Influence of joint stiffness on the free vibrations of a marine riser conveying fluid

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Influence of Joint Stiffness on the Free Vibrations of a Marine Riser Conveying Fluid

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ABSTRACT

The marine risers are generally used as the main transport means for economic materials and resources discovered undersea. In general, the marine risers are secured at either an offshore platform or a vessel. Both ends of the riser are to be adjusted to obey with the design criteria but in many cases their rotational stiffness is hardly set free for the hinge connection. This paper integrates the analytical investigation and the design consideration for the effects of end supports on the free vibrations of a marine riser conveying fluid. It is well known that resonances of marine risers/pipes can cause excessive stress and strain responses, leading to ultimate fatigue failure of the riser system. To avoid a resonance resulting in the sustainable use of the riser, the study to evaluate the influence of end rotational stiffness on the natural frequency of a marine riser conveying fluid is imperative. Using variational principle, the riser model formulation includes both the bending rigidity and the axial elasticity. Using Galerkin finite element method, the natural frequencies and their corresponding mode shapes are determined. The highlight in this paper is the free vibration behaviours in the transition from hinge to fully fixed ends, which can indicate a health monitoring methodology of the risers. The design concepts for the marine risers/pipes conveying fluid to minimize the influence of support conditions are discussed herein as a guideline for structural offshore engineers.

KEY WORDS: Marine risers; pipes; Free vibrations; Natural frequency; Rotational joint stiffness; internal fluid; Galerkin finite element method.

INTRODUCTION

Offshore structures are built mostly to suit the energy and mining industry. The natural resources discovered under sea bed are transported to the sea surface in order to store and process the raw materials. Marine risers/pipes are such main component connected to the floating offshore platforms. There are a number of different types of platforms such as jacking platform, tensioned leg platform, or even ship vessels. Those platforms will be the temporary storage of the hydrocarbon resources drilled from the sub sea, before shipping to the refinery or other manufacturing processes. In construction, the typical connection between the vessel and the top end of the risers is arranged as close as a hinge support, which is adopted for the riser analysis and design. However, in certain circumstances, the connection is restrained by fastening system and/or shock absorber. This has led to the semi-rigid connection behaviour at the top end, which would be different from the earlier analysis and design of the risers. This constraint can also be caused by the welded connection and the improper design of bolt patterns. Although these semi-rigid joints have been often observed in a number of offshore civil constructions (Lennon, 2008), it is found that the investigations related to the dynamic behaviours are inadequate.

Typically, the marine riser consists of steel pipes attached to the floating support by a constant top tension. It is usually kept almost straight or nearly vertical as shown in Fig. 1. In two dimensional space, it is commonly modeled as a beam-column-like structure with internal fluid flowing inside. In practice, the riser is installed along the vertical position. After the riser subjected to its own weight, internal and external pressures, and current and wave forces, it deforms to an equilibrium position (Kaewunruen et al., 2005). After the static deformation, the vibration of the riser can occur due to surrounding excitations afterwards, e.g. vortex shedding, hydrodynamic force, etc. It is imperative to investigate the vibrational behaviour of the marine risers in order to avoid excessive displacements due to a resonance (Leklong et al., 2008). These large displacements yield excessive stresses along the riser, resulting in the decrement of service life of the risers. This would not allow the riser to be reused in other projects or applications. The sustainability in design and construction cannot thus be achieved.

A number of previous studies show the significance of the riser dynamics to public safety. Irani et al. (1987) performed the dynamic analyses of risers with steady internal flow and nutation dampers in three dimensions using an energy approach and the finite element method. The results showed that the internal flow reduced the overall stiffness of the marine risers. It also gave a negative damping mechanism to the riser dynamic behaviour. A similar study was done using asymptotic approach together with a finite element method (Moe and Chucheepsakul, 1988). It was found that the natural frequencies were slightly reduced at a low internal flow speed but significantly trend could be observed at very high flow speed.
Huang (1993) derived the governing equation of kinetics of transported mass inside an extensible riser in three approaches: Lagrange, Euler, and Coriolis. This derivation has led to a number of investigations related to large displacements of the marine risers (Chucheepsakul and Huang, 1994; Chucheepsakul et al., 1999; 2002). It was found that the natural frequencies of the riser decrease as the internal flow and static displacement increase. Also, the effect of transported fluid is more pronounced in the highly extensible risers than in the lowly extensible ones. Wu et al. (1991) carried out a mathematical model for lateral motion of a marine riser considered the effects of bending rigidity and internal flow. The results were obtained using the perturbation method and they showed that the rigidity has more importance to the dynamic responses of risers at high internal flow rates. Up to date, it is found that the eigenvalue problems of the marine risers have not been thoroughly investigated.

