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Complementary Properties of Hadamard Matrices

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Abstract—This paper demonstrates and compares the complementary properties of two types of Hadamard matrices, i.e., the Walsh-Hadamard matrix and the Golay-paired Hadamard matrix. A simple process is also proposed to construct mutually orthogonal Walsh-Hadamard matrices and mutually orthogonal Golay-paired Hadamard matrices, which represent respectively a collection of mutually orthogonal complementary sets of sequences. Examples are given to verify the relationships between the two types of Hadamard matrices and illustrate the completeness and scalability of the constructed mutually orthogonal complementary sets of sequences.

I. INTRODUCTION

Mutually orthogonal sequences, in which every two sequences have zero cross-correlation at zero offset, are widely used for signal representation, coding, modulation, and spread spectrum applications [1-3]. Although an ideal autocorrelation (i.e., zero at any non-zero offset) is impossible for a finite length sequence, the sum of the respective autocorrelation functions in a set of sequences can be ideal. Such a set of sequences is referred to as complementary set of sequences [4]. Furthermore, although the cross-correlation between two sequences can not be ideal (i.e., zero at any offset), the sum of respective cross-correlation functions between two complementary sets of sequences can be always zero. Such complementary sets of sequences are said to be mutually orthogonal [5-8]. Complementary set of sequences and mutually orthogonal complementary sets of sequences have attracted more and more attention in recent years for applications in digital signal processing and communication systems [9-11,14].

Mutually orthogonal sequences can be easily provided by Hadamard matrices. The most popular one is the Walsh-Hadamard matrix. Complementary sets of sequences can be also constructed using some Hadamard-type recursions [5,8] with properly chosen initial complementary sequences. However, the complementary properties of the Walsh-Hadamard matrix itself are sometimes overlooked. This paper is to identify these complementary properties and thus to find a simple way for the construction of mutually orthogonal complementary sets of sequences.

Golay-paired Hadamard matrix and mutually orthogonal Golay-paired Hadamard matrices have been proposed to provide mutually orthogonal complementary sets of sequences [12,13]. It will be interesting to compare the complementary properties of the two types of Hadamard matrices. Also, their relationships with Walsh-Hadamard matrix need to be further studied, which will lead to simpler implementations of these complementary sets of sequences.

The rest of this paper is organized as follows. In Section II, the relationship between Walsh-Hadamard matrix and Golay-

paired Hadamard matrix is revealed through the closed-form expressions of the two types of Hadamard matrices. Then, the complementary properties of the Walsh-Hadamard matrix are compared with those of the Golay-paired Hadamard matrix. In Section III, a process with a vivid illustration is proposed to construct the mutually orthogonal Walsh-Hadamard matrices which represent a class of mutually orthogonal complementary sets of sequences. It is also shown that any of these mutually orthogonal Hadamard matrices is simply obtained from the original Walsh-Hadamard matrix by a bit-inverted permutation. The same process is used to produce the mutually orthogonal Golay-paired Hadamard matrices, which demonstrate further scalability in addition to their completeness. These complementary properties and some alternative matrix implementations are demonstrated using examples in Section IV. Finally, conclusions are drawn in Section V.

II. HADAMARD MATRIX AND COMPLEMENTARY SET OF SEQUENCES

A Hadamard matrix of order N (an even integer) is a square $N \times N$ matrix of binary elements with the property that any row (or column) differs from any other row (or column) in exactly $N/2$ positions. If we denote the elements by $+1$ and -1 , then the rows (or columns) of a Hadamard matrix are mutually orthogonal. For $N = 2^n$ (n is any positive integer), the well-known Walsh-Hadamard matrix is generated by the recursion

$$W_N = \begin{bmatrix} W_{N/2} & W_{N/2} \\ W_{N/2} & -W_{N/2} \end{bmatrix} \quad (1)$$

with the initial matrix $W_1 = [+1]$. By permuting all the rows in the upper half of W_N with the respective rows in the lower half of W_N , a commuted version of W_N can be defined as

$$\tilde{W}_N = \begin{bmatrix} W_{N/2} & -W_{N/2} \\ W_{N/2} & W_{N/2} \end{bmatrix}. \quad (2)$$

