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Symposium of the International Society for Rock Mechanics

Modelling Dynamic Fracture Propagation in Rock

Gaetano Venticinque* and Jan Nemcik

Faculty of Engineering, University of Wollongong, Australia

Abstract

Fracture propagation in brittle rock is very fast and highly dynamic. Typically this process consists of fracture initiation, propagation and termination. Growth of micro-fractures is conceptually and numerically well established, however, current practices to model fracture propagation in rock employs slow evolving static regimes that do not represent the true nature of fracture propagation in the laboratory or the field. This paper presents a newly developed numerical approach using Micro-Brittle Dynamics theory to model the propagation of fractures through rock in real time. This work presented here is based on a newly developed Dynamic Rock Fracture Model (DRFM^{2D}) and validated against laboratory experiments.

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Keywords: Dynamic fracture propagation; Numerical modelling

1. Introduction

Current advances in computing technology have enabled solutions of many complex geotechnical problems. However when a presence of large fractures dominate strata behaviour, these solutions may not be correct. Available numerical software can be generally divided into three groups: Elastic, Inelastic Time-Dependent Implicit and Inelastic Time-Dependent Explicit. These are briefly discussed in Table 1.

Whilst available Inelastic Time-Dependent Explicit Methods are currently used to simulate ground failure, they are in most cases not employed correctly. Most researchers will use the Quasi-Static formulation and slow evolving material damage to resolve highly dynamic fracture phenomena, consequently they are not capable of capturing true real time dynamic brittle effects occurring within the rock mass.

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Table 1. Classification of numerical methods [1].

Method	Description	Commercial Software
Elastic Methods:	Linear elastic mechanical response models. These are the most simplest and inaccurate variety of geotechnical models as rock masses are treated to linearly strain forever without failure. Application of such models is limited to estimating stress redistributions around geometries with an assumption of a perfectly elastic rock mass that never yields.	Map3D Examine3D
Inelastic Time-Dependent Implicit Methods:	Are implicit (meaning matrix oriented solution schemes) time dependent models with coupled elastic-inelastic mechanical response. Despite exhibiting both time dependent and post failure-yielding capabilities, implicit simultaneous-solving matrix orientation processes is unable to simulate non-linear material changes, progressive fractures or dynamic stress evolution through rock mass.	Phase2 (RS2) Phase3 (RS3) Abaqus/Implicit
Inelastic Time-Dependent Explicit Methods:	Are explicit (meaning continually algebraically evolving schemes) time dependent models with coupled elastic-inelastic mechanical response. These models are best suited for geotechnical analysis enabling simulation of non-linear material behaviours as well as progressive fracture development and dynamic stress redistribution ahead of failure surface. These methods are recommended for most if not all geotechnical analysis.	FLAC ^{2D} FLAC ^{3D} UDEC 3DEC Abaqus/Explicit

New development of a 2-dimensional dynamic rock fracture model (DRFM^{2D}) in FLAC^{2D} [1] challenges the use of numerical methods, demonstrating superior computational efficiencies towards accurately modelling the dynamic brittle fracture propagation through rock in real time. Our currently developed DRFM^{2D} model is focused on the practical application of fracture dynamics in engineering. An overview of the natural physical processes and the governing constitutive laws is outlined here.

2. Micro-Brittle Dynamics (MBD)

2.1. Micro-Scale Dynamic Response

The dynamic action of the smallest finite mass in a continuum model represents the discrete dynamic response of a single element. In general, mechanical stress-strain response of any brittle or ductile rock past the elastic limit can follow one of four behaviours; Strain Hardening, Elastic-Plastic, Strain Softening or Perfectly Brittle. These rock failure types are illustrated in Fig. 1a. In Micro-Brittle Dynamics (MBD), where brittle materials are unable to absorb plastic deformation energy prior to fracture, the dynamic micro-scale interactions exhibit an ideal brittle failure response. This occurs at the elastic strength limit taking the brittle failure path curve in time as illustrated in Fig. 1b.

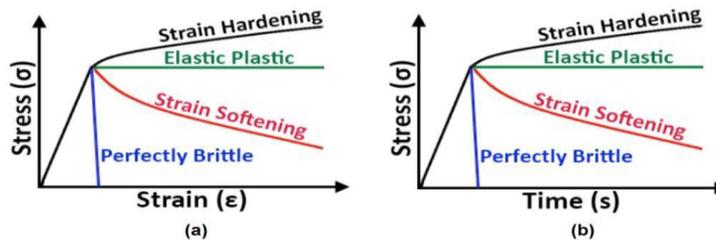


Fig. 1. Four types of rock post-failure behaviour plotted as (a) The stress-strain used in the conventional classification of material failure response; (b) New definition extending classification of material failure response dynamically with respect to time [1].