**Strain Energy due to Bending Deformation**

According to Fig.1, let the displacement \( u \) be a function of variable \( z \) and time \( t \). The strain energy due to bending is written as

\[
U_b = \frac{H}{2} E u'^2 dz
\]

where \( H \) is initial riser length or the instant sea depth and \( u \) is the lateral displacement.

This equation is admissible for a vertical riser for which the strain is assumed to be small and the top end moves laterally with small amount of static offset, \( x_h \).

**Strain Energy due to Axial Deformation**

The strain energy due to axial deformation is occurred by two causes; one is by the axial tension and another is by hydrostatic pressure. In this study, the hydrostatic pressure is blended with the riser’s tension as the external forces in which provide the effective axial tension. Thus, the virtual strain energy due to axial deformation is

\[
\delta U_t = \delta \left[ \frac{H}{2} E A \varepsilon^2 dz \right] = \int_0^H E A \delta \varepsilon dz
\]

\[
\varepsilon = \frac{T}{EA} = \frac{ds}{dz} = \frac{dw}{dz} + \frac{1}{2} \left( \frac{dv}{dz} \right) + \left( \frac{du}{dz} \right)^2
\]

where \( \varepsilon \) is nonlinear strain undergoing large deformation (Bhashyam and Prathap, 1980; Sarma and Varadan, 1982; McDonald, 1991).

**Virtual Work due to External Forces**

There are the external forces acting on the riser such as current and wave forces, inside and outside pressures, its weight, and buoyancy. The velocity of current \( V_c \) is decomposed into two components: in normal direction \( V_n \) and in tangential direction \( V_t \). The forces due to current and wave loads in normal direction \( F_n \) and tangential force \( F_t \) so-called the hydrodynamic drag force which are given per unit length by

\[
F_n = 0.5 \rho_c D_0 C_{Dn}(V_n^2 - u^2) \left| V_n - u \right| + \rho_c A_{in} C_{mn} \dot{V}_n - \rho_c A_{in} C_{mn} \dot{u}
\]

\[
F_t = 0.5 \rho_c \pi D_0 C_{Dt}(V_t^2 - u^2) \left| V_t - u \right| + \rho_c A_{it} C_{mt} \dot{V}_t - \rho_c A_{it} C_{mt} \dot{u}
\]
where \( \rho_v \) is density of sea water; \( D_0 \) is outside diameter of riser; \( C_{Dw}, C_{Dz} \) are normal and tangential drag coefficients; \( C_M \) is inertia coefficient; \( V_n, V_t \) are normal and tangential components of the wave particle velocity; \( A_{off} \) is hydrodynamic cross-sectional area.

The tangential component \( w \) is small and then \( F_t \) is neglected in static analysis. The inside and outside pressures of the riser are derived to statically equivalent forces. The remaining forces are effective weight of riser per unit length \( W_0 \), which consists of the buoyant force, the inside fluid force, and the riser weight \( W \) (Fellipa and Chung, 1981; Chakrabati and Frampton, 1982). The effective expression is

\[
W_0 = W + \gamma_f A - \gamma_o A_0
\]

(7)

where \( \gamma_f \) and \( \gamma_o \) are the specific weights of the internal and external fluids. \( A_f \) and \( A_o \) are the inner and outer cross-sectional areas of the riser. Thus, the transverse force \( q \) is

\[
q = F_0 = 0.5 \rho D C_n(V_n - \bar{u})(V_n - \bar{u}) + \rho A \bar{u} - \rho u A_0 C_n \bar{u}
\]

(8)