Denoting an element at the i th row and the j th column of W_N as $w_N(i, j)$ and expressing the integer i and j in

binary forms, i.e., $i = i_{n-1}i_{n-2} \dots i_0 = \sum_{r=0}^{n-1} i_r 2^r$ and

$j = j_{n-1}j_{n-2} \dots j_0 = \sum_{r=0}^{n-1} j_r 2^r$ where i_r and j_r ,

$r = 0, 1, \dots, n-1$, are the r th binary bits of i and j respectively, the Walsh-Hadamard matrix has the closed-form equation

$$w_N(i, j) = (-1)^{\sum_{r=0}^{n-1} i_r j_r} \quad (3)$$

which is also known as the Walsh function. From (2), an element in $\tilde{W}_N$ can be derived as

$$\tilde{w}_N(i, j) = (-1)^{\sum_{r=0}^{n-2} i_r j_r + \tilde{i}_{n-1} j_{n-1}} \quad (4)$$

where $\tilde{i}_{n-1}$ denotes the binary inverted bit of i_{n-1} .

Another type of Hadamard matrix, called Golay-paired Hadamard matrix, has been introduced in [12,13], which is constructed by the recursion

$$G_N = \begin{bmatrix} G_{N/2} & \tilde{G}_{N/2} \\ G_{N/2} & -\tilde{G}_{N/2} \end{bmatrix} \quad (5)$$

with initial matrices $G_1 = \tilde{G}_1 = [1]$, where $\tilde{G}_N$ is the commuted version of G_N , defined by

$$\tilde{G}_N = \begin{bmatrix} G_{N/2} & -\tilde{G}_{N/2} \\ G_{N/2} & \tilde{G}_{N/2} \end{bmatrix}. \quad (6)$$

Elements at the i th row and the j th column of G_N and $\tilde{G}_N$ are respectively given by

$$g_N(i, j) = (-1)^{\sum_{r=0}^{n-2} (j_{r+1} \oplus i_r) j_r + i_{n-1} j_{n-1}} \quad (7)$$

$$\tilde{g}_N(i, j) = (-1)^{\sum_{r=0}^{n-2} (j_{r+1} \oplus i_r) j_r + \tilde{i}_{n-1} j_{n-1}} \quad (8)$$

where $\oplus$ denotes the modulo-2 sum of two binary bits. We call $g_N(i, j)$ the Golay function.

From (3), (4), (7) and (8), the relationship between the Walsh-Hadamard matrix and the Golay-paired Hadamard matrix can be found as

$$g_N(i, j) = g_N(0, j) w_N(i, j) \quad (9)$$

$$\tilde{g}_N(i, j) = \tilde{g}_N(0, j) w_N(i, j) \quad (10)$$

which shows that the Golay-paired Hadamard matrix is a scrambled Walsh-Hadamard matrix with the basic Golay function $g_N(0, j)$ (here scrambling means element-by-element multiplying each row of the Walsh-Hadamard matrix with $g_N(0, j)$).

A Hadamard matrix not only represents a set of mutually orthogonal sequences, but also demonstrates some complementary properties. For the Walsh-Hadamard matrix, all the sequences given by rows of this matrix constitute a set of complementary sequences, i.e., the sum of all the respective autocorrelation functions of these sequences is an ideal impulse (zero at all non-zero offset). This means that a Walsh-Hadamard matrix of order N represents a set of N

complementary sequences of length N . A Golay-paired Hadamard matrix demonstrates even stronger complementary properties. All the sequences given by rows of the Golay-paired Hadamard matrix of order N not only represent a set of N complementary sequences of length N , but also can be grouped into $N/2$ complementary sequence pairs (i.e., Golay pairs). Each pair is a set of two complementary sequences of length N with one sequence given by a row in the upper half of the matrix and the other sequence by the corresponding row in the lower half of the matrix.

Complementary property also exists between a Hadamard matrix and its commuted version. That is, a Hadamard matrix (either Walsh-Hadamard matrix or Golay-paired Hadamard matrix) and its commuted version are complementary mates of each other (the sum of all cross-correlation functions between corresponding sequences given by the respective rows of the two Hadamard matrices is always zero for any offset [5]). For a

Golay-paired Hadamard matrix, the $N/2$ complementary pairs can be further grouped into $N/4$ pairs of mates. Together with its commuted version, there are total $N/2$ pairs of mates.

