Behavior Study for a Self-mixing Laser Diode with Undamped Relaxation Oscillation and Its Sensing Applications

Bin Liu

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Behavior Study for a Self-mixing Laser Diode with Undamped Relaxation Oscillation and Its Sensing Applications

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Supervisor:

A/Prof. Yanguang Yu

This thesis is presented as part of the requirement for the conferral of the degree:

Doctor of Philosophy

From

UNIVERSITY OF WOLLONGONG

School of Electrical, Computer and Telecommunications Engineering

October 2018
Dedicated to my family
Declaration

This is to certify the work reported in this thesis was carried out by the author, unless specified otherwise, and that no part of it has been submitted in a thesis to any other university or similar institution.

Bin Liu

October 2018
Abstract

Self-mixing interferometry (SMI), or also called optical feedback interferometry (OFI) is a promising non-contact sensing technology. It is based on the self-mixing effect which occurs when a fraction of light back-reflected or back-scattered by an external target re-enters the laser internal cavity. A sensing system by using SMI technique consists of a laser diode (LD), a photodiode (PD) packaged at the rear of the LD, a lens and a target. The LD is called self-mixing laser diode (SMLD). This configuration reflects a minimum part-count scheme, which is useful for engineering implementation. Compared to traditional interferometry, e.g., Michelson or Mach-Zehnder interferometry, SMI has the advantages of simplicity in system structure, low cost in implementation, and ease in optical alignment. Using these merits, SMI technology has been developed for various applications, such as measurement of displacement, vibration, velocity, imaging, material related parameters, laser related parameters, etc.

Most of the SMI-based applications and behavior study on SMLD system are based on the analytical SMI model, which is derived from the steady-state solution of the Lang and Kobayashi (L-K) equations, or from the classical three-mirror model, by assuming the system operates in stable mode, i.e. both the electric field and carrier density in an LD with a stationary external cavity can reach constant state after transient period. However, undamped relaxation oscillation (RO) may occur under some operation conditions, e.g. it is found an SMLD in moderate feedback regime exhibits undamped RO. The moderate feedback regime is quite commonly employed by researchers. Based on our in-depth study, the behavior of an SMLD system with undamped RO cannot be described by the existing analytical SMI model. The laser intensity (called as sensing signal) from such SMLD system shows some new characteristics. In order to differentiate the conventional SMI signals, we name the SMI signals with undamped RO as RO-SMI signals. Usually, the PD packaged inside the LD is used for detecting SMI signals. Such PD has limited bandwidth usually less than 1GHz. However, RO frequency of an LD can be higher than several GHz. Hence, using the existing SMI configuration for detection of RO-SMI signal, many
frequency components cannot be observed. Therefore, it is of great interest and significance to reveal these phenomena and find their potential applications.

In this thesis, in Chapter 2, the features and waveforms of an SMLD with undamped RO are theoretically investigated. Firstly, an improved stability boundary of an SMLD system is obtained. The influence of injection current and initial external cavity length on the stability boundary is analyzed. Based on the stability boundary analysis, intensive simulations by numerically solving L-K equations are performed to conclude the features of an SMLD system under different conditions. The influence of the photodetector bandwidth on a RO-SMI signal is discussed. Furthermore, an analytical expression for describing RO-SMI signals when the SMLD system operates in period-one oscillation is derived, which clearly describes the features of RO-SMI signals and indicates that such signal has the potential for displacement measurement.

In Chapter 3, an experimental system is implemented for experimentally investigation on the behavior of an SMLD system with undamped RO. The details of each part in the experimental system are presented firstly. Intensive experiments are conducted for verifying the theory presented in Chapter 2. In addition, the influence of photodetector bandwidth on the captured RO-SMI signals is experimentally studied. The experimental results are consistent with the theoretical analysis in Chapter 2.

Two new displacement sensing methods by using an SMLD with undamped RO are proposed in Chapter 4. Firstly, a micro-displacement sensing method with very high resolution is presented. The influence of the injection current and the initial external cavity on the sensing performance is investigated. Secondly, displacement sensing by employing both the time-domain RO-SMI signal and its RO frequency is developed, showing this method has large measurement range, high sensitivity and resolution.

In Chapter 5, an all-fiber SMLD system is built to demonstrate that an SMLD in moderate feedback regime with a high injection current and long external cavity can be always stable. Then, this system is used to measure the acoustic emission events successfully, contributing to a novel compact system in structure health monitoring.

All the results presented in this thesis are confirmed by both simulations and experiments, which unveil the behavior of an SMLD system in moderate or strong
feedback regime with undamped relaxation oscillation. Several sensing applications are proposed and verified by both theory and experiment. The results in this thesis provide useful guidance for developing an SMLD sensing system.
**Acronyms**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>SMI</td>
<td>self-mixing interferometry</td>
</tr>
<tr>
<td>SMLD</td>
<td>self-mixing laser diode</td>
</tr>
<tr>
<td>SME</td>
<td>self-mixing effect</td>
</tr>
<tr>
<td>OFI</td>
<td>optical feedback interferometry</td>
</tr>
<tr>
<td>L-K</td>
<td>Lang-Kobayashi</td>
</tr>
<tr>
<td>EOF</td>
<td>external optical feedback</td>
</tr>
<tr>
<td>LD</td>
<td>laser diode</td>
</tr>
<tr>
<td>PD</td>
<td>photodiode</td>
</tr>
<tr>
<td>RO</td>
<td>relaxation oscillation</td>
</tr>
<tr>
<td>BS</td>
<td>beam splitter</td>
</tr>
<tr>
<td>VA</td>
<td>variable attenuator</td>
</tr>
<tr>
<td>PZT</td>
<td>piezoelectric transducer</td>
</tr>
<tr>
<td>DFB</td>
<td>distributed feedback</td>
</tr>
<tr>
<td>OSC</td>
<td>oscilloscope</td>
</tr>
<tr>
<td>FBG</td>
<td>fiber Bragg grating</td>
</tr>
<tr>
<td>AE</td>
<td>acoustic emission</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-noise ratio</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width at half maximum</td>
</tr>
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</table>
Acknowledgement

PhD study has been a challenging, exciting and amazing experience, which is a milestone in my life. During the four-year study, it would not have been possible without the kind help and support from many people. I would like to extend thanks to them who generously contributed their valuable assistance in my PhD study.

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Finally, but not least, thanks go to my beloved wife, parents-in-law and parents, for almost unbelievable support. They are the most important people in my world and I dedicate this thesis to them.
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Chapter 1  Introduction

1.1 Background

Self-mixing interferometry (SMI), also called optical feedback interferometry (OFI), is an emerging and promising non-contact sensing technique, which uses the self-mixing effects (SME) that occurs when a fraction of light back-reflected or back-scattered by an external target reenters the laser inside cavity [1-3]. Similar to Michelson interferometry, the laser intensity in an SMI system is modulated in the form of interference fringes due to mixing of the intra-cavity electromagnetic wave with an emitted electromagnetic wave re-injected into the laser cavity after interaction in the external cavity. In this case, the steady-state intensity of the lasing light is modulated due to a varying external optical feedback phase. Usually, the modulated laser intensity is captured as the SMI signal [4-7]. Recently, it was pointed out that the voltage across the laser anode and cathode can also be utilized as the SMI signal [8-10].

The basic structures of a typical SMI system and Michelson interferometry system are depicted in Figure 1-1, where (a) is the SMI system and (b) is the traditional two-beam interferometry, e.g. Michelson interferometry system. It can be found that there is only one optical path in the SMI system, while two optical paths exist in traditional two-beam interferometry system. Moreover, in SMI system, a photodiode (PD) packaged in the rear of the laser diode (LD) rather than an external PD in the traditional two-beam interferometry is used to detect the interferometric signal. In an SMI system, the laser source is usually an LD. In this case, it is also named as self-mixing laser diode (SMLD) or optical feedback laser diode (OFLD). The advantages of SMI-based sensing scheme are summarized as below [11-13]:
• Optical part-count is minimal, i.e. no extra optical interferometer external to laser source is needed, leading to a compact and part-count-saving set-up.
• The interferometric signal can be achieved everywhere on the beam.
• Very weak external optical feedback, e.g. less than 0.1% of the laser emitting intensity, is sufficient to cause self-mixing effect, which means SMI technology can be used for rough diffusive surfaces.
• Sensitivity is high. The resolution of half laser wavelength can be achieved with fringe-counting and sub-wavelength can be achieved with analog processing.

![Schematic diagram of interferometry system](image)

Figure 1-1 Schematic diagram of interferometry system,
(a) self-mixing interferometry, (b) traditional two-beam interferometry

The configuration of SMI system reflects a minimum part-count scheme, which is useful for engineering implementation. By using its unique advantages, various SMI-based applications have been developed in the industrial and laboratory environment, which can be classified as below:

• Metrology: displacement [1, 14-17], absolute distance [18-21], vibration [4, 7, 22], velocity [23-26], angle [27], etc.
• Physical quantities: thickness [28], refractive index [29], roughness [30], etc.
Young’s modulus [31, 32], mechanic resonance [33], strain [34, 35], etc.

- Laser parameter: laser linewidth [36], linewidth enhancement factor [37-40]
- Sensing: confocal microscopy sensor [41, 42], biological motility [24, 43], acoustic emission sensor [44], CD/scroll sensors [45], remote echoes sensors [46], etc.
- Imaging [47-49]

All above applications are based on the analytical SMI mathematic model, which is derived from the steady-state solution of the Lang and Kobayashi (L-K) equations, or from the classical three-mirror model, by assuming the SMLD system operates in stable mode, i.e. both the electric field and carrier density in an LD with a stationary external cavity can reach a constant state after a transient period. However, undamped relaxation oscillation (RO) may occur in an SMLD system for some operation conditions [50-52], e.g. it was found the SMLD in moderate feedback regime exhibits undamped RO [53, 54]. The moderate feedback regime is quite commonly employed by researchers in this research field. In this case, the behavior of an SMLD system cannot be described by the existing analytical SMI model. The laser intensity (called as sensing signal) from such SMLD system shows some new characteristics and the signal waveform looks complicated. In order to differentiate the conventional SMI signals, we name the SMI signals with undamped RO as RO-SMI signals. Usually, the PD packaged inside the LD is used for detecting the SMI signal, but it has limited bandwidth less than 1GHz. However, the RO frequency of an LD can even higher than several GHz. Hence, using the existing SMI configuration for detection of RO-SMI signal, many frequency components of RO-SMI signals cannot be observed although they exist there in reality. Therefore, it is of great interest and significance to reveal these phenomena and find their potential applications.

This chapter presents an introduction and background of this thesis. Section 1.1 gives the background of this thesis, including the basic structure, advantages and various applications of SMLD system. The rest part of this chapter is organized as following. Section 1.2 gives a literature review for the SMLD, mainly focusing on the features of signal presented by an SMLD system at different operation conditions.
Through literature review, in Section 1.3, the existing problems for investigating the SMLD behavior are recognized, and thus the topic ‘behavior study of an SMLD with undamped RO and its sensing applications’ is proposed as the research work of this thesis. The overall structure of this thesis is shown in Section 1.4

1.2 Literature Review

In 1960s, it was reported that when a fraction of the light back-reflected or back-scattered by a remote target is allowed to re-enter the laser cavity, both the amplitude and the frequency of the lasing field can be modulated [55]. On one hand, the external optical feedback may influence the laser properties, degrading the modulation response characteristics or enhancing laser intensity noise [56, 57]. On the other hand, this phenomenon contributes to the discovery of a new class of laser interferometry which is named self-mixing interferometry or optical feedback interferometry [58]. In the early SMI-based applications in 1970s, gas lasers were used as the laser source and Doppler shift caused by a moving remote target was detected by using SMI technology in gas laser [58]. In 1980s, sensing applications based on SMI technology in low-cost commercial LDs were reported [23, 59]. Since then, various SMLD-based applications appeared, and SMI technology enters a new era.

1.2.1 Analytical Model for a Self-mixing Laser Diode (SMLD)

In order to analyze the behavior of an SMLD system, researchers have proposed different approaches. In 1988, Petermann [60] presented an three-mirror model, which shows that the basic structure of an SMLD system can be elegantly illustrated by the three-mirror model as depicted in Figure 1-2. After that, the three-mirror model has been adapted by many authors [34, 61-66]
As shown in Figure 1-2, the internal laser cavity is formed by mirrors $M_1$ and $M_2$ with length $L_{in}$, refractive index $n_{in}$. The intensity amplitude reflectivity of $M_1$ and $M_2$ is $r_1$ and $r_2$ respectively. The laser emitted through $M_2$ hits the external target which has an amplitude reflectivity of $r_3$. Then, a portion of the laser re-enters the laser internal cavity through the mirror $M_2$. The external cavity length $L$ is the distance between the $M_2$ and the external target. And the refractive index of the external cavity length is denoted by $n_{ext}$.

Assuming the laser diode is a Fabry-Perot (FP) type laser, the SMLD can be equivalent to a laser with rear facet $M_1$ and front facet $M_{eff}$ as shown in Figure 1-3 [60], where mirror $M_{eff}$ has an equivalent amplitude reflectivity $r_{eff}$.

Assuming $|r_2 r_3| \ll 1$, i.e. multi-reflection within the external cavity is neglected. Then, the equivalent amplitude reflectivity $r_{eff}$ of $M_{eff}$ (the equivalent laser front facet) can be expressed as [60-62]:

\[
|r_{eff}| \ll 1
\]
\[ r_{\text{eff}} = A_{\text{eff}} e^{j\phi_{\text{eff}}} = r_2 + (1 - |r_2|^2) r_2 e^{-j\omega_\tau} \]  \hspace{1cm} (1.1)

where \( \omega_\tau \) the laser angular frequency with optical feedback and \( \tau \) is the roundtrip time of the light in the external cavity, expressed as \( \tau = 2n_{\text{ext}} L / c \), where \( c \) is the speed of light in vacuum. Introducing a parameter \( \kappa \) for describing the coupling rate of re-injected light into the internal cavity length, named by optical feedback strength, which is expressed as:

\[ \kappa = \eta (1 - r_2^2) r_1 / r_2 \]  \hspace{1cm} (1.2)

where \( \eta \) is the coupling efficiency which accounts for possible loss on re-injection, e.g. mode mismatch, finite coherence length. When \( \kappa << 1 \), from Eq. (1.1), we can get:

\[ A_{\text{eff}} = r_2 \left[ 1 + \kappa \cos(\omega_\tau \tau) \right] \]  \hspace{1cm} (1.3)

\[ \phi_{\text{eff}} = \kappa \sin(\omega_\tau \tau) \]  \hspace{1cm} (1.4)

since \( \kappa \sin(\omega_\tau \tau) << r_2 \left[ 1 + \kappa \cos(\omega_\tau \tau) \right] \) and \( \arctan(x) \approx x \) for small \( x \) [65].

As the round-trip phase in the laser cavity must be equal to an integer multiple of \( 2\pi \), we obtain the phase condition of compound cavity of the three mirror model shown as below:

\[ -\alpha (g_c - g_{\text{th}}) \delta + \tau_{\text{in}} (\omega_\tau - \omega_0) + \phi_{\text{eff}} = 2\pi q \]  \hspace{1cm} (1.5)

where \( q \) is an integer, \( \alpha \) is the linewidth enhancement factor or called Henry factor of the laser. \( \tau_{\text{in}} \) is the laser roundtrip time in the internal cavity, expressed as \( \tau = 2n_{\text{in}} L_{\text{in}} / c \) and \( \omega_0 \) is the angular frequency of the solitary laser without feedback. \( g_c \) and \( g_{\text{th}} \) are the threshold gain of the laser with and without external feedback respectively [60, 62]. \( g_{\text{th}} \) can be expressed as:

\[ g_{\text{th}} = a_s + d^{-1} \ln \left[ (r_1 r_2)^{-1} \right] \]  \hspace{1cm} (1.6)
where \( a_s \) accounts for any optical loss in the internal cavity. Additionally, \( g_c \) must satisfy the amplitude condition of the compound cavity [60, 61]:

\[
r_1 A_{eff} e^{[g_c - a_s]d} = 1
\]  

(1.7)

Inserting Eq. (1.3) and (1.6) into (1.7), we can obtain:

\[
g_c - g_m = -\frac{\kappa}{d} \cos(\omega_c \tau)
\]  

(1.8)

Then, inserting Eq. (1.4) and (1.8) into (1.5) and letting \( q=0 \), i.e. the phase difference is 0, the phase equation of the SMLD system is give as below:

\[
\omega_c \tau = \omega_c \tau - \frac{\kappa}{\tau_{in}} \tau \sqrt{1 + \alpha^2} \sin\left[ \omega_c \tau + \arctan(\alpha) \right]
\]  

(1.9)

Some interesting results can be explained by using the three-mirror model. However, some details related to the physical setting of the phenomena may not be explained [2].

In order to investigate the more detailed phenomena of the SMLD system, another approach to achieve the mathematic model of an SMLD is to solve the steady-state solution of the well-known Lang Kobayashi (L-K) equations which are the modified Lamb’s equations [2]. Compared to the three-mirror model, the L-K equations are point-independent equations. The dynamic behavior of a single mode LD with external optical feedback (EOF) is able to be elegantly described by the L-K equations. The L-K equations [67] were first proposed by Lang and Kobayashi in 1980, which can be written in the following form:

\[
\frac{dE(t)}{dt} = \frac{1}{2} \left\{ G[N(t), E(t)] \frac{1}{\tau_p} - \frac{1}{\tau_p} E(t) + \frac{\kappa}{\tau_{in}} E(t - \tau) \right\} \cos[\omega_c \tau + \phi(t) - \phi(t - \tau)]
\]  

(1.10)

\[
\frac{d\phi(t)}{dt} = \frac{1}{2} \alpha \left\{ G[N(t), E(t)] \frac{1}{\tau_p} - \frac{1}{\tau_p} E(t) - \frac{\kappa}{\tau_{in}} E(t - \tau) \right\} \sin[\omega_c \tau + \phi(t) - \phi(t - \tau)]
\]  

(1.11)

\[
\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - G[N(t), E(t)] E^2(t)
\]  

(1.12)
where \( G[N(t),E(t)] = G_N \left[ N(t) - N_0 \right] \left[ 1 - \alpha E^2(t) \right] \) is the modal gain per unit of time [51, 68]. There are three main variables in L-K equations for describing the LD with EOF, i.e. the amplitude of the electric field \( E(t) \), the phase of the electric field \( \phi(t) \) and the carrier density \( N(t) \). The laser intensity or output power is denoted by \( P(t) = E^2(t) \).

