, Chatjigeorgiou (2004) investigated the parametric excitation of vertical elastic slender structures and the damping effect in marine applications. The two-to-one resonance behaviour to the lowest natural frequency of the riser was determined (Chatjigeorgiou and Mavrakos, 2005).

In this study, the marine risers/pipes are modeled as the beam pipes in two dimensional spaces whereas the dynamic excitations at the top end could be simulated by the coupled harmonic displacements. The variational approach used to formulate and solve the problem of riser dynamics is a highlight in this paper. The top end excitation effects on riser responses are then determined using the numerical methods.

**VARIATIONAL APPROACH**

The dynamic responses of the marine risers/pipes are considered from the virtual displacements along the deformed risers. Fig. 1 shows a typical marine riser structure in different deformed states. The marine risers can be simulated by a beam-column-like structure. They are subjected to external forces in either lateral or longitudinal directions. Both ends of the riser are attached to ground anchorage at the bottom and to floating balloon at the top. The support boundary conditions are thus modeled by a non-fictional hinge at the bottom end, while the top end is considered as a free support. The static analysis of the riser was carried out through a previous study (Chucheepsakul et al., 1999). The riser mass, static top tension and the buoyancy were considered in the analysis. The static offset of the riser was predicted from the external wave and hydrostatic loading. At this stage, the top end support redistributes the static bending moment due to external forces and could be assumed as a pinned support, in order to obtain the equilibrium configurations of the riser. The initial static displacements in lateral and longitudinal are obtained from the energy method as shown in Appendix 1.

**Assumptions**

In this study, these assumptions have been made.

- The material property of the riser is homogeneous and linearly elastic.
- Due to the very high aspect ratio of the riser, the cross section of the riser is uniform.
- The effect of shear, torsion, rotational inertia, and Poisson’s ratio is negligible.
- The internal fluid fully flows inside along the pipeline.
- The risers behave within its elastic range.

**Environmental Conditions and External Forces**

The actions due to current drag force and sea surface wave loading are depicted in Fig. 2. To estimate the total external forces acting on the riser, the environmental conditions are considered, including the current velocity, density of sea water, temperature, sea water viscosity, surface wind speed, and so on.
Deformed Configuration

Considering a position of riser as shown in Fig. 3, the riser deforms to a new position when subjected to a perturbation. In this study, the coordinate system is referenced to the centerline of the riser at static equilibrium position. The new deformed configuration read

\[ r'(s) = r_0(s) + u\hat{n} + w\hat{i} \tag{1} \]

![Fig. 3 Motion state of marine risers/pipes](image)

Strain and Displacement Relationships

Using Euler-Bernoulli beam theory, the strain and displacement relationships can be written as

\[ \kappa = \frac{d^2u}{ds^2} + \kappa u + \frac{d\kappa}{ds} w \tag{2} \]

\[ \varepsilon = \frac{dw}{ds} - \kappa u + \frac{1}{2} \left( \frac{dv}{ds} - \kappa w \right)^2 + \left( \frac{du}{ds} + \kappa w \right)^2 \tag{3} \]

Energy Method

**Total strain energy** consists of the strain energy due to axial and flexural deformation, which can be written as

\[ U_s = U_a + U_b \tag{4} \]

Where

\[ U_s \] = Total strain energy  
\[ U_a \] = Axial strain energy  
\[ U_b \] = Bending strain energy

**Virtual axial strain energy** is the strain energy due to axial deformation, which reads

\[ U_a = \frac{1}{2} \int_0^l E\varepsilon^2 \, ds \tag{5} \]

Where

\[ E \] = Modulus of elasticity of riser  
\[ A \] = Cross sectional area of riser

**Virtual bending strain energy** is formulated from the bending curvature of riser, which reads

\[ U_b = \frac{1}{2} \int_0^l EI\kappa'^2 \, ds \tag{6} \]

