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Does Mathematical Study Develop Logical Thinking? Testing the Theory of Formal Discipline

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Abstract

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Book Reviews

Does Mathematical Study Develop Logical Thinking? Testing the Theory of Formal Discipline

Matthew Inglis and Nina Attridge
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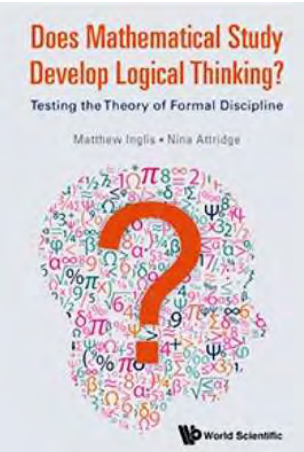
Mathematics has long had acknowledgement as a worthwhile area of study, including that of many outside mathematics itself. Plato, Francis Bacon, John Locke and Simone Weil are just a few of those who have who expressed views along the lines that the study of mathematics advantages students outside the immediate knowledge that they learn. Today we might put this by saying that mathematics develops generic skills such as logical thought, a capacity for analysing issues based on reason, and an awareness of how underlying assumptions may affect the conclusions of analysis or thought. Such a position is referred to by the authors of this book as 'The Theory of Formal Discipline' (TFD). The TFD predicts that mathematics study increases the level of normative logical responses — that is, responses corresponding to the standard logic underlying mathematics. It would seem that the TFD is endorsed widely by the mathematical community, even if mostly at an implicit level. However, psychological research has been sceptical of the TFD and of the possibility of transferring skills from one context to another (see [2], and subsequent literature). The main rival to the TFD is the Filtering Hypothesis, which says that better reasoning skills in a student are due to an initial propensity for mathematical study rather than a consequence of it. However, the TFD has continued to be influential on education policy, although every so often it is challenged at the public and policy level.

The research described in this book aims to test the validity of the TFD by statistical techniques and, in large part, it describes the research undertaken by the authors to this end. As there are difficulties in testing the TFD using randomised groups, the authors used longitudinal studies, starting from a common base point, to try and eliminate bias in the analyses. Further steps were taken to eliminate bias and avoid the chicken-or-egg situation in the Filtering Hypothesis, by controlling the data for general ability and the thinking disposition of the student. The method for measurement of the latter derives from research that indicates that students of high ability not infrequently have difficulty with relatively simple questions that nevertheless require a degree of analytical thought [3]. An example of such a question is: 'a pencil and a sheet of paper cost $1.10. The pencil costs $1.00 more than the sheet of paper. How much does the sheet of paper cost?' A person resistant to analytical thought may simply jump to the conclusion that the answer is 10 cents.

Following consultation with prominent people in the mathematics community, the authors tested using three reasoning tasks: the extent to which the validity of the conclusion in a syllogism is affected (say) by the believability of the conclusion rather than the logic of the syllogism, accuracy in reasoning with conditional statements, and Wason's selection task [4]. The latter is widely studied in psychology, and involves selecting the minimum number of cards from a given set of cards in order to confirm or deny a given statement about the cards. Each card has a letter on one side and a number on the other, the cards are laid out and the statement to be confirmed or denied could be: 'if a card has D on one side it has 3 on the other'. So, this is a rather more complicated test about how people reason with conditional statements of the form 'if p then q'.



Not surprisingly, critics of the type of testing in the authors' research claim that abstract or decontextualized reasoning is not necessary or useful in day-to-day life. In the jargon of today, we would say it is not concerned with 'real world' problems. If so, that could mean that even if the study of mathematics accentuates such skills as abstract reasoning, this should not be considered as justification for wide participation in mathematical study beyond immediate practical skills such as arithmetic. However, the authors provide an interesting argument (pp. 128–129), based on research, which concludes that a better performance on seemingly abstract tasks, such as the understanding of conditional statements, predicts that day-to-day decision making is better over and above general cognitive capacity [1]. As society is placing more and more reliance upon abstract rules and procedures one might expect that this effect could become more useful and important for raising general mathematical awareness.

The authors say that at the outset of their research they had no definite view of the truth or falsity of the TFD. However, their conclusions are to a large extent favourable to the TFD, although they express certain reservations. They conclude (p. 130): '... the evidence we have presented provides good reasons to suppose that studying advanced mathematics does indeed develop some aspects of conditional reasoning, notably the ability to reject invalid inferences. In that sense, we have found evidence that is highly consistent with at least one version of the TFD'. However, they consider that the picture concerning general conditional reasoning is more nuanced, although a study carried out by them in Cyprus suggests that this could be a function of the curriculum content (see pp. 80–86).

Mathematics in general, and conditional statements in particular, present a greater cognitive load than statements of fact, or listings of information. Is the aim of mathematics teaching to promote genuine understanding and appreciation of the material for the bulk of students we have in front of us, or is it to present more capable students with a wide range of topics and ideas, so as to prepare them for study in other disciplines or possible future careers as mathematicians? This type of problem is not as immediate in less analytical disciplines, and there is a balance that has to be to be struck between promoting understanding and the range of mathematics in a given curriculum. Understanding takes time and effort, on the part of the teacher and the student, and this is sometimes sacrificed so as to cover more material and more topics perceived to be necessary for later study.

This book is recommended. It is quite technical and detailed, but its methods and conclusions are in keeping with the complexity of testing the TFD. It is essentially a book reporting research, and it is clear and well written. However, many questions concerning the TFD remain. For example, how influential is the curriculum upon the development of logical skills? What about the quality of teaching? Should mathematics be taught in a way that specifically encourages logical thought and general reasoning, instead of leaving them as implicit skills that may be picked up incidentally along the way? Should logical thought and reasoning be thought of in the abstract only, or should they be incorporated much more into day-to-day contexts and decision making? To what extent should school mathematics, or university mathematics for that matter, be taught as a 'formal discipline'? Not everyone thinks that logical thought conforms or should conform to the standard mathematical and logical norms, and criticisms of the worth of mathematical study are now quite common, so the answers to such questions could be more important in the future for the health of mathematical study than they have been in the past. Mathematical culture may be well advised to consider them more seriously than has been the case.

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