The dynamics of the LD with an EOF are governed by three controllable parameters, i.e. the injection current density \( J \), feedback strength \( \kappa \) and laser round-trip time \( \tau \) in the external cavity. The other parameters in Eq. (1.10) - (1.12) are associated to the solitary LD itself, and they are constants for a specific LD. The physical meanings of the symbols appearing in Eq. (1.10)- (1.12) and the values of the parameters used in the simulations of this thesis are shown in Table 1-1, which are adopted from [50, 69].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Physical Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>time</td>
<td></td>
</tr>
<tr>
<td>( E(t) )</td>
<td>amplitude of the intra-cavity electric field</td>
<td></td>
</tr>
<tr>
<td>( N(t) )</td>
<td>carrier density</td>
<td></td>
</tr>
<tr>
<td>( \phi(t) )</td>
<td>phase of the intra-cavity electric field, ( \phi(t) = [\omega(t) - \omega_0]t )</td>
<td></td>
</tr>
<tr>
<td>( \omega(t) )</td>
<td>laser angular frequency with feedback</td>
<td></td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>laser angular frequency without feedback</td>
<td>( 2.42 \times 10^{15} ) rads(^{-1} )</td>
</tr>
<tr>
<td>( \lambda_0 )</td>
<td>laser wavelength without feedback</td>
<td>( 780nm )</td>
</tr>
<tr>
<td>( G_N )</td>
<td>modal gain coefficient</td>
<td>( 8.1 \times 10^{-13} ) m(^3)s(^{-1} )</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>carrier density at transparency</td>
<td>( 1.1 \times 10^{24} ) m(^3)</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>nonlinear gain compression coefficient</td>
<td>( 2.5 \times 10^{-23} ) m(^3)</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>confinement factor</td>
<td>( 0.3 )</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>photon life time</td>
<td>( 2.0 \times 10^{-12} ) s</td>
</tr>
<tr>
<td>( \tau_s )</td>
<td>carrier life time</td>
<td>( 2.0 \times 10^{-9} ) s</td>
</tr>
<tr>
<td>( \tau_{in} )</td>
<td>internal cavity round-trip time</td>
<td>( 8.0 \times 10^{-12} ) s</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>line-width enhancement factor</td>
<td></td>
</tr>
<tr>
<td>( J_{th} )</td>
<td>threshold injection current density</td>
<td></td>
</tr>
<tr>
<td>( J )</td>
<td>injection current density</td>
<td></td>
</tr>
<tr>
<td>( \kappa )</td>
<td>feedback strength</td>
<td></td>
</tr>
<tr>
<td>( L )</td>
<td>external cavity length</td>
<td></td>
</tr>
<tr>
<td>( \tau )</td>
<td>light roundtrip time in the external cavity, ( \tau = 2L/c )</td>
<td></td>
</tr>
</tbody>
</table>

In 1995, Donati et al. [1] obtained the analytical SMI model by solving the
steady-state solutions of the L-K equations. Introducing \( E_s \), \( \omega_s \) and \( N_s \) as the steady-state solutions of electric filed amplitude, angular frequency and carrier density, by setting \( \frac{dE(t)}{dt} = 0 \), \( \frac{d\omega(t)}{dt} = \omega_s - \omega_0 \) and \( \frac{dN(t)}{dt} = 0 \), it can be obtained

\[
E(t) = E(t - \tau) = E_s \quad , \quad N(t) = N_s \quad \text{and} \quad \phi(t) = (\omega_s - \omega_0)t .
\]

Substituting them into Eq. (1.10)-(1.12), the well-known steady-state solutions is obtained as follow [1, 2, 13, 51, 67, 70]:

\[
\omega_s \tau = \omega_0 \tau - \frac{\kappa}{\tau_{in}} \tau \sqrt{1 + \alpha^2} \sin(\omega_0 \tau + \arctan \alpha)
\]  

(1.13)

\[
N_s = N_0 + \frac{1}{\tau_p G_N} - \frac{2\kappa \cos(\omega_0 \tau)}{\tau_{in} G_N}
\]  

(1.14)

\[
E_s^2 = \frac{J N_s / \tau_s}{G_N(N_s - N_0)}
\]  

(1.15)

Introducing \( \phi_0 = \omega_0 \tau \), \( \phi_s = \omega_s \tau \) and \( C = \frac{\kappa}{\tau_{in}} \tau \sqrt{1 + \alpha^2} \), (1.13) is expressed as:

\[
\phi_s = \phi_0 - C \sin[\phi_s + \arctan(\alpha)]
\]  

(1.16)

where \( \phi_0 \), \( \phi_s \) are the optical phase associated with the external cavity length with and without EOF respectively, \( C \) is the optical feedback factor. Eq. (1.16) is the phase equation of the SMI model which is the core part of the existing SMI model, describing the behavior of an SMLD system, and it is same as Eq. (1.9) derived from the three-mirror model. Inserting Eq. (1.14) into (1.15) and considering the case of \( \kappa < 0.01 \), it can be obtained:

\[
P = E_s^2 \approx E_{s0}^2 + \tau_p (J - N_0 / \tau_s) \frac{2\kappa \tau_p}{\tau_{in}} \cos \omega_0 \tau
\]  

(1.17)

where \( E_{s0}^2 \) is the laser intensity without feedback, which is determined by the injection current density \( (J \) ) expressed as below:

\[
E_{s0}^2 = \tau_p [J - (N_0 + 1 / G_N \tau_p) / \tau_s]
\]  

(1.18)
Normalizing Eq. (1.17), the normalized variation of the LD output power (it is usually called as SMI signal $g$) is expressed as:[1]:

$$g = \cos(\phi_s)$$  \hspace{1cm} (1.19)

Eq.(1.16) and (1.19) constitute the existing SMI model which has been widely accepted to describe the waveforms of SMI signals [1, 14, 61, 70-75].

1.2.2 Signal Waveforms Described by Analytical Self-mixing Interferometry (SMI) Model

Most of the existing work on behavior study on an SMLD is mainly based on the analytical SMI model, i.e. Eq.(1.16) and (1.19). In this SMI model, the parameter $C$ (called optical feedback factor) is of great significance, which characterizes the waveform shape of the SMI signal. It has been widely accepted that the operating regime of an SMLD system can be divided into three regimes based on the value of $C$, i.e. weak feedback regime for $C < 1$, moderate feedback regime for $1 < C < 4.6$ and strong feedback regime for $C > 4.6$[12, 74].

In weak feedback regime with $C < 1$, a unique mapping between $\phi_0$ and $\phi_s$ can be found based on Eq.(1.16). Even in some work, when the SMLD is in weak feedback, the approximation of $\phi_s \approx \phi_0$ have been taken [6, 19, 63, 76]. Figure 1-4(a) shows the relationship between $\phi_0$ and $\phi_s$ when $C=0.3$ and $\alpha=3$. In weak feedback regime, the SMI signals induced by a moving target have the similar fringe shape as the traditional two-beam interferometry, and also each fringe in SMI signals corresponds to half wavelength displacement for the external target. Figure 1-4(b) shows the relationship between SMI signal $g$ and $\phi_0$. 
Supposing an continuous sinusoidal displacement is applied on the target with
$L(t) = L_0 + \Delta L \cdot \sin(2\pi ft)$, where $L_0$, $\Delta L$ and $f$ are the initial external cavity
length, the amplitude of the displacement and frequency respectively, which are
chosen as $L_0 = 0.24m$, $\Delta L = 1.5 \lambda_0$ and $f = 400kHz$. In this case, the variation of
the initial optical phase can be expressed as: $\Delta \phi_0(t) = 4\pi \Delta L \sin(2\pi ft) / \lambda_0$. Figure 1-5
shows the time-varying optical phase and its corresponding SMI signal. It can be
found the SMI signal in Figure 1-5 (b) has the similar fringe shape as the traditional
two-beam interferometry.
In moderate feedback regime with $1 < C < 4.6$, three possible $\phi_s$ may be yielded from Eq.(1.16), two are stable and one is unstable. In this case, the SMI signals show hysteresis and producing sawtooth-like fringes. Varieties of SMI-based applications have set the SMLD system in moderate feedback regime, and the behavior of SMLD in this regime have been investigated intensely [1, 5, 37, 59, 70]. Figure 1-6 shows the relationship between $\phi_0$ and $\phi_s$ as well as $g$ and $\phi_0$ when $C = 2.5$ and $\alpha = 3$.

Based on the behavior analysis in [1, 37, 70], when $\phi_0$ increases, that $\phi_s$ and $g$ will track the route $A \rightarrow B \rightarrow B_1$ and it will vary following the route $B_1 \rightarrow A_1 \rightarrow A$ when $\phi_0$ decreases. When $\phi_0$ locates between point A and B, $\phi_0$ will yields three $\phi_s$, and the one located on the line A1-B is unstable. Similarly, we suppose a continuous sinusoidal displacement is applied on the target as same as in Figure 1-5(a), then we can get the corresponding SMI signal $g(t)$. As shown in Figure 1-7 (b)
In strong feedback regime with \( C > 4.6 \), five or seven or even more possible \( \phi_i \) may be yielded from Eq.(1.16). In this situation, the SMI signals may experience fringe loss [22, 65, 72]. Additionally, when \( C \) increases to certain values, the shape of the SMI signals may closely replicates that of the external movement [65, 77]. In 2009, Y. Yu et al [72] studied the behavior of SMLD system with a large \( C \), and gave the details about the mode jumping rules, i.e. for a given \( \phi_0 \), which \( \phi_i \) should be
chosen. Figure 1-8 shows the relationship between $\phi_0$ and $\phi_s$ as well as $g$ and $\phi_0$ when $C = 6$ and $\alpha = 3$. Based on the behavior analysis in [72], when $\phi_0$ increases, that $\phi_s$ and $g$ will track the route $B \rightarrow B_1 \rightarrow D \rightarrow D_1$ and it will vary following the route $C_1 \rightarrow C \rightarrow A_1 \rightarrow A$ when $\phi_0$ decreases. Similarly, we suppose a continuous sinusoidal displacement is applied on the target as same as in Figure 1-4(a), then we can get the corresponding SMI signal $g(t)$ as shown in Figure 1-9(b).

Figure 1-8 (a) Relationship between $\phi_0$ and $\phi_s$, (b) relationship between $g$ and $\phi_0$ when $C = 6$ and $\alpha = 3$.

Figure 1-9 (a) Time-varying optical phase induced by a moving target (b) corresponding SMI signal when $C = 6$ and $\alpha = 3$. 

-20 0 20
\Delta \phi_0(t) [\text{rad}]
\begin{array}{c}
\hline
0 0.5 1 1.5 2 2.5
\end{array}
t [\text{us}]

-1 0 1
g(t) [\text{a.u.}]
\begin{array}{c}
\hline
0 0.5 1 1.5 2 2.5
\end{array}
t [\text{us}]
1.2.3 Signal Waveforms Described by L-K Equations

All the related work of analyzing the behavior of an SMLD in Section 1.2.1 and 1.2.2 are based on an assumption that the SMLD system operates in stable mode, i.e. both the electric field and carrier density in an LD with a stationary external cavity can reach a constant state after a transient period. However, undamped relaxation oscillation may occur in an SMLD system for some operation conditions. Moreover, these operation conditions are quite common in the existing literatures, e.g. it was found the SMLD in moderate feedback regime may exhibit undamped RO, which means the behavior of an SMLD system cannot be described by the existing analytical SMI model. In this case, we need to start from the original L-K equations to investigate the behavior of an SMLD system.

In 2012, Teysseyre et al. [78] found damped oscillation in sawtooth-like SMI signals as shown in Figure 1-10 by solving the L-K equations. In this work, the external target is located more than one meter away from the LD. It was found that damped oscillation appearing on the discontinuous point of the SMI signal when the SMLD system operates in moderate feedback regime and this kind of damped oscillation contains information on the target distance and reflectivity. The work in [78] indicates that the SMI signal in moderate feedback regime is more complex than that described by the widely-accepted analytical SMI model. It shows that the oscillation frequency in the SMI signal in this condition is determined by the round-trip time of the light in the SMLD external cavity. For an external target is less than 1 m away from the LD, the round-trip time is less than 7 ns, inducing an oscillation frequency higher than 100 MHz. As the oscillation frequency is determined by the external cavity length, this kind oscillation is called as external cavity mode oscillation. The detection system in that work only has a bandwidth of 90MHz and it can only be used to detect the oscillation signal when the LD is more than 1 m away from the external target. However, for most of the SMI-based applications, the target...
is less than 1 m away from the LD. Additionally, due to the bandwidth limitation of the detection system, the dynamics observed in [78] is mainly due to the external cavity mode oscillation. The effects of relaxation oscillation is still not analyzed and discussed.

![Figure 1-10](image1.png)  
(a) Time-varying optical phase induced by a moving target (b) corresponding SMI signal when $C = 4$, $\alpha = 3$, $L_m = 3 m$

In 2014, Yuanlong Fan et al. [54] presented a dynamic stability analysis for an SMLD system and found an SMLD system may operate in three regions, i.e. stable region, semi-stable region and unstable region, as shown in Figure 1-11. Figure 1-12 shows the SMI signal waveforms described by L-K equations and existing analytical SMI model. The left column shows the SMI signals from L-K equations and the right column shows ones from existing analytical SMI model. Each row is obtained under the same laser operation conditions but from different models. Here, Figure 1-12(b), (g) are the signals in stable region, (c) (d) (h) and (i) are in semi-stable region, (e) and (j) are in unstable region. It can be found that only in stable region, the behavior of an SMLD system is able to be analyzed by using the existing analytical SMI model, i.e. the simulated SMI signals from the L-K equations and analytical SMI model under same operation conditions are similar. In semi-stable and unstable region, simulated
SMI signals from the L-K equations are different from the analytical SMI model. In semi-stable region, the RO is undamped. The SMI signals exhibit complicated waveform but still contain the displacement information of the external target. However, the details of the SMI signals in semi-stable region and related experimental results are not given in [54].

Figure 1-11 The stability boundary of SMI system when \( J=1.3J_0 \), \( L_0 = 0.24 \) m, \( \alpha = 3 \)

Figure 1-12 SMI signals described by the L-K Equations and the existing analytical SMI model respectively, (a) and (f): movement trace of the external target, (b)-(e): SMI signals obtained by the L-K model with \( C = 1.5 \), \( C = 2.5 \), \( C = 4.0 \) and \( C = 9.0 \) respectively. (g)-(j) SMI signals obtained by the existing SMI model with \( C = 1.5 \), \( C = 2.5 \), \( C = 4.0 \) and \( C = 9.0 \) respectively.
1.3 Outstanding Issues and Objectives

In Section 1.2, we reviewed the related literatures on the behavior study on an SMLD system, from which the outstanding research issues can be drawn as below:

1. Most of the existing work, e.g. [1, 5, 6, 12-14, 19, 22, 37, 59, 70-76], on behavior study of an SMLD system is based on the analytical SMI mathematic model, which is derived from the steady-state solution of the L-K equations, or from the classical three-mirror model, by assuming the SMLD system operates in stable mode, i.e. both the electric field and carrier density in an LD with a stationary external cavity can reach a constant state after a transient period. However, undamped RO may occur in an SMLD system for some operation conditions. Moreover, these conditions are quite common in the existing literatures, e.g., for the case when the feedback level factor C is around 2.5, which is quite commonly reported for the moderate feedback level in the literature. In this case, the behavior of an SMLD system cannot be described by the existing analytical SMI model.

2. The seminal work [54, 78] investigated the behavior of an SMLD by solving the original L-K equations. However, [78] only considered the long external cavity length (more than 1m) when the SMLD is in moderate feedback regime. Whereas, for typical SMI-based applications the external cavity length is less than 1m. Thus it is of necessity to analyze behavior of an SMLD system in moderate feedback regime with relatively short external cavity length. Additionally, due to the bandwidth limits of the detection system, the dynamics observed in [78] is mainly due to the external cavity mode oscillations. The effects of relaxation oscillation is still not analyzed and discussed. Ref. [54] described a dynamic stability boundary for an SMLD system, and defined three operating region for an SMLD system, i.e. stable region, semi-stable region and unstable region. It shows that the existing analytical SMI model is only valid in stable region, The SMLD system exhibits undamped RO in semi-stable region and unstable region. However, due to the
main goal of [54] is to investigate the system stability, it does not give details of the behavior analysis on an SMLD system in semi-stable or unstable region. Moreover, it does not show the related experimental results in [54].

3. The work in [54] finds the SMI signal in semi-stable region still contains information of the external cavity, which means the SMI signals in semi-stable region may be used for sensing. However, there are no related application details shown in [54].

The main objectives of this thesis are: (1) investigating the behavior of an SMLD system with undamped RO by numerically solving L-K equations with the aid of MATLAB; (2) deriving an analytical approximation from L-K equations to further analyze the features of RO-SMI signals; (3) building an experimental setup to capture and analyze the behavior of an SMLD system operating with undamped RO; (4) proposing sensing applications based on the behavior study of an SMLD system.

1.4 Thesis Organization

This thesis consists of six chapters and is organized as below.

Chapter 1 gives a brief background and introduction for this thesis. In Section 1.1, the background of the SMI technology is introduced which contains the basic system structure, its advantages compared to the traditional two-beam interferometry, and its applications. In Section 1.2, the literature related to behavior study of an SMLD system is reviewed. Specifically, the analytical SMI model derived from both three-mirror model and L-K equations is presented, then the behavior analysis on an SMLD system based on the analytical SMI model and original L-K equations is reviewed. Based on the literature review in Section 1.2, existing problems on behavior study on an SMLD system, and the corresponding research goals of this thesis are summarized. At the end of Chapter 1, the organization of this thesis is given.

Chapter 2 presents theoretical analysis on behavior of an SMLD systems operating with undamped RO. In Section 2.1, a new stability boundary of an SMLD system is
obtained in the plane of optical feedback factor $C$ and variation of optical phase $\Delta \phi$. Then the influence of the injection current density and initial external cavity length on the stability boundary is investigated. In Section 2.2, the RO-SMI signal is analyzed by numerically solving the L-K equations and the effects of the bandwidth of detection circuits on the detected RO-SMI signals are then analyzed. In Section 2.3, an approximate analytical expression for RO-SMI signals when the SMLD system operates in period-one oscillation is derived. Finally, in Section 2.4, the features of the RO-SMI signals are discussed and summarized.

Chapter 3 gives the information of experimental setup for analyzing the SMLD behavior and the related experimental results. In Section 3.1, the structure of experimental setup is introduced in details. Then the related experimental results and discussions are given in Section 3.2 and Section 3.3 respectively.

Chapter 4 shows displacement sensing by an SMLD operating with undamped RO. In Section 4.1, a micro-displacement sensing method by using the RO frequency is introduced. In this Section, the influences of operation parameters, e.g. injection current and initial external cavity length on the sensing performance is investigated. Both the simulation and experiment verify the proposed displacement sensing method. In Section 4.2, displacement sensing by simultaneously using the time-domain RO-SMI signal and its frequency is developed, showing this method has large measurement range, high sensitivity and resolution.

Chapter 5 shows an all-fiber SMLD system which is always stable in moderate feedback regime and its application. In Section 5.1, the all-fiber SMLD system is built to demonstrate that an SMLD system in moderate feedback regime with a high injection current and long external cavity can be always stable, which verifies the analysis in Chapter 2. Then this all-fiber SMLD is used to measure the acoustic emission (AE) events successfully. The related introduction of AE measurement, theoretical operation principle, and experiments are presented in Section 5.2-5.4. The results in this chapter contribute to a novel compact sensing system in structure health monitoring.

Chapter 6 summarizes the contributions of this thesis and suggests the potential future research topics.
Chapter 2 Behavior Analysis on an SMLD with Undamped Relaxation Oscillation (RO)

Most of the SMI-based application are based on the analytical SMI mathematic model, which is derived from the steady-state solution of the L-K equations, or from the classical three-mirror model, by assuming the SMLD system operates in stable mode, i.e. both the electric field and carrier density in an LD with a stationary external cavity can reach a constant state after a transient period. However, undamped RO may occur in an SMLD system for some operation conditions. Moreover, these operation conditions are quite common in the existing literatures, e.g. it is found the SMLD in moderate feedback regime may exhibit undamped RO. In this case, the behavior of an SMLD system cannot be described by the analytical SMI model and the SMLD system shows some novel features. In order to differentiate the conventional SMI signals, the SMI signal with undamped RO is named as RO-SMI signal in this thesis.

In this chapter, starting from the analysis on dynamic stability boundary for an SMLD system, theoretical analysis on behavior of an SMLD operating with undamped RO is conducted. The analysis is firstly based on the numerical simulation of L-K equations. Then, theoretical modelling is carried out to further investigate its behavior.

2.1 Stability Boundary of an SMLD system

The stability boundary of an SMLD system was derived in [54] and depicted in the plane of optical feedback factor $C$ and variation of optical phase $\Delta \phi_0$, based on the work in [50, 51], shown as below:
\begin{equation}
C \left\{ \cos(\phi_s) \left[ 1 - 2 \left( \frac{\Omega}{\omega_R} \right)^2 \right] - \alpha \sin(\phi_s) \right\} \leq \left( \frac{\Omega}{\omega_R} \right)^2 \frac{\tau \sqrt{1 + \alpha^2}}{2 \tau_R \sin^2(\Omega \tau / 2)}
\end{equation}

(2.1)

where \( \omega_R , \tau_R \) are respectively the relaxation oscillation angular frequency and damping time of the solitary LD, \( \Omega \) is the relaxation oscillation angular frequency of the LD with EOF operating on its stability boundary. These three parameters can be obtained by the following equations.

\begin{equation}
\omega_R = \sqrt{\frac{G_N E_{s0}}{\tau_p}}
\end{equation}

(2.2)

\begin{equation}
\frac{1}{\tau_R} = \frac{1}{\tau_s} + \left( \frac{\tau_p}{\tau_s} + \frac{\alpha \tau}{G_N} \right) \omega_R^2
\end{equation}

(2.3)

\begin{equation}
\Omega^2 - \omega_R^2 = \frac{\Omega}{\tau_R} \cot \left( \frac{\Omega \tau}{2} \right)
\end{equation}

(2.4)

In Eq. (2.1)-(2.4), \( \phi_s, E_{s0} \) are the same as in Eq. (1.16) and (1.18). \( \phi_s \) is the stable optical phase of the LD with EOF which can is determined by \( \phi_0 \) following the Eq. (1.16), and \( E_{s0} \) is the stationary electric filed amplitude of the solitary LD which is determined by the injection current density \( J \) following the Eq. (1.18). Therefore, for an SMLD system, with given values for the injection current density \( J \) and initial external cavity length \( L_0 \), the stability boundary of an SMLD system can be determined. Figure 2-1(a) is a typical boundary of an SMLD system when \( J = 1.1 J_{th}, L_0 = 0.35 m, \alpha = 3 \) obtained from Eq. (2.1) , which the same as Fig.2 in [54]. Three regions are defined as shown in Figure 2-1. In stable region, the LD is stable and the RO is damped. In semi-stable region, the SMI signals contains high frequency oscillation corresponding to RO frequency but still have regular shape. In this case the SMI signals are called RO-SMI signals in this thesis. In unstable region, SMI signals show chaotic oscillation.
Eq. (2.1) is derived based on the assumptions of $\omega_R^2 >> \kappa / (\tau_{in} \tau_R)$ and $\omega_R^2 >> (\kappa / \tau_{in})^2$. In the following, an improved stability boundary is obtained by numerically solving the L-K equations without assumptions taken. In order to get the stability boundary in the plane of optical feedback factor $C$ and variation of optical phase $\Delta \phi_0$, $C$ and $\Delta \phi_0$ is taken 200 points equally with range of [0,10] and [-3\pi, 3\pi]. Then, we have 40000 pairs of $C$ and $\Delta \phi_0$. For each pair, we numerically solve the L-K equations using 4th Runge-Kutta integration algorithm. For each $\Delta \phi_0$, we record the value of $C$ when the laser intensity starts to oscillate with undamped RO. Finally, the stability boundary can be achieved. Figure 2-1(b) shows the new stability boundary obtained under same conditions as in Figure 2-1(a). From Figure 2-1(a) and (b), slightly difference on the shape of the boundary. However, periodic fluctuation with period of 2\pi can be seen from both figures. Also, an SMLD system in moderate feedback regime, i.e. $1< C<4.6$, may fall into the semi-stable region in this operation condition.