Where

\[ I \] = Moment of inertia of the riser

**Virtual work done due to external forces** is due to effective weight of risers (Eq. 7), hydrodynamic drag force (Eq. 8), and inertial force (Eq. 9). Those work done components read (Morison et al., 1950; Leklong, 2008):

\[ \delta W_e = -\int_0^l w_e \delta w \, ds \tag{7} \]

\[ \delta W_H = -\int_0^l \left[ (F_n)\delta u + (F_i)\delta w \right] ds \tag{8} \]

\[ \delta W_I = -\int_0^l \left[ (m_r a_{rx} + m_f a_{fx})\delta u + (m_r a_{rz} + m_f a_{fz})\delta w \right] ds \tag{9} \]

Where

\[ w_e \] = Effective weight of the riser  
\[ F_n \] = Drag force in lateral direction  
\[ F_i \] = Drag force in longitudinal direction  
\[ m_r \] = Mass of riser  
\[ m_f \] = Mass of internal fluid  
\[ a_{rx} \] = Acceleration of the riser in x direction  
\[ a_{rz} \] = Acceleration of the riser in z direction  
\[ a_{fx} \] = Acceleration of the internal fluid in x direction  
\[ a_{fz} \] = Acceleration of the internal fluid in z direction

**Total kinetic energy** consists of the kinetic energy due to riser and internal flow movements, which can be written as

\[ T_s = T_r + T_f \tag{10} \]

Where

\[ T_s \] = Total kinetic energy  
\[ T_r \] = Kinetic energy due to riser movement  
\[ T_f \] = Kinetic energy due to fluid movement

Work-Energy Functional

The total work energy functional includes the total strain energy, virtual work done, and total kinetic energy (Langhaar, 1962). Taking variation on total work energy functional yields

\[ \delta \Pi = \int_0^l \left[ E\varepsilon^* \delta u^* + E\varepsilon_j \delta \varepsilon_j \right] ds \tag{11} \]
By applying the Hamilton’s principle, the equations of motion can be
achieved as follows:

\[
\begin{align*}
& \left[ m_r + m_f \right] \ddot{u} + 2m_r V \dot{w} + m_f V^2 \dot{w} - \left[ EA (\varepsilon_0 + \varepsilon_1) u \right] + E h u'' + F_{D,0} = 0 \quad (12) \\
& \left[ m_r + m_f \right] \ddot{w} + 2m_r V \dot{u} + m_f V^2 \dot{u} - \left[ EA (\varepsilon_0 + \varepsilon_1) (1 + w) \right] = F_{D,0} = 0 \quad (13)
\end{align*}
\]

Where

\[
F_{D,0} = \text{Hydrodynamic drag force in normal direction}
\]

\[
F_{D,0} = \text{Hydrodynamic drag force in tangential direction}
\]

The mass matrix can be written as

\[
[M] \{\ddot{D}\} + [G] \{\dot{D}\} + [K] \{D\} = \{R\} \quad (18)
\]

The gyroscopic matrix due to added mass can be written as

\[
[G] = \sum_{i=1}^{N} \left[ \int [N]^T [B] [N'] dz \right]
\]

The stiffness matrix reads

\[
[K] = \sum_{i=1}^{N} \left( \int [N]^T [C] [N'] dz + \int [N]^T [D] [N'] dz \right)
\]

The external force matrix is

\[
\{R\} = \sum_{i=1}^{N} \left( \int [N]^T \{f\} dz \right)
\]

**FINITE ELEMENTS**

The riser structure shown in Fig. 2 can be discretized into finite elements as illustrated in Fig. 4. Each element is divided equally along the arc length of the riser. Using the cubic polynomial displacement approximation, the displacement vectors \{u\} in terms of nodal degree of freedom \{d\} and shape functions \{N\} are

\[
\{u\} = \left\{ \begin{array}{cc} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \\ u_4 \\ w_4 \end{array} \right\} \cong \{N\} \{d\} \quad (14)
\]

\[
\{N\} = \begin{bmatrix} N_1 & N_2 & 0 & 0 & N_1 & N_4 & 0 & 0 \\
0 & 0 & N_1 & N_2 & 0 & 0 & N_3 & N_4 \end{bmatrix}
\]

\[
\{d\} = \left\{ \begin{array}{cc} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \\ u_4 \\ w_4 \end{array} \right\}
\]