Since considering the hydrostatic pressure for any underwater structures, the tension \( T \) is substituted by the effective tension \( T_e \) as

\[
T_e = E A_0 \epsilon = T_{new} + p_n A_0 - p_t A_f
\]

(9)

where \( T_{new} \) is actual tension in the riser; and \( p_n, p_t \) are the external and internal pressures, respectively. Thus, the virtual work done due to external forces is defined by

\[
\delta W_e = -\int_0^h q \delta u dz + \frac{1}{2} \int_0^h (W_0 + F_0) \delta w dz
\]

(10)

Virtual Work done by Inertia force

Virtual work done by inertia force is caused by both transverse and longitudinal movements. Inertia terms include the terms of internal fluid flow and riser movement. The virtual work done by inertia forces is written as,

\[
\delta W_i = -\vec{F} \cdot \delta \vec{r}
\]

(11)

\[
\delta \vec{r} = \frac{\delta u \vec{i} + \delta w \vec{k}}{}
\]

(12)

in which force \( F \) can be expressed by the Newton’s second law,

\[
\vec{F} = -\left( m \ddot{u} + M \ddot{r} \right) dz
\]

(13)

where \( m, a \) is the unit mass and the acceleration of the riser; \( M, a_r \) is the unit mass and the acceleration of the internal fluid; \( r \) is displacement vector. Since considering that the nearly vertical riser moves laterally, incorporating to longitudinal response, the velocity components of the internal fluid can be expressed as (Langhaar, 1962; Irani et al., 1987; Moe and Chuuchepsakul, 1988),

\[
x - \text{component} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial z} \right) u
\]

(14)

\[
z - \text{component} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial z} \right) w
\]

(15)

The lateral acceleration of internal fluid by taking the second derivative on the velocity equations (14, 15) are as follows,

\[
x - \text{component} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial z} \right)^2 u
\]

(16)

\[
z - \text{component} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial z} \right)^2 w
\]

(17)

The time-independent virtual work done by inertia force from internal fluid flow in two dimensions has been evaluated, and written as

\[
\delta W_i = -\int_0^h \left[ M \dot{u}^2 + M \dot{w}^2 \right] du + \left[ M \ddot{u}^2 + M \ddot{w}^2 \right] \delta w dz
\]

(18)

Total Kinetic Energy

Free vibration is determined through the virtual oscillations along the marine riser. The riser is slightly displaced from the static configuration (with \( \epsilon_0 \)) to the directions of \( u \) and \( w \). Due to the static displacements are small, the formulation is based on the initial state \( \delta \). All kinetic energy terms are caused by the riser movement, and by moving of the internal fluid flow relatively to the displacements. The total kinetic energy of the system is determined from the oscillations in which are translations \( u \) and \( w \) (Chakrabati and Frampton, 1982). The kinetic energy of the riser is written as

\[
T_r = \frac{1}{2} \int_0^h M \left[ (\dot{u}^2 + \dot{w}^2) \right] dz
\]

(19)

Similar to the static analysis, the velocity components of the internal fluid can be expressed as in equations (14) and (15). The kinetic energy of internal fluid flow in two dimensions can be written as (Paidoussis, 1998)

\[
T_{fl} = \int_0^h \frac{1}{2} M \left[ (\dot{u}^2 + \dot{w}^2) \right] dz
\]

(20)

Variational form of the total kinetic energy is written as

\[
\delta T = \int_0^h \left[ m \delta \ddot{u} + \delta \dot{w} \dot{w} \right] dz
\]

(21)

Total Work Energy Functional

Taking the variation on total work-energy functional accounted from equations (1), (2), and (10), one obtains

\[
\delta W = \int_0^h \left[ E u^2 \delta u'' + E A \delta u \delta x \right] dz
\]

(22)

where

\[
\delta \vec{x} = \vec{\epsilon}_0 + \vec{\epsilon}_u \vec{u} + \frac{1}{2} \left[ \left( \frac{\partial \vec{w}}{\partial \vec{x}} \right)^2 + \left( \frac{\partial \vec{u}}{\partial \vec{x}} \right)^2 \right]
\]

(23)

Hamilton’s Principle

By applying the Hamilton’s principle, the governing differential equations of motion are evaluated from equations (21) and (22), and
then the governing equation of motion in transverse direction is shown in equation (24).