Figure 2-1 shows the SMLD system under moderate feedback regime may work in semi-stable region. We then investigate the influence of injection current density and
initial external cavity length on the stability boundary. Figure 2-2 shows the stability boundaries for different injection current density and initial external cavity length, from which, the features of the stability boundary can be summarized as below:

1. The stability boundary shows periodic fluctuation with a period of $2\pi$ equivalent to a half wavelength movement of the external cavity.

2. The SMLD system in moderate regime may be also in semi-stable region under certain operating conditions (e.g. $L_0=20 \text{ cm, } J=1.3J_{th}$).

3. For a relatively high injection current and long initial external cavity length, the SMLD system in moderate regime is easier in stable region.

![Figure 2-2 Stability boundaries](image)

Figure 2-2 Stability boundaries for different injection current densities and initial external cavity lengths, (a) For a fixed $L_0=0.20 \text{ m}$ with different $J$, (b) for a fixed $J=1.3J_{th}$ but different $L_0$.

### 2.2 Behavior Study by L-K Equations

In this Section, the behavior of an SMLD system is investigated by numerically solving the L-K equations. Its features in both time and frequency domain are presented. We then analyze influence of photodetector bandwidth on the SMI signal.

#### 2.2.1 Features of an SMLD with Undamped RO

By using the method described in Section 2.1, we can get a stability boundary as shown in the solid line in Figure 2-3 when the injection current density is 1.3 times the threshold current ($J=1.3J_{th}$) and initial external cavity ($L_0$) is 0.16 m long.
From Figure 2-3, we can see three regions, i.e., stable, semi-stable and unstable respectively.

In the following simulations, all the parameters are set according to Table 1-1. Under the same injection current \((J=1.3J_{th})\) and initial external cavity length \((L_0=0.16\ m)\), we vary the values of \(C\) so that the SMI system operates in the stable, semi-stable and unstable regions, e.g., \(C = 1.5, 2.5, 3.5, 5\) and \(9\) corresponding to \(C1, C2, C3, C4\) and \(C5\) indicated in Figure 2-3. As we can see, \(C1\) locates in the stable region, \(C2-C4\) in the semi-stable and \(C5\) in the unstable region. For all the cases, the external target moves in a simple harmonic form. Suppose that the initial cavity length is \(L_0=0.16\ m\), and the displacement of the target is

\[
\Delta L(t) = \Delta L_m \sin(2\pi f_0 t), \quad \text{where} \quad f_0 = 400 \text{ KHz}, \quad \Delta L_m = 1.5\lambda_0, \quad \lambda_0 = 780 \text{ nm}.
\]

The corresponding waveforms of laser intensity \(E^2(t)\) (called SMI/RO-SMI signals) are numerically solved from the L-K equations and shown in Figure 2-4, where the laser intensity is scaled by \(E^2(t)/10^{20}\). We then apply fast Fourier transform (FFT) on the signals to calculate their magnitude spectra as shown in the right column of Figure 2-4. Note that we removed the DC component from each signal before applying the FFT. Figure 2-4a shows the displacement of the target; Figure 2-4f shows the conventional SMI signal located in the stable region which is commonly seen in the literature \([4, 13, 37, 72, 79]\); Figure 2-4c–e show the RO-SMI signals in a semi-stable region; Figure 2-4b shows the signals in unstable region; and Figure 2-4g–k are spectra.
corresponding to the signals in Figure 2-4b–f respectively. From Figure 2-4, we can see the following:

1. The RO-SMI signals exhibit the form of high frequency oscillation with its amplitude modulated by a slow-varying signal. Interestingly, the slow-varying envelopes are similar to the conventional SMI signal characterized by the same fringe structure. It can be seen from the left column in Figure 2-4, that there are nearly six fringes corresponding to the peak-peak displacement (3λ₀) of the target. That is, each fringe in the RO-SMI signals also corresponds to a target displacement λ₀/2, and hence the RO-SMI signal can also be used to measure the displacement with the same resolution as the conventional SMI signal in the stable region.

2. Although having the same fringe structure, the RO-SMI signals are very different from the conventional SMI signals in their frequencies. The right column in Figure 2-4, shows that, the spectrum of a conventional SMI signal (Figure 2-4f) locates in the relatively low frequency range, stopping at 0.2 GHz for the case with C = 1.5. However, the dominate frequency components associated with a RO-SMI signal locate in a much higher frequency range, with a central frequency of 2.3 GHz (denoted by fロー). fロー is generated due to the relaxation oscillation when the SMI system operates above its stability boundary.

3. The Peak-Peak (P-P) value of a conventional SMI signal (in Figure 2-4f) located in the stable region is around 0.012 while the P-P values of the RO-SM signals in semi-stable region are about 6.64, 5.04 and 4.12 respectively as shown in Figure 2-4c–e. Hence the RO-SMI signals are much stronger (more than 300 times stronger) than the conventional SMI signal. This implies that an SMI system working at semi-stable region has potential for achieving sensing with improved sensitivity.

4. When the SMI system enters the unstable region shown in Figure 2-4b,g, the laser output will be unstable, characterized by a much wider frequency spectrum. In this case there is not an obvious relationship between the target movement and the laser output, and hence the system is not suitable for such waveform-based sensing.
Figure 2-4 Modulated laser intensity at different regions and their corresponding spectra, where the laser intensity is scaled by $E^2(t)/10^{20}$ (a) displacement of the external target; (b–f) laser intensity when $C = 9$, $C = 5$, $C = 3.5$, $C = 2.5$, and $C = 1.5$ respectively; (g–k) spectra corresponding to (b–f) respectively.

### 2.2.2 Bandwidth Analysis on RO-SMI Signals

From Section 2.1 and 2.2.1, it can be found the SMLD in semi-stable region exhibits undamped relaxation oscillation. On the other hand, the work in [78] shows that another kind of oscillation corresponding to the external cavity mode happens when the SMLD is in moderate or strong feedback regime. Meanwhile, an SMLD system in moderate feedback regime may fall into the semi-stable region for some cases as shown in Figure 2-2. As a result, these two kinds of oscillation may simultaneously exist in the RO-SMI signals. Due to the limit of the bandwidth of the practical detection circuit, incomplete RO-SMI signals may be captured and considered as the ‘true’ SMI signals. Therefore, in the following, we will investigate the influence of bandwidth of detection circuit on the RO-SMI signals.

Considering the case in Figure 2-4e, i.e. $J=1.3J_{th}$, $L_0=0.16$ m, $\alpha=6$, and
\( C = 2.5 \), the simulation results is depicted in Figure 2-5, where (a) is the displacement of the external target as the same in Figure 2-4, (b) is the corresponding RO-SMI signal. From Figure 2-4, it can be found the RO frequency in the RO-SMI signal in Figure 2-5 is about 2.3 GHz. Meanwhile, based on [78], the corresponding frequency of the external cavity mode can be calculated as \( 1/(2L_0 / c + \tau_{\text{offset}}) \), where \( c \) is the light speed in vacuum, \( \tau_{\text{offset}} \) is off-set which depends on the feedback strength, with typical values of several times of 0.1ns, According to the method in [78], we get \( \tau_{\text{offset}} = 0.5 \text{ ns} \) in this condition. Thus, the frequency of the external cavity mode here is about 680 MHz.

![Figure 2-5 Simulation results for the modulated intensity scaled as \( E^2(t)/10^{20} \) when \( J=1.3J_{\text{at}} \).](image)

\( L_0 = 0.16 \text{ m}, \alpha = 6 \) and \( C = 2.5 \). (a) Displacement of target, (b) Corresponding RO-SMI signal

The SMI signal is usually detected by the photodiode packaged at the rear of the LD together with a trans-impedance amplifier. The rising time of the packaged PD is usually several nano seconds, corresponding to a bandwidth of several hundred MHz. The bandwidth of the trans-impedance amplifier is usually from several hundred kHz to several ten MHz, e.g. the detection circuit for conventional SMI signals in this
thesis has a bandwidth of 10 MHz. Therefore, we apply a low-pass filter with cut-off frequency of 800 MHz and 10 MHz respectively on the RO-SMI signal in Figure 2-5 to investigate the influence of the detection system bandwidth on the detected SMI signal. Figure 2-6 shows the filtered RO-SMI signals with different cut-off frequency, where Figure 2-6a is the original RO-SMI signal when \( J=1.3J_a, \ L_0=0.16 \ m, \alpha = 6 \) and \( C = 2.5 \). Applying a low-pass filter with cut-off frequency of 800 MHz on Figure 2-6a, we can obtain the filtered signal as shown in Figure 2-6b. Figure 2-6c is the enlargement of the ‘T area in (b)’. From (b) and (c), damped oscillation with an frequency corresponding to the external cavity mode (around 700 MHz ) is clearly shown at discontinuities of the SMI signal. Additionally, the maximum peak-peak value of the external cavity mode oscillation is about 0.056 as shown in Figure 2-6c, whereas the maximum peak-peak value of the relaxation oscillation is about 3.179 as shown in Figure 2-6a, more than 50 times larger than the external cavity mode oscillation. Figure 2-6d is the filtered RO-SMI signal with a low-pass filter with cut-off frequency of 10 MHz, it can be seen from which both the relaxation oscillation and external cavity mode oscillation are filtered, showing a conventional SMI as in the literature.

From above analysis, it can be concluded that there are two kinds of high-frequency oscillations in the RO-SMI signals. One is the relaxation oscillation and the other is the external cavity mode oscillation. Furtherly, the magnitude of the relaxation oscillation is much stronger than the external cavity mode oscillation. By applying a low-pass filter with proper cut-off frequency, the SMI signal containing damped
external cavity mode oscillation as in [78] can be got. Finally, the conventional SMI signals can be obtained by applying a low-pass filter with cut-off frequency lower than the relaxation oscillation and external cavity mode oscillation frequency on the RO-SMI signals.

![Graphs and Diagrams]

Figure 2-6 Filtered RO-SMI signal with different cut-off frequency, (a) the original RO-SMI signal, (b) filtered RO-SMI signal with cut-off frequency of 800 MHz, (c) the enlargement of ‘T area’ in (b), (d) filtered RO-SMI signal with cut-off frequency of 10 MHz.

2.3 Modelling an SMLD in Period-one Oscillation

In Section 2.2, we have analyzed the RO-SMI signals by numerically solving the L-K equations. In order to further investigate the features of the RO-SMI signals, we intend to derive an analytical expression for RO-SMI signals based on the L-K equations. The external cavity mode oscillation is neglected in the following derivation because as discussed in Section 2.2, the external cavity mode oscillation is much weaker than the relaxation oscillation, and it is damped, which can be neglected. In this section, we mainly focus and assume the SMLD operating at period-one oscillation, which is a part of the semi-stable region.
2.3.1 Modelling and Analyzing

In order to show the derivation clearly, we re-write the L-K equations as below:

\[
\frac{dE(t)}{dt} = \frac{1}{2} \left( G \left[N(t), E^2(t)\right] - \frac{1}{\tau_p} \right) E(t) + \frac{\kappa}{\tau_m} \cdot E(t - \tau) \cdot \cos[\omega_0 \tau + \phi(t) - \phi(t - \tau)] \tag{2.5}
\]

\[
\frac{d\phi(t)}{dt} = \frac{1}{2} \alpha \left( G \left[N(t), E^2(t)\right] - \frac{1}{\tau_p} \right) - \frac{\kappa}{\tau_{in}} \cdot \frac{E(t - \tau)}{E(t)} \cdot \sin[\omega_0 \tau + \phi(t) - \phi(t - \tau)] \tag{2.6}
\]

\[
\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - G \left[N(t), E^2(t)\right] E^2(t) \tag{2.7}
\]

The nonlinear gain of the laser is neglected in the following. Based on the numerical simulation of L-K equations and work in [80], when a SMLD operates in period-one oscillation, the expression of \( E(t) \), \( N(t) \) and \( \phi(t) \) can be written as below:

\[
E(t) = \bar{E} + \Delta E \cos(\omega_r t + \theta_e) \tag{2.8}
\]

\[
N(t) = \bar{N} + \Delta N \cos(\omega_n t + \theta_n) \tag{2.9}
\]

\[
\phi(t) = (\bar{\phi} - \omega_0) t + \frac{\Delta \phi}{2} \cos \omega_r t \tag{2.10}
\]

where, \( \bar{E} \), \( \bar{N} \) and \( \bar{\phi} \) are respectively the mean values of \( E(t) \), \( N(t) \) and optical frequency, \( \Delta E \), \( \Delta N \) and \( \Delta \phi / 2 \) are the modulation amplitudes, \( \omega_r \) is the relaxation oscillation angular frequency when the SMLD is in period-one oscillation, \( \theta_e \), \( \theta_n \) are the initial phase of \( E(t) \) and \( N(t) \) respectively. With a similar treatment as done in [80], we set \( \omega_\tau = (2m+1) \cdot \pi \), where \( m \) is an integer. Note that, in the case \( \omega_\tau = 2m \cdot \pi \), the SMLD will be not in period-one oscillation [81]. We will need to express the mean values and the modulation amplitudes as well as the initial phases using the parameters related to the LD and its external cavity listed in Table 1.
Regarding $\bar{E}$, $\bar{N}$, they can be determined by solving Eq. (2.5)-(2.7), by setting
d$E(t)/t = 0, dN(t)/t = 0, d\phi(t)/dt = \omega - \omega_0$, shown as below:

$$
\bar{E} = \frac{J - \bar{N}/\tau_s}{G_N \cdot (\bar{N} - N_0)}
$$

(2.11)

$$
\bar{N} = N_0 + \frac{1}{G_N \tau_p} - \frac{2k \cos(\omega \tau)}{\tau_m G_N}
$$

(2.12)

$$
\omega \tau = \omega_0 \tau - \frac{k \tau \sqrt{1 + \alpha^2}}{\tau_m} \sin(\omega \tau + \arctan \alpha)
$$

(2.13)

where $\omega_0$ is the optical frequency when the OFLD is stable. In what follows, terms
with $\Delta E \cdot \Delta N$, $\Delta E^2$ and $\Delta N^2$ are neglected, which is justified for $\Delta E/\bar{E} \ll 1$,
$\Delta N/N \ll 1$.

Firstly, we insert Eq.(2.8) and (2.9) into (2.7), and then obtain:

$$
2G_N (\bar{N} - N_0) \bar{E} \Delta E \cos(\omega \tau + \theta_e)
$$

$$
= J - \frac{\bar{N}}{\tau_s} - G_N \cdot (\bar{N} - N_0) \cdot \bar{E}^2
$$

(2.14)

$$
+ \Delta N \sqrt{\omega_0^2 + \left(\frac{1}{\tau_s} + G_N \bar{E}^2\right)^2} \sin(\omega \tau + \theta_e - \arctan[1 / \tau_s + G_N \bar{E}^2 / \omega_e])
$$

As Eq. (2.14) is an identical equation, let the corresponding terms in both sides are
equal, we then have:

$$
2G_N (\bar{N} - N_0) \bar{E} \Delta E = \Delta N \sqrt{\omega_0^2 + (G_n \bar{E}^2 + 1/\tau_s)^2}
$$

(2.15)

$$
\theta_e = \theta_n - \arctan(\frac{\tau_s}{\omega_n}) - \frac{\pi}{2}
$$

(2.16)

$$
J - \frac{\bar{N}}{\tau_s} - G_N \cdot (\bar{N} - N_0) \cdot \bar{E}^2 = 0
$$

(2.17)

Next considering Eq. (2.5), we substitute Eq. (2.8)-(2.10) into (2.5), and then the
advantage of choice of $\omega \tau = (2m + 1) \cdot \pi$ is evident. We can get,
\[-\omega \Delta E \sin(\omega, t + \theta)
= \left\{ \frac{1}{2} \left[ G_N \cdot (\bar{N} + \Delta N \cos(\omega, t + \theta_n) - N_0) - \frac{1}{\tau_p} \right] \right\} \cdot \left[ \bar{E} + \Delta E \cos(\omega, t + \theta) \right] \] 
\(2.18\)

\[+ \frac{\kappa}{\tau_{in}} \cdot [\bar{E} - \Delta E \cos(\omega, t + \theta_e)] \cdot \cos(\bar{\omega} \tau + \Delta \phi \cos(\omega, t)) \]

After expansion and reorganization of Eq. (2.18), we have:

\[-\omega \Delta E \sin(\omega, t + \theta)
= \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) - \frac{1}{\tau_p} \right] \cdot \bar{E} + \frac{\kappa}{\tau_{in}} \cdot \bar{E} \cos(\omega, t + \theta) \] 
\(2.19\)

\[+ \frac{1}{2} G_N \Delta N \cos(\omega, t + \theta_n) + \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) - \frac{1}{\tau_p} \right] \cdot \Delta E \cos(\omega, t + \theta_e) \]

\[-2 \frac{\kappa}{\tau_{in}} \cdot \bar{E} \sin(\omega, \tau) \cdot J_1(\Delta \phi) \cos(\omega, t) \]

where \(J_0(\Delta \phi)\) and \(J_1(\Delta \phi)\) are the 0th-order and 1st-order Bessel function respectively.