Under the newly recognised dynamic context “ideal brittle behaviour” is defined as a sudden fracture that propagates within the smallest finite unit of body at the mechanical wave speed (information speed within a material) given in general form by Newton Laplace’s mechanical wave:

$$C = \sqrt{\frac{E}{\rho}} \quad (1)$$

where: C = Mechanical Wave Speed (m/s)
 E = Young's Modulus (Pa)
 ρ = Material Density (kg/m³)

Hence for an ideal brittle material, Tensile Fracture by definition propagates at the mechanical shear wave speed, given by:

$$C_s = \sqrt{\frac{G}{\rho}} \quad (2)$$

where: C_s = Mechanical Shear Wave Speed (m/s)
 G = Shear Modulus (Pa)
 ρ = Material Density (kg/m³)

Likewise Shear Fracture propagates at the mechanical compression wave speed, given by Eq. (3).

$$C_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \quad (3)$$

where: C_p = Mechanical Compression Wave Speed (m/s)
 K = Bulk Modulus (Pa)
 G = Shear Modulus (Pa)
 ρ = Material Density (kg/m³)

The above definitions emerge directly from conservation of energy equations by equating the potential energy in the form of elastic strain with the kinetic energy of a unit volume material having centroid mass as defined by Eq. (4) and (5).

$$U = \frac{\sigma^2}{2E}V \quad (4)$$

where: U = Potential Strain Energy (J)
 σ = Stress (Pa)
 V = Volume (m³)
 E = Young's Modulus (Pa)

$$KE = \frac{1}{2}mv^2 \quad (5)$$

where: KE = Kinetic Energy (J)
 m = Mass (kg)
 v = Mass Velocity (m/s)

Noting also that mass re-written in terms of density and volume is given by:

$$m = \rho V \quad (6)$$

Equating (4) and (5) and further substituting with (6):

$$U = KE$$

$$\frac{\sigma^2}{2E}V = \frac{1}{2}mv^2$$

$$\frac{\sigma^2}{2E}V = \frac{1}{2}(\rho V)v^2$$

Which simplifies to:

$$\sigma^2 = E\rho v^2$$

We can substitute the Young's Modulus E from (1):

$$E = C^2\rho$$

Hence

$$\sigma^2 = (C^2\rho)\rho v^2$$

Simplifying:

$$\sigma = \rho v C$$

Furthermore:

$$\Delta\sigma = \rho\Delta v C \tag{7}$$

where: $\Delta\sigma$ = Change in Stress (Pa)
 ρ = Density (kg/m³)
 v = Particle Velocity (m/s)
 C = Mechanical Wave Speed (m/s)

Eq. (7) indicates that the speed of a stress wave through a material is related to the mechanical wave speed, a material constant. Subsequently, the maximum absolute speed of any brittle fracture propagation is limited to the mechanical wave speed C . In the numerical model, fracture through individual elements are assumed to propagate at the maximum wave speed limit as they are identified to behave in an ideally brittle manner. This behaviour is consistent with expectations of how rock fractures should propagate as observed and proposed by notable other researchers [2, 3].

2.2. Micro-Scale Heterogeneous Concept

In rock mechanics various failure criteria are conventionally represented on diagrams such as the Mohr-Coulomb circle diagram by a single linear or non-linear line defining the strength limit of rock mass at any stress state. This classical representation assumes ideal homogenous rock strength over the whole rock mass. In nature however, ideal homogeneity is non-existent regardless of scale or size.

In micro-scale heterogeneous concepts all rock masses regardless of scale or size are recognized to consist of smaller heterogeneous components as shown in Fig. 2. Statistical rock strength distribution is therefore introduced to describe a heterogenic strength envelope as shown in Fig. 3. Compared to classical failure criteria the new heterogeneous strength concept illustrated in Fig. 3b offers more realistic synergies with nature and is indispensable for simulating the dynamic fracture mechanism.