\[
\frac{d^4 u}{dx^4} + (2MU^2/EI) \frac{d^2 u}{dx^2} + (M+M^2)u + E(\frac{d^2 u}{dx^2} + \frac{\partial}{\partial z}(Ea_0 + e) \frac{\partial u}{\partial z}) = 0
\]  

(24)

Also, the governing equation of motion in vertical direction is

\[
\frac{d^4 w}{dx^4} + (2MU^2/EI) \frac{d^2 w}{dx^2} + (M+M^2)w + E(\frac{d^2 w}{dx^2} + \frac{\partial}{\partial z}(Ea_0 + e) \frac{\partial w}{\partial z}) = 0
\]  

(25)

Linearization of the equation (24) and (25) yields the governing equation of motion in matrix form as,

\[
\begin{bmatrix}
A & C \\
B & D
\end{bmatrix} \begin{bmatrix}
\ddot{w} \\
\dddot{u}
\end{bmatrix} + \begin{bmatrix}
C & D
\end{bmatrix} \begin{bmatrix}
\dddot{u} \\
\dddot{w}
\end{bmatrix} + \begin{bmatrix}
\dddot{u} \\
\dddot{w}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(26)

where,

\[
\begin{bmatrix}
\ddot{u} \\
\dddot{w}
\end{bmatrix} = \begin{bmatrix}
u \\
w
\end{bmatrix}
\]  

(27)

and,

\[
A = \begin{bmatrix}
m + M & 0 \\
0 & m + M
\end{bmatrix}, \quad B = \begin{bmatrix}
2MU & 0 \\
0 & 2MU
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
E(a_0 - MU^2) & 0 \\
0 & E(a_0 + 1) -MU^2
\end{bmatrix}, \quad D = \begin{bmatrix}
EI & 0 \\
0 & 0
\end{bmatrix}
\]

(28a, b, c, d)

For pined-pined connection, the boundary conditions are;

\[
\{\ddot{u}\}_{x=0} = \{0\}, \quad \{\ddot{w}\}_{x=H} = \{0\}
\]  

(29a, b)

Due to the effect of hydrodynamic drag force, the mass matrix is additionally included by the added mass.

\[
\{F\} = -\rho \sum A_{hi} C_{di} \begin{bmatrix}
u \\
w
\end{bmatrix}
\]

(30)

Hence, the new mass matrix is

\[
\begin{bmatrix}
A & C \\
B & D
\end{bmatrix} \begin{bmatrix}
\ddot{w} \\
\dddot{u}
\end{bmatrix} + \begin{bmatrix}
C & D
\end{bmatrix} \begin{bmatrix}
\dddot{u} \\
\dddot{w}
\end{bmatrix} + \begin{bmatrix}
\dddot{u} \\
\dddot{w}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(31)

let,

\[
\begin{bmatrix}
\ddot{u} \\
\dddot{w}
\end{bmatrix} = \begin{bmatrix}
u \\
w
\end{bmatrix}
\]

(32)

\[
\begin{bmatrix}
u \\
w
\end{bmatrix} = \begin{bmatrix} u_1 \ u_2 \ w_1 \ w_2 \ u_1' \ u_2' \ w_1' \ w_2'
\end{bmatrix}
\]

(33)

and

\[
\begin{bmatrix}
\ddot{u} \\
\dddot{w}
\end{bmatrix} = \begin{bmatrix}
u \\
w
\end{bmatrix}
\]

(34)

The cubic polynomial shape function is used for determining the fundamental frequencies of marine risers. Through the Galerkin finite element approach, the free vibration equation (26) becomes

\[
\begin{bmatrix}
M & A & C & D \\
A & C & D & D \\
C & D & D & D \\
D & D & D & D
\end{bmatrix} \begin{bmatrix}
\ddot{u} \\
\dddot{w}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(35)

where the mass and gyroscopic matrices are

\[
[M] = \sum_{j=1}^{n} \int_0^h [N]^T \begin{bmatrix} 1 & 1 \end{bmatrix} [N] dz
\]

(36a, b)

\[
[G] = \sum_{j=1}^{n} \int_0^h [N]^T \begin{bmatrix} 1 & 1 \end{bmatrix} [N'] dz
\]

(37)

\[
[K] = \sum_{j=1}^{n} \int_0^h [N]^T \begin{bmatrix} C & D \\
D & D
\end{bmatrix} [N'] dz + \int_0^h [N]^T \begin{bmatrix} 1 & 1 \end{bmatrix} [D] [N'] dz
\]

(38)

Thus, the eigenproblem (38) is solved numerically by QZ-algorithm (Bathe, 1982).