The relationship between arc length and height discretions can be estimated by

\[
ds = \sqrt{1 + x^2} \, dz \quad (17)
\]

\[
K = K + a_n M + a_C
\]

\[
R_{\text{ext}} = R_{\text{ext}} + M \left( a_n \dot{U} + a_{n+1} \dot{U} + a_{n+2} \ddot{U} \right) + C \left( a_n \dot{U} + a_{n+1} \dot{U} + a_{n+2} \ddot{U} \right)
\]

The displacement over a time can be computed from

\[
\ddot{K} + a_n R = R_{\text{ext}}
\]

It should be noted that at each time step the iteration method is employed in order to cope with the nonlinear drag force pattern. The acceleration and velocity functions of the risers can be determined using the following equations.

\[
\ddot{a}_n \dot{U} = a_n \left( \ddot{a}_n \dot{U} - \dot{a}_n \dot{U} \right) - a_{n+1} \dot{U} - a_{n+2} \ddot{U}
\]

\[
\dot{a}_n \dot{U} = \dot{U} + a_{n+1} \ddot{U} + a_{n+2} \ddot{U} = a_n \left( \ddot{a}_n \dot{U} - \dot{a}_n \dot{U} \right) - a_{n+1} \dot{U} - a_{n+2} \ddot{U}
\]
The boundary conditions at the bottom end are set as

\[ u_{\text{bottom}} = w_{\text{bottom}} = 0 \]  

(29)

In contrast, the top end is treated by the excitation functions as follows.

\[ F_x^u(t) = F_y^u \sin(\omega t) \]  

(30a)

\[ F_y^u(t) = F_x^u \sin(\omega t) \]  

(30b)

The initial conditions for determining the dynamic responses of the marine risers read

\[ \ddot{u} = 0 \]  

(31a)

\[ \dot{u} = 0 \]  

(31b)

\[ \dot{u} = 0 \]  

(31c)

MODEL VALIDATIONS

The initial conditions for determining the dynamic responses of the marine risers read

\[ \ddot{u} = 0 \]  

(31a)

\[ \dot{u} = 0 \]  

(31b)

\[ \dot{u} = 0 \]  

(31c)

The initial conditions for determining the dynamic responses of the marine risers read

\[ \ddot{u} = 0 \]  

(31a)

\[ \dot{u} = 0 \]  

(31b)

\[ \dot{u} = 0 \]  

(31c)

MODEL VALIDATIONS

The numerical validations of the model contain two parts: numerical and analytical results (Moe and Chucheepsakul, 1988; Kaewunruen and Chucheepsakul, 2004). The effect of internal flow friction is not considered in this study as it clearly reduces the flow speed.