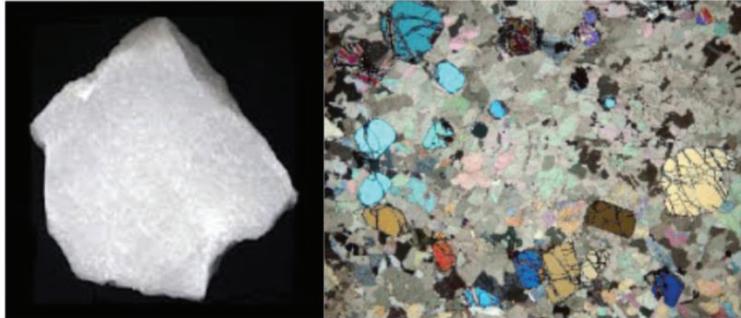


Fig. 2. Homogenous intact marble rock composed of finer heterogeneous grains [4].

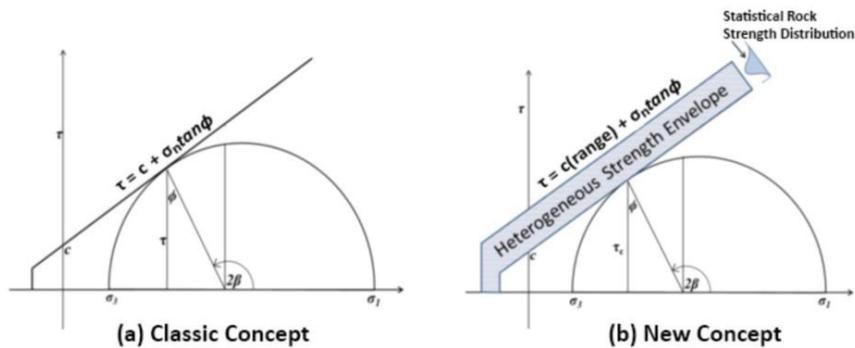


Fig. 3. (a) Classical Mohr-Coulomb failure criteria; (b) Newly revised failure criteria accommodating statistical rock strength.

3. Dynamic Rock Fracture Model (DRFM^{2D})

The Dynamic Rock Fracture Model (DRFM^{2D}) introduced here is a new two-dimensional continuum model capable of simulating the dynamic real time propagation of brittle fractures within rock mass. Dynamic solution scheme provided by FLAC [5] is inherited directly into the (DRFM^{2D}) model; having full dynamic equations of motion included in its formulation. Rock heterogeneity is introduced by statistical distribution of cohesive strength properties assigned within discrete element zones, illustrated in Fig. 4.

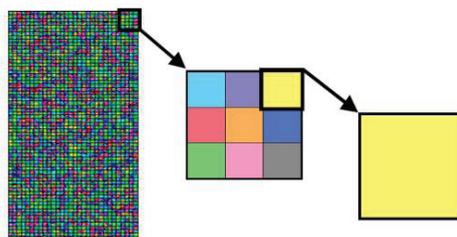


Fig. 4. Rock heterogeneity introduced through statistical distribution of cohesion strength properties within the continuum mesh depicted by various colours.

3.1. Discrete Element Failure Criterion

In the numerical model, when the stress state of an element satisfies failure criterion, initiation of a fracture occurs.

The Mohr Coulomb Shear Criterion with Tensile cut off can be expressed as follows:

$$\tau_{mc} = \begin{cases} c_E + \sigma_3(\tan \phi) & ; \sigma_3 > -T_E \\ 0 & ; \sigma_3 \leq -T_E \end{cases} \quad (8)$$

where: τ_{mc} = Mohr Coulomb Shear Failure Envelope (Pa)
 c_E = Intact Element Cohesion Strength (Pa)
 T_E = Intact Element Tensile Strength (Pa)
 σ_3 = Minor Principal Stress (Pa)
 ϕ = Internal Angle of Friction (Degrees)

For tensile Failure

$$t_f = \frac{a}{c_s \sqrt{2}} \quad (9)$$

For shear Failure

$$t_f = \frac{a}{c_p \sqrt{2}} \quad (10)$$

where: t_f = Fracture Time-Step (s)
 a = Side length for a square grid element (m)
 c_p = Mechanical Compressional Wave Speed (m/s)
 c_s = Mechanical Shear Wave Speed (m/s)

Moreover, both tension (T) and cohesion (c) dynamically reduce to zero as a function of change in global time (Δt) following after initial detection of failure within an element.