JOINT STIFFNESS CONDITIONS

In this study, the semi-rigid connection is applied at both ends; one by one as illustrated in Fig. 2. The rotational spring simulating semi-rigid connection is applied to top end by \(k_1\) and to the bottom end by \(k_2\). The additional generalized stiffness formulation has been added to the elemental stiffness matrices at the first and the last elements of the riser structure (Cheng et al., 1995; Cook et al., 1998).

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & u \n 0 & k_1 & 0 & 0 & u' \\
0 & 0 & 0 & 0 & w
\end{bmatrix} = \begin{bmatrix} M_{dof} \\
\end{bmatrix}
\]

Fig. 2 Semi-rigid connections
RESULTS AND DISCUSSION

The numerical examples are pursued to validate the model in both static and dynamic analyses. This mathematical model is validated by the results from Moe and Chucheepsakul (1988) and the finite element model developed by Chucheepsakul et al. (1989). The parameters used in the calculation are $E = 207 \times 10^6 \text{kN/m}^2$, $D = D_{OH} = 0.26 \text{ m}$, $D_i = 0.20 \text{ m}$, $H = 300 \text{ m}$, $X_H = 10 \text{ m}$, $T_H = 476.198 \text{ kN}$, $\rho_s = 7850 \text{ kg/m}^3$, $\rho = 998 \text{ kg/m}^3$, $V_i = 0 \text{ m/s}$, $V_f = 0 \text{ m/s}$, $C_{pa} = 0.7$, $C_{pa} = 0.03$, $C_i = 1$, $C_f = 2$, $\nu = 0.5$. The static result is found that in good agreement with the literature (Kaewunruen and Chucheepsakul, 2004). The free vibration behaviours are calibrated against the previous analytical studies.

Dynamic Model Verification

Table 1 validates the fundamental natural frequencies $\omega_0$ (rad/sec) computed from this study comparing with solutions from literature by neglecting flexural rigidity under various internal flow $U$ (m/sec). The fundamental frequencies, for which the flexural stiffness is neglected, are in good agreement. The GFEM solution employed 40 elements and indicated that the instability would occur at the internal flow speeds of 38 m/s, which causes the negative combined tension to the riser bottom leg. From the result, the in-plane vibrations (u-mode) show the dominance on the fundamental mode shapes. In this paper, the flexural rigidity, which stabilizes the riser system, is considered in the analyses.

<table>
<thead>
<tr>
<th>U (m/s)</th>
<th>Analytical solution</th>
<th>FEM</th>
<th>This study</th>
<th>%Difference from [a]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2878</td>
<td>0.2891</td>
<td>0.2890</td>
<td>0.4170</td>
</tr>
<tr>
<td>5</td>
<td>0.2881</td>
<td>0.2881</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>0.2838</td>
<td>0.2853</td>
<td>0.2852</td>
<td>0.4933</td>
</tr>
<tr>
<td>15</td>
<td>0.2804</td>
<td>0.2803</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td>0.2706</td>
<td>0.2731</td>
<td>0.2730</td>
<td>0.8869</td>
</tr>
<tr>
<td>25</td>
<td>0.2627</td>
<td>0.2627</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>30</td>
<td>0.2413</td>
<td>0.2478</td>
<td>0.2478</td>
<td>2.6937</td>
</tr>
<tr>
<td>35</td>
<td>0.2224</td>
<td>0.2221</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>37</td>
<td>-</td>
<td></td>
<td>0.1962</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>0.1710</td>
<td>-</td>
<td>unstable</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>unstable</td>
<td>-</td>
<td>unstable</td>
<td></td>
</tr>
</tbody>
</table>

[a] Moe and Chucheepsakul; [b] Chucheepsakul et al.