Note that higher harmonic components of \(\omega_e\) are neglected as done in [80]. Then, we have:

\[\frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) \right] - \frac{1}{\tau_p} \] 
\(\cos(\omega, t + \theta) \cdot J_0(\Delta \phi) = 0 \) 
\(2.20\)

\[\omega \Delta E \sin(\omega, t + \theta) = 2 \frac{\kappa}{\tau_{in}} \cdot \bar{E} \sin(\omega, \tau) \cdot J_1(\Delta \phi) \cos(\omega, t) \]

\[-\frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) \right] \cdot \Delta E \cos(\omega, t + \theta) - \frac{1}{2} G_N \bar{E} \Delta N \cos(\omega, t + \theta_n) \] 
\(2.21\)

After expansion and reorganization of Eq. (2.21), we have:

\[\omega \Delta E \cos(\theta_e) \sin(\omega, t) + \omega \Delta E \sin(\theta_e) \cos(\omega, t) = \]

\[\left\{ \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) \right] - \frac{1}{\tau_p} \right\} \cdot \Delta E \sin(\theta_e) + \frac{1}{2} G_N \bar{E} \Delta N \sin(\theta_n) \] 
\(\sin(\omega, t) \) 
\(2.22\)

\[\left\{ \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) \right] - \frac{1}{\tau_p} \right\} \cdot \Delta E \cos(\theta_e) - \frac{1}{2} G_N \bar{E} \Delta N \cos(\theta_n) \] 
\(\cos(\omega, t) \)

\[+ 2 \frac{\kappa}{\tau_{in}} \cdot \bar{E} \sin(\bar{\omega} \tau) \cdot J_1(\Delta \phi) \]

From Eq. (2.22), we then obtain:
\[ \omega, \Delta E \cos \theta = \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_o) - \frac{1}{\tau_p} \right] \cdot \Delta E \sin \theta + \frac{1}{2} G_N \bar{E} \Delta N \sin \theta \] (2.23) \]

Inserting Eq. (2.16) into (2.23), and then yield:

\[ \cot \theta = \frac{\omega_r^2 - G_N^2 \bar{E}^2 (\bar{N} - N_o) - \Gamma \frac{K}{\tau_{in}} J_o (\Delta \phi) \cos \bar{\omega} \tau}{\Gamma + \frac{K}{\tau_{in}} J_o (\Delta \phi) \cos \bar{\omega} \tau} \] (2.24) \]

Then we consider phase equation in L-K equations, i.e. Eq. (2.6). Insert Eq. (2.8) - (2.10) into (2.6), we get:

\[ \bar{\omega} - \omega_0 - \frac{\Delta \phi}{2} \omega \sin \omega t \]

\[ = \left[ \frac{1}{2} \alpha \left[ G_N \cdot (\bar{N} + \Delta N) \cos(\omega \tau + \theta) - N_o \right] - \frac{1}{\tau_p} \right] \]

\[ - \frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \sin(\omega_r \tau + \frac{\Delta \phi}{2}) \{ \cos \omega_r t - \cos[\omega_r (t - \tau)] \} \] (2.25) \]

Since \( \Delta E / \bar{E} \ll 1 \), we have \( \frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \approx 1 \), thus the Eq. (2.25) can be simplified and rewritten as:

\[ \bar{\omega} - \omega_0 - \frac{\Delta \phi}{2} \omega \sin \omega t = -\frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \sin(\omega_r \tau + \frac{\Delta \phi}{2}) \{ \cos \omega_r t - \cos[\omega_r (t - \tau)] \} \]

\[ \frac{1}{2} \alpha G_N \cdot \Delta N \cos \theta - 2 \frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \sin(\omega_r \tau + \frac{\Delta \phi}{2}) \{ \cos \omega_r t - \cos[\omega_r (t - \tau)] \} \cos \omega_r t \] (2.26) \]

With the similar treatment in Eq. (2.14) and Eq. (2.22), we can obtain from (2.26) as below:

\[ \bar{\omega} - \omega_0 = -\alpha \frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \sin(\omega_r \tau + \frac{\Delta \phi}{2}) \{ \cos \omega_r t - \cos[\omega_r (t - \tau)] \} \]

\[ = -\alpha \frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \sin(\omega_r \tau + \frac{\Delta \phi}{2}) \{ \cos \omega_r t - \cos[\omega_r (t - \tau)] \} \]

\[ = -\frac{1}{2} \alpha G_N \cdot \Delta N \cos \theta - 2 \frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \sin(\omega_r \tau + \frac{\Delta \phi}{2}) \{ \cos \omega_r t - \cos[\omega_r (t - \tau)] \} \cos \omega_r t \] (2.27) \]

\[ \frac{1}{2} \alpha G_N \cdot \Delta N \cos \theta - 2 \frac{\bar{E} + \Delta E \cos[\omega_r (t - \tau) + \theta_r]}{\bar{E} + \Delta E \cos(\omega_r t + \theta_r)} \sin(\omega_r \tau + \frac{\Delta \phi}{2}) \{ \cos \omega_r t - \cos[\omega_r (t - \tau)] \} = 0 \] (2.28) \]
\[
\frac{\Delta \phi}{2} = \frac{1}{2} \alpha G_N \cdot \Delta N \sin \theta_n 
\] (2.29)

Combining Eq. (2.12) and (2.20), we get:

\[
\cos \omega \tau \cdot J_0(\Delta \phi) = \cos \omega \tau 
\] (2.30)

With the Taylor expansion of the Bessel functions, we have the approximations:

\[
J_0(\Delta \phi) \approx 1 - \frac{\Delta \phi^2}{4}, \quad J_1(\Delta \phi) \approx \frac{\Delta \phi}{2} - \frac{\Delta \phi^3}{16} 
\] Solving Eq. (2.24), (2.28), (2.29) and (2.30), we get:

\[
\cot \theta_n = \frac{1}{\omega_r} \cdot \frac{\omega_r^2 - G_N \bar{E}^2(N_s - N_0) - (G_N \bar{E}^2 + \frac{1}{\tau_s}) \frac{K}{\tau_m} \cos \omega_s \tau}{G_N \bar{E}^2 + \frac{1}{\tau_s} + \frac{K}{\tau_m} \cos \omega_s \tau} 
\] (2.31)

\[
\Delta \phi = 2 \sqrt{\frac{\omega_r \tau_m \cot \theta_n - 2 \kappa \cos \omega_s \tau}{\omega_r \tau_m \cot \theta_n - \kappa \cos \omega_s \tau}} 
\] (2.32)

\[
\Delta N = \frac{2 \omega_r}{\alpha G_N \sin \theta_n} \sqrt{\frac{\omega_r \tau_m \cot \theta_n - 2 \kappa \cos \omega_s \tau}{\omega_r \tau_m \cot \theta_n - \kappa \cos \omega_s \tau}} 
\] (2.33)

Inserting Eq. (2.33) into Eq. (2.15), we get:

\[
\Delta E = \frac{\omega_r}{\alpha G_N \sin \theta_n} \sqrt{\frac{\omega_r \tau_m \cot \theta_n - 2 \kappa \cos \omega_s \tau}{\omega_r \tau_m \cot \theta_n - \kappa \cos \omega_s \tau}} \left( \frac{\omega_r^2 + (G_N \bar{E}^2 + \frac{1}{\tau_s})^2}{G_N (\bar{N} - N_0) \bar{E}} \right) 
\] (2.34)

Because \( \omega_r \gg \frac{1}{\tau_s} + G_N \bar{E}^2 \), [82], Eq. (2.34) can be simplified as:

\[
\Delta E = \frac{\omega_r^2}{\alpha G_N \sin \theta_n} \sqrt{\frac{\omega_r \tau_m \cot \theta_n - 2 \kappa \cos \omega_s \tau}{\omega_r \tau_m \cot \theta_n - \kappa \cos \omega_s \tau}} \cdot \frac{1}{G_N (\bar{N} - N_0) \bar{E}} 
\] (2.35)

Thus, we get \( E(t) = \bar{E} + \Delta E \cos(\omega_r t + \theta_e) \) and the laser intensity can be written as:

\[
P(t) = E^2(t) = \bar{E}^2 + 2 \bar{E} \Delta E \cos(\omega_r t + \theta_e) + \Delta E^2 \cos^2(\omega_r t + \theta_e) 
\] (2.36)

35
From Eq. (2.12), we have $G_N(N - N_0) = \frac{1}{\tau_p} - \frac{2\kappa}{\tau_{in}} \cos \omega_\tau$, in the case of $\kappa < 0.01$, and neglecting the terms with $\Delta E^2$, Eq. (2.36) can be approximated as follows:

$$P(t) = \bar{E}^2 + 2 \frac{\tau_p \omega_s^2}{\alpha G_N \sin \theta_n} \sqrt{\frac{\omega_s \tau_{in} \cot \theta_n - 2\kappa \cos \omega_\tau}{\omega_s \tau_{in} \cot \theta_n - \kappa \cos \omega_\tau}} \cos(\omega t + \theta_e) \quad (2.37)$$

where $\bar{E}^2$ is as shown in Eq. (2.11), which can be rewritten as:

$$\bar{E}^2 = E_{S0}^2 + 2(E_{S0}^2 + \frac{1}{G_N \tau_s}) \kappa \cos(\omega \tau) \quad (2.38)$$

where $E_{S0}^2$ is the laser intensity of the LD without optical feedback, which is determined by the injection current. Inserting Eq. (2.11) and ((2.12) into (2.31), we can get:

$$\cot \theta_n = \frac{\tau_s(\omega_s^2 - \omega_{r0}^2)}{\omega_r} \quad (2.39)$$

where $\omega_{r0}$ is the RO frequency of the LD without optical feedback, which is determined by the injection current. Substituting Eq. (2.38) and (2.39) into (2.37), considering the case of $\kappa < 0.01$, we obtain:

$$P(t) = E_{S0}^2 + 2(E_{S0}^2 + \frac{1}{G_N \tau_s}) \kappa \cos(\omega \tau)$$

$$+ 2 \frac{\tau_p \omega_s}{\alpha G_N} \sqrt{\omega_r^2 + \frac{\tau_s(\omega_s^2 - \omega_{r0}^2)}{\tau_{in}} \cos \omega_\tau} \cos(\omega t + \theta_e) \quad (2.40)$$

Therefore, Eq. (2.40) is the laser intensity when the SMLD system is in period-one oscillation. When the external cavity is moving, it will cause a varying feedback phase $\phi_F \ (\phi_F = \omega_s \tau)$ [1, 2] as well as $\omega_r$ [83]

We re-write Eq. (2.40) as below to include displacement sensing:

$$\Delta P(L, t) = P(t) - E_{S0}^2 = \Delta P_{SMF}(L) + \Delta P_{RO-SMF, \phi_p}(L) \cos[\omega_r(L) \cdot t + \theta_\tau] \quad (2.41)$$

where ,
\[ \Delta P_{\text{SMI}}(L) = 2 \left( E_{0}^2 + \frac{1}{G_{N} \tau_{s}} \right) \frac{\kappa \tau_{r}}{\tau_{in}} \cos[\phi_r(L)] \] (2.42)

\[ \Delta P_{\text{RO-SMI-Ep}}(L) = 2 \frac{\tau_{p} \omega_{r}(L)}{\alpha G_{N}} \sqrt{\omega_{r}^2(L) + \tau_{r} \left[ \omega_{r}^2(L) - \omega_{r0}^2 \right] - \frac{\kappa}{\tau_{in}} \cos[\phi_r(L)]} \] (2.43)

Eq. (2.42) is conventional SMI signal reported in the literature [3, 37, 65]. This signal is in the form of sinusoidal (at weak feedback case) or sawtooth-like (moderate or above feedback level) waveform when the external cavity length \( L \) changes. Each periodical variation (called a fringe) corresponds to displacement with half laser wavelength \( \frac{\lambda_0}{2} \) [65]. For a varying external target, the external cavity length can be expressed as \( L = L_0 + \Delta L \), where \( L_0 \) is the initial external cavity length, \( \Delta L \) is the displacement.

We are more interested in the last term appeared in Eq. (2.41). Since this term also contains the information of displacement \( \Delta L \), we can make use of this term to achieve displacement sensing. Regarding \( \omega_r \) contained in Eq. (2.41) and (2.43), the work in [83] shows that \( \omega_r \) changes with displacement \( \Delta L \) in the form of a saw-tooth waveform with a period of \( \frac{\lambda_0}{2} \) when the SMLD system is in period-one oscillation. Thus, we have \( \omega_r(\Delta L) = \omega_r(\Delta L + \frac{\lambda_0}{2}) \). If we denote the maximum and minimum value of \( \omega_r \) within a period by \( \omega_{r\text{max}} \) and \( \omega_{r\text{min}} \) respectively, the expression for \( \omega_r \) within one period (for \( 0 < \Delta L < \frac{\lambda_0}{2} \)) is as below:

\[ \omega_r(\Delta L + N \frac{\lambda_0}{2}) = \omega_{r0} + \omega_{r\text{offset}} + \frac{2(\omega_{r\text{max}} - \omega_{r\text{min}})}{\lambda_0} \Delta L \] (2.44)

where \( N \) is integer, \( \omega_{r0} \) is the angular RO frequency of the solitary laser diode, which is determined by injection current density, \( \omega_{r\text{offset}} \) is the offset value which is determined by the initial external cavity length and feedback level. \( \omega_{r0} \) and
\( \omega_{r, \text{offset}} \) are constant when the operation conditions of an LD are fixed.

Now, let us have a close look at Eq. (2.43). As both \( \omega_r \) and \( (\kappa / \tau_m) \cos[\phi(L)] \) are periodical with same period of \( \lambda_0 / 2 \) for a varying external cavity length, we have:

\[
\Delta P_{\text{RO-SMI-Ep}}(L + N \frac{\lambda_0}{2}) = 2 \frac{\tau_e \omega_r (L + N \frac{\lambda_0}{2})}{\alpha G_N} \sqrt{\alpha^2 (L + N \frac{\lambda_0}{2}) + [\tau_e (\omega_r^2 (L + N \frac{\lambda_0}{2}) - \omega_r)] - \frac{\kappa}{\tau_m} \cos(\phi_r (L + N \frac{\lambda_0}{2}))^2} \quad (2.45)
\]

So we say \( \Delta P_{\text{RO-SMI-Ep}}(L) \) also periodically change with displacement \( \Delta L \) by \( \lambda_0 / 2 \). Figure 2-7(a) shows an example of the relationship between the angular RO frequency and displacement of external target when \( L_0 = 0.24 \, m \), \( J = 1.3 J_{\text{th}} \) and \( C = 4.5 \). Figure 2-7 (b) shows the waveform of \( \Delta P_{\text{RO-SMI-Ep}}(L) \) for a varying external cavity with the same operation conditions.

![Graph](image-url)
In the following, we only consider the last term in Eq. (2.41) and call it as RO-SMI signal denoted by $\Delta P_{RO-SMI}(L,t)$, and rewrite it as below:

$$\Delta P_{RO-SMI}(L,t) = \Delta P_{RO-SMI-Ep}(L)\cos[\omega_r(L) \cdot t + \theta_r]$$

(2.46)

Now, we can describe the RO-SMI signal as amplitude modulated and frequency modulated (AM-FM) signal. Both the envelope ($\Delta P_{RO-SMI-Ep}(L)$) and frequency of the signal contain displacement information. So we can explore a new sensing approach by using this signal. We denote the maximum and minimum value of $\Delta P_{RO-SMI-Ep}(L)$ as $\Delta P_{RO-SMI-Ep-max}$ and $\Delta P_{RO-SMI-Ep-min}$.

Based on the simulation and work in [83], $\omega_{r-offset}$ and $\omega_{r_{max}} - \omega_{r_{min}}$ is much smaller than $\omega_{r_0}$ and we have $\omega_r \approx \omega_{r_0}$. In the case of $\kappa < 0.01$, we get:

$$\Delta P_{RO-SMI-Ep-min} \approx 2 \frac{\tau_p \omega_{r_0}^2}{\alpha G_N}$$

(2.47)

$$\Delta P_{RO-SMI-Ep-max} \approx 2 \frac{\tau_p \omega_{r_0}^2}{\alpha G_N} \sqrt{1 + [2\tau_s(\omega_{r-offset} + \omega_{r_{max}} - \omega_{r_{min}})]^2}$$

(2.48)

Thus the peak-peak variation in $\Delta P_{RO-SMI-Ep}(L)$ (denoted by $\Delta P_{RO-SMI-Ep-pp}$) is expressed as:

$$\Delta P_{RO-SMI-Ep-pp} = 2 \frac{\tau_p \omega_{r_0}^2}{\alpha G_N} (\sqrt{1 + [2\tau_s(\omega_{r-offset} + \omega_{r_{max}} - \omega_{r_{min}})]^2} - 1)$$

(2.49)

Making a comparison on the magnitude of $\Delta P_{RO-SMI-Ep}(L)$ and $\Delta P_{SMI}(L)$ as below:

$$\frac{\Delta P_{RO-SMI-Ep-pp}}{\Delta P_{SMI-pp}} = \frac{2 \frac{\tau_p \omega_{r_0}^2}{\alpha G_N} (\sqrt{1 + [2\tau_s(\omega_{r-offset} + \omega_{r_{max}} - \omega_{r_{min}})]^2} - 1)}{4(E_{SO}^2 + \frac{1}{G_N \tau_s}) \frac{\kappa T_p}{\tau_{in}}}$$

$$\approx \frac{\tau_{in}}{2\alpha \kappa T_p} \left(\sqrt{1 + [2\tau_s(\omega_{r-offset} + \omega_{r_{max}} - \omega_{r_{min}})]^2} - 1\right)$$

(2.50)
Based on the simulation and work in [83], $\omega_{r\text{-offset}}$ and $\omega_{r\text{max}} - \omega_{r\text{min}}$ are usually in the order of $10^6 \text{rad/s}$ e.g. in the case in Figure 2-8(e), $\omega_{r\text{-offset}} + \omega_{r\text{max}} - \omega_{r\text{min}} \approx 6 \times 10^6 \text{rad/s}$, We substitute the typical values in Table 1 into Eq. (2.50), i.e. $\tau_i = 8.0 \times 10^{-12}\text{s}$, $\tau_p = 2.0 \times 10^{-12}\text{s}$, $\tau_s = 2.0 \times 10^{-9}\text{s}$, $\alpha = 3$ and $\kappa = 0.007$ in the case in Figure 2-8(e), and get $\Delta P_{\text{RO-SMI-pp}} / \Delta P_{\text{SMI-pp}} \approx 154$. As a result, if we use the envelope to detect displacement, it can be seen the envelope is much more sensitive to displacement compared to using conventional SMI signal.

2.3.2 Verification

Figure 2-8 shows the simulation results by numerically solving the original L-K equations (2.5)-(2.7) and the approximate analytical expression of Eq. (2.46) when the external cavity length has a linear displacement of $3\lambda_0$, the initial external cavity length $L_0 = 0.24\text{ m}$, and the injection current $J = 1.3 J_n$. Note that $\omega_r \tau \approx 23\pi$ in this case, which satisfies the requirement for Eq. (2.46). Figure 2-8 (a) is the displacement; (b) and (c) is the numerically simulation results of L-K equations with $C = 2.8$ and $C = 4.5$; (d) and (e) is the simulation results of the Eq. (2.46) with the operation conditions the same as in (b) and (c). For the simulation of Eq. (2.46), we firstly obtain the relationship between $\omega_r$ and $\Delta L$ by numerically solving the L-K equations as shown in Figure 2-7(a).

From Figure 2-8, it can be concluded that:

(1) The simulations results from the L-K equations and the approximate analytical expression derived above are closely similar, showing the approximate expression is valid when the SMLD is in period-one oscillation.

(2) The slow-varying envelop signal of the RO-SMI signals exhibit the similar fringe waveform as the conventional SMI signals, showing the same resolution as the
conservational SMI signals for displacement measurement by using fringe counting method, i.e. each fringe corresponding to half wavelength $\lambda_0/2$.

(3) The in Figure 2-8 (b) and (d), there are regions between the fringes where the OFLD is stable. Therefore, it is essential to ensure that the OFLD system is always in period-one oscillation with the change of external cavity length when using RO frequency for displacement sensing. The method to achieve this requirement is detailed in [83], which will be discussed in Section 4.2.

![Figure 2-8](image)

Figure 2-8 Verification of RO-SMI signal (Note $\Delta P_{\text{RO-SMI}}$ is scaled by $\Delta P_{\text{RO-SMI}}/10^{20}$) expressed in Eq. (2.46) by the numerical results from L-K Equations. (a) displacement of the external target; (b) and (c) results from L-K equations with $C=2.8$ and $C=4.5$ respectively; (d) and (e) results from Eq. (2.46) when $C=2.8$ and $C=4.5$ respectively.

### 2.4 Summary

In this chapter, theoretical analysis on behavior of an SMLD operating with undamped RO has been conducted. Firstly, an improved dynamic stability boundary for an SMLD compared to that in [54] is obtained by numerically solving the L-K equations. The influence of injection current density $J$ and initial external cavity
length $L_0$ on the stability boundary has been investigated. Based on the stability boundary analysis, three operation regions for an SMLD system, i.e. stable, semi-stable, and unstable region is defined. It is found that an SMLD system in moderate feedback regime may fall into the semi-stable region for some operation conditions. Long external cavity length and high injection current can make the SMLD system in moderate feedback regime locate in stable region easier. Then, RO-SMI signals are investigated from time domain and frequency domain by numerically solving the L-K equations. After that, through analyzing the influence of detection circuit bandwidth on the RO-SMI signals, it is found, apart from the relaxation oscillation, external cavity mode oscillation exists in the RO-SMI signals even though it is much weaker than the relaxation oscillation. Finally, an approximate analytical expression for RO-SMI signals when the SMLD system operates in period-one oscillation is derived and its validity is verified by simulations. According to the analysis, it has been found that both the amplitude and frequency of the RO-SMI signal is modulated by the external target displacement.
Chapter 3 Experimental Study on an SMLD with Undamped RO

The SMI signal is usually detected by the photodiode packaged at the rear of the LD together with a trans-impedance amplifier. The rising time of the packaged PD is usually several nano seconds, corresponding to a bandwidth of several hundred MHz. The bandwidth of the trans-impedance amplifier is usually from several hundred kHz to several ten MHz, e.g. the detection circuit for conventional SMI signals in this thesis has a bandwidth of 10 MHz. However, based on the theoretical analysis and simulation in Chapter 2, RO-SMI signals exhibit high-frequency oscillations with frequency as high as several GHz. Therefore, in order to capture RO-SMI signals, a fast photodetector and oscilloscope are needed. The structure of the existing experimental setup is required to adjust.

In this chapter, we experimentally analyze the behavior of an SMLD operating with undamped relaxation oscillation. Firstly, the experimental setup is built and introduced. Then, the detailed experimental procedure and results are presented. Lastly, we analyze and discuss the experimental results and summarize the whole chapter.

3.1 System Design

The schematic diagram of the experimental system for observing the RO-SMI signal is depicted Figure 3-1. The laser emitted from the LD is collimated by a lens and split by a beam splitter (BS) with ratio of 50:50. A portion of the laser beam is directed to the external target and then reflected back to the LD and causes self-mixing effect, whereas the other part of the laser beam is to an external fast photodetector for detection. The external target is a piezoelectric transducer (PZT) with a mirror affixed on its surface, which is driven by a PZT driver to generate
displacement. A variable attenuator is placed between the beam splitter and PZT. A fast oscilloscope is used to observe and record the RO-SMI signals. The details of the key components in the experimental setup are introduced in the following part.