Table 1 Analysis input data

<table>
<thead>
<tr>
<th>Description and notation</th>
<th>Quantity</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top tension</td>
<td>( T_T )</td>
<td>476.198 kN</td>
</tr>
<tr>
<td>Number of riser elements</td>
<td>( N_{el} )</td>
<td>40 Element</td>
</tr>
<tr>
<td>Sea level</td>
<td>( H )</td>
<td>300 m</td>
</tr>
<tr>
<td>Offset</td>
<td>( x_0 )</td>
<td>0 m</td>
</tr>
<tr>
<td>Riser weigh in air</td>
<td>( W )</td>
<td>7850 kg/m</td>
</tr>
<tr>
<td>Outer diameter of riser</td>
<td>( D_0 )</td>
<td>0.26 m</td>
</tr>
<tr>
<td>Thickness of riser</td>
<td>( t )</td>
<td>0.03 m</td>
</tr>
<tr>
<td>Sea density</td>
<td>( \rho_s )</td>
<td>1025 kg/m³</td>
</tr>
<tr>
<td>Internal flow density</td>
<td>( \rho_i )</td>
<td>998 kg/m³</td>
</tr>
<tr>
<td>Modulus of elasticity of riser</td>
<td>( E )</td>
<td>2.07x10⁸ kg/m²</td>
</tr>
<tr>
<td>Added mass coefficient</td>
<td>( C_M )</td>
<td>2.0</td>
</tr>
<tr>
<td>Drag force coefficient in normal direction</td>
<td>( C_{Dn} )</td>
<td>0.7</td>
</tr>
<tr>
<td>Drag force coefficient in tangential direction</td>
<td>( C_{Dt} )</td>
<td>0.03</td>
</tr>
<tr>
<td>Current velocity</td>
<td>( V )</td>
<td>0-35 m/s</td>
</tr>
<tr>
<td>Internal flow rate</td>
<td>( V )</td>
<td>0-35 m/s</td>
</tr>
<tr>
<td>Time Step</td>
<td>( \Delta t )</td>
<td>0.05 s</td>
</tr>
<tr>
<td>Excitation frequency range</td>
<td>( \Delta \omega )</td>
<td>0.02 rad/s</td>
</tr>
<tr>
<td>Forced Amplitude</td>
<td>( F_\alpha )</td>
<td>100 N</td>
</tr>
<tr>
<td>Newmark’s integration parameter</td>
<td>( \delta )</td>
<td>0.50</td>
</tr>
<tr>
<td>Newmark’s step parameter</td>
<td>( \alpha )</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The calibration data are tabulated in Table 1. These data are identical to previous study by Kaewunruen and Chucheepsakul (2004). To compare the results, the riser is excited by a wide range of forcing frequencies as shown in Fig 5. The resonances of the riser responses can then be determined as shown in Table 2. It is found that the fundamental frequencies of the marine risers in this study are in very good agreement with previous studies. Analogously, it should be noted that other modes of vibration found in this study are also in very good agreement with previous studies (Leklong, 2008).

Table 2 Fundamental frequency of marine risers conveying fluid

<table>
<thead>
<tr>
<th>Internal flow speed (m/s)</th>
<th>Numerical solution [A]</th>
<th>This study</th>
<th>%Difference from [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2878</td>
<td>0.3005</td>
<td>0.6026</td>
</tr>
<tr>
<td>5</td>
<td>0.2979</td>
<td>0.2996</td>
<td>0.5707</td>
</tr>
<tr>
<td>10</td>
<td>0.2956</td>
<td>0.2990</td>
<td>1.1502</td>
</tr>
<tr>
<td>15</td>
<td>0.2916</td>
<td>0.2956</td>
<td>1.3717</td>
</tr>
<tr>
<td>20</td>
<td>0.2858</td>
<td>0.2870</td>
<td>0.4199</td>
</tr>
<tr>
<td>25</td>
<td>0.2782</td>
<td>0.2798</td>
<td>0.5751</td>
</tr>
<tr>
<td>30</td>
<td>0.2683</td>
<td>0.2685</td>
<td>0.0745</td>
</tr>
<tr>
<td>35</td>
<td>0.2558</td>
<td>0.2590</td>
<td>1.2510</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Internal flow speed (m/s)</th>
<th>Analytical solution [B]</th>
<th>This study</th>
<th>%Difference from [B]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2878</td>
<td>0.2825</td>
<td>-1.8416</td>
</tr>
<tr>
<td>5</td>
<td>0.2869</td>
<td>0.2814</td>
<td>-1.9170</td>
</tr>
<tr>
<td>10</td>
<td>0.2843</td>
<td>0.2789</td>
<td>-1.8994</td>
</tr>
<tr>
<td>15</td>
<td>0.2797</td>
<td>0.2780</td>
<td>-0.6078</td>
</tr>
<tr>
<td>20</td>
<td>0.2728</td>
<td>0.2770</td>
<td>1.5396</td>
</tr>
<tr>
<td>25</td>
<td>0.2628</td>
<td>0.2599</td>
<td>-1.1035</td>
</tr>
<tr>
<td>30</td>
<td>0.2476</td>
<td>0.2430</td>
<td>-1.8378</td>
</tr>
</tbody>
</table>

Fig. 5 Displacement responses of risers in frequency domain; a) 0 m/s; b) 5 m/s; c) 10 m/s; d) 15 m/s; e) 20 m/s; f) 25 m/s