Effects of Boundary Conditions

This numerical model has been ensured that it would be sufficient model for ocean engineering applications. Aspects of each parameter are studied here by using the above input data.

The sets of specific boundary conditions are applied to the vertical riser model as to evaluate the effects of the boundary conditions. First, the pinned-pinned supports at both ends are considered. The riser’s free vibration behaviour under the pinned-pinned supports whereas only translations are restrained is presented in Table 2.

The flexural rigidity increases the fundamental frequencies of risers about 3.36-27.37% with the speed of internal flow 0-30 m/s and much more percent in higher speeds (Kaewunruen and Chucheepsakul, 2004).

Considering the vibrations neglecting the flexural stiffness in Table 1, the tolerant speeds of internal flow are limited to some lower figures while the speeds for which take flexural stiffness into account are much higher.

Table 2 Natural frequencies (rad/s) of a pined-pined vertical riser with various internal flow (considering bending rigidity)

<table>
<thead>
<tr>
<th>U (m/s)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2987</td>
<td>0.6284</td>
<td>0.9937</td>
<td>1.4018</td>
<td>1.8578</td>
</tr>
<tr>
<td>5</td>
<td>0.2979</td>
<td>0.6270</td>
<td>0.9919</td>
<td>1.3997</td>
<td>1.8555</td>
</tr>
<tr>
<td>10</td>
<td>0.2956</td>
<td>0.6230</td>
<td>0.9867</td>
<td>1.3934</td>
<td>1.8484</td>
</tr>
<tr>
<td>15</td>
<td>0.2916</td>
<td>0.6162</td>
<td>0.9778</td>
<td>1.3829</td>
<td>1.8365</td>
</tr>
<tr>
<td>20</td>
<td>0.2858</td>
<td>0.6065</td>
<td>0.9653</td>
<td>1.3681</td>
<td>1.8197</td>
</tr>
<tr>
<td>25</td>
<td>0.2782</td>
<td>0.5938</td>
<td>0.9490</td>
<td>1.3489</td>
<td>1.7980</td>
</tr>
<tr>
<td>30</td>
<td>0.2683</td>
<td>0.5779</td>
<td>0.9287</td>
<td>1.3250</td>
<td>1.7711</td>
</tr>
<tr>
<td>35</td>
<td>0.2558</td>
<td>0.5583</td>
<td>0.9041</td>
<td>1.2964</td>
<td>1.7390</td>
</tr>
</tbody>
</table>

Table 3 shows the natural frequencies of the marine riser subject to pinned-fixed supports. In this case, the bottom end is simulated as a hinge connection whereas only the translations are constrained, whilst the top end’s rotations and translations are fully restrained as a fixed support. It is found that the natural frequencies in the fundamental mode increase about two percent, while they increase up to three percent for the fifth mode of vibration.

Table 3 Natural frequencies (rad/s) of a pined-fixed vertical riser with various internal flow (considering bending rigidity)

<table>
<thead>
<tr>
<th>U (m/s)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3046</td>
<td>0.6417</td>
<td>1.0166</td>
<td>1.4364</td>
<td>1.9063</td>
</tr>
<tr>
<td>5</td>
<td>0.3038</td>
<td>0.6403</td>
<td>1.0148</td>
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<td>1.0005</td>
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<td>1.8680</td>
</tr>
<tr>
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<td>0.6066</td>
<td>0.9713</td>
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<td>30</td>
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<td>0.5904</td>
<td>0.9508</td>
<td>1.3591</td>
<td>1.8193</td>
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<tr>
<td>35</td>
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<td>0.5705</td>
<td>0.9260</td>
<td>1.3303</td>
<td>1.7871</td>
</tr>
</tbody>
</table>

The natural frequencies of the marine riser subject to fixed-fixed supports are tabulated in Table 4. In this case, both of the bottom and the top ends are fully restrained of rotations and translations as a fixed support. The results reveal that the natural frequencies remarkably due to the support constraint.

In comparison with the results of the marine riser under the pined-pinned support condition, the fundamental frequencies increase about 20 percent, while the increments of natural frequencies of the other modes of vibration vary from 10 to 15 percent. In addition, it is noticeable that, for all modes of vibration, the differences of the natural frequencies increase with the internal flow speeds.