![Experimental setup for observing the RO-SMI signals](image)

**Figure 3-1** Experimental setup for observing the RO-SMI signals, LD, laser diode; BS, beam splitter; PZT, piezoelectric transducer

In this thesis, three commercial single-mode laser diodes are used as the laser source, which are HL8325G, DL5032-001 and DL4140-001S. All the LDs are from Thorlabs. The detailed optical and electrical characteristics of these three LDs are shown in Table 3-1. The LD is assembled in a temperature-controlled laser diode mount TCLDM9. During the experiments, The LD is driven by a laser controller (LDC 2000, Thorlabs, Newton, NJ, USA) with the injection current being above the threshold. The laser controller can operate with anode- or cathode-grounded LDs and PDs with a maximum driving current of 2A. With the LDC 2000, LDs can be driven in constant current or constant power mode. The temperature of the LD is stabilized at the room temperature by the temperature controller (TED 200, Thorlabs, Newton, NJ, USA).
Table 3-1 Optical and electrical characteristics of the laser diode in the experiment

<table>
<thead>
<tr>
<th></th>
<th>HL8325G</th>
<th>DL5032-001</th>
<th>DL4140-001S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lasing Wavelength</td>
<td>830 nm</td>
<td>830 nm</td>
<td>785 nm</td>
</tr>
<tr>
<td>Output Power</td>
<td>40 mW</td>
<td>30 mW</td>
<td>25 mW</td>
</tr>
<tr>
<td>Threshold Current</td>
<td>40 mA</td>
<td>30 mA</td>
<td>30 mA</td>
</tr>
<tr>
<td>Operating Temperature</td>
<td>-10 ~ 60°C</td>
<td>-10 ~ 60°C</td>
<td>-10 ~ 60°C</td>
</tr>
<tr>
<td>Operating Current</td>
<td>≤120 mA</td>
<td>≤90 mA</td>
<td>≤90 mA</td>
</tr>
</tbody>
</table>

Table 3-2 Specifications of the laser diode mount TCLDM9

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lasers Supported</td>
<td>5.6 mm and 9 mm</td>
</tr>
<tr>
<td>Max Laser Current</td>
<td>2 A</td>
</tr>
<tr>
<td>RF Modulation Frequency</td>
<td>100 kHz-500 MHz</td>
</tr>
<tr>
<td>RF Input Impedance</td>
<td>50 Ω</td>
</tr>
<tr>
<td>Max RF Power</td>
<td>200 mW</td>
</tr>
</tbody>
</table>

Since the PD packaged in the LD does not have a sufficiently fast rising time for detecting the RO-SMI signal, an external fast photodetector PDA8GS from Thorlabs is employed. It is an InGaAs fiber-coupled amplified photodetector, and its specifications are shown in Table 3-3. PDA8GS has a bandwidth of 9.5 GHz, which is sufficient for most of the cases as the RO frequency of the typical commercial LD is usually several GHz. In order to couple the laser from the beam splitter into the fiber-coupler photodetector, a fiber-port coupler (Thorlabs, PAF2P-11B) is applied. Figure 3-2 shows the physical setup for the coupler and photodetector. The laser coupled by the fiber-port coupler transmits through a multi-mode fiber into the photodetector.
To observe the SMI/RO-SMI signals, the external target needs to be applied a continuous displacement. In this thesis, a PZT (PAS-009) from Thorlabs is used. It has a maximum travel length of 40um and a displacement resolution of 40nm, which is driven by a PZT driver (Thorlabs, MDT694) with maximum driving output current of 60mA. The detailed specification of the PZT is shown in Table 3-4. During the experiments, the PZT is assembled on a linear translation stage as shown in Figure 3-3. The operating frequency range of the PZT is determined by three factors, i.e. the required PZT voltage (denoted by $V_{pp}$) for a certain displacement, the capacitance of the PZT (denoted by $C_{PZT}$) and the maximum output current of the PZT controller ($I_{max}$), which is 7.2 $\mu$F and 60 mA respectively for the latter two. For example, to achieve a sinusoidal displacement of 5300 nm, the maximum frequency for the PZT will be $265 \text{ Hz} \left( \frac{I_{max}}{(\pi V_{pp} C_{PZT})} \right)$.
Table 3-4 Specifications of the PZT PAS-009

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Length</td>
<td>40 um</td>
</tr>
<tr>
<td>Length</td>
<td>57 mm</td>
</tr>
<tr>
<td>Resolution</td>
<td>40 nm</td>
</tr>
<tr>
<td>Capacitance</td>
<td>7.2 uF</td>
</tr>
<tr>
<td>Input Voltage Range</td>
<td>0-75 V</td>
</tr>
</tbody>
</table>

Figure 3-3 Assembled PZT and linear stage

As the RO-SMI contains high-frequency oscillation, the common data acquisition card is not able to capture and record the RO-SMI signals. In this work, a fast digital oscilloscope (DSA7804) from Tektronix is employed. The key specifications of the DSA70804 are summarized in Table 3-5. This oscilloscope has an analog bandwidth of 8 GHz, a real-time sampling rate of 25 GS/s, a maximum recording length of 200M sampling points. Additionally, the function of spectrum analyzing is integrated. Therefore, it is capable to be used investigate the RO-SMI signals in both time and frequency domain.

Table 3-5 Specifications of the oscilloscope DSA70804

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>DC-8 GHz</td>
</tr>
<tr>
<td>Analog Channels</td>
<td>4</td>
</tr>
<tr>
<td>Sampling Rate</td>
<td>25 GS/s</td>
</tr>
<tr>
<td>Recording Length</td>
<td>200M points</td>
</tr>
</tbody>
</table>
According to the schematic diagram in Figure 3-1, the details of all the parts in the experiments setup are introduced in the above of this section. The final physical experimental system is depicted in Figure 3-4. The LD is driven by a laser controller with the injection current being above the threshold. The temperature of the LD is stabilized at $T=22\pm0.01^\circ C$ by the temperature controller. A mirror is used as the external cavity which is affixed on the surface of a PZT actuator in order to provide sufficient optical feedback to the LD. The PZT actuator is driven by a PZT controller. An attenuator is used to adjust the optical feedback level of the SMI system. A beam splitter (BS) with a splitting ratio of 50:50 is used to direct a part of the self-mixing light into the external fast PD through the fiber port coupler. The detected SMI/RO-SMI signals are then collected and recorded by the digital oscilloscope.

Figure 3-4 Physical experimental setup for investigating the behavior of an SMLD system, LD: laser diode; VA: variable attenuator; BS: beam splitter; PZT: piezoelectric transducer; OSC: oscilloscope
3.2 Experimental Investigation

3.2.1 Experimental Observations for Different LDs

Firstly, an LD HL8325G has been used as the laser source. It is difficult to conduct the experiments covering all three regions, i.e. stable region, semi-stable region and unstable region, under the condition of the same injection current and external cavity length. Hence, we designed two groups of experiments to confirm the phenomena predicted in the Chapter 2. In the first group, the injection current to the LD is 44 mA, and the initial external cavity length is 25 cm. In this case, we are able to observe the phenomena happened at semi-stable and unstable region. The relevant experimental steps are described as below:

1. Initially, set the PZT stationary, and adjust the attenuator so that periodic oscillation signal can be observed by the oscilloscope.
2. Apply a control signal to the PZT actuator using the PZT controller. The signal is a 30.0 V DC offset superposed with a sinusoidal voltage signal (200 Hz and 3.9 V P-P). The corresponding displacement (P-P) generated by the PZT is 2.08 μm.
3. Adjust the attenuator to vary the feedback strength from low to high, and record three RO-SMI waveforms at different feedback levels, as shown in the left column in Figure 3-5. Meanwhile, for each case, the frequency spectrum is measured using the function provided by the oscilloscope. The corresponding frequency spectra are shown in the right column.

Figure 3-5a shows the control signal for the PZT actuator; Figure 3-5b,c show the RO-SMI signals when the LD is in a semi-stable region; Figure 3-5d shows the signal in the unstable region; and Figure 3-5e–g are the spectra corresponding to Figure 3-5b–d, respectively. From these waveforms we can see that the P-P value grows with the increase of the feedback strength. It can be seen that about five fringes can be found which correspond to the PZT displacement with 2.08 μm P-P. Hence, each fringe corresponds to 416 nm, which is about $\lambda_0/2$ of the LD. Note that, due to the use of a very high sampling frequency (i.e., 12.5 GS/s), we are only able to take a very short time duration (indicated in Figure 3-5b) of the signals for FFT. Hence, the spectrum in Figure 3-5 can only show where the central frequency ($f_{RO}$) is. At the semi-stable
region, a dominant frequency component can be found at the location of 2.19 $GHz$, as shown in Figure 3-5e,f. The envelope in the signal disappears in Figure 3-5d and its corresponding spectrum is significantly broadened. In this case, the SMI system is working in the unstable region. The observed phenomena from the experiments are consistent with the simulations results. Note that there exists a frequency component at 1.19 $GHz$ across all the spectra shown in Figure 3-5 and Figure 3-6; the reason for its existence will be explained below.

Figure 3-5 Experimental signals and their spectra in semi-stable and unstable regions. (a) PZT control signal; (b,c) RO-SMI signals in semi-stable region; (d) SMI signal in unstable region; (e–g) Spectra corresponding to (b–d) respectively.

In the second group, we aim to compare the features of RO-SMI and conventional SMI signals in both time and frequency domains. By shortening the external cavity length and increasing the injection current, both stable and semi-stable regions can be achieved. In the experiments, we set the initial cavity to 14.5 $cm$ and the injection current to 53 $mA$. The PZT is controlled in the same way as in group 1 of the experiments. The attenuator is adjusted to change the feedback level. Figure 3-6 shows the experimental signals recorded, from which we can see that the peak-peak value of a RO-SMI signal is much higher than that of the SMI signal in Figure 3-6 b. The fringe number in a RO-SMI is the same as the one in the conventional SMI signal. Therefore, we confirm that the RO-SMI signals have the same displacement resolution with conventional SMI signals.
We also chose a short piece from each signal (marked in the left column in Figure 3-6) and measured the corresponding spectrum. It can be seen that a dominant frequency component located at \( f_{RO} = 3.1 \, GHz \) can be found in the spectrum of each RO-SMI signal but not in the conventional SMI signal. Again, the amplitude of RO-SMI signals exhibits a similar fringe structure and hence carries the information of the target movement. As mentioned above, the frequency component at 1.19 GHz always exists in the spectrum, and it can be observed even when there is no input to the oscilloscope. Therefore, this 1.19 GHz component must be from the oscilloscope or other external disturbances rather than from the self-mixing effect.

Figure 3-6 Experimental signals and their spectra in stable and semi-stable region. (a) PZT control signal; (b) conventional SMI signals at stable region; (c,d) RO-SMI signals at semi-stable region; (e–g) the spectra corresponding to (b–d) respectively.

Another point we want to mention is that the signals in Figure 3-5 and Figure 3-6 are displayed by setting the oscilloscope at 10 \( mV/div \), and hence the signals, including the 3.1 GHz and 1.19 GHz components, are all very weak indeed. This is because only a small amount light from the LD can be detected due to mismatch among the LD (HL8325G), the fiber coupler and the PD. In order to obtain stronger RO-SMI signals, we also carried experiments using another two laser diodes. The first one was DL5032-001 ( \( J_{th} = 35 \, mA \), \( \lambda_0 = 830 \, nm \), \( P = 30 \, mW \) ) and we do the
experiments under the conditions of $J = 46 \ mA$ and $L_0 = 12.5 \ cm$ for both stable and semi-stable cases. The experimental results are presented in Figure 3-7. For the same target movement, we noticed that RO-SMI signals are always much stronger (about 93 times higher) than a conventional SMI signal.

![Figure 3-7](image)

Figure 3-7 Experimental signals with $J = 46 \ mA$ and $L_0 = 12.5 \ cm$ for DL5032-001 (a) PZT control signal; (b) conventional SMI signals at stable region; (c,d) RO-SMI signals at semi-stable region.

Another laser diode we tested was DL4140-001s ($J_{th} = 30 \ mA$, $\lambda = 785 \ nm$, $P = 20 \ mW$). The results are shown in Figure 3-8. The conventional SMI signal is obtained with $J = 60 \ mA$ and $L_0 = 11 \ cm$, and the RO-SMI signal with $J = 46 \ mA$ and $L_0 = 14 \ cm$. It shows that the P-P value of RO-SMI signal is 27 times that of the conventional SM signal. The spectrum obtained for a small piece from the RO-SMI is shown in Figure 3-8e with $f_{RO} = 4.3 \ GHz$. In the semi-stable region, we also recorded the RO signal and its spectrum is shown in Figure 3-8d,f, respectively, when the target is stationary. It can be seen that $f_{RO}$ is same as the one in Figure 3-8e. This
implies that an RO-SMI signal has a central frequency that is the same as the relaxation oscillation of the LD system with optical feedback.

Figure 3-8 Experimental signals for DL4140-001S. (a) PZT control signal; (b) conventional SM signals at stable region; (c) RO-SM signals at semi-stable region; (d) the laser intensity when the target is stationary; (e, f) the spectra corresponding to (c, d) respectively.

3.2.2 Influence of Photodetector Bandwidth

In Chapter 2, it has been demonstrated that, for some operation conditions, SMLD system in moderate feedback regime may enter semi-stable region. In this case, high-frequency oscillation corresponding to RO and external cavity mode oscillation happens in the SMI signals. However, existing SMI-based applications all assume that the SMI signals they used are “true”, or in other words stable, the SMI signals detected by the PD packaged at the rear of the SL still look like “stable” SMI signals. Such a phenomenon is caused by the limit in the rising time (also known as cut-off frequency, or bandwidth) of the detection system, which consists of the PD packaged at the rear of the LD and the trans-impedance amplifier. In Section 2.2.2, the influence of the bandwidth of detection circuit on the SMI signal has been analyzed by simulations. In this section, we experimentally investigate this point.
In order to investigate the influence of the bandwidth of detection circuit on the SMI signal, we slightly change the experimental setup in Figure 3-1 and the new setup is shown in Figure 3-9. It has been demonstrated that the SMI signals can be obtained wherever on the light path although there may exist phase difference [84]. As a result, we simultaneously use both the internal photodiode (PD) packaged in the rear of the LD with a trans-impedance amplifier (we call this detection system as internal photodetector) and an amplified external fast photodetector to observe the SMI signals. The bandwidth of the internal photodetector is designed as 10MHz in this experiment. Its bandwidth may slightly change with different photodiodes. The bandwidth of the external photodetector is 9.5 GHz. The laser diode in this experiment is DL4140-001s and the experimental conditions is set as $J = 50 \, mA$ and $L_0 = 12 \, cm$.

Figure 3-9 Experimental setup for investigating the influence of detection circuit bandwidth on SMI signals

Figure 3-10 shows the experimental results from two photodetectors with different bandwidths. The left column in Figure 3-10 are the results from the external fast photodetector with a bandwidth of 9.5 GHz, while the right column are the results captured by the internal photodiode packaged at the rear of the LD together with a 10 MHz trans-impedance amplification detection circuit. Each row is obtained under same operation conditions but different photodetectors. Figure 3-10 (a) and (e) are the
driving signals applied on the PZT. For the PZT in the experiments, each 0.1 \( V \) contributes to 53 \( nm \) displacement. The peak-peak value of the voltage in (a) and (e) is about 3.8 \( V \), i.e. 2014 \( nm \) displacement. All the RO-SMI and conventional SMI signals in Figure 3-10 show 5 fringes in one oscillation period, i.e. each fringe corresponding to about 403 \( nm \) displacement. The laser wavelength in this experiment is 785 \( nm \), which verifies each fringe in RO-SMI signal corresponding to a displacement of half laser wavelength. Each row in the figure are captured under the same laser operation conditions, i.e. same injection current, initial external cavity length and optical feedback level. From top to bottom, the optical feedback level increases. It can be found only when the feedback level is low, i.e. in (b) and (f), the signals from the fast and slow detection systems are similar. In this case, the SMLD system is in stable region, and the stable mathematic model can be used to describe its behavior. With the increase of feedback level, the signals in (c) and (d) show the pattern as the simulation results in Figure 2-8. In this case, if the bandwidth of the photodetector is not high enough, the signals in (g) and (h) will be captured and considered as the ‘true’ SMI signals.

![Graph](image-url)  
**Figure 3-10** SMI signals from internal photodiode and external photodetector. (a) and (e) the controlling signals to the PZT, (b)-(d): SMI signals from external fast photodetector, (e)-(f): SMI signals from internal photodiode with an external amplifier.
The experimental results verify the analysis in Section 2.2.2. The external cavity mode oscillation are not found in Figure 3-10, which is mainly due to the base noise of the fast photodetector is relatively high. Based on the analysis in Section 2.2.2, the magnitude of the external mode oscillation is much weaker than the relaxation oscillation in RO-SMI signals. On the other hand, the bandwidth of the internal photodiode and trans-impedance amplifier is too low to capture the external mode oscillation. So, external cavity mode oscillations are not observed in (f), (g) and (h) either. Nonetheless, the results in Figure 3-10 demonstrate that the existing SMI model derived from the stationary solutions of the L-K equations can be only used to describe the behavior of an SMLD system in stable region. The bandwidth of the photodetector is of great significance for detecting the completed and true SMI signal.

3.3 Analysis and Discussion

Based on the experimental results in Section 3.2, the behavior of an SMLD system can be summarized as below:

1. The existing SMI mathematic model derived from the stationary solutions of the L-K equations can be only used to describe the behavior of an SMLD system in stable region. When the SMLD system is in semi-stable region, uncompleted SMI signals may be captured and considered as the ‘true’ signal due to the limit of the bandwidth of the photodetector.

2. When the SMLD system is in semi-stable region, the movement information of the external target can be still found. We call the SMI signal in this case as RO-SMI signal. The RO-SMI signals exhibit the form of high frequency oscillation with its amplitude modulated by a slow-varying signal. Interestingly, the slow-varying envelopes are similar to the conventional SMI signal characterized by the same fringe structure. Each fringe in the RO-SMI signals also corresponds to a target displacement $\lambda_0 / 2$, and hence the RO-SMI can also be used to measure the displacement with the same resolution as the conventional SMI operating in the stable region.
3. Although having the same fringe structure, the RO-SMI signals are very different from the conventional SMI signals in their frequencies. The main high-frequency components in the RO-SMI signals are from the relaxation oscillation. Although the theoretical analysis in Chapter 2 illustrates that external cavity mode may also exist in the RO-SMI signals. However, the external cavity mode oscillation is much weaker than the relaxation oscillation. If the bandwidth of the photodetector is low or the output of the photodetector has a low signal-noise ratio (SNR), the external cavity mode oscillation may not be observed during the experiments.

4. The RO-SMI signals are much stronger (more than 100 times stronger) than the conventional SMI signal. This implies that an SMI system working at a semi-stable region has potential for achieving sensing with improved sensitivity.

5. When the SMI system enters the unstable region, the laser output is unstable, characterized by a much wider frequency spectrum. In this case there is not an obvious relationship between the target movement and the laser output, and hence the system is not suitable for such waveform-based sensing.

3.4 Summary

In this chapter, the behavior of an SMLD system operating with undamped relaxation oscillation has been experimentally investigated. Firstly, the details of each part in the experimental SMLD system including the laser diodes, photodetector, external target, and data-recording oscilloscope are presented. Using the built system, the SMLD system behavior is analyzed. The influence of the bandwidth of the photodetector on the RO-SMI signals are discussed, which is followed by the summary of the features of RO-SMI signals. The experimental results are consistent with the theoretical analysis in Chapter 2.
Chapter 4 Displacement Measurement by an SMLD with Undamped RO

In Chapter 2 and 3, it is found that an SMLD system in moderate or strong feedback regime may enter semi-stable region, where the relaxation oscillation is undamped. The features of the RO-SMI signals in this case have been analyzed, showing the RO-SMI signals still contains the information of the external target, e.g. the displacement of the target. In this chapter, based on the analysis in Chapter 2 and 3, displacement sensing by an SMLD system with undamped RO are proposed.

4.1 Micro-displacement Measurement with Ultra-high Resolution

The properties and behavior of a laser diode (LD) can be significantly affected by external optical feedback. Such optical feedback has been thought as a serious problem that degrades the performance of a laser. On the other hand, various applications have been found, such as imaging and measurement of absolute distance, angle, displacement, velocity, etc. One typical example is self-mixing interferometry (SMI), or called laser feedback interferometry, where the steady-state intensity of a laser is modified by the optical feedback from an external target [3, 6, 8, 19, 24, 32, 37, 53, 79, 85, 86]. Another is to use the laser dynamics, which are more sensitive to optical feedback than the steady-state intensity [87-92].