III. MUTUALLY ORTHOGONAL HADAMARD MATRICES

From [5] we know that, for a collection of more than two complementary sets of sequences, if any two sets in the collection are mates of each other, then these complementary sets of sequences are mutually orthogonal. Extending this concept to the Hadamard matrices, we can define a set of mutually orthogonal Hadamard matrices if the complementary sets of sequences represented by respective Hadamard matrices in the set are mutually orthogonal. In this Section, we propose a process to construct such a set of mutually orthogonal Hadamard matrices by alternately applying the recursions for generating a Hadamard matrix and its commuted version.

Let us first describe the process of constructing the mutually orthogonal Walsh-Hadamard matrices. Denoting the k th Walsh-Hadamard matrix of order $N = 2^n$ in a mutually orthogonal set as $W_N^{(k)}$, the construction will take n steps.

Step 1, we construct $W_2^{(0)}$ and $\tilde{W}_2^{(0)}$ using (1) and (2) respectively with the initial matrices $W_1^{(0)} = \tilde{W}_1^{(0)} = [1]$, and then denote $\tilde{W}_2^{(0)}$ as $W_2^{(1)}$ to form a set of mutually orthogonal Walsh-Hadamard matrices of order 2. Step 2, we construct $W_4^{(0)}$ and $W_4^{(1)}$ from $W_2^{(0)}$ and $W_2^{(1)}$ using (1), their commuted versions $\tilde{W}_4^{(0)}$ and $\tilde{W}_4^{(1)}$ using (2), and then denote the two commuted versions as $W_4^{(2)}$ and $W_4^{(3)}$ to expand the set to four matrices. This expansion can proceed successively until a desired order of the Walsh-Hadamard

matrix is reached. After n steps, a set of N mutually orthogonal Walsh-Hadamard matrices of order N are constructed. This process can be vividly illustrated by a flow graph, called the *construction pyramid*, as shown in Fig. 1, where the solid-lined arrow represents the recursion from $W_{N/2}^{(k)}$ to $W_N^{(k)}$ and the dashed-lined arrow represents the recursion from $W_{N/2}^{(k)}$ to $\tilde{W}_N^{(k)}$ which constitutes the second half of the set as $W_N^{(k+N/2)}$.

From the construction pyramid, we can also derive the closed-form expression of $W_N^{(k)}$. Obviously, the matrix $W_N^{(0)}$ is the same as the Walsh-Hadamard matrix W_N constructed by always using the recursion (1). Putting the binary representation of the matrix index $k = k_{n-1}k_{n-2}...k_0$ onto the construction pyramid as shown in Fig. 2, we see that a bit $k_r = 1$ indicates that a recursion (2) is performed (i.e., a dashed-lined arrow is gone through) at step $r + 1$, which means that a permutation of rows of the matrix obtained using recursion (1) is taken place. This permutation is to swap the rows of the matrix indexed $i_r i_{r-1}...i_0$ with the rows indexed $\bar{i}_r i_{r-1}...i_0$ respectively (i.e., to permute rows with inverted bit i_r). At step n , the number of bits which are 1s in $k_{n-1}k_{n-2}...k_0$ indicates how many such permutations have been performed. Thus, $W_N^{(k)}$ is obtained after permuting

W_N 's rows by inverting the bits in the original row index i 's binary form $i_{n-1}i_{n-2}...i_0$ at positions where the bits in $k_{n-1}k_{n-2}...k_0$ are 1s. We call this process *bit-inverted permutation*.

Fig. 2. Binary representation of matrix index k on the construction pyramid.

The bit-inverted permutation is illustrated for $N = 8$ (i.e., $n = 3$) in Fig. 3, where the row indexes after bit-inverted permutation under all different k are shown. To determine which bit or bits in the binary representation of i need to be inverted for a given k , we can simply use k to mask i . Then, all pairs of rows with inverted bit or bits are permuted respectively. For example, if k is 001 in binary form, the least significant bit (LSB) in i 's binary representation needs to be inverted. Thus the two rows indexed $xx0$ and $xx1$ (x means either 0 or 1) are to be permuted. If $k = 011$, the two LSBs in i 's binary representation need to be inverted. Thus the rows $x00$ and $x11$ as well as the rows $x01$ and $x10$ are to be permuted respectively. In general, the row index after bit-inverted permutation under mask k is the bitwise modulo-2 sum of i and k , i.e., $i \oplus k = i_{n-1} \oplus k_{n-1}, i_{n-2} \oplus k_{n-2}, ..., i_0 \oplus k_0$.