Table 4 Natural frequencies (rad/s) of a fixed-fixed vertical riser with various internal flow (considering bending rigidity)

<table>
<thead>
<tr>
<th>U (m/s)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3395</td>
<td>0.7025</td>
<td>1.0970</td>
<td>1.5337</td>
<td>2.0196</td>
</tr>
<tr>
<td>5</td>
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<td>0.7013</td>
<td>1.0954</td>
<td>1.5317</td>
<td>2.0173</td>
</tr>
<tr>
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<td>0.6977</td>
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<td>1.5257</td>
<td>2.0105</td>
</tr>
<tr>
<td>15</td>
<td>0.3335</td>
<td>0.6916</td>
<td>1.0822</td>
<td>1.5157</td>
<td>1.9990</td>
</tr>
<tr>
<td>20</td>
<td>0.3287</td>
<td>0.6829</td>
<td>1.0705</td>
<td>1.5016</td>
<td>1.9829</td>
</tr>
<tr>
<td>25</td>
<td>0.3225</td>
<td>0.6717</td>
<td>1.0553</td>
<td>1.4833</td>
<td>1.9621</td>
</tr>
<tr>
<td>30</td>
<td>0.3146</td>
<td>0.6576</td>
<td>1.0365</td>
<td>1.4607</td>
<td>1.9363</td>
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</table>
The effect of semi-rigid joints is evaluated into two phases as illustrated in the Appendix. The first case is to apply the rotational spring to the top end support. Then, the other rotational spring is additionally mounted at the bottom end of the riser model. The effects of joint top end support. Then, the other rotational spring is additionally mounted at the bottom end of the riser model. The effects of joint stiffness on the free vibration behaviour of the risers are presented in terms of a non-dimensional parameter, $k/EI$.

**Rotational Constraint at Top End**

Fig. 3 shows the influences of the rotational constraint at the top end ($k_1 \neq 0$) while the bottom end is kept as a hinge ($k_2 = 0$). The normalized natural frequency is the ratio between the frequencies under the specific condition and those of the riser with pined-pined condition as given in Table 2.

It should be noted that when $k/EI$ equals to zero, the connection behaviour is identical to that of a hinge. The results show that the joint stiffness at the top end has little influence on the natural frequencies of the marine risers/pipes conveying fluid. This is because the riser curvature at the top end region is very small. Overall, the frequencies change about 2 to 3 percent as the top end stiffness increases. When the rotational stiffness is about five thousands times of the bending rigidity ($k/EI \rightarrow 5,000$), the connection joint tends to behave like a fixed end. It is noticed that the fifth mode of vibration tends to be more sensitive to the joint stiffness variations than any other modes.

When the internal flow rate increases, the normalized frequencies tend to increase but in a very small amount. Therefore, the effect of the internal flow speeds, which generally affect the top tension, is negligible.

<table>
<thead>
<tr>
<th>$U$</th>
<th>$k_1/EI$</th>
<th>$k_2/EI$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$0.00E+00$</td>
<td>$0.00E+00$</td>
</tr>
<tr>
<td>$5$</td>
<td>$2.14E+03$</td>
<td>$2.14E+03$</td>
</tr>
<tr>
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<td>$15$</td>
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<td>$4.23E+04$</td>
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<td>$2.14E+03$</td>
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<tr>
<td>$40$</td>
<td>$3.35E+06$</td>
<td>$3.35E+06$</td>
</tr>
<tr>
<td>$50$</td>
<td>$4.23E+04$</td>
<td>$4.23E+04$</td>
</tr>
</tbody>
</table>
Rotational Constraint at Bottom End