Recently, sensing and measurement by using the laser relaxation oscillation frequency have been reported [93, 94]. The RO frequency of a LD with optical feedback can be determined from stability boundary analysis [50, 51, 69, 95, 96]. In 1980, Lang and Kobayashi [67] proposed the well-known L-K equations to describe the dynamics of such lasers. Based on the L-K equations, Tromborg et al. [50] presented a stability boundary in 1984, from which the RO frequency can be
estimated by using an approximate formula (Eq. (46) of [20]). This formula gives a cluster of curves, and each indicates that the RO frequency approximately decreases with the external cavity length in the range from 0 to about 30 cm. In 1998, Ye et al. [95] experimentally confirmed the results in [50]. In their experiment, the cavity is changed from 0 to 230 mm with a step-size of 0.18 mm. The result shows that the RO frequency changes with the cavity length periodically with a period of 46.4 mm. These works show that the RO frequency carries information associated with the external cavity, and thus has potential use for sensing and instrumentation.

Sensing by making use of a laser exhibiting RO was reported first for microchip solid-state lasers [87-93]. In 1999, Lacot et al. [87] proposed a laser optical feedback imaging (LOFI) solution where the optical frequency shift is resonant with the RO frequency of a solid-state laser. In this case, the modulation of the laser intensity is highly sensitive to the reflectivity of the external target surface. The works in [88-91] use the LOFI solution for different applications including biological tissues imaging, vibration and 3D profile measurements. In 2001, Lacot et al. [93] presented another work using a YAG microchip laser, where the laser RO frequency is modified by the reflectivity of a target surface. An approximate quadratic relationship between the variation of RO frequency and the reflectivity is presented and applied to imaging. The work shows that using RO can achieve several orders of magnitudes higher sensitivity than using the perturbation of the laser intensity.

In 2013, Cohen et al. [94] presented a subwavelength position sensing system using an LD with optical feedback, where the LD is set to operate at quasi-periodic oscillation. A unique mapping between the variation of the RO frequency and 2D position (100 nm*100 nm) is presented. The system can achieve an average 2D resolution of ~λ/160.

The RO frequency of an LD can be influenced by the laser operation conditions including the injection current, feedback level and external cavity length [82]. In this section, we investigate the influence of the external parameters on the RO frequency,
from which the operation conditions optimizing the sensing performance of an LD can be determined.

In the rest part of this section, we first present how to obtain an accurate relationship between the RO frequency and the external cavity length by numerically solving the L-K equations. We then investigate the influence of the injection current and the initial external cavity length on the relationship. We show that a linear relationship which can be used for achieving high-sensitivity sensing performance can be achieved by setting suitable injection current and initial cavity length. Finally, we experimentally verify the results from the simulations.

4.1.1 Relationship between RO Frequency and External Cavity Length

The dynamics of an LD with optical feedback can be described by the L-K equations. Once a set of parameters shown in Table 1-1 are given, the LD intensity $E^2(t)$ can be obtained by solving the L-K equations. For a fixed external cavity length ($L$) and injection current density ($J$), the laser output may evolve from the stable status into the chaotic status via the periodic and quasiperiodic status as the optical feedback level is increased [50, 51, 69, 82, 95, 96]. We consider the special case of semi-stable region, i.e. the period-1 oscillation. In this case, the dominant frequency of the period-one oscillation is considered as the RO frequency [95], denoted by $f_{RO}$. For example, Figure 4-1 shows an LD operates in period-1 oscillation with $J=1.5J_{th}$, $L=160 \text{ mm}$, $\kappa=0.0051$, where $J_{th}$ is the threshold current density of the laser. Figure 4-1 (a) shows the intensity $E^2(t)$, while Figure 4-1 (b) shows the corresponding spectrum with a peak at $f_{RO} = 3.17 \text{ GHz}$.
Figure 4-1 Numerical results of temporal waveforms of the laser intensity when \( J = 1.5J_{\text{in}} \),

\( L = 160 \text{ mm} \), and \( \kappa = 0.0051 \). (a) Intensity, (b) Spectrum

The stability boundary of an LD with optical feedback has been investigated in Section 2.1, where a boundary can be described in a plane of parameters ( \( \kappa, L \) ) for an LD under a certain injection current density \( J \). In [24], a boundary was presented for an external cavity \( L \) varying in a large range covering a few tens of centimeters. For micro displacement sensing, we care more about how the stability boundary varies if the external cavity varies in a micrometer scale. In this case, let us express the external cavity length as \( L = L_0 + \Delta L \), where \( L_0 \) denotes the initial cavity length and \( \Delta L \) the variation of the cavity length. Using the approach described in section 2.1, we obtain the stability boundary with \( \Delta L \) varying from 0 to \( 1.5\lambda_0 \), as shown in Figure 4-2.

For a given set of \( L \) and \( J \), the period-one oscillation starts on the boundary and ends when \( \kappa \) increases to a certain value. We choose a \( \kappa \) indicated by the dotted line in Figure 4-2, which enables the period-one oscillation to always exist over the range of the movement of the external target (i.e., \( \Delta L \)). With the chosen \( \kappa \), we obtain the laser intensity waveform \( E^2(t) \) by numerically solving the L-K equations for each given value of \( \Delta L \), from which the corresponding RO frequency \( f_{\text{RO}} \) is
determined. Hence the relationship between $f_{RO}$ and $\Delta L$ can be established, as shown in Figure 4-3. It can be seen that $f_{RO}$ varies with $\Delta L$ in a sawtooth-like form with a period of half wavelength ($\lambda_0 / 2$). Within each period, there is a good linear relationship between $f_{RO}$ and $\Delta L$ with a slope ($f_{RO} / \Delta L$) of $70 \, MHz/\lambda_0$. In our simulations, $\lambda_0$ is chosen as $780 \, nm$, which means $f_{RO} / \Delta L = 0.09 \, MHz / nm$. This relationship tells us a possible sensing strategy for measuring a moving target using the value of $f_{RO}$. In this thesis, we use an oscilloscope with spectrum resolution of $62.5 \, kHz$ to record $f_{RO}$. Hence, the displacement resolution is $\lambda_0 / 1120$. However, the resolution bandwidth (RBW) of the commercial spectrum analyzers can be higher than $62.5 \, kHz$, e.g. the RBW of the Tektronix RSA5000 series spectrum analyzer is $1 \, Hz$-$5 \, MHz$. In addition, the spectrum resolution can be improved by using signal processing methods. With a higher frequency resolution, a higher displacement resolution can be achieved.

![Figure 4-2 Stability boundary of an LD with optical feedback, when $J = 1.5J_b$, $L_0 = 160 \, mm$](image-url)
The RO frequency $f_{RO}$ is determined by the operation parameters of the laser. In the following section, we will study how the relationship between $f_{RO}$ and $\Delta L$ is influenced by the parameters. We will focus on the influence of the injection current density $J$, the initial external cavity $L_0$ and the feedback strength $\kappa$.

### 4.1.2 Influence of Operation Parameters

Firstly, the influence of the injection current density $J$ is analyzed respectively. Fixing the initial external cavity length to $L_0 = 160 \text{ mm}$, four different injection current densities $J = 1.1J_{th}$, $J = 1.3J_{th}$, $J = 1.5J_{th}$, and $J = 1.9J_{th}$ are assumed. For each case, we establish the relationship between $f_{RO}$ and $\Delta L$ using the approach described in the above and present the results in Figure 4-4. From Figure 4-4, the following observations are made:

1. The injection current density has effect on the linearity of the relationship. Low injection current density can cause a large non-linear area in each sawtooth period. The non-linear area is reduced by increasing $J$, and a good linearity can be achieved when $J$ goes up to a certain value.
2. For the cases without non-linearity, the slope \( \left( \frac{f_{RO}}{\Delta L} \right) \) of the sawtooth is nearly the same, i.e., about 0.09 MHz/nm. However, the ranges of \( f_{RO} \) are different, with a central value varying from about 1.3 GHz to 4.2 GHz.

Based on the above observations, we recommend choosing a relatively high injection current density for using such systems for sensing and measurement.

![Graphs showing the relationship between \( f_{RO} \) and \( \Delta L \) for different injection current density with fixed initial external cavity length of \( L_0 = 160 \text{ mm} \) (a) \( J = 1.1J_{th} \), (b) \( J = 1.3J_{th} \), (c) \( J = 1.5J_{th} \), (d) \( J = 1.9J_{th} \).](image)

Figure 4-4 Relationship between \( f_{RO} \) and \( \Delta L \) for different injection current density with fixed initial external cavity length of \( L_0 = 160 \text{ mm} \) (a) \( J = 1.1J_{th} \), (b) \( J = 1.3J_{th} \), (c) \( J = 1.5J_{th} \), (d) \( J = 1.9J_{th} \).

Secondly, let’s examine the influence of initial cavity length \( L_0 \) on \( f_{RO} \). We have set four different initial cavity length \( L_0 = 73 \text{ mm} \), \( L_0 = 122 \text{ mm} \), \( L_0 = 220 \text{ mm} \) and \( L_0 = 317 \text{ mm} \) with a fixed injection current \( J = 1.5J_{th} \). The results are shown in Figure 4-5. The influence can be summarized as below:

1. Under different initial external cavity length i.e. for \( L_0 = 73 \text{ mm} \), \( L_0 = 122 \text{ mm} \), \( L_0 = 220 \text{ mm} \) and \( L_0 = 317 \text{ mm} \) the values of \( f_{RO} \) are all around 3 GHz.
2. The shorter the external cavity length is, the larger the slope \( \frac{f_{RO}}{\Delta L} \) is. It can be seen from Fig.6, for \( L_0 = 73 \text{mm}, \ L_0 = 122 \text{mm}, \ L_0 = 220 \text{mm} \) and \( L_0 = 317 \text{mm} \), the slope is about \( 192 \text{MHz}/\lambda_0, 120 \text{MHz}/\lambda_0, 50 \text{MHz}/\lambda_0, \) and \( 26 \text{MHz}/\lambda_0 \), respectively. Obviously the shorter cavity case is preferred for achieving a sensitive measurement.

3. When the external cavity length less than a certain length, it may also cause non-linear area, as shown in Figure 4-5(a).

![Figure 4-5](image)

Figure 4-5 Relationship between the RO frequency and ΔL for different initial external cavity lengths with a fixed injection current density \( J = 1.5J_{th} \), (a) \( L_0 = 73 \text{mm} \), (b) \( L_0 = 122 \text{mm} \), (c) \( L_0 = 220 \text{mm} \), (d) \( L_0 = 317 \text{mm} \)

The period-one oscillation happens during a range of feedback level for a fixed external cavity length and injection current. We then investigate the change of RO frequency for different feedback strength when the laser diode is within the periode-one oscillation for a fixed external cavity length and injection current, as shown Figure 4-6. In Figure 4-6 (a) and (b), the optical feedback strength decreases, but the laser diode is always in period-one oscillation state when \( J = 1.3J_{th} \),
\( L = 16 + 0 \lambda_0 \) and \( J = 1.3J_{th} \), \( L = 16 + 0.6\lambda_0 \) respectively. It is found that during the period-one oscillation status for a fixed external cavity length and injection current, the oscillation frequency decrease with the increase of the feedback level.

\[ L = 16 + 0 \lambda_0 \] and \( J = 1.3J_{th} \), \( L = 16 + 0.6\lambda_0 \) respectively. It is found that during the period-one oscillation status for a fixed external cavity length and injection current, the oscillation frequency decrease with the increase of the feedback level.

The non-linear area is unwanted for sensing. We make a further look at the non-linear case by choosing different \( \kappa \). The obtained relationships are shown in Figure 4-7. It can be seen that the feedback level \( \kappa \) has little influence on the linearity of the curve. For different feedback strength \( \kappa = 0.0030 \), \( \kappa = 0.0032 \), \( \kappa = 0.0034 \) and \( \kappa = 0.0036 \), the non-linearity always exists in the relationship between \( f_{RO} \) and \( \Delta L \).
From the above simulation results, for achieving a good linearity and high sensitivity when using the RO frequency for micro-displacement measurement, we recommend to set a high injection current density ($J > 1.3J_{th}$ in our simulations) and choose a short external cavity. A critical initial cavity length can be experimentally determined by examining the relationship between $f_{RO}$ and $\Delta L$.

Figure 4-7 Relationship between $f_{RO}$ and $\Delta L$ for different feedback level with fixed initial external cavity $L_o = 160\, mm$ length and injection current $J = 1.1J_{th}$, (a) $\kappa = 0.0030$, (b) $\kappa = 0.0032$, (c) $\kappa = 0.0034$, (d) $\kappa = 0.0036$

Note that $f_{RO}$ has a value of several GHz, which requires very fast signal processing to obtain its value. Actually, we do not need to know the absolute value of $f_{RO}$ but are more interested in its variation with respect to $\Delta L$. By mixing the RO signal $E^2(t)$ with a reference sinusoidal signal with the central RO frequency (e.g. 3 GHz in Figure 4-5 (b)) and then applying a low-pass filter with cut-off frequency of 100 MHz, the RO signal $E^2(t)$ can be down-converted to the low-frequency regime. The relationship between the down-converted frequency (denoted by $\Delta f_{RO}$) and $\Delta L$ is shown in Figure 4-8 by the solid line. This will make it more convenient to implement the proposed sensing system. Another interesting
point is that the periodical sawtooth-like form exists in the relationship. Exploiting the inclination direction of the sawtooth, we can unwrap the periods and obtain a longer region of displacement over which $\Delta f_{RO}$ is linear with $\Delta L$, as shown by the dashed line in Figure 4-8. This will extend the range of displacement that can be measured to beyond a half laser wavelength.

![Figure 4-8 Relationship between $\Delta f_{RO}$ and $\Delta L$, when $J = 1.5J_0$, $L_0 = 122$ mm, $\kappa = 0.0078$](image)

**4.1.3 Experiment**

To verify the relationship presented in the above simulations, we use the experimental setup built in Chapter 3, shown as in Figure 3-1. The laser diode is a single-mode quantum well laser diode (Hitachi, HL8325G) with a wavelength of 830 nm and maximum output power of 40 mW. The LD is driven by a laser controller (Thorlabs, LDC2000) at injection current of 50 mA (threshold current is 40 mA), and operates at the temperature of $T=25\pm0.01^\circ C$ by using a temperature controller (Thorlabs, TED 200). A mirror is used as the external target which is glued on the surface of a PZT actuator (Thorlabs, PAS009), working together with the attenuator to achieve different feedback strengths. The actuator is driven by a PZT controller (Thorlabs, MDT 694) to provide a displacement signal. A beam splitter (BS) with a splitting ratio of 50:50 is used to direct a part of light into the fast external photodetector (Thorlabs, PDA8GS) through a fiber port coupler. A high-speed digital oscilloscope (Tektronix DSA 70804) with a sampling rate of 25 GHz is used to observe the
laser output to ensure the laser intensity oscillates in period-1 status, and record the RO frequency $f_{RO}$.

The experiments were carried out following the steps below:

1. Experimentally determine a proper initial external cavity length and the injection current as suggested in the simulation parts. Injection current of 50 mA and initial external cavity length of 156 mm are chosen for this experiment.

2. Use the PZT controller to decrease the voltage signal (denoted by $V_{PZT}$) applied onto the PZT from 29.0 V to 25.0 V with a minimum step of 0.1 V. Note that the PZT can produce a 53 nm displacement for a 0.1 V voltage change applied on it. So a 2120 nm displacement signal is applied to the above experimental system.

3. Adjust the attenuator to obtain a proper feedback level, which enables the LD to operate at the period-1 oscillation over the range of displacement generated by the PZT.

4. For each change from the controlling voltage ($V_{PZT}$), use the Tektronix DSA 70804 oscilloscope to record the laser intensity waveform and thus obtain the corresponding peak frequency ($f_{RO}$) by using the FFT function provided by the oscilloscope. In total, we measured 40 pairs of $f_{RO}$ and $V_{PZT}$. Figure 4-9 shows an example of the laser intensity waveform and the corresponding spectrum.

![Figure 4-9](image)

Figure 4-9 Experimental waveform of laser intensity when injection current is 50mA, initial external cavity length is 156mm, $V_{PZT} = 26.9$ V  (a) Intensity, (b) Spectrum
Figure 4-10 illustrates the experimental results for the relationship between $f_{RO}$ and $V_{PZT}$. As the PZT used in the experiment can only provide 53nm displacement resolution, we are only able to provide 8 experimental points within each period. It can be seen that the measured RO frequency $f_{RO}$ is a sawtooth-like function of the PZT voltage $V_{PZT}$. The period of $f_{RO}$ is 0.8V, which corresponds to 424 nm, approximately a half laser wavelength ($\lambda_0=830$ nm). Though the experiment presented only limited experimental data, the results still confirmed the periodical and linear relationship between $f_{RO}$ and $V_{PZT}$.

![Figure 4-10 Experimental results for the relationship between $f_{RO}$ and $V_{PZT}$](image)

From both the simulation and experimental results, we have revealed that there is a linear relationship between $f_{RO}$ and $V_{PZT}$ within each period. The influence of the injection current, the initial external cavity and feedback strength on the relationship is investigated. It is pointed out that the non-linear area can be avoided by choosing a relatively high injection current density (e.g. $J > 1.3J_{th}$) and a high sensitivity can be achieved by using a critical initial external cavity. This critical cavity length can be experimentaly determined. The non-linearity of the relationship is not able to be improved by changing the feedback strength. However, it is found that for a certain injection current density and initial external cavity length, there is a range of feedback strength within which period-one oscillation always happens. Additionally, the RO
frequency decrease with the increase of the feedback strength.

The frequency resolution of the oscilloscope in the experiments is 62.5 kHz, while the variation of \( f_{RO} \) in each period is measured as 40 MHz. In theory, if a perfect linearity can be guaranteed, the proposed experimental system can achieve a resolution of \( \lambda_0 / 1280 \) for the displacement measurement, which is 0.65 nm for \( \lambda_0 = 830 \) nm. To facilitate reading \( f_{RO} \) while using the proposed sensing system, a signal processing block can be further added to the experimental system by connecting the block to the output end of the external photodetector. This block will have the functionalities including down-converter, low pass filter and period unwrapping for achieving the result presented in Figure 4-8. This will be part of our future design for the proposed sensing system.

**4.2 Displacement Sensing with Large Measurement Range and High Resolution**

In Section 4.1, micro-displacement sensing by using RO frequency of LD with optical feedback operating in period-one oscillation is proposed. However, the method in Section 4.1 is for small displacement range of less than half laser wavelength. In this Section, we introduce a displacement sensing method with large measurement range, high sensitivity and resolution by simultaneously using the RO-SMI time-domain waveform and its frequency.

**4.2.1 Measurement Principle**

As discussed in Section 2.3, both the amplitude and frequency of an RO-SMI signal are modulated by the external target displacement. Both the envelope \( \Delta P_{RO-SMI-Ep}(L) \) and frequency of the signal contain displacement information. So we can explore a new sensing approach by using this signal. In this section, a displacement sensing
method of the external target by using both time-domain waveform and the RO frequency contained in the signal when the SMLD operates in period-one oscillation is presented.