Table 2 Fundamental frequency of marine risers conveying fluid

<table>
<thead>
<tr>
<th>Internal flow speed (m/s)</th>
<th>Numerical solution [A]</th>
<th>This study</th>
<th>%Difference from [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2878</td>
<td>0.3005</td>
<td>0.6026</td>
</tr>
<tr>
<td>5</td>
<td>0.2979</td>
<td>0.2996</td>
<td>0.5707</td>
</tr>
<tr>
<td>10</td>
<td>0.2956</td>
<td>0.2990</td>
<td>1.1502</td>
</tr>
<tr>
<td>15</td>
<td>0.2916</td>
<td>0.2956</td>
<td>1.3717</td>
</tr>
<tr>
<td>20</td>
<td>0.2858</td>
<td>0.2870</td>
<td>0.4199</td>
</tr>
<tr>
<td>25</td>
<td>0.2782</td>
<td>0.2798</td>
<td>0.5751</td>
</tr>
<tr>
<td>30</td>
<td>0.2683</td>
<td>0.2685</td>
<td>0.0745</td>
</tr>
<tr>
<td>35</td>
<td>0.2558</td>
<td>0.2590</td>
<td>1.2510</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Internal flow speed (m/s)</th>
<th>Analytical solution [B]</th>
<th>This study</th>
<th>%Difference from [B]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2878</td>
<td>0.2825</td>
<td>-1.8416</td>
</tr>
<tr>
<td>5</td>
<td>0.2869</td>
<td>0.2814</td>
<td>-1.9170</td>
</tr>
<tr>
<td>10</td>
<td>0.2843</td>
<td>0.2789</td>
<td>-1.8994</td>
</tr>
<tr>
<td>15</td>
<td>0.2797</td>
<td>0.2780</td>
<td>-0.6078</td>
</tr>
<tr>
<td>20</td>
<td>0.2728</td>
<td>0.2770</td>
<td>1.5396</td>
</tr>
<tr>
<td>25</td>
<td>0.2628</td>
<td>0.2599</td>
<td>-1.1035</td>
</tr>
<tr>
<td>30</td>
<td>0.2476</td>
<td>0.2430</td>
<td>-1.8378</td>
</tr>
</tbody>
</table>
RESULTS AND DISCUSSION

Using the data in Table 1 as the initial input, the dynamic responses of the marine risers/pipes transporting fluid to top end excitation can be attained. Interestingly, both resonant and beat phenomena have been detected. These phenomena have been also found in the deep-ocean mining pipelines (Chung et al. 1997a; 1997b).

**Dynamic Top Tensions**

The dynamic top tensions are determined from the dynamic strains at the top of the riser. The main contribution is from the first derivative of longitudinal displacements. It is found that the resonances to the natural frequencies of the riser occur at the frequency magnifications between 82 and 85, or about 54.7742, 65.7733, 109.622, 153.471 rad/s for the first, second, third, and fourth modes of vibration, respectively.

Figs. 6 presents the dynamic top tensions in time domain due to different excitation frequencies. It is found that the dynamic top tensions at the excitation frequency of 65.7733 rad/sec prevail the beating phenomena.

![Fig. 6 Dynamic top tensions of marine risers/pipes in time domain: a) 0.1147 rad/s; b) 0.4 rad/s; c) 0.8 rad/s; d) 54.7742 rad/s; e) 65.7733 rad/s; f) 100 rad/s](image)

**Effect of Bending Rigidity**

To identify the effect of bending rigidity on the dynamic responses of the marine risers, the internal flow speed is kept at zero. The dynamic behaviours of the riser taking into account the bending stiffness and without such stiffness are considered, as illustrated in Fig. 7. The frequency ratio $FR = \omega_r / \omega_f$ varies from 0.5 to 2.0. The results show that the maximum displacements of the risers with bending rigidity are slightly lower than those without the bending rigidity. However, the resonant behaviours of the risers taken into account the bending rigidity tend to be higher than those without the bending rigidity.

![Fig. 7 Displacement responses of marine risers/pipes in time domain: a) FR = 0.5; b) FR = 0.8; c) FR = 0.9; d) FR = 1.0; e) FR = 1.1; f) FR = 1.2; g) FR=1.5; h) FR=2.0](image)

**Effect of Internal Flow**

The internal flow speeds ($V$) from 0 to 35 m/s are considered in the parametric studies. Losses due to internal flow frictions in the pipe system clearly deter the internal flow speed and are not considered in this investigation. The dynamic responses of the marine risers transporting the internal fluids are illustrated in Fig. 8. The results clearly exhibit that the displacement responses of the risers with bending rigidity tend to converge onto steady state response amplitudes. It is noticeable that when the internal flow speeds increase, the steady-state convergence time durations decrease. Table 3 summarizes the convergence time onto the steady states of the marine risers conveying different internal flow speeds.
Table 3 Convergence time onto the steady states of the marine risers

<table>
<thead>
<tr>
<th>Internal flow speed (m/s)</th>
<th>Time(sec), $FR = 0.8$</th>
<th>Time(sec), $FR = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>&gt; 2000</td>
<td>&gt; 2000</td>
</tr>
<tr>
<td>10</td>
<td>1340</td>
<td>1495</td>
</tr>
<tr>
<td>15</td>
<td>1030</td>
<td>1165</td>
</tr>
<tr>
<td>20</td>
<td>730</td>
<td>930</td>
</tr>
<tr>
<td>25</td>
<td>555</td>
<td>625</td>
</tr>
<tr>
<td>30</td>
<td>421</td>
<td>580</td>
</tr>
<tr>
<td>35</td>
<td>285</td>
<td>480</td>
</tr>
</tbody>
</table>

**Effect of Elastic Modulus and Top Tension**

Using the main data in Table 1, the elastic moduli are varied from $2.07\times10^8$ to $2.01\times10^{12}$ while the top tensions are considered in the range from 450 to 2050 kN. The dynamic responses of the marine risers transporting the internal fluids are used for the determination of the resonant frequencies. Fig. 9 shows that the resonant frequencies vary significantly with the elastic modulus and the top tension.
CONCLUSIONS

The dynamic responses of marine risers/pipes transporting fluid subjected to harmonic excitation at top end are presented in this paper. The dynamic analyses focus on both elastic and bending effects. Based on the virtual work-energy functional of the marine risers/pipes, the structural model developed consists of the strain energy due to axial and bending deformations. Virtual work created by effective tension, and inertial and hydrodynamic drag forces is presented as well as the kinetic energy due to both the riser and internal fluid motions. Nonlinear equations of motion due to the effect of a nonlinear Morison type term coupled in axial and transverse displacements are derived through the Hamilton’s principle. To determine the dynamic responses of the marine risers, the finite element method is implemented for which the Newmark Average Acceleration method is used for direct numerical integration. Beating and resonant phenomena are observed via the dynamic responses of the risers. The effects of internal flow, top tensions, hydrodynamic forces, and modulus of elasticity are investigated and found to influence marine riser dynamic behaviours. The internal flow rate and the hydrodynamic drag force have a major impact on the amplitude of dynamic displacements as to remain in the steady state condition. The top tension and modulus of elasticity play a key role in the increment of natural frequencies of the marine risers.

ACKNOWLEDGEMENTS

The authors are grateful to acknowledge the financial support by Thailand Research Fund under the Grant No RTA/03/2543.

REFERENCES


APPENDIXES

To study riser statics, the time-independent functions are considered. By applying the stationary potential energy to the conservative systems, the first variation for the integration of the subtraction between total potential energy and work-done functionals approaches zero. Hence, the governing differential equations are evaluated from equation (4), (10), (12), and (13). The derivation for these equations yields the governing equilibrium equations in transverse direction and vertical direction as follows

$$\left(\frac{M\dot{C}}{}\right)\ddot{C} + E\left(\frac{\dot{C}^2}{\ddot{C}^2}\right) - \dot{C} + \left(\frac{T}{\ddot{C}}\frac{\dot{C}}{\ddot{C}}\right) + M\dot{C} = 0$$

$$\frac{\dot{T}}{\ddot{C}} + W = 0$$

The static solution can be generally found by using the finite element method (Chucheepsakul et al., 1999).