It is found that the rotational spring transform the boundary conditions from a pinned onto a fixed support. In this study, the effect to a rotational spring at the bottom end in addition to the top end is determined. Fig. 4 shows the influences of the rotational constraint at the bottom end \((k_2 \neq 0)\) while the top end is fully restrained \((k_1 \rightarrow \infty)\). The normalized natural frequency is the ratio between the frequencies under the specific condition and those of the riser with pinned-fixed condition as given in Table 3. Clearly, the numerical results reveal that the joint stiffness at the bottom end play a significant role on the natural frequencies of the marine risers transporting fluid. Due to its own self-weight, the bottom end play a significant role on the natural frequencies of the marine risers, especially due to the bottom end constraints are presented in Figs. 5 and 6. The parametric results reveal that the joint stiffness plays a remarkably role on the natural frequencies of the marine risers transporting fluid. Due to its own self-weight, the curvature of the riser at the bottom region becomes large. As a result, the restraint of the bottom end is remarkable. It should be noted that when \(k_2/EI\) equals to zero, the bottom joint acts like a hinge. When the joint stiffness increases, the joint transforms into a fixed connection. Overall, the frequencies considerably change from 6 to 17 percent as the bottom end stiffness increases.

In addition, the results show that when the rotational stiffness is about 10 thousands times of the bending rigidity \((k_2/EI \rightarrow 10,000)\), the connection behaves like a fixed end. However, it can be seen that the fundamental modes of vibration are more sensitive to the joint stiffness variations than any other modes. The higher the mode of vibration, the lower the sensitivity to the joint stiffness.

In this case, when the internal flow rate increases, the normalized frequencies tend to increase remarkably. At the low internal flow rates between 0 and 15 m/s, the natural frequencies change between 5 and 12 percent. The effect of the joint stiffness increases at the higher internal flow whereas the frequencies vary from 6 to 17 percent. It is also found that at the low internal flow rate, the frequencies vary by gentle degrees with the rotational stiffness. In comparison, the frequency increments are very sensible to the joint stiffness when the internal flow rate is high.

DESIGN CONSIDERATION OF MARINE RISER

As a guideline for offshore engineers to consider the effects of joint stiffness in the construction, the average frequency shifts due to the support constraints are presented in Figs. 5 and 6. The parametric results reveal that the joint stiffness plays a remarkably role on the natural frequencies of the marine risers, especially due to the bottom end support restrain. The riser designer should adequately consider the arrangements of the support conditions of the risers (Cheng et al., 1995).

Fig. 5 clearly shows that if the top end support is not treated properly, the semi-rigid connection may easily occur and affect the increment of resonant frequencies of the risers. Although the bottom end has larger influence on the free vibration behaviour of the risers as depicted in Fig. 6, the semi-rigid connection tends to gradually affect the resonances of the marine risers conveying fluid. Consequently, it is imperative that the dynamic responses to hydrodynamic forces and vortex shedding must have been considered in the riser design as to avoid the excessive displacements and bending moments. The displacements and curvatures of the risers can be controlled by the top tension, static offset, and the joints. The designer must be aware that if the top tension is high or the static offset is small, the transverse displacements and the curvatures of the risers become insignificant and then the dynamic effect of semi-rigid joints is diminished. This insight will help the designers to address the sustainability in the design and construction of the marine riser/pipe systems.

CONCLUSIONS

The free vibration analysis of a nearly vertical marine riser transporting fluid is investigated in this paper using the finite element method accompanied with Galerkin technique. The nonlinear equations of motion are derived through a variational method. The riser model has been verified against the previous studies. The natural frequencies are presented to demonstrate the behaviors of the riser under the effects of joint stiffness, which is caused by the improper construction method. Based on the numerical simulations, the results indicate that the joint stiffness has influence on the riser behaviors. The severity of such effects is depending on the curvatures of the risers. The key findings show that the dynamic effect of top end restraint is insignificant when comparing with that of the bottom end. This can be explained by the fact that the riser displaces and bends remarkably in the bottom region due to its accumulative own weight. In contrast, the increased internal flow could considerably affect the riser dynamics by incorporating the action of tension and centrifugal forces, hence reducing the accumulative self-weight of the riser. On this ground, the designers can adjust or reduce the dynamic influences of joint stiffness by controlling the riser displacements and curvatures.

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Thailand are gratefully acknowledged.

REFERENCES


APPENDIX

In these investigations, the structural models of the marine risers are presented as follows:

\[ \begin{align*}
\text{Case 1: } & k_1 \neq 0 & k_2 &= 0 \\
\text{Case 2: } & k_1 \rightarrow \infty & k_2 &\neq 0
\end{align*} \]