Let us express this displacement to be measured as below:

\[ \Delta L = \Delta L_t + \Delta L_f \]  \hspace{1cm} (4.1)

where \( \Delta L_t \) is measured by fringe counting with \( \lambda_0/2 \) resolution by using the envelop in RO-SMI signal. \( \Delta L_f \) is measured by the RO frequency for the fractional displacement within \( \lambda_0/2 \). As an example, Figure 4-11 shows an RO-SMI signal and the corresponding displacement information contained in it when a linear displacement of \( 3\lambda_0 \) is applied on the external target, when \( L_0=24cm \), \( J=1.3J_\text{th} \) and \( C=4.5 \)

![Figure 4-11 RO-SMI signal and displacement corresponding to \( \Delta L_t \) and \( \Delta L_f \) when \( L_0=24cm \), \( J=1.3J_\text{th} \) and \( C=4.5 \)]

It can be seen that an RO-SMI signal contains complete fringes and may contain an incomplete fringe (or called fractional fringe) at the beginning or end part of the waveform. The measurement of the fractional fringe enables a displacement measurement with much higher resolution than \( \lambda_0/2 \). As described in Section 4.1, there is a certain linear relationship between \( \omega_r \) and the displacement within each
fringe. As in Eq. (2.44), the maximum and minimum RO frequency within $\frac{\lambda_0}{2}$ are $\omega_{r_{\text{max}}}$ and $\omega_{r_{\text{min}}}$. We denote the RO angular frequency at the starting and ending point in a fractional fringe by $\omega_{r_1}$ and $\omega_{r_2}$ respectively. Then the displacement corresponding to fractional fringes can be obtained as:

$$\Delta L_f = \frac{\lambda_0}{2} \cdot \frac{\omega_{r_2} - \omega_{r_1}}{\omega_{r_{\text{max}}} - \omega_{r_{\text{min}}}}$$

(4.2)

where, for the first fractional fringe, $\omega_{r_1}$ is the RO angular frequency when the displacement starts, $\omega_{r_2} = \omega_{r_{\text{max}}}$; for the last fractional fringe, $\omega_{r_1} = \omega_{r_{\text{min}}}$ and $\omega_{r_2}$ is the RO angular frequency when the displacement ends. Due to $\omega_{r_{\text{max}}}$ and $\omega_{r_{\text{min}}}$ is fixed for a LD with certain operation conditions, the displacement corresponding to fractional fringes can be determined by only measuring the RO frequency of the SMLD when displacement starts and ends. Note that the first fringe is always considered as a fractional fringe. Before measurement, we first get the relationship between RO frequency and external target displacement with the same operation conditions as in Figure 4-11, shown in Figure 4-12

Figure 4-12 Relationship between RO frequency and external target displacement when $L_0 = 24\text{cm}$, $J = 1.3J_0$ and $C = 4.5$

A detailed procedure of retrieving the displacement from an RO-SMI signal is
presented as below. Assuming the external target has a linear movement with 
\[ \Delta L = 3.000 \lambda_0 \] as shown in Figure 4-13 (a), the corresponding RO-SMI signal can be obtained as shown in Figure 4-13 (b). The upper envelope of the RO-SMI signal can be retrieved by upper-peak detection shown in Figure 4-13 (c). Then, applying differentiation on Figure 4-13 (c), we can get pulse train shown in Figure 4-13 (d). After pulse counting we can obtain 
\[ \Delta L_i = (N - 1) \cdot \lambda_0 / 2 \] (here \( N \) is the pulse train number). It can be seen from Figure 4-13 (d), there are 6 pulses. Thus 
\[ \Delta L = 2.5 \lambda_0 \]. Then we need to obtain the displacement determined by the first fractional fringe and last fractional fringe denoted by \( \Delta L_{f1} \) and \( \Delta L_{f2} \) as shown in Figure 4-13. In order to get \( \Delta L_{f1} \) and \( \Delta L_{f2} \), the RO angular frequency when displacement starts and ends should be obtained firstly. Before the displacement starts and after the displacement ends, for the stationary target, we get the laser intensity and then the RO angular frequency by applying FFT on the intensity. Using this method, we get the RO angular frequency at these two time points are the same with \( 1.399 \times 10^{10} \, rad/s \). According to Figure 4-12, \( \omega_{r_{\max}} = 1.436 \times 10^{10} \, rad/s \) and \( \omega_{r_{\min}} = 1.394 \times 10^{10} \, rad/s \). For \( \Delta L_{f1} \), \( \omega_{r_{1}} = 1.436 \times 10^{10} \, rad/s \), \( \omega_{r_{1}} = 1.399 \times 10^{10} \, rad/s \), whereas for \( \Delta L_{f2} \), \( \omega_{r_{2}} = 1.399 \times 10^{10} \, rad/s \), \( \omega_{r_{2}} = 1.394 \times 10^{10} \, rad/s \). Accordingly, Based on Eq. (4.2), we can get 
\[ \Delta L_{f1} = 0.441 \lambda_0 , \Delta L_{f2} = 0.059 \lambda_0 \]. Thus, \( \Delta L = \Delta L_i + \Delta L_{f1} + \Delta L_{f2} = 3 \lambda_0 \), which is the same as the pre-set value. Therefore, displacement of the external target can be obtained by the RO-SMI signals which just needs to measuring the RO frequency when the displacement starts and ends.
Figure 4-13 Displacement reconstruction procedure from the RO-SMI signal, (a) the pre-set displacement, (b) corresponding RO-SMI signal, (c) upper envelope of RO-SMI signal, (d) envelope differentiation.

### 4.2.2 Experiment

By using the experimental setup built in Chapter 3, we verify the proposed method in the following. In this experiment, the laser diode is HL8325G and a PZT (PI, P-841.20) with an integrated displacement sensor is used to generate the displacement. The PZT has a displacement resolution of 9 nm with a traveling range up to 30 μm. By using the integrated sensor, the displacement of the PZT can be compared with the value measured by the proposed method. Choosing proper initial external cavity length and injection current as referred to in Section 4.2, we can ensure the system operating in period-one oscillation. Here, We choose \( J = 50 \text{ mA}, L_0 = 18 \text{ cm} \).

Before staring the measurement, we firstly need to get the relationship between RO frequency \( (\omega_r) \) and external target displacement \( (\Delta L) \) when the external cavity length changing \( \lambda_0 / 2 \). By using the method in Section 4.2, we get the results as below:
Then we apply a dynamic linear displacement on the PZT by applying a controlling voltage through the PZT driver. Figure 4-15 (a) is the PZT controlling signal applied on the PZT, Figure 4-15 (b) is the corresponding RO-SMI signal. The displacement generated by the PZT can be read from its internal integrated sensor which indicates that the displacement is $\Delta L = 2304 \text{ nm}$.

From the experimental RO-SMI signal in Figure 4-15 (b), we process it by procedure in Figure 4-13 to get the displacement. 5 complete fringes can be found which corresponds to $\Delta L_c = 5\lambda_0 / 2$. In order to obtain the displacement of the first and last fractional fringes, we need to obtain the RO frequency at the beginning and ending point. By using the method in Section 4.1, the RO frequency at these two points are measured as $2.634 \text{ GHz}$ and $2.595 \text{ GHz}$ respectively. From Figure 4-14, it can be seen the variation range of $f_{ao}$ in each period is from $2.579 \text{ GHz}$ to $2.645 \text{ GHz}$. Thus, we can get $\Delta L_{f1} = 0.083\lambda_0$ and $\Delta L_{f2} = 0.121\lambda_0$ based on Eq. (4.2). Thus, $\Delta L = \Delta L_c + \Delta L_{f1} + \Delta L_{f2} = 2.704\lambda_0 (2244 \text{ nm for } \lambda_0 = 830 \text{ nm})$, which matches...
the value from the integrated sensor in the PZT. The difference between the value from the PZT integrated sensor and the proposed method comes from the accuracy of the PZT integrated sensor, laser wavelength shift induced by temperature or other operation conditions, and the accuracy of the measured RO frequencies.

The resolution of this method is determined by the measured RO frequency and its variation range within $\lambda_0/2$. The frequency resolution of the oscilloscope is 62.5 kHz and the variation of $f_{ro}$ in each period is measured as 66 MHz. Hence, the proposed experimental system can achieve a displacement resolution of $\lambda_0/2112$ (0.39 nm for $\lambda_0$=830 nm), which may be affected by the linearity. In order to get a good linearity, a suitable injection current and initial external cavity length need to be set. The non-linearity in the experiments may be from accuracy-linearity-hysteresis of the PZT. Therefore, the proposed method can be used for displacement measurement with large measurement range, high sensitivity and resolution.

4.3 Summary.

In this Chapter, displacement sensing by using an SMLD with undamped RO is introduced. Firstly, a micro-displacement within half wavelength sensing method based on the RO frequency is proposed. It is found that the RO frequency varies with the displacement $\Delta L$ in a sawtooth-like form with the period of half laser wavelength. Within each period, the relationship exhibits a linear nature. Then the influence of the injection current and the initial external cavity on the relationship is investigated. It is pointed out that the non-linear area can be avoided by choosing a relatively high injection current density and a high sensitivity can be achieved by using a critical initial external cavity. This critical cavity length can be experimentally determined. After that, displacement sensing by simultaneously using the time-domain RO-SMI signal and RO frequency contained in this signal is developed, showing this method has large measurement range, high sensitivity and resolution.
Chapter 5  Acoustic Emission Measurement by an SMLD in Stable Mode

In Chapter 2, we have analyzed the influence of the injection current and initial external cavity length on the stability boundary of an SMLD system. It is found that with high injection current and long initial external cavity length, an SMLD system in moderate or even in strong feedback regime can be always in stable region. In this case, the existing analytical SMI model can be used to describe such SMLD system. For some LDs, the operation current range may be not very wide but the initial external cavity length can be long. A fiber SMLD system is a such system. In this Chapter, an all fiber stable SMLD system is built and acoustic emission is measured by this system.

5.1 Design of Stable All-fiber SMLD

The system is depicted in Figure 5-1. The LD is a distribute feedback (DFB) laser diode with pigtail (LP1550-SAD2, Thorlabs, Newton, NJ, USA) with a wavelength of 1550 nm and maximum output power of 2 mW. The LD is driven by a combined laser diode and temperature controller (ICT4001, Thorlabs, Newton, NJ, USA), operating at injection current of 25 mA (threshold current is 10 mA) and temperature of \( T = 20 \pm 0.01 \, ^\circ\text{C} \). An optical variable attenuator (VOA50, Thorlabs, Newton, NJ, USA) is applied to adjust the optical feedback level. The PD attached to the LD is used to convert the laser intensity to an electrical signal and then the signal is further amplified by the trans-impedance amplifier and then sent to the oscilloscope for display. In the experiments, we use a PZT to generate the dynamic strain on the FBG. One end of the FBG is glued on a fixed base and the other end is glued on a PZT. The PZT (PAS009, Thorlabs, Newton, NJ, USA) is controlled by a PZT controller (MDT694, Thorlabs, Newton, NJ, USA) which can be used to generate dynamic longitudinal strains along the FBG. We simultaneously use both the internal photodiode (PD) packaged at the rear of the LD with a trans-impedance amplifier and an amplified external fast photodetector (PDA8GS, Thorlabs, Newton, NJ, USA) to observe the SMI signals.
The initial external cavity length in this system is 3m and the LD operates at injection current of 25 mA (threshold current is 10 mA) and temperature of $T = 20 \pm 0.01 \degree C$. Figure 5-2 (a) and (e) is the driving voltage from the PZT controller applied on the PZT. The left column in Figure 5-2, i.e. (b), (c) (d) is the corresponding SMI signal obtained by internal PD with trans-impedance amplifier, whereas the right column, (f), (g) (h) by is the SMI signal from the external fast photodetector. Each row in Figure 5-2 is obtained under the same laser operation conditions, e.g. (b) and (f) are the SMI signals under weak feedback regime. It can be seen that all the SMI signals in weak, moderate and even in strong feedback regime show the stable SMI pattern, which demonstrates that the SMLD system is always in stable region, not showing undamped RO. There is phase difference in the left column and right column in Figure 5-2, which has been explained in [84]. Note that, the results in Figure 5-2 (b), (c) and (d) show a much larger magnitude than that in (f), (g) and (h), which is due to the amplification gain of the external fast photodetector is much smaller than that of the trans-impedance amplifier. Therefore, in the following experiments, we just use SMI signals from the internal PD with trans-impedance amplifier.
5.2 Introduction of Acoustic Emission

Acoustic emissions (AEs) are the transient elastic waves within a material, caused by the rapid release of localized stress energy. AE testing is a well-known technique in detecting stress/strain waves generated by structural defects, allowing continuous structural monitoring during the service life of an infrastructure [97-99]. Traditionally, piezoelectric crystal transducers are used to detect AE events. In recent decades, optical fiber based AE detection is getting wide acceptance because fiber-optic sensors offer many benefits compared with their electric counterparts, e.g., immunity to electromagnetic interference, low cost, and capability of directly attaching or embedding in the host structure without modifying the host’s properties and functions while maintaining the structural integrity [100]. Amongst the varieties of fiber-optic sensors, fiber Bragg gratings (FBGs) are considered as the most popular technology for implementing health-monitoring systems because in addition to the advantages mentioned above, FBG offers some other advantages, e.g., ease of multiplexing, and simultaneous measurement of several parameters such as temperature and strain [101, 102]. Traditionally, an FBG interrogation system is needed to detect the reflected wavelength shift induced by external parameters on the FBG such as strain and temperature. There are two main challenges when using FBG for AE testing. Firstly, the typical frequency of an AE event is from a few kHz to several MHz which requires
a wide dynamic measurement range. Secondly, the related strain level caused by an AE event is normally in the micro-strain scale \([103]\). However, conventional FBG interrogation systems have relatively small measurement frequency bandwidth and the strain sensitivity is not sufficient to measure the AE induced events. To measure the AE induced strains in FBG, high sensitivity and bandwidth interrogation systems are required. Recently, Rajan et al. \([102]\) demonstrated the capability of a commercial distributed fiber optic acoustic emission sensor (FAESense) interrogation system to detect the acoustic emission in ballast crack activities. The above system requires an expensive interrogator which makes the overall system sophisticated and costly.

Recently, some low cost FBG-based AE systems were reported for different applications. C. Baldwin et al. presented an FBG-based acoustic emission crack detection system by using a matched FBG as a passive optical filter for the reflected signal from the sensing FBG \([104]\). J.R. Lee et al. designed an FBG–AE sensor head for mechanical test with a wide temperature operation range \([105]\). H. Tsuda et al. measured the AE during a pressure test of carbon fiber-reinforced plastic tank by using an FBG \([106]\). N. Mabry et al. measured the AE Felicity ratio in a carbon composite structure by using FBG \([107]\). Raju et al. presented an FBG-based AE detection technique for the failure characterization of the top-hat stiffener \([108]\). Q. Wu designed a phase-shifted FBG balancing sensing system \([109]\), which was used for detecting AE induced by damages in CFRP laminates \([110, 111]\). The sensing and measurement in these systems are based on the variation in laser intensity induced by the Bragg wavelength shift. In order to achieve a good linearity and wide measurement range, an operation point for these systems, e.g., the 3-dB position of the absorption filters transmittance is needed to be set. Additionally, for these systems, an external photodetector is required.

By combing self-mixing interference (SMI) and FBG technology, we aimed to build a new compact and cost-effective system meanwhile capable of achieving a wide dynamic measurement range. In this work, a 3 m long fiber with an FBG attached to its one end is used as the external cavity of an LD. FBG is the target and the dynamic strain source replicates an acoustic emission event and the self-mixing LD converts the strain within the fiber containing the FBG to an SMI signal.

Regarding the strain measurement by SMI method, D. Tosi reported a chaotic SMI system \([112, 113]\). M. Suleiman et al. \([34, 114]\) demonstrated an SMI system with weak feedback level for measuring dynamic strains. By considering the features of AE events
and based on our previous design on AE measurement [102], we describe a complete measurement model to show the relationship between an SMI signal and the equivalent dynamic deformation applied on an FBG caused by an AE signal. The model described is suitable for different feedback levels including a weak, moderate, and even a strong feedback case. Meanwhile, a varying refractive index of the FBG is considered and included in the model, which makes the measurement model more accurate. By correctly setting the LD and making its emitting spectrum match the reflective spectrum of the FBG, high quality SMI signals were obtained by the experimental system designed in this paper. We also designed the system to detect AE waves. In the experiments, a 40 kHz ultrasonic transducer and pencil lead breaking were used as the AE source. In this design, the broad band laser source and interrogation system required by existing FBG-based AE measurement system can be removed and a wide dynamic measurement range can be achieved.

5.3 Theoretical Analysis

Figure 5-3 shows the schematic of a typical SMI system (Figure 5-3a) and an FBG–SMI system (Figure 5-3b). A typical SMI system consists of an LD, a photodiode (PD) attached to the LD, a lens and an external target. Instead of the free space between the laser front facet and the target in a typical SMI system, in an FBG–SMI system, a piece of fiber with length of $L_0$ with an FBG constitutes the external cavity. Because the most basic form of an FBG is a periodic modulation of the refractive index along a single mode fiber, the light phase in an FBG–SMI system will be different from that in the typical SMI system. The FBG is a distributed reflector, which usually acts as a narrowband reflection filter. The maximum reflectivity of the FBG occurs at a wavelength matching the Bragg condition [101]:

$$\lambda_B = 2n_{eff}\Lambda$$

(4.3)

where $\lambda_B$ is the peak reflected wavelength, $n_{eff}$ is the effective refractive index of the fiber, and $\Lambda$ is the grating pitch [115, 116]. FBG sensors are usually used for temperature or strain measurement. When the temperatures varies or a longitudinal stain is imposed on the FBG, both the $n_{eff}$ and $\Lambda$ will be changed, resulting in the wavelength shift. By monitoring the shift of $\lambda_B$, the temperature change or strain can
be measured. In the following derivation of the measurement model, we assume that an AE event causes a dynamic strain change and the environmental temperature is constant.

![Schematic diagram of a self-mixing laser diode (SMLD) system](image)

As we discussed above, the FBG-SMLD in this thesis is always stable in moderate feedback regime. Thus, we can just use the existing analytical SMI model to describe its behavior. The analytical SMI model is rewritten as below:

\[
\phi_0(t) = 4\pi n(t)/\lambda_0 \quad (4.4)
\]

\[
\phi_F(t) = \phi_0(t) - C \sin(\phi_F(t) + \arctan(\alpha)) \quad (4.5)
\]

\[
g(t) = \cos(\phi_F(t)) \quad (4.6)
\]

\[
P(t) = P_0(1 + m \times g(t)) \quad (4.7)
\]

where, \(\lambda_0\) is laser wavelength without external optical feedback, \(\alpha\) is the linewidth enhancement factor, \(\phi_F(t)\) and \(\phi_0(t)\) are the external light phases at the location of the target for the LD with and without feedback respectively, \(n=1\) for Figure 5-3a, which is the refractive index of air. \(L(t)\) is the instant external cavity length, which is expressed as \(L(t) = L_0 + \Delta L(t)\), where, \(L_0\) is the initial external cavity length, and \(\Delta L(t)\) is the varying part caused by a physical quality to be measured, e.g., displacement, velocity, or vibration. From an SMI system, \(P(t)\) can be observed and is called the SMI signal. In Equation (4.7), \(P_0\) is the power emitted by the free running LD, \(m\) is the modulation index (with typical values \(\sim 10^{-3}\)). The normalized SMI signal
can be obtained from $P(t)$ by normalizing it. Since a physical quantity to be measured is generally linked to $\Delta L(t)$, thus causing a phase change in $\phi_0(t)$ shown by (4.4). For an SMI based sensing scheme, $\phi_0(t)$ needs to be extracted from $P(t)$ following the procedure: $P(t) \rightarrow g(t) \rightarrow \phi_r(t) \rightarrow \phi_0(t)$. In (4.5), $C$ is the feedback coefficient, which is defined by (4.8):

$$C = \eta (1 - r_2^2) \left( \frac{r_s}{r_2} \right) \sqrt{1 + \alpha^2 \frac{\tau}{\tau_{in}}}$$

(4.8)

where, $r_2$ is the amplitude reflection coefficient of the front facet of the LD, and $r_s$ is the amplitude reflection coefficient of the front facet of the external target, $\tau_{in}$ is the internal roundtrip time determined by the length and refractive index of laser internal cavity, $\tau$ is the external roundtrip time of light transmitting in the external cavity. $\eta$ is the coupling efficiency and accounts for possible loss on re-injection. In the case of optical fiber as the external cavity, the length of the fiber is long and it makes $C$ larger [117]. Therefore FBG–SMLD system can easily enter moderate feedback case with $C > 1$. For an FBG–SMLD system, when a strain is applied on the FBG, $r_s$ may change, which may lead to the feedback coefficient $C$ changing. Thus, it may cause fluctuation in the amplitude of the SMI signals which is similar to the effect of speckle as shown in [16, 118, 119]. In this case, extra signal processing like that in [118] is needed to eliminate the fluctuation. In order to reduce the effect of strain-induced FBG wavelength shift, we choose the FBG with a wider full width at half maximum (FWHM) than the emitting spectrum of the LD. Additionally, we make the LD spectrum locate at the center of the reflective spectrum of the FBG to obtain a wide flat response range. By doing so, we can keep $C$ nearly constant during the measurement.

For the FBG–SMLD system shown in Figure 5-3b, the laser is coupled into an optical fiber. The external cavity is the optical fiber with the FBG at one end. The gauge length is the length of the fiber with the FBG glued on the plate for sensing the AE-induced strain, denoted by $L_{gauge}$. The length of the fiber including the FBG is the initial external cavity length, denoted by $L_0$. Once AE occurs, a corresponding dynamic strain on the FBG (denoted by $\varepsilon(t)$) is expressed by $\varepsilon(t) = \Delta L(t)/L_{gauge}$, where $\Delta L(t)$ is the dynamic deformation of the gauge fiber with FBG. The deformation will make changes in both physical length and refractive index of the
gauge fiber with FBG. The two factors will thus modify the optical phase $\phi_0(t)$ as below. Suppose the original effective refractive index of the FBG is $n_{\text{eff}0}$, the instant $n_{\text{eff}}(t)$ is expressed as $n_{\text{eff}}(t) = n_{\text{eff}0} + \Delta n_{\text{eff}}(t)$, where $\Delta n_{\text{eff}}(t)$ is the varying part caused by a dynamic strain. The instant external cavity is $L(t) = L_0 + \Delta L(t)$.

$$
\phi_0(t) = \frac{4\pi n_{\text{eff}0}(L_0 - L_{\text{gauge}})}{\lambda_0} + \frac{4\pi(L_{\text{gauge}} + \Delta L(t))(n_{\text{eff}0} + \Delta n_{\text{eff}}(t))}{\lambda_0}
$$

(4.9)

where the last term $4\pi\Delta L(t)\Delta n_{\text{eff}}(t)/\lambda_0$ is much smaller than the other terms, thus it can be neglected. Thus the optical phase can be written as:

$$
\phi_0(t) \approx \frac{4\pi n_{\text{eff}0}L_0}{\lambda_0} + \frac{4\pi\Delta L(t)n_{\text{eff}0}}{\lambda_0} + \frac{4\pi L_{\text{gauge}}\Delta n_{\text{eff}}(t)}{\lambda_0} + \frac{4\pi\Delta L(t)\Delta n_{\text{eff}}(t)}{\lambda_0}
$$

(4.10)

Due to the photo-elastic effect, the refractive index of the fiber containing the FBG under a strain can be expressed as (4.11) assuming the temperature as constant [98, 99]:

$$
n_{\text{eff}}[\varepsilon(t)] = n_{\text{eff}0}[1 - \mu \varepsilon(t)]
$$

(4.11)

$$
\Delta n_{\text{eff}}(t) = -p_e \Delta L(t)/L_{\text{gauge}}
$$

(4.12)

where, $\mu$ is the Poisson’s ratio, $p_{11}$, $p_{12}$, are Pockel’s strain-optic tensor coefficients, which are all constants for a specific FBG, $p_e = n_{\text{eff}0}^2[p_{12} - \mu(p_{11} + p_{12})]/2$ is known as the effective photo-elastic constant. Substituting (4.12) to (4.10), we get:

$$
\phi_0(t) = \frac{4\pi n_{\text{eff}0}L_0}{\lambda_0} + \frac{4\pi n_{\text{eff}0}(1 - p_e)}{\lambda_0} \Delta L(t)
$$

(4.13)

where, $\phi_{00}$ is the initial light phase for a stationary target, $\Delta \phi_0(t)$ is the varying part in the light phase correlated to the dynamic strain applied on the FBG. Therefore, Equations (4.5)–(4.7) and (4.13) are considered as the measurement model for an FBG–SMLD system. The AE induced dynamic strain is applied on the FBG–SMLD system and causes a modulated laser intensity (called SMI signal) following this procedure:
\[ \varepsilon(t) \rightarrow \Delta L \rightarrow \phi_0(t) \rightarrow \phi_c(t) \rightarrow g(t) \rightarrow P(t) . \]

We can then retrieve the strain through the observed \( P(t) \).

From (4.13), it can be seen that the change of the equivalent optical path length in a FBG–SMLD system is \( n_{\text{eff}0}(1-p_c)\Delta L(t) \). When the equivalent light path length change is \( \lambda_0 / 2 \), that is \( n_{\text{eff}0}(1-p_c)\Delta L(t)=\lambda_0 / 2 \), the corresponding SMI signal \( g(t) \) has a fringe change. For the FBG we used, we have the parameters as \( p_{11} = 0.113 \), \( p_{12} = 0.252 \), \( \mu = 0.16 \) and \( n_{\text{eff}0} = 1.48 \) [116], so we can have each fringe change corresponding to the deformation of \( \Delta L(t) \) with 0.429\( \lambda_0 \). It can be seen from Equation (4.4) that ignoring the varying refractive index can result in an absolute error with 0.091\( \lambda_0 \) in each fringe for deformation measurement (\( \Delta L(t) \)) as seen in the work presented in [34, 114]. Therefore, it can be argued that including the change of refractive index in the measurement model presented in this paper is more accurate than the existing model. The LD we used has its wavelength \( \lambda_0 \) with 1550 nm. The gauge length \( L_{\text{gauge}} \) is 15 cm. We can say the strain resolution is 4.4 \( \mu \varepsilon \) by the fringe counting method. The resolution can be further improved by the waveform reconstruction algorithm reported in [5, 75]. Regarding the dynamic response, the maximum response speed for an SMI system depends on the photodiode (PD) and the related detection circuit. Our current physical experimental system has a bandwidth with 10 MHz. This fits well for AE measurement [97].

Simulations are made in Figure 5-4 to show that the proposed FBG–SMI sensor has a wide frequency response and good sensing relationship between the strain measured and the SMI signal. The left column in Figure 5-4, i.e., (a), (c), and (e), shows the normalized SMI signals for a dynamic strain with the same magnitude of 20.7 \( \mu \varepsilon \) but at different frequencies respectively with 200 Hz, 20 kHz, and 2 MHz. It can be seen that the FBG–SMI has the same response for this set of strain signals covering a large range of frequency. The right column in Figure 5-4 shows the normalized SMI signals corresponds to a set of strain signals operating at same frequency of 200 kHz but different magnitudes respectively with 10.3 \( \mu \varepsilon \), 20.7 \( \mu \varepsilon \), and 31.0 \( \mu \varepsilon \). The fringe number in the SMI signal increases with the strain magnitude. There is a linear relationship between the strain magnitude and the fringe number.
Figure 5-4 Responses of the FBG–SMI system to strains with different frequencies and amplitudes, (a,c,e) show the normalized SMI signals for a dynamic strain with 20.7 με magnitude respectively at frequency of 200 Hz, 200 kHz, and 2 MHz; (b,d,f) show the normalized SMI signals for a dynamic strain at 200 kHz but with the magnitudes as 10.3 με, 20.7 με, and 31.0 με respectively.

5.4 Experiment

To obtain a clear SMI signal from an FBG–SMLD system and have a signal with approximate constant amplitude, first, the reflective spectrum of the FBG should have a wider FWHM than the emitting spectrum of the LD. The peak of the LD spectrum should locate at the center of the reflective spectrum of the FBG. This can be achieved by carefully and accurately adjusting the injection current to the LD and testing its emitting spectrum. Meanwhile, an initial strain can be applied on the FBG to adjust its reflective spectrum so that the two spectra can achieve an optimal match, as shown in Figure 3. In our experiments, a distributed feedback (DFB) laser diode with pigtail with a wavelength of 1550 nm was chosen as the laser source. The length of the FBG used was 3 mm with an FWHM of ~0.5 nm and a reflectivity greater than 80%. The typical values of other parameters for the FBG are: \( p_{11} = 0.113 \), \( p_{12} = 0.252 \), \( \mu = 0.16 \), and \( n_{eff} = 1.48 \). As shown in Figure 3, a pre-strain of 80 με is applied to the FBG to ensure that the LD signal lies at the center of the FBG signal with an injection current of 25 mA applied to the LD at a temperature of 20 °C.
Using the experimental setup in Figure 5-1, we firstly verify the proposed model for the FBG-SMLD system. The PZT controller is used to adjust the control voltage signal (denoted by $V_{\text{PZT}}$) of the PZT. Each 1 V change in $V_{\text{PZT}}$ causes the PZT to have a 530 nm displacement. The maximum displacement of the PZT is 40 μm with a resolution of 40 nm. In the experiments, the initial external cavity length $L_0$ is set as $L_0 = 3$ m, and the gauge length $L_{\text{gauge}}$ is set as $L_{\text{gauge}} = 15$ cm. The control voltage signal applied on the PZT is a sinusoidal signal, i.e., $V_{\text{PZT}} = 4\sin(200\pi t)$. The observed SMI signal is shown in Figure 5-6, (a) is the PZT control signal, and (b) is the corresponding SMI signal. From Figure 5-6, it can be seen that there are 6.5 fringes within a half oscillation period, corresponding to 4322 nm ($0.429 \times 1550 \times 6.5$, as we discussed in Section 4.3.2 by using the typical values of the FBG’s parameters, each fringe corresponds to $0.429\lambda_0$) which is equivalent to a strain of 28.8 με.

Figure 5-6 SMI signal of the FBG-SMLD system when $L_0 = 3$ m, $I = 25$ mA, (a) Control voltage applied on PZT, (b) SMI signal.
We also changed the amplitude of the control voltage signals on the PZT. Figure 5-7 shows the SMI signals when the PZT control voltage has a frequency of 100 Hz, but for different amplitudes, (a)–(c) $V_{PZT}$ is 3.6 V, 8.0 V, and 11.0 V respectively. From Figure 5-7, it can be found that the number of fringes has a linear relationship with the amplitude of $V_{PZT}$, i.e., about 0.8 fringes per volt, which coincides with our previous analysis.

![Figure 5-7 SMI signals for $V_{PZT}$ with frequency of 100 Hz but different amplitudes, (a) $V_{PZT} = 3.6$ V, (b) $V_{PZT} = 8.0$ V, (c) $V_{PZT} = 11.0$ V.](image)

Then, we made a comparison with a commercial FBG interrogation system (IMON 256, Ibsen Photonics, Farum, Denmark) for dynamic measurement. The system is depicted in Figure 5-8. A broadband light source is connected to the FBG via a circulator and the reflected signal is directed to the interrogator. Figure 8 shows an example of the measured results by using the commercial interrogation system when the PZT control voltage signal is the same as in Figure 5-6. As shown in Figure 5-9, the variation of the Bragg wavelength is in sinusoidal form with a frequency of 100 HZ and a wavelength variation (peak-peak) of 0.035 nm. The strain sensitivity of the typical silica FBG is 1.2 pm/$\mu$ε [116], the strain measured by the interrogation system is 29.2 $\mu$ε, which coincides with the results shown in Figure 5-6 obtained from the proposed FBG–SMLD system.

![Figure 5-8 Traditional FBG-based strain measurement system](image)
Regarding the FBG strain caused by the PZT, the value of the strain can be estimated using the PZT data sheet according to the control voltage applied on the FBG. We refer these values to test our measurement results obtained from the proposed FBG–SMLD system. In addition, we employed the commercial I-MON 256 interrogation system to verify our results. Table 5-1 presents the comparison for the cases with different strains caused by the PZT. Our results coincide with the ones measured by the commercial system. The above experimental results verify the accuracy of the measurement model in this work. It can be concluded that the FBG–SMLD signal can be used for dynamic strain measurements in FBG. With the aid of the algorithm we developed in [75] for recovering the displacement from the SMI signal, a real-time dynamic strain measurement can be obtained. Theoretically, for our FBG-SMLD system, the displacement resolution (by fringe counting) is 663 nm, which is equivalent to 4.4 με. The resolution can be further improved by using displacement recovery (e.g., the algorithm reported in [75]) or fringe subdivision. In our experiments, the amplitude of the SMI signal is 60 mV (peak-peak) with a noise of about 3 mV (peak-peak), which means the measurement resolution is 33 nm (0.428λ0 × 3/60) by using the algorithm in [75], corresponding to a strain of 221 nε. The resolution may be improved by using a denoising algorithm. The above experimental results verify the accuracy of the derived measurement model in this work.

Table 5-1 Comparison of the strain measured by the FBG–SMLD system and I-MON 256 system

<table>
<thead>
<tr>
<th>PZT (piezoelectric transducer)</th>
<th>Strain Caused by PZT (με)</th>
<th>Strain Measured by FBG–SMLD (με)</th>
<th>Strain Measured by I-MON256 (με)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Voltage V_{PZT} (V)/freq. (Hz)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.6/100</td>
<td>12.7</td>
<td>12.2</td>
<td>12.5</td>
</tr>
<tr>
<td>8.0/100</td>
<td>28.3</td>
<td>28.7</td>
<td>29.2</td>
</tr>
<tr>
<td>11.0/100</td>
<td>38.9</td>
<td>39.8</td>
<td>38.3</td>
</tr>
</tbody>
</table>
To demonstrate the feasibility of using the proposed FBG–SMI system for AE measurement, we modified the setup shown in Figure 5-1 by introducing an AE source and removing the related PZT. The new experiment setup is depicted in Figure 5-10. The FBG is glued on a $24 \times 12 \times 0.25$ cm aluminum plate using super glue.

An ultrasonic transducer with a resonant frequency of 40 kHz is adhered on the surface of the aluminum plate to launch ultrasonic waves. The transducer is driven by a customized power driver board. Figure 5-11a shows the driving signal to the transducer (a sinusoidal signal with frequency of ~38 kHz and amplitude of 10 V). Figure 5-11b is the corresponding SMI signal captured by our FBG–SMI system. It can be seen from the received SMI signal that about two fringes are generated corresponding to one sinusoidal period of the AE driving signal which indicates that the AE wave induced a displacement of about 665 nm in the FBG.
Furthermore, we generated an AE signal by pencil lead breaking, to test out system. In this experiment, the most representative Hsu-Niesen broken lead method was employed, in which the pencil lead had a diameter of 0.5 mm, length of 3.0 mm, and hardness of HB [103]. The position of the pencil lead breaking is 20.0 mm away from the FBG. Figure 5-12 shows the test result for the AE wave generated by pencil lead breaking. To get a clear AE-induced SMI signal and remove the unwanted low-frequency disturbance, a band-pass filter with bandwidth of 10 kHz–1 MHz [109] is applied on the raw SMI signal shown on Figure 5-12a and a clear one after filtering is shown in Figure 5-12b. This experiment demonstrates that the proposed FBG–SMLD system is able to capture the pencil-lead-breaking-induced AE wave.

![Figure 5-12 SMI signal corresponding to the AE wave generated by pencil lead breaking, (a) a raw detected SMI signal, (b) an SMI signal after filtering.](image)

5.5 Summary.

In this Chapter, we build an all fiber SMI system where the external target is an FBG. It is firstly demonstrated the proposed FBG-SMLD system with a high injection current and long external cavity can be always stable in moderate feedback regime. Then the new FBG-SMLD system is used to detecting AE induced dynamic strain. We analyzed the SMI theory model for an FBG–SMLD system. Afterwards, the preliminary experimental results verified the feasibility of the proposed method. Compared with the existing traditional FBG interrogation system for AE measurement, the FBG–SMLD system has a compact structure, resulting in a new low-cost option for FBG-based AE measurement. One point that needs to be noted is that the wavelength of the laser and FBG should match each other to guarantee that the laser can be reflected into the laser cavity. Then the FWHM of the FBG should be wider than that of the laser.
to guarantee the feedback coefficient $C$ to be close to constant. The readers should be reminded that the influence of the temperature on the FBG has not been discussed in this work. Nonetheless, the proposed sensor in this paper combines the advantages of fiber FGB and SMI, contributing to a novel system in structure health monitoring which can be used to measure AE signals to enable the early detection of failure of structures.
Chapter 6  Conclusion and Future Work

Self-mixing interferometry is well-developed and promising technology for non-destructive sensing and measurement. The configuration of an SMLD system reflects a minimum part-count scheme, which is useful for engineering implementation. Various SMI-based applications have been developed. Most of the applications are based on the analytical SMI mathematic model by assuming the SMLD system operates in stable mode, i.e. both the electric field and carrier density in an LD with a stationary external target can reach a constant state after a transient period. However, undamped RO may occur in an SMLD system for some operation conditions, e.g. it is found an SMLD in moderate feedback regime exhibits undamped RO. The moderate feedback regime is quite commonly employed by researchers in this research field. Based on our in-depth study, the behavior of an SMLD system operating with undamped RO cannot be described by the existing analytical SMI model. The laser intensity (called as sensing signal) from such SMLD system shows some new characteristics and the signal waveform looks complicated. Usually, the PD packaged inside the LD is used for detecting the SMI signal, but it usually has limited bandwidth less than 1GHz. However, the RO frequency of an LD can even higher than several GHz. Hence, using the existing SMI configuration for detection of RO-SMI signal, many frequency components cannot be observed although they exist there in reality. Therefore, it is of great necessity and significance to reveal these phenomena and find their potential applications. In this Chapter, the research contributions of this thesis are summarized in Section 6.1, based on which, future research topics are advised in Section 6.2.
6.1 Research Contributions

The contributions of this thesis are summarized as below:

1. The behavior of an SMLD operating with undamped RO is analyzed by numerically solving the L-K equations. In order to differentiate the conventional SMI signals, we name the SMI signals with undamped RO as RO-SMI signals. It is found that RO-SMI exhibits the form of high frequency oscillation with its amplitude modulated by a slow-varying signal. Interestingly, the slow-varying amplitude envelopes are similar to the conventional SMI signal characterized by the same fringe structure. Each fringe in the RO-SMI signals also corresponds to a target displacement of $\frac{\lambda_0}{2}$. Additionally, RO-SMI signals are much stronger than the conventional SMI signal (e.g. more than 300 times for the simulation results in Figure 2-4). Furthermore, it is found that the RO frequency varies with the displacement $\Delta L$ in a sawtooth-like form with the period of half laser wavelength ($\frac{\lambda_0}{2}$). Within each period, the relationship exhibits a linear nature. [Publication, J2, J3]

2. An approximate analytical expression for describing the RO-SMI signals is derived based on the L-K equations by assuming the SLMD system operates in period-one oscillation. The numerical simulations for original L-K equations are conducted to verify the derived analytical expression. According to the expression, the features of the RO-SMI signals are clearly described. It shows that an RO-SMI signal consists of a low-frequency part and high-frequency part. The high-frequency component has a much larger magnitude than the low-frequency one (e.g. more than 150 times as calculated in Section 2.3). The low-frequency part corresponds to the conventional SMI signal described by existing analytical SMI model, while the high frequency part shows its amplitude and frequency are modulated by the external target
displacement, showing potential of using this part for displacement sensing with large measurement range, high sensitivity and resolution. [Publication, J4]

3. An experimental setup for investigating the behavior of an SMLD with undamped RO is built. In order to observe the complete RO-SMI signals, an external photodetector with bandwidth of 9.5GHz is employed because the RO frequencies of LDs are usually several GHz. By using this system, the experimental RO-SMI signals are captured and the features are analyzed, which verify the theoretical analysis. [Publication, J3, C3]

4. Displacement sensing method based on the RO-SMI signal waveform and RO frequency when the SMLD system operates in period-one oscillation is proposed. For displacement less than half laser wavelength, it can be measured by RO frequency with ultra-high resolution (e.g. the experimental results in Figure 4-10 shows it can achieve resolution of $\lambda_0/1280$). For displacement larger than half laser wavelength. By simultaneously using the RO-SMI signal time-domain waveform and its frequency, displacement can be measured with larger measurement range, high resolution and sensitivity. [Publication, J2, J4, C1,C2]

5. The influence of the bandwidth of photodetector on the captured SMI signals is analyzed from both simulation and experiment. It is found that if the bandwidth of the photodetector is not high enough, distorted SMI signals may be considered as the ‘true’ and complete RO-SMI signal. In a RO-SMI signal, apart from the RO, another oscillation corresponding external cavity mode may exist but the external cavity mode oscillation is much weaker than the RO (e.g. weaker than 50 times in Figure 2-6), which makes it difficult to be experimentally observed in RO-SMI signals. If we apply a low-pass filter with cut-off frequency less than RO frequency and external cavity mode oscillation, a conventional SMI signal described by the existing analytical
SMI model can be obtained [Publication, J3, C3]

6. An improved stability boundary of an SMLD system is obtained and the influence of the injection current and initial external cavity length on the stability boundary is analyzed. The results show that high injection current density and long external cavity length can make an SMLD system in moderate regime is always in stable region. An experimental all-fiber SMLD system is built to verify it. Then, acoustic emission (AE) is measured by the all-fiber SMLD system, contributing to a novel system in structure health monitoring. [Publication, J1]

6.2 Suggested Future Research Topics

Subsequent to the investigations described in this thesis, conducting future research into the following topics would be interesting:

1. For an SMLD, when the RO is undamped, the LD may be in period-one, quasi-periodic or chaotic oscillation. The analytical expression for the RO-SMI signals in this thesis is under the assumption that the SMLD system operates in period-one oscillation with the special case of $\omega \tau = (2m+1) \cdot \pi$. It can be used to describe the features of the RO-SMI signals to some extent, but a more common expression considering other cases may be more useful for explaining the behavior of a SMLD system operating with undamped relaxation oscillation.

2. The base noise of the external fast photodetector in this thesis is large and the amplification gain is relatively low, which makes the captured RO-SMI signals have a low SNR. In this case, some details in RO-SMI signals may be lost, e.g. the external cavity mode oscillation. A photodetector with high bandwidth and output SNR might be used to in the experiments to analyze the features of the RO-SMI signals more clearly.

3. The RO frequency of the LDs is usually in order of GHz, which requires very
fast signal processing to obtain its value. Actually, we do not need to know the absolute value of $f_{RO}$ but are more interested in its variation with respect to $\Delta L$.

By mixing the RO intensity signal $E^2(t)$ with a reference sinusoidal signal with the central RO frequency and then applying a low-pass filter, the RO intensity signal $E^2(t)$ can be down-converted to the low-frequency regime. This will make it more convenient to implement the proposed sensing system.

4. Finally, acoustic emission (AE) events from the ultrasonic transducer and pencil-lead breaking are captured by using the all-fiber SMLD system in this thesis, which provides a new option for AE measurement by fiber Bragg grating (FBG), but the influence of the temperature on the FBG has not been discussed. Also, the FWHM of the FBG in the system is wider than that of the laser to guarantee the feedback factor $C$ to be close to constant. In this case, in influenced of the wavelength shift of the FBG on the SMI signals is not discussed. If the spectrum of the FBG is not located in the center of the laser but in the edge area, the influence of the wavelength shift of the FBG on the SMI signals should be discussed.
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