$$T(\Delta t) = \begin{cases} T_E & ; \frac{1}{2} \cdot \left(\frac{\Delta t}{t_f}\right) < 0.5 \\ T_E \left(2 - \frac{\Delta t}{t_f}\right) & ; 0.5 \leq \frac{1}{2} \cdot \left(\frac{\Delta t}{t_f}\right) \leq 1 \\ 0 & ; \frac{1}{2} \cdot \left(\frac{\Delta t}{t_f}\right) > 1 \end{cases} \quad (11)$$

$$c(\Delta t) = \begin{cases} c_E & ; \frac{1}{2} \cdot \left(\frac{\Delta t}{t_f}\right) < 0.5 \\ c_E \left(2 - \frac{\Delta t}{t_f}\right) & ; 0.5 \leq \frac{1}{2} \cdot \left(\frac{\Delta t}{t_f}\right) \leq 1 \\ 0 & ; \frac{1}{2} \cdot \left(\frac{\Delta t}{t_f}\right) > 1 \end{cases} \quad (12)$$

where: $T(\Delta t)$ = Tension Strength as a function of Global Time change Δt (Pa)
 T_E = Initial Element Tensile Strength (Pa)
 $c(\Delta t)$ = Cohesion Strength as a function of Global Time change Δt (Pa)
 c_E = Initial Element Cohesion Strength (Pa)
 t_f = Fracture Time-Step (s)
 Δt = Global Time change following detection of failure within an element (s)

To account for energy losses in the elastic range, a generalised 2% fraction of critical damping is employed using FLAC's dynamic relaxation method.

4. Validation of DRFM^{2D} model

4.1. Mesh Sensitivity Analysis

The propagation of fractures within the Dynamic Rock Fracture Model (DRFM^{2D}) was investigated for mesh size and shape dependency giving good accuracies for density ratios greater than 5,000 and mesh distortions within the normal modelling range [1]. The model is considered robust and resilient against mesh size and shape dependant effects.

4.2. Dynamic Fracture Propagation in Brazilian Tensile Test

Numerical simulation replicating indirect tensile laboratory Brazilian test of homogenous marble rock core 24 mm thick and 54 mm diameter is shown in Fig. 5. High mesh density ratio of 40,000 was used to ensure fine accuracy to correlate the data with the laboratory observations.

Table 2. Mechanical properties of Marble Rock.

Rock	Density	Bulk Mod	Shear Mod	Internal friction	UCS	Tensile Strength
Marble	2700	37 GPa	20 GPa	33°	71.7 MPa	7 MPa

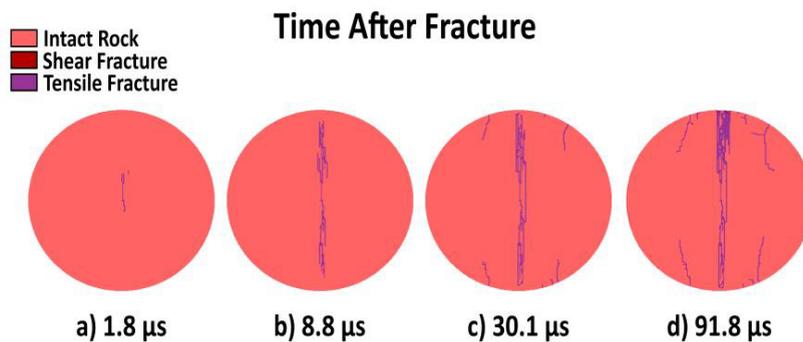


Fig. 5. Dynamic fracture propagation through the cylindrical marble rock in time.

Progressive fracturing is reportedly aligned against well-established primary, secondary and tertiary fracturing, notably observed amongst other researches [6].

4.3. Triaxial Compression Test using Heterogeneous Rock Strength concept

Using the DRFM^{2D} model, triaxial compression tests of two rock types were modelled and compared to the laboratory rock tests. Laboratory testing was performed on siltstone and carbonaceous siltstone samples sourced from an underground coal mine. The choice of two rock types was based on a known difference between their heterogeneities. Compared to siltstone the presence of much weaker carbonaceous material within the carbonaceous siltstone attributed a wider heterogenic strength envelope. Because of this, dynamic fracture propagation through the carbonaceous siltstone rock was expected to occur in an isolated sporadic manner with fractures interrupted by unyielding higher strength zones, dissipating dynamic stresses from the tips of initiating

fractures and hindering the propagation of any continual dynamic fractures. In contrast, more homogenous strength distribution within the intact siltstone rock was expected to produce failure in a sudden brittle fashion throughout the entire rock sample.