$k \rightarrow$	000	001	010	011	100	101	110	111
i	000	001	010	011	100	101	110	111
$\downarrow$	001	000	011	010	101	100	111	110
	010	011	000	001	110	111	100	101
	011	010	001	000	111	110	101	100
	100	101	110	111	000	001	010	011
	101	100	111	110	001	000	011	010
	110	111	100	101	010	011	000	001
	111	110	101	100	011	010	001	000

Fig. 3. Illustration of bit-inverted permutation.

Since $W_N^{(k)}$ is obtained from W_N after bit-inverted permutation under mask k , its closed-form expression is easily found to be

$$\begin{aligned} w_N^{(k)}(i, j) &= w_N(i \oplus k, j) = (-1)^{\sum_{r=0}^{n-1} (i_r \oplus k_r) j_r} \\ &= w_N(i, j) w_N(k, j). \end{aligned} \quad (11)$$

Thus, a set of N mutually orthogonal Walsh-Hadamard matrices is simply constructed by the bit-inverted permutation of W_N under masks $0, 1, \dots, N-1$. As defined in [7], a collection of mutually orthogonal complementary sequence sets is complete if the number of sets in this collection is equal to the number of sequences in each set. We see that the collection of complementary sets of sequences represented by the N mutually orthogonal Walsh-Hadamard matrices of order N is complete.

A set of mutually orthogonal Golay-paired Hadamard matrices can be constructed following exactly the same procedure as described above. That is, the k th Golay-paired Hadamard matrix $G_N^{(k)}$ in the set is obtained from G_N by the bit-inverted permutation under mask k , with closed-form equation

$$\begin{aligned} g_N^{(k)}(i, j) &= g_N(i \oplus k, j) \\ &= (-1)^{\sum_{r=0}^{n-2} (j_{r+1} \oplus i_r \oplus k_r) j_r + (i_{n-1} \oplus k_{n-1}) j_{n-1}}. \end{aligned} \quad (12)$$

According to (9) and (11), (12) can be also expressed as

$$\begin{aligned} g_N^{(k)}(i, j) &= g_N(0, j) w_N(i \oplus k, j) \\ &= g_N(k, j) w_N(i, j) \\ &= g_N(i, j) w_N(k, j). \end{aligned} \quad (13)$$

The collection of mutually orthogonal complementary sets of sequences represented by the N mutually orthogonal Golay-paired Hadamard matrices of order N is also complete.

Furthermore, in addition to the completeness, the mutually orthogonal Golay-paired Hadamard matrices demonstrate superb scalability. That is, the set of N mutually orthogonal Golay-paired Hadamard matrices of order N consists of four

subsets, each of which is a set of $\frac{N}{2}$ mutually orthogonal

Golay-paired matrices of dimension $\frac{N}{2} \times N$. Each subset

consists of four sub-subsets, each of which is a set of $\frac{N}{4}$

mutually orthogonal Golay-paired matrices of dimension $\frac{N}{4} \times N$. This decomposition process can be continued until

the dimension of a Golay-paired matrix is scaled down to $2 \times N$. As a whole, the set of N mutually orthogonal Golay-

paired Hadamard matrices of order N consists of $\left(\frac{N}{K}\right)^2$

subsets of K mutually orthogonal Golay-paired matrices of dimension $K \times N$ for $K = \frac{N}{2}, \frac{N}{4}, \dots, 2$. With this scalability, once a set of mutually orthogonal Golay-paired Hadamard matrices is constructed, all the subsets of mutually orthogonal Golay-paired matrices of lower dimensions are constructed at the same time.

Among the $\left(\frac{N}{K}\right)^2$ subsets of K mutually orthogonal

Golay-paired matrices of dimension $K \times N$, only $\frac{N}{K}$ of

them are different from each other. The rest of them are those subsets which consist of the same set of Golay-paired matrices but with different ordering (i.e., the position of a Golay-paired matrix in the set is different, but the matrix remains the same).

The K Golay-paired matrices of dimension $K \times N$ in the

l th subset, $l = 0, 1, \dots, \frac{N}{K} - 1$, can be also formulated as

$$\begin{bmatrix} g_N^{(k+lK)}(0, j) \\ g_N^{(k+lK)}(1, j) \\ \vdots \\ g_N^{(k+lK)}\left(\frac{K}{2}-1, j\right) \\ g_N^{(k+lK)}\left(\frac{N}{2}, j\right) \\ g_N^{(k+lK)}\left(\frac{N}{2}+1, j\right) \\ \vdots \\ g_N^{(k+lK)}\left(\frac{N}{2}+\frac{K}{2}-1, j\right) \end{bmatrix}, \text{ for } k = 0, 1, \dots, K-1. \quad (14)$$

IV. EXAMPLES

The Walsh-Hadamard matrix of order $N = 8$ obtained using recursion (1) or the closed-form equation (3) is

$$W_8 = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{bmatrix}. \quad (15)$$

It can be verified that all the sequences given by rows of W_8 constitute a complementary set of sequences. The Golay-paired Hadamard matrix of order 8 is

$$G_8 = \begin{bmatrix} +1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 \\ +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 \\ +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 \end{bmatrix}, \quad (16)$$

which can be constructed by either the recursion (5) or the closed-form equation (7). Also, according to (9), G_8 can be obtained by element-by-element multiplying W_8 with the sequence $[+1 \ +1 \ +1 \ -1 \ +1 \ +1 \ -1 \ +1]$ given by the row indexed 0 in G_8 . Four complementary sequence pairs are found in G_8 .

The mutually orthogonal Walsh-Hadamard matrices of order 8 are shown in Fig. 4 (a). We see that $W_8^{(0)}$ is the same as W_8 and the other matrices are obtained from W_8 by the bit-inverted permutation as illustrated in Fig. 3. The mutually orthogonal Golay-paired Hadamard matrices of order 8 is shown in Fig. 4 (b). Similarly, the first matrix $G_8^{(0)}$ is the same as G_8 and the other matrices are the permuted versions of G_8 according to the bit-inverted permutation. Alternatively, according to (13), any matrix in the set can be obtained by element-by-element multiplying W_8 with a corresponding row in G_8 , or, by element-by-element multiplying every row of G_8 with a corresponding row in W_8 .

Fig. 4 (b) also illustrates the scalability of the mutually orthogonal Golay-paired Hadamard matrices of order 8. There are total 4 collections of complete complementary sets of order 4 ($K = 4$); 2 of them are different. According to (14), the first collection ($l = 0$) includes 4 4×8 Golay-paired matrices which are enclosed in the two solid-lined boxes in Fig. 4 (b). There are also total 16 collections of complete complementary sets of order 2 ($K = 2$); 4 of them are different. According to (14), the first collection ($l = 0$) is composed of 2 2×8 Golay-paired matrices which are enclosed in the two dashed-lined boxes in Fig. 4 (b).

Fig. 4. (a) Mutually orthogonal Walsh-Hadamard matrices and (b) mutually orthogonal Golay-paired Hadamard matrices of order 8 (+ and - denote the elements +1 and -1 respectively).

V. CONCLUSIONS

The complementary properties of Walsh-Hadamard matrix and Golay-paired Hadamard matrix are demonstrated and compared in this paper. A unified process for constructing mutually orthogonal Walsh-Hadamard matrices and mutually orthogonal Golay-paired Hadamard matrices is also proposed. It is shown that each of the constructed mutually orthogonal Hadamard matrices is a bit-inverse permuted version of the original Hadamard matrix. The mutually orthogonal Walsh-Hadamard matrices represent a collection of complete complementary sets of sequences, whereas the mutually orthogonal Golay-paired Hadamard matrices also demonstrate superb scalability in addition to their completeness. For applications which use fixed sequence length and fixed size of complementary sequence set, the mutually orthogonal Walsh-Hadamard matrices provide a simple solution to generate such a collection of complete complementary sets of sequences. However, if variable sequence length and/or adaptive size of complementary sequence set are required, the mutually orthogonal Golay-paired Hadamard matrices will be advantageous.

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