Using a 82 by 128 grid with Mesh Density of 7,680 and material properties for siltstone and carbonaceous siltstone rock material listed in Table 3, triaxial compression testing with 10 MPa confinement was simulated in the DRFM2D model.

Table 3. Properties for Siltstone and Carbonaceous Siltstone Rock.

Rock	Density	Bulk Mod	Shear Mod	Internal friction	UCS
Siltstone	2476 kg/m ³	6 GPa	4.5 GPa	33°	80 MPa
Carbonaceous Siltstone	2476 kg/m ³	3.3 GPa	2.5 GPa	33°	51.8 MPa

Simulated fracturing and output stress/strain response from the DRFM^{2D} model are reported as closely matching results produced by laboratory triaxial compression testing of siltstone and carbonaceous siltstone rock core samples. Validation is presented in Fig. 6 below.

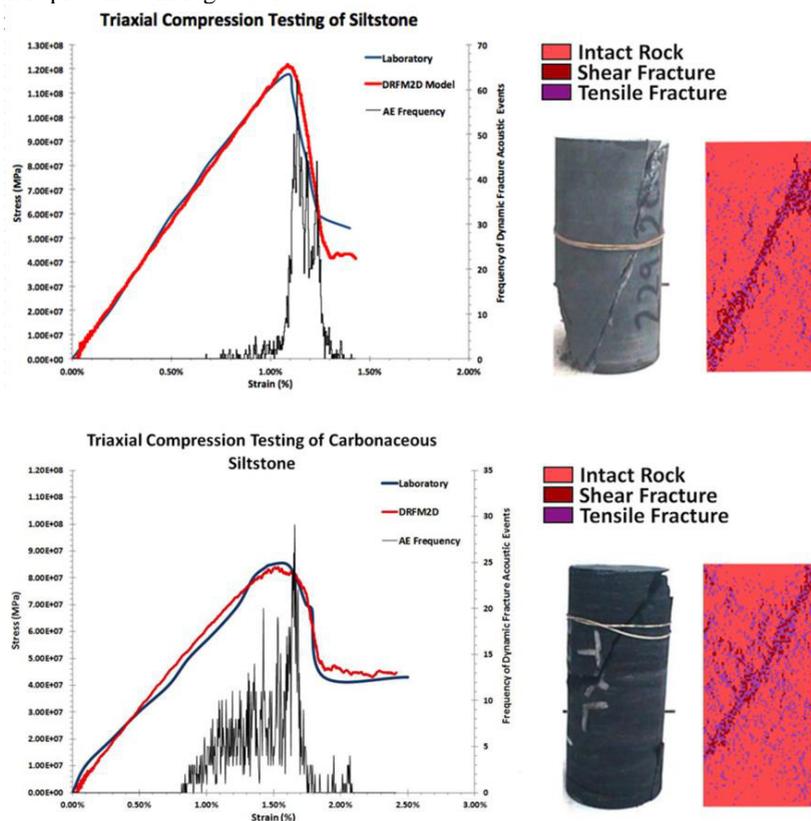


Fig. 6. Comparison of triaxial compression tests for siltstone and carbonaceous siltstone rock samples under 10 MPa confinement [1].

Analysis of the above results show the influence of strength heterogeneity that is in-line with expected dynamic rock failure response. In carbonaceous siltstone, dynamic fractures initiated and dispersed in a sporadic fashion indicated by the activity range of dynamic emission occurrences simulated in the FLAC model prior to abrupt failure. Additionally a notably more brittle behavior occurred within the intact siltstone material, characterised

by a narrower heterogenic strength envelope. This was further validated from similarity of resulting fracture distribution produced.

5. Conclusions

Through logical assumptions and classical physics of dynamic wave propagation in the elastic media, the mathematically derived conditions of dynamic fracture propagation were devised. This theory was used to construct a new computational model to simulate initiation and propagation of dynamic fractures in rock. The new dynamic rock fracture model (DRFM^{2D}) in FLAC^{2D} captures this mechanism and demonstrates superior computational accuracy and efficiency when compared to the existing yield based fracture models for rock material. Our unique method demonstrated ability to captures the dynamic propagation of fractures through rock in real time as it occurs in